# Some Well-known Distributions

이 상 화

# Geometric Distribution

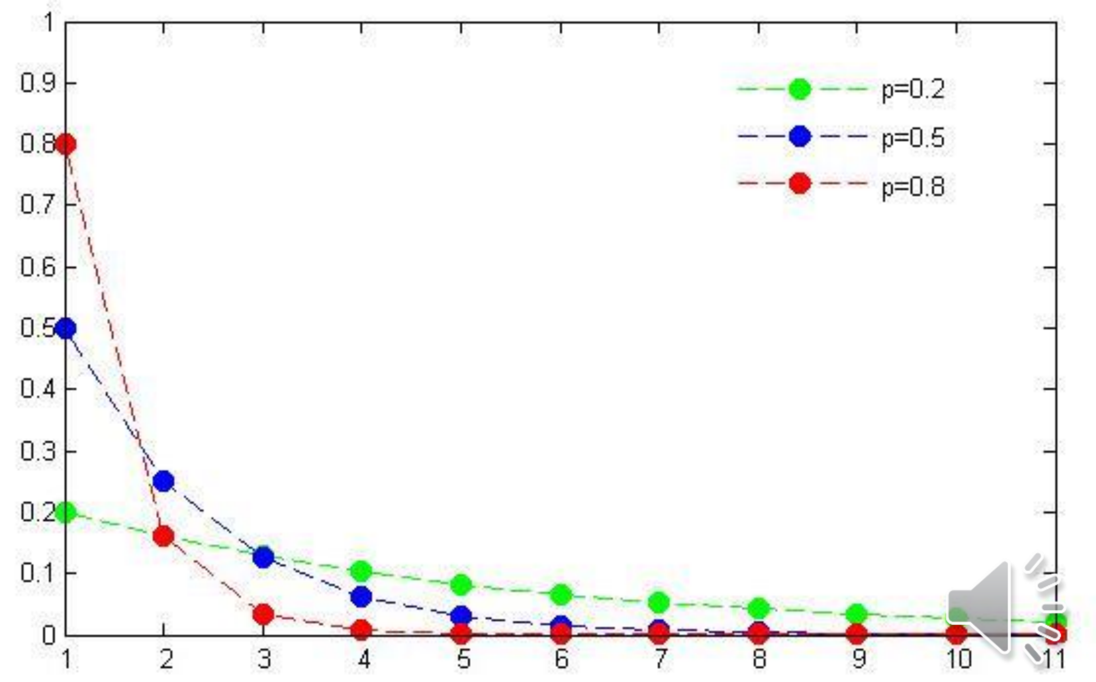
- *X*: number of trials until the 1st success
  - p : probability of success for a trial
- Applications
  - Network server access trials until success

