Linear Algebra
Basis, EigenVectors

이 상 화

Basis (1/2)

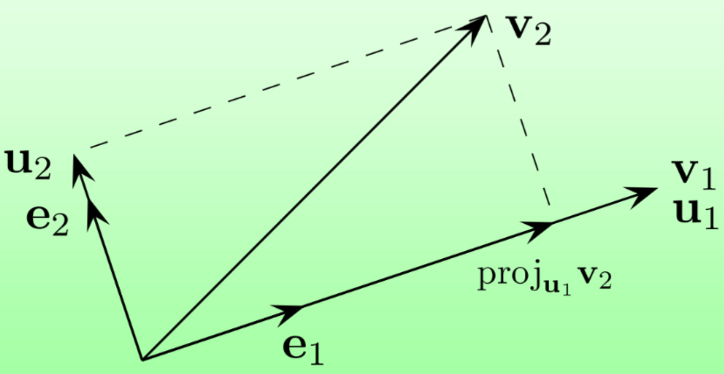
- Set of vectors (or functions) to represent the space
  - Vector space – basis vectors
  - Function space – Basis functions
- Main features
  - Elements of the space
  - Linearly independent
    - Can not be represented by the linear combinations of other vectors (or functions)
  - Usually, orthogonal
    - Projection

Basis (2/2)

- Example: 3D vector space
  - (1,0,0), (0,1,0), (0,0,1) – orthogonal basis
  - (1,0,0), (1,1,0), (1,1,1) – non-orthogonal
  - Basis is not unique for a space !!
- Example: Function space
  - Taylor series
    - Non-orthogonal polynomial functions are bases
  - Fourier series
    - Orthogonal sine, cosine functions are bases

How to find Bases ? (1/2)

- Gram-Schmidt orthogonalization
  - Find orthogonal bases from an initial vector iteratively
  - Different bases w.r.t different initial vector

