GALAXY FORMATION FROM TEPID INFLATION

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Abstract. It is argued that the primordial density fluctuations needed for galaxy formation may have been of thermal origin in a quasi-exponential state of inflation.

1. Introduction

The total entropy of the Universe amounts to 10^{87}. In the standard model this number would be an initial value, but for that the number is highly unnatural. Inflationary models are capable to produce $\geq 10^{87}$ entropy (with $\geq 10^{29}$ times expansion) additionally explaining the flatness, horizon, and monopole problems too (Guth, 1981), but, unless accepting still fairly unnatural initial conditions and/or parameters, only at the expense of the following drawbacks:

(1) *Too high primordial local density fluctuations are predicted compared to the need of galaxy formation theory.* Fine tuning, extreme parameter values or many extra degrees of freedom, undetermined by low-energy experience, are then invoked to reproduce a single fluctuation amplitude (Linde, 1984; Blau and Guth, 1986). *Principle of Occam's razor is violated.*

(2) *Global homogeneity and isotropy is lost.* Highly irregular structure is predicted (Linde, 1984) outside the present horizon, so observable cosmic order can only be explained away by a supposed but absolutely unobservable disorder. *Cosmological principle is violated.*

(3) *Almost exactly critical density is required.* In order to get rid of inhomogeneities on scales still observable a very long exponential expansion is needed leading to very close balance between the velocity and density terms, respectively, in the Friedmann equation; according to the present observational indications such a specialization is not favoured. *Principle of minimal specialization is violated.*

Here a picture is presented free of these three problems and conform with the approach of classical cosmology.

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2. Coincidence?

In order to explain galaxy formation by nonlinear processes relative density fluctuations $\delta e/e \sim 10^{-5}$ roughly independent of scale are needed when the extragalactic scales re-enter the horizon in the radiation-dominated era (Linde, 1984).

Thermal fluctuations at the GUT phase transition temperature would be just in the same order of magnitude. To see this, let us use thermodynamic phenomenology. For charge symmetry

$$s = s(e)\,, \tag{1}$$

where s is the entropy density, e is the energy density. Then

$$\left(\frac{\delta e}{e}\right)^2 = -\frac{1}{V}\left(\frac{\mathrm{d}^2 s}{\mathrm{d}e^2}\right)^{-1} \tag{2}$$

(Einstein, 1910; Diósi and Lukács, 1985a). For a radiation field $e = N(\pi^2/30)T^4$, at horizon size,

$$\frac{\delta e}{e} = \left(\frac{8\pi}{3}\right)\left(\frac{N\pi}{5}\right)^{1/4}\left(\frac{T}{M}\right)^{3/2}, \tag{3}$$

where M is the Planck mass, 1.22×10^{19} GeV. Substituting $N \sim 100$ and $T_{\mathrm{GUT}}/M \sim 10^{-4}$, we have

$$\frac{\delta e}{e} \simeq 2.5 \times 10^{-5}\,, \tag{4}$$

in agreement with the above requirement. However, these thermally generated fluctuations are not transmitted without change from the GUT era to the recombination era in a usual inflationary model, because:

(a) *Generally inflation is supposed to happen at deep supercooling*, when of course $\delta e/e \ll 10^{-5}$;

(b) Equation (3) *yields the initial value of fluctuation*, at the first horizon crossing, during inflation, which is still to be multiplied by a factor $\sim e/(e+p)$ to get the *final* value at the second horizon crossing (Brandenberger, 1985, 1986).

3. Tepid Inflation

Both the above problems are simultaneously resolved if there is no deep supercooling with violent reheating but rather a continuous entropy production from delayed phase transition, so *tepid inflation is suggested*. Then $\delta e/e$ does not decrease too much, and $e/(e+p) = e/Ts$ is not too high.

Such models exist, and indeed exhibit fairly constant temperature during inflation. The cooling is limited because the transition is not forbidden, only slow at the beginning. Therefore, there is a gradual release of latent heat (Kämpfer *et al.*, 1987). For illustration the simplest model is the isothermal one. There the *assumptions* and *approximations* are as follows:

(a) *The transition happens isothermally*, at some $T_0 < T_{eq}$ (T_{eq} is the phase *equilibrium* temperature).