$$P(X = n) = p(1-p)^{n-1}$$

$$P(X > n) = (1-p)^n$$

$$\text{Mean } \mu = \frac{1}{p}$$

$$\text{Variance } \sigma^2 = \frac{1-p}{p^2}$$

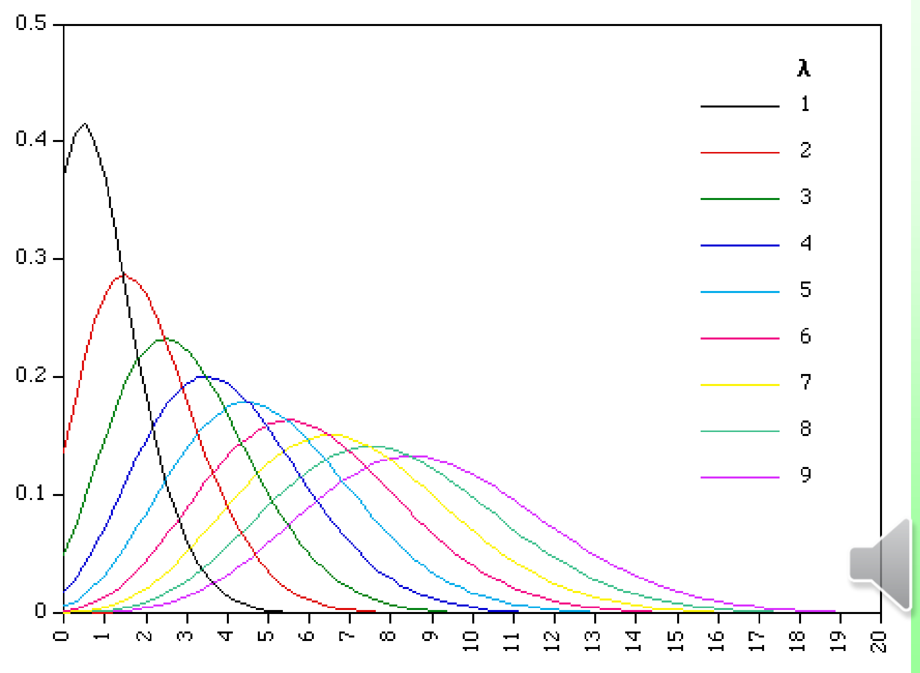


# Poisson Distribution

- ❑ *X*: number of occurrences in a time interval
  - $\lambda$ : mean, variance
- ❑ Applications
  - Network server access trials for a time interval
  - Service system queues: Bank counter, Tollgate

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P_X(k) = \frac{e^{-(\lambda t)} * (\lambda t)^k}{k!}$$



# Exponential Distribution

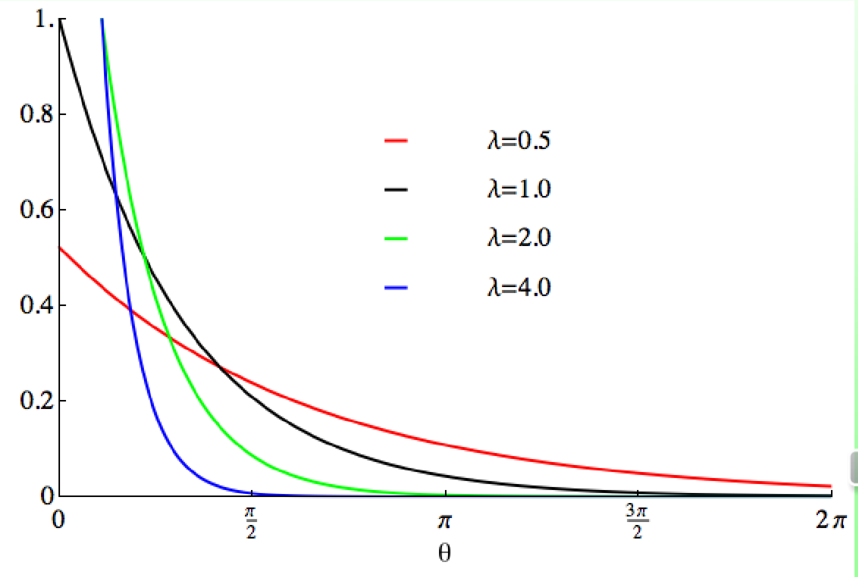
- *X*: interval (or distance) between occurrences
  - $\lambda$ : rate (mean of counts in a unit interval)
- Applications
  - Time interval between successive occurrences of independent identical distributed events.
    - EX: time interval between guests in the bank