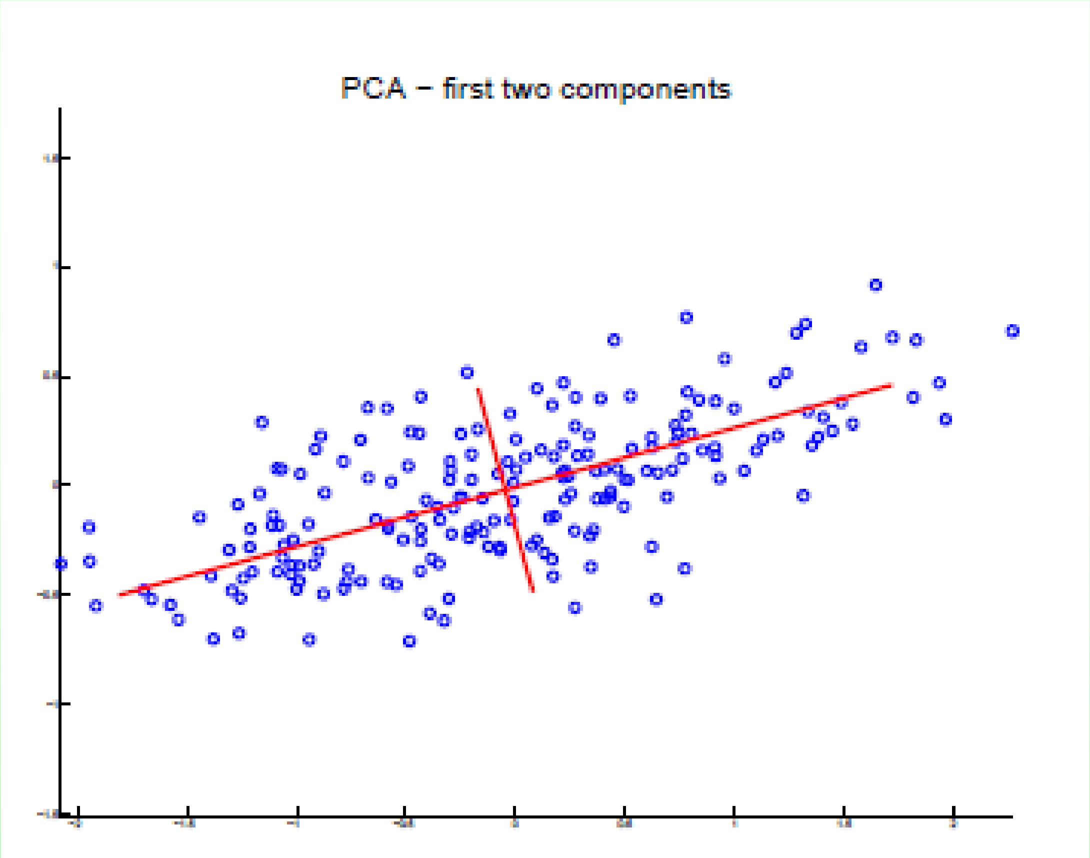


$$
\begin{aligned}
\mathbf{u}_1 &= \mathbf{v}_1, & \mathbf{e}_1 &= \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} \\
\mathbf{u}_2 &= \mathbf{v}_2 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_2), & \mathbf{e}_2 &= \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|} \\
\mathbf{u}_3 &= \mathbf{v}_3 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_3) - \mathrm{proj}_{\mathbf{u}_2}(\mathbf{v}_3), & \mathbf{e}_3 &= \frac{\mathbf{u}_3}{\|\mathbf{u}_3\|} \\
\mathbf{u}_4 &= \mathbf{v}_4 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_4) - \mathrm{proj}_{\mathbf{u}_2}(\mathbf{v}_4) - \mathrm{proj}_{\mathbf{u}_3}(\mathbf{v}_4), & \mathbf{e}_4 &= \frac{\mathbf{u}_4}{\|\mathbf{u}_4\|} \\
&\vdots & &\vdots \\
\mathbf{u}_k &= \mathbf{v}_k - \sum_{j=1}^{k-1} \mathrm{proj}_{\mathbf{u}_j}(\mathbf{v}_k), & \mathbf{e}_k &= \frac{\mathbf{u}_k}{\|\mathbf{u}_k\|}.
\end{aligned}
$$

How to find Bases ? (2/2)