(b) *Asymptotic equations of state* (Diósi *et al.*, 1985a, b) are used as

$$p_1 = -C + (5/24)\mu^2 T^2 + \left[\frac{142}{90}\pi^2 - \frac{35\lambda_1 + 15\lambda_2}{288}\right] T^4 , \tag{5a}$$

$$p_2 = \tfrac{113}{90}\pi^2 T^4 , \tag{5b}$$

$$C = \tfrac{5}{16}\mu^4\Gamma - 3[\varepsilon + (\varepsilon^2 + 4\Gamma)^{1/2}]^2 [\varepsilon^2 + \varepsilon(\varepsilon^2 + 4\Gamma)^{1/2} + 6\Gamma] , \tag{5c}$$

$$\Gamma = 30\lambda_1 + 7\lambda_2 ; \tag{5d}$$

where ε, λ, and μ are parameters of the Higgs potential

$$V_0(\phi) = C - \tfrac{1}{2}\mu^2 Tr\phi^2 + \tfrac{1}{3}\varepsilon\mu Tr\phi^3 + \tfrac{1}{4}[\lambda_1(Tr\phi^2)^2 + \lambda_2 Tr\phi^4] . \tag{6}$$

(c) *The actual equation is a weighted average*

$$e = xe_1 + (1 - x)e_2 , \tag{7}$$

$$p = xp_1 + (1 - x)p_2 , \tag{8}$$

where x is an evolution parameter of the transition, going from 1 to 0.

The Einstein equation governs $R(t)$ and $x(t)$ for any value of T_0. The solution (cf. Kämpfer *et al.*, 1987; Kämpfer, 1986) is of the form

$$R = R_0\left[\frac{L(e_1 - B)}{2e_2 B}(1 + \cos\phi)\right]^{L/[4(L-B)]} , \tag{9}$$

$$x = \frac{e_2}{L - B}\left[\frac{2B}{L}(1 + \cos\phi)^{-1} - 1\right], \tag{10}$$

where

$$\phi = w(t - t_0) + \alpha_0 , \tag{11}$$

with

$$L \equiv e_1 - e_2 , \tag{12a}$$

$$B \equiv C - \tfrac{5}{48}\mu^2 T^2 , \tag{12b}$$

$$w = \sqrt{\frac{8\pi}{3}}\,\frac{1}{M}\,\frac{4}{L}\sqrt{e_2 B(L - B)} , \tag{13a}$$

$$\alpha_0 = \arccos\left(\frac{2e_2 B}{L(e_1 - B)} - 1\right). \tag{13b}$$

Obviously L plays the rôle of latent heat and B is some vacuum energy. In these formulae e_1 and e_2 are meant at $T = T_0$. According to Equations (5) and the thermodynamic formula $e = Tp_T - p$,

$$e_1 = C + (5/24)\mu^2 T^2 + \left[\frac{142}{30}\pi^2 - \frac{35\lambda_1 + 15\lambda_2}{96}\right] T^4 , \tag{14a}$$

$$e_2 = \frac{113}{30}\pi^2 T^4 . \tag{14b}$$

The equilibrium temperature can be calculated from the equality of pressures as

$$T_{\text{eq}}^2 = \frac{1}{2Q}\left[\sqrt{4CQ + \tfrac{25}{576}\mu^4} - \tfrac{5}{24}\mu^2\right] , \tag{15a}$$

$$Q \equiv \frac{29}{90}\pi^2 - \frac{35\lambda_1 + 15\lambda_2}{288} . \tag{15b}$$

Hence, any relevant quantity can be analytically expressed as a function of the Higgs parameters μ, ε, λ_1, and λ_2 and the relative supercooling $(T_{\text{eq}} - T_0)/T_{\text{eq}}$.