$$f(x) = \lambda e^{-\lambda x}$$

$$F(x) = 1 - e^{-x\lambda}$$

$$E(X) = 1/\lambda$$

$$V(X) = 1/\lambda^2$$



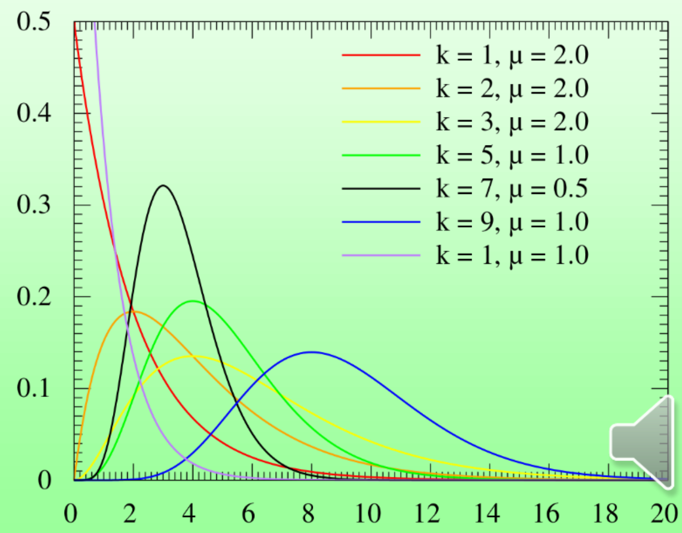
# Erlang-r Distribution

- $X$: interval between $r$ successive occurrences
  - $\lambda$ : rate (mean of counts in a unit interval)
  - $r-1$ convolutions of exponential distributions
- Applications
  - Time interval between $r$ successive occurrences of independent identical distributed events.
    - EX: time interval between guests in the bank

$$P(X > x) = \sum_{k=0}^{r-1} \frac{e^{\lambda x}(\lambda x)^k}{k!} = 1 - F(x)$$

Now differentiating $F(x)$:

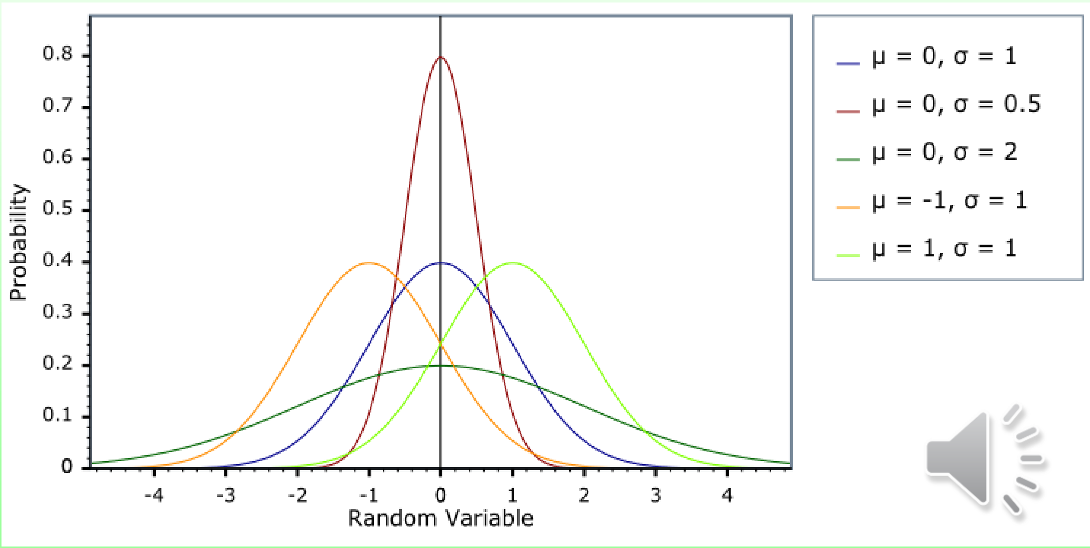
$$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!} \quad \text{for } x > 0 \text{ and } r = 1, 2, \ldots$$



