# Sub-GAN: An Unsupervised Generative Model via Subspaces

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## Outline





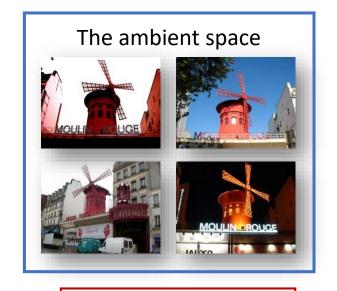
- Problem and Related Work
- Motivation
- Main Idea
- The Proposed Sub-GAN method
- Experiments

## **Problems**





## *In an ideal generative model:*



High-dimensional!



Need large-scale training data and deep architecture to model the ambient space



Mode Collapse!

#### Problems to be solved in this paper:

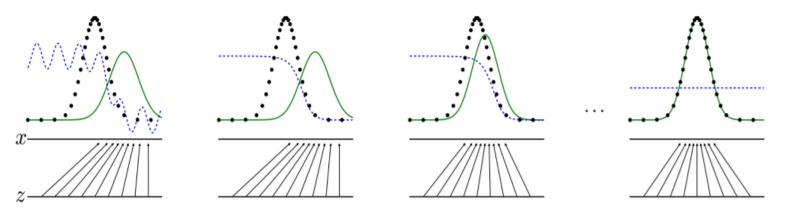
- Disentangling the low-dimensional subspaces of the ambient space
- Generating diverse samples without any supervision

## Related Work

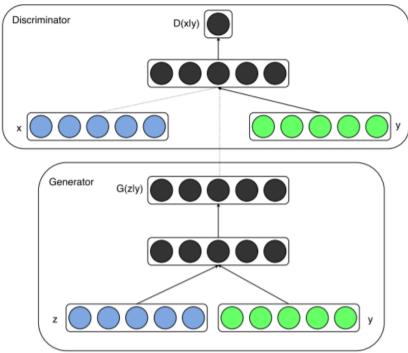




#### Generative Adversarial Networks (GAN):



Training process of the GAN.



Framework of the conditional GAN.

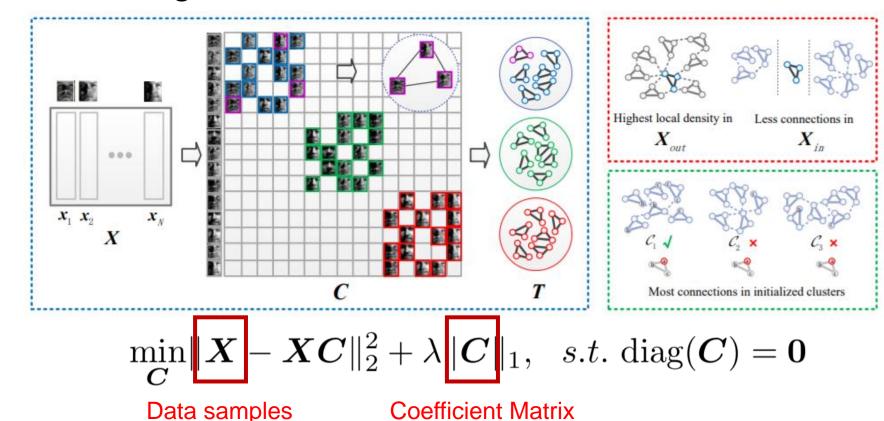
Goodfellow et. al, Generative Adversarial Nets, NIPS 2014. Mirza et. al, Conditional Generative Adversarial Nets.

## Related Work





#### Subspace Clustering:



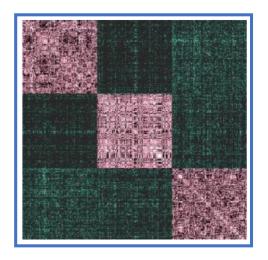
Elhamifar et. al, Sparse Subspace Clustering, CVPR 2009 Yang et. al, Automatic Model Selection in Subspace Clustering via Triplet Relationships, AAAI 2018

## Motivation

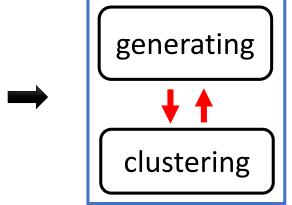




1. To disentangle the low-dimensional subspaces

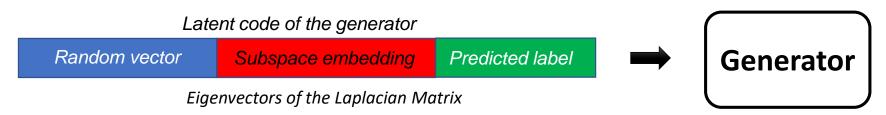


The Coefficient Matrix in Subspace Clustering is block-diagonal. Each entry reflects the similarity between two samples.



Interaction between generating module and clustering module. Finally, Both modules are improved.

2. To control the diversity of the generated samples without supervision

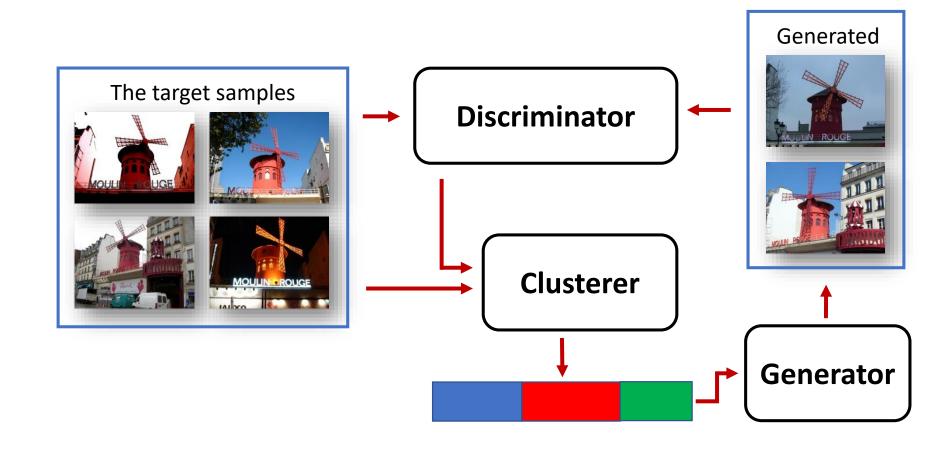


The subspace embedding and the predicted label are iteratively updated.