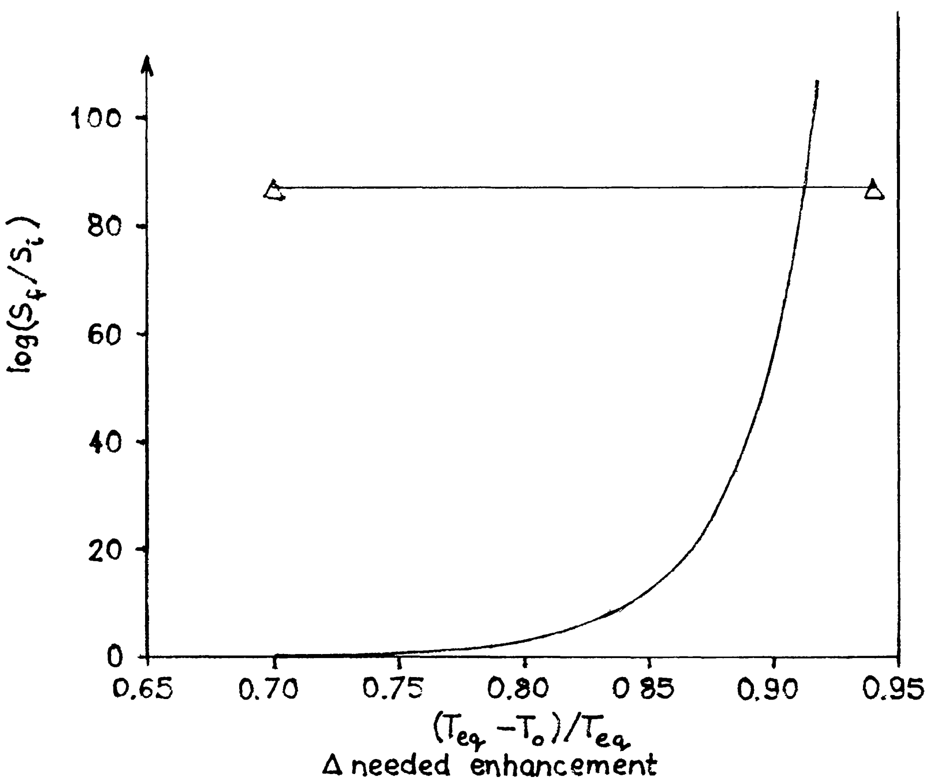


Fig. 1. Entropy enhancement versus relative supercooling.

The phase transition ends at $x(t_f) = 0$; whence

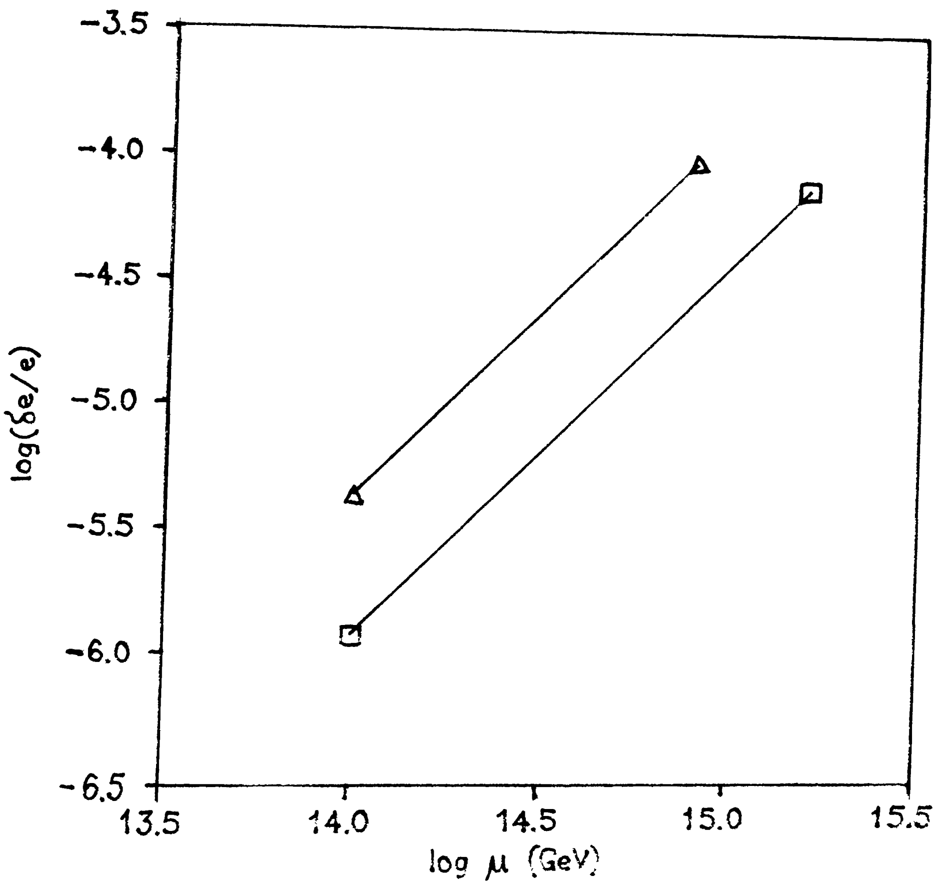
$$t_f - t_i = \frac{1}{w}\left[\arccos\left(\frac{2B}{L} - 1\right) - \alpha_0\right], \tag{16}$$