# Gaussian Distribution (1/2)

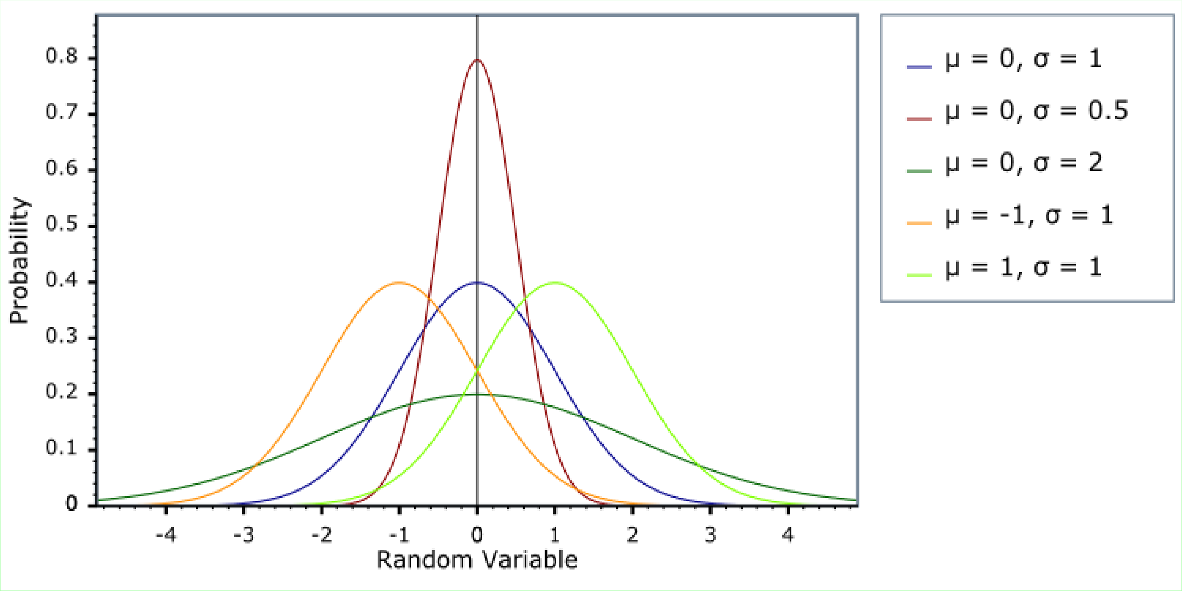
- *X*: sum of independent identical distributed random variables
  - $X = X_1 + X_2 + X_3 + \ldots\ldots X_n \quad (n \to \infty)$
  - Convolutions of pdf of $X_i$
- Applications
  - General distribution for any signals
  - Noise

$$\mathcal{N}(x\ ;\ \mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}(x-\mu)^2/\sigma^2\right]$$



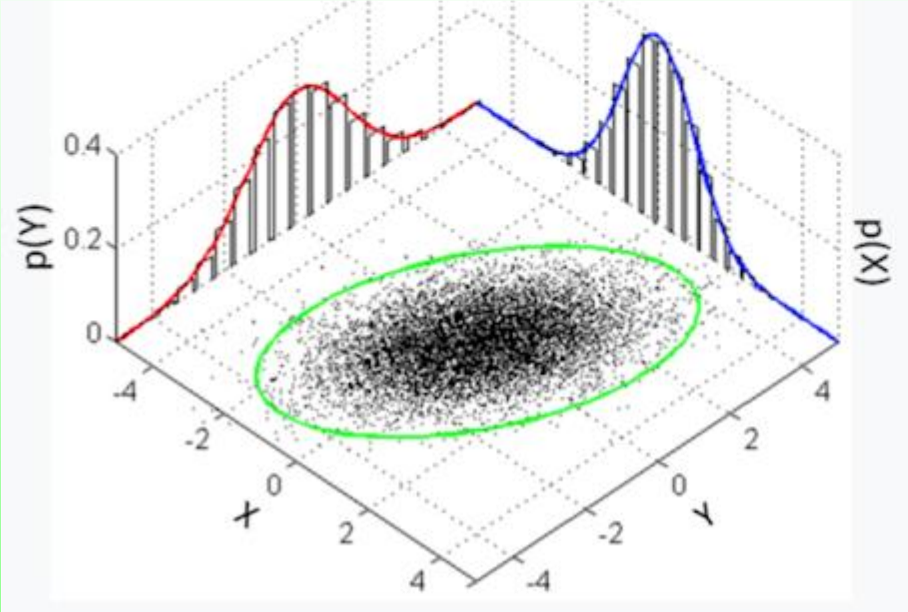
# Gaussian Distribution (2/2)