## Main Idea







The three modules are mutually optimized during training.

# Algorithm





#### Clusterer:

Optimizing a Self-Representation problem:

$$\min_{\boldsymbol{C}} \|\boldsymbol{X} - \boldsymbol{X}\boldsymbol{C}\|_2^2 + \lambda \|\boldsymbol{C}\|_1, \quad s.t. \ \mathrm{diag}(\boldsymbol{C}) = \boldsymbol{0}, \tag{1}$$
 Data samples Coefficient Matrix

Calculating the eigenvectors of the Laplacian matrix:

$$[\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_K] = \operatorname{eig}(\boldsymbol{M}),$$
 (3)

Laplacian Matrix

Each of the K eigenvectors reflects the embedding of a subspace.

# Algorithm





#### Generator:

The latent code of G is composed of three vectors:

#### Random

$$oldsymbol{l} = oldsymbol{z} \oplus oldsymbol{e} oldsymbol{\hat{y}}.$$
 Predicted label

Subspace embedding

#### Discriminator:

1) Distinguish between real and fake samples; 2) Classify the inputs into subspaces.

Minimax objective:

$$\min_{G,C} \max_{D} \ \mathcal{L}(D,G,C),$$

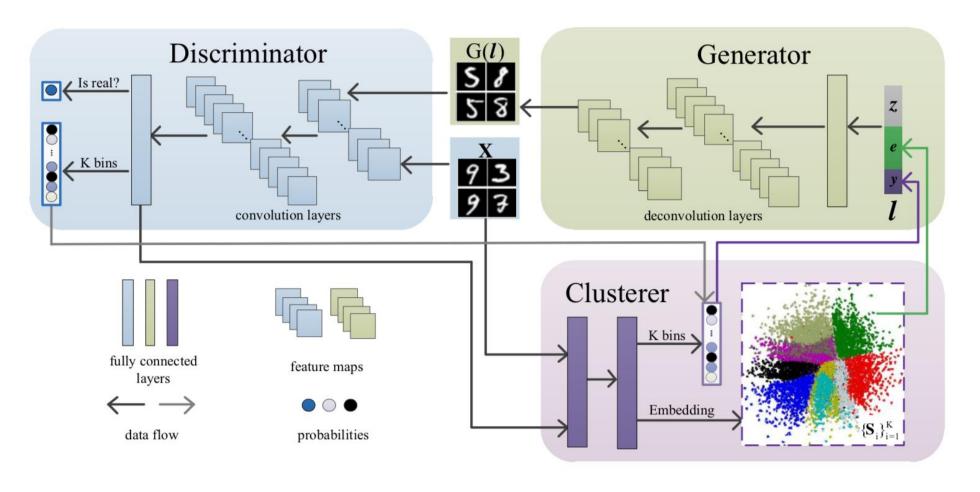
$$\mathcal{L}(D, G, C) = \mathbb{E}_{\boldsymbol{x} \sim p_{\boldsymbol{S}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{l} \sim p_{\boldsymbol{L}}(\boldsymbol{l})}[\log(1 - D(G(\boldsymbol{l})))] + \mathrm{KL}(\boldsymbol{Q}_{\boldsymbol{S}}||\boldsymbol{P}_{\boldsymbol{S}}).$$
Loss of D
Loss of G
Loss of C

# Algorithm





### Pipeline of the proposed Sub-GAN:







#### Setup

**Datasets**: MNIST and CIFAR-10

#### **Contrastive Methods**

Generation: CGAN, Improved GAN, Improved WGAN, DCGAN and InfoGAN.

Clustering: K-means, SSC, LSR, SMR, NSN, SSC-OMP, ORGEN, iPursuit, DEC, CatGAN and InfoGAN.

#### **Evaluation Metrics**

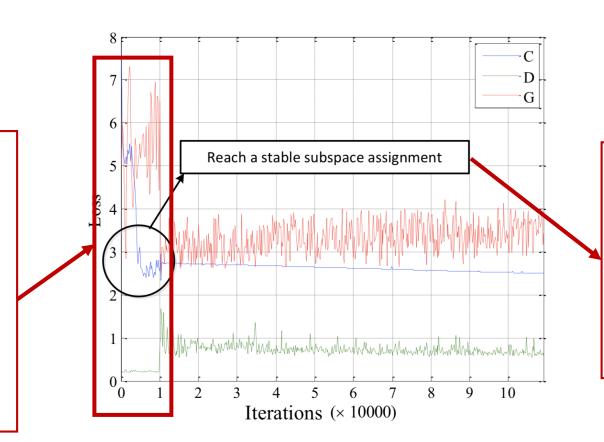
Inception Score
Diversity Score
Adjusted Accuracy for Clustering





#### Training Process:

The loss of the Clusterer demonstrates a downward trend before around 10, 000-th iteration. Sequentially, the training of the Generator and Discriminator is unstable, e.g., D can easily discriminate the fake images from the real one so that the loss of discriminator is low.



After C reaches a stable subspace assignment, the framework begins a normal adversarial training of the three modules G, D and C.

Optimization losses of three modules.