with the entropy enhancement

$$\zeta \equiv \frac{(sR^3)_f}{(sR^3)_i} = \frac{s_2(T_0)}{s_1(T_0)}\left(\frac{e_1 - B}{e_2}\right)^{3L/[4(L-B)]} . \tag{17}$$

One can see that the expansion is not strictly exponential. It would seem a power expansion, but it is rather something between the above two pure cases, because $R(t)$ is a very high power of a *transcendental* function. The quantity in which we are most interested is the entropy enhancement ratio $\zeta \equiv (sR^3)_{\rm final}/(sR^3)_{\rm initial}$. Now, from Equation (17),

$$(sR^3)_{\rm final}/(sR^3)_{\rm initial} \to \infty \qquad \text{if} \quad (T_0/T_{\rm eq}) \to 0 .$$



□ $\varepsilon = 1$, $\lambda_1 = \lambda_2 = 0.1$, △ $\varepsilon = 1$, $\lambda_1 = \lambda_2 = 0.01$

Fig. 2. Thermal density perturbations versus GUT energy scale μ.

Entropy enhancement ζ versus $(T_{eq} - T_0)/T_{eq}$ is shown in Figure 1.

Note that 92% *supercooling is enough* for all the above-mentioned purposes of an inflationary cosmology (Linde, 1984) (at any reasonable choice of Higgs parameters).

It follows that *if the release of latent heat is spread over the whole inflation, then matter remains tepid.* Therefore, it is reasonable indeed to calculate *thermal fluctuations.*

Let us do it for 92% supercooling: for $\varepsilon = 1$, $\lambda_1 = \lambda_2 = 0.1$, and $\lambda_1 = \lambda_2 = 0.01$, see $(\delta e/e)_f$, as a function of μ, in Figure 2.

Thermal perturbations are, therefore, in the good range when $\mu \sim (10^{14}\text{–}10^{15})$ GeV, *so they are indeed capable to explain galaxy formation.* However, still there are quantum fluctuations as well, and in usual models *they* tend to be too high (at the second horizon crossing, because of an amplification factor $\sim e/(e+p) \sim 10^{20}$ for cold inflation (Brandenberger, 1985, 1986)). Nevertheless, our amplification factor is much smaller, $e/(e+p) = e/Ts \sim 100$, but, at the same time, the quantum fluctuations occur at $T_0 \neq 0$, where they may be higher, so one has to discuss their possible influence. First, one can consider the quantum fluctuations which are present even at $T = 0$. However, as can be seen from traditional cold inflationary models, these fluctuations arrive at roughly order of magnitude 1 only after 10^{20} amplification (Brandenberger, 1985, 1986). Since our amplification is only cca. 10^2, these fluctuations can be neglected. Now, in the second step, the quantum fluctuations may be excited by temperature. This possibility was investigated by Pagels (1983) who calculated the one-loop quantum fluctuations in an Einstein–de Sitter universe at a finite temperature T_G characteristic to the GUT era. His result was that at the horizon

$$\frac{\delta e}{e} = (2\pi)^3 \sqrt{\frac{28N}{135}}\, \frac{T_G T_H}{M^2}\,, \tag{18}$$

where T_H is the Hawking temperature, $T_H \sim \mu^2/M \sim 10^{11}$ GeV. Now, comparing this to the 'genuine' thermal fluctuations given in Equation (3), one obtains

$$\left(\frac{\delta e}{e}\right)_q \bigg/ \left(\frac{\delta e}{e}\right)_{th} = \frac{3}{8\pi}\,(2\pi)^3 \sqrt{\frac{28}{135}} \left(N\,\frac{5}{\pi}\right)^{1/4} \frac{T_H}{\sqrt{TM}}\,. \tag{19}$$