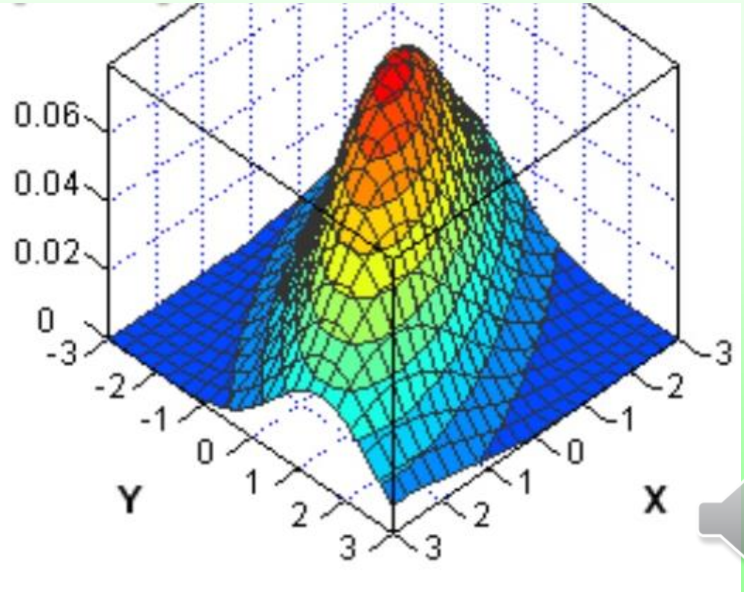
- Single mode
  - There is one value with the maximum probability.
- Symmetric at the mean
  - Maximum probability => mean
- Finding the mean is the best for Gaussian RV.



# Multi-variate Distributions (1/3)

- Two or more random variables are considered simultaneously.
  - Influenced or dependent on each other
- Discrete RVs: P(X=$x$, Y=$y$)
- Continuous RVs: Joint pdf: $f(x, y)$

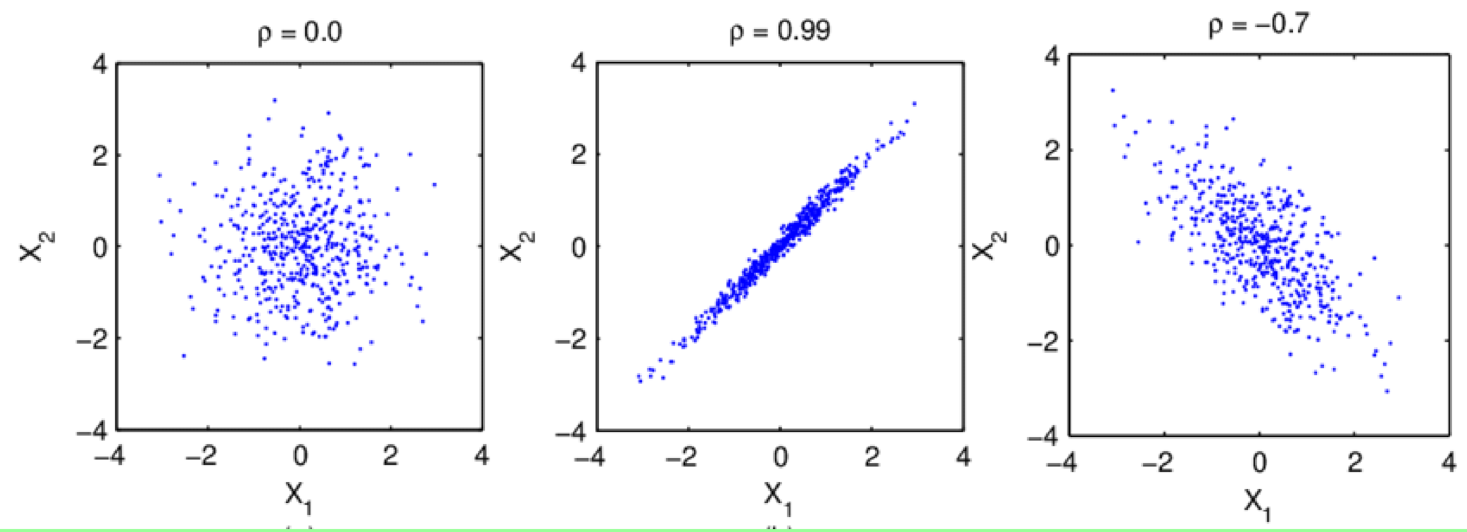


# Multi-variate Distributions (2/3)

- Correlation of 2 random variables
  - Similarity measure of two random variables
  - Normalized measure: −1 ~ +1