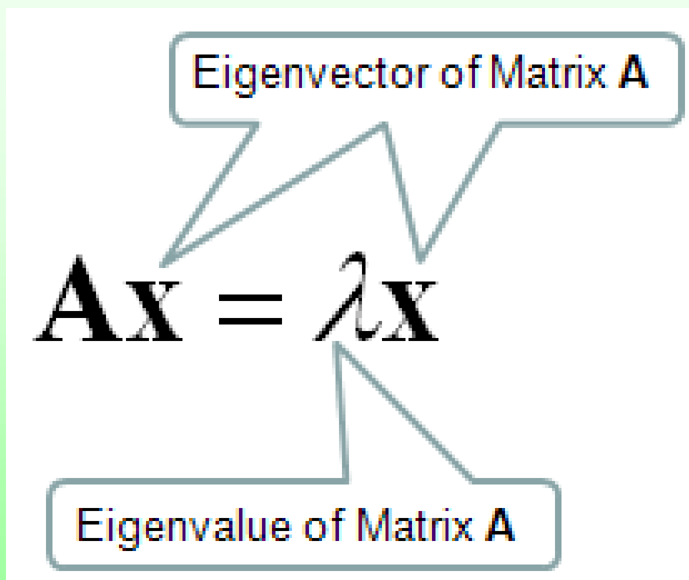
❑ Eigenvectors

- Find orthogonal bases w.r.t dominant components



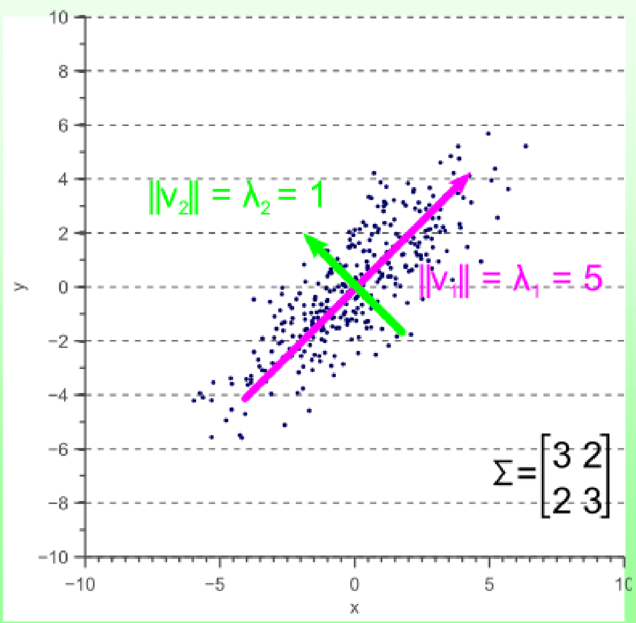
Eigenvalues and Eigenvectors (1/2)

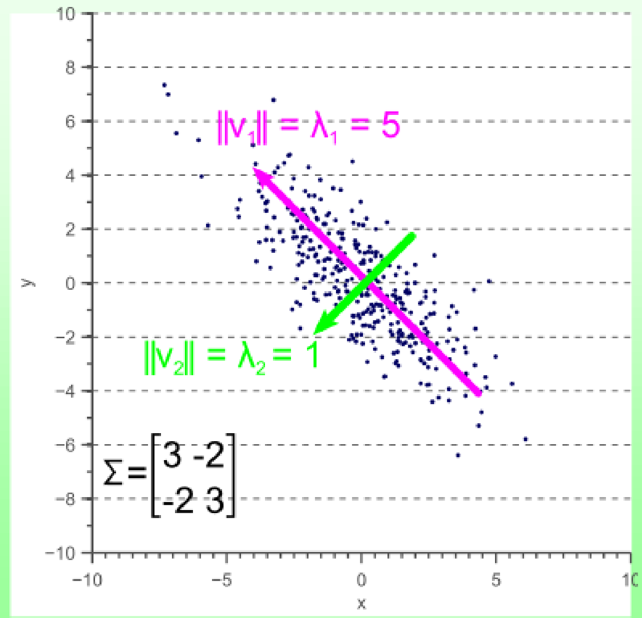
- Do not change the vector(or function) shapes for a transform (system input)
  - Just scalar multiplication
  - A: square matrix, system, transformation



Eigenvalues and Eigenvectors (2/2)

- For symmetric matrix, A^TA (covariance matrix)
  - Eigenvalues are non-negative
    - power of corresponding eigenvectors
  - Eigenvectors are orthogonal bases of column space of A





KOCW 강의 수강

- http://www.kocw.net/home/search/kemView.do?kemId=977757

차시	강의 제목	관련 내용
5차시	벡터의 선형독립과 기저벡터	Basis
8차시	1차 연립 방정식 풀이와 직교벡터 구하기	Orthogonal Basis
	일반 최소 제곱법과 QR 분할	
11차시	고유값과 고유벡터 및 대각화	Eigen vectors

HW#4 (1/2)

- Summary of the OCW lectures
  - 1~2 pages in A4 size paper
  - Handwriting
- Submit to blackboard
  - Scanning and upload a pdf file
  - Taking pictures and upload a pdf file
  - Due: 9/30 (Wed), 22:00

HW#4 (/2)

- Curve fitting using Least squares
  - For the 2D points, find the 2^{nd}-order curve using least squares
  - $y = ax^2 + bx + c$
- Sampling 6 2D points out of total 8 points
  - Randomly select 6 points
  - Twice curve fitting
  - Compare two curves
- A 행렬 직접 생성후 inverse 등의 최소 과정만 라이브러리 이용
  - Pseudo-inverse 사용

2D points
(-2.9, 35.4)
(-2.1, 19.7)
(-0.9, 5.7)
(1.1, 2.1)
(0.1, 1.2)
(1.9, 8.7)
(3.1, 25.7)
(4.0, 41.5)

5주차: 추석 연휴

- No Lecture, No Homeworks
- 맑고 깨끗한 가을의 정취, 휴식의 시간으로...
- 방역 수칙 철저!!