Hence, at $\mu \sim 10^{15}$ GeV and $T_0 \sim 0.1\mu$ one obtains still a relative factor $\sim 10^3$ in favour of the thermal fluctuations. Although ours is not a strictly Einstein–de Sitter universe, this suggests that the thermal fluctuations dominate, as could have been expected from the fact that $T = T_0 \gg T_H$, the latter being characteristic for the quantum degree of freedom.

4. Results and Conclusions

Numerical calculations show that for tepid inflation very simple and suggestive relations hold: namely,

(1) 10^{87} *entropy and* 10^{29} *times expansion appears at* 92% *supercooling* for all reasonable Higgs parameters.

(2) *For this supercooling* and the range $10^{14} < \mu < 10^{15}$ GeV, favoured by low-energy data of particle physics, *there is no 'fluctuation problem'*, since there always exist such $\varepsilon \sim O(1)$, $0.01 < \lambda < 1$ values that $\delta e/e$ be in the proper range $\simeq 10^{-5}$ – and scale-invariant. (Compare this to $\lambda \leq 10^{-12}$ of cold inflationary models.) Thus *density perturbations may be due to thermal rather than quantum fluctuations*; *no very weakly coupled inflaton field is needed* (Turner, 1987), the scalar field can be in thermal contact with the radiation (Ellis and Steigman, 1980; Steigman, 1983), and the thermal history of the Universe may be monotonous from T_{GUT} to the present background temperature.

(3) The amplification factor $e/(e+p) = O(10^2)$ for any reasonable Higgs number parameter set independently of μ (compare this to the factor 10^{20} of cold inflation).

(4) For proper $\delta e/e$ always $T_0 \sim 10^{14}$ GeV, $T_{\mathrm{eq}} \sim 10^{15}$ GeV.

(5) The thermodynamical coherence length of thermal fluctuations (Diósi and Lukács, 1984, 1985b) is $\sim 1/T_0 \ll l_{\mathrm{horizon}}$ since $T_0 \gg T_{\mathrm{Hawking}}$.

The phenomenologic picture supported by the above coincidences corresponds to a 'continuous creation' of coherent domains and a quasi-homogeneous vacuum decay during the whole inflation. Not a single coherent region is inflated to encompass the observable part of the Universe and, $e \simeq e_{\mathrm{crit}}$ is not needed. So the strongest restriction of inflationary scenarios on cosmologic observables is avoided. Inflation and standard hot FRW evolution may produce expansions comparable to each other – both being about $(M_{\mathrm{Planck}}/M_{\mathrm{proton}})^{3/2} \simeq 10^{29}$ times. *The cosmological principle may hold true even behind the present horizon.* Of course, then the large-scale regularity of the Universe is still without a direct explanation by homogenizing processes, however, it may be a consequence of the quantum birth of the Universe, as an indivisible unit at $t \sim t_{\mathrm{Planck}}$ (as suggested, e.g., Vilenkin, 1983, 1985; Hartle and Hawking, 1983). Perhaps regularities of the *entire* Universe rather than irregularities of its parts on galactic scales ought to be attributed to quantum effects. At the same time the incoherent substratum is not completely smoothed out by the moderate inflation, so on supercluster scales the Universe need not be completely featureless. Consequently, some difficulties, familiar in inflationary context when trying to explain large-scale inhomogeneities both in matter distribution and in velocity field, for which observational evidence has been found long since (Paál, 1971) and is becoming fashionable only today (see, e.g., for peculiar velocities, Collins *et al.*, 1986; and for voids and froths, White, 1986), do not emerge here.

Without specifying a model for just this amount of supercooling and the possible quasi-homogeneous evaporation of vacuum energy, we think that *these results suggest the possibility of a classical phenomenologic explanation for primordial density perturbations* as a reasonable alternative to more exotic ones (quantum fluctuations, superstrings, topological defects, etc.) necessary only if the classical explanations fail.

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