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

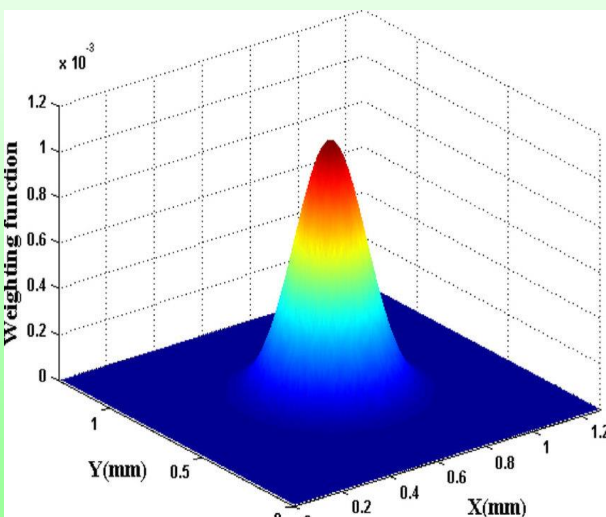


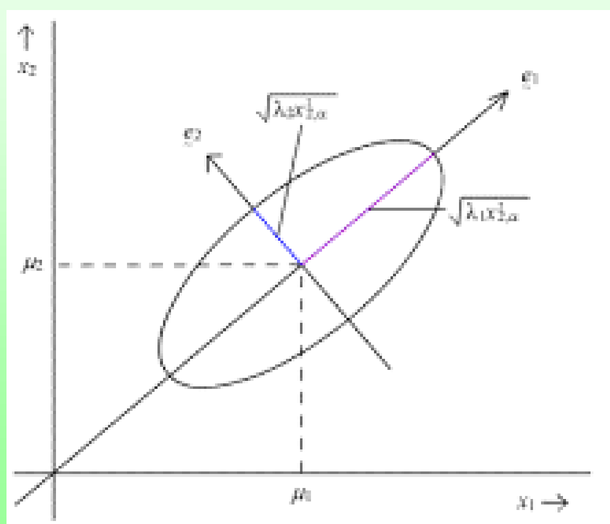
# Multi-variate Distributions (3/3)

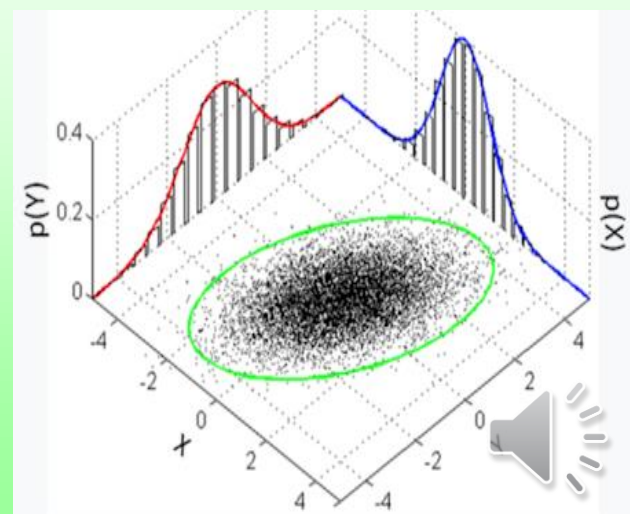
## Bivariate Gaussian

- Joint 2 Gaussian RVs

$$f_{XY}(x, y; \sigma_X, \sigma_Y, \mu_X, \mu_Y, \rho) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(x-\mu_X)^2}{\sigma_X^2} - \frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right]\right\} \quad (5-$$



# HW#11 (1/2)

- Find the correlation coefficients between color components in the images
  - G–R in RGB space
  - G–B in RGB space
  - R–B in RGB space
  - Y–U in YUV space
  - Y–V in YUV space
  - U–V in YUV space
- Due date
  - 11/18 (Wed.) 22:00

# HW#11 (2/2)

- ❑ Collect 10 colorful images
- ❑ Use RGB components
- ❑ Use YUV components
  - Any YUV format is OK

$$\begin{bmatrix} Y' \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.14713 & -0.28886 & 0.436 \\ 0.615 & -0.51499 & -0.10001 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.13983 \\ 1 & -0.39465 & -0.58060 \\ 1 & 2.03211 & 0 \end{bmatrix} \begin{bmatrix} Y' \\ U \\ V \end{bmatrix}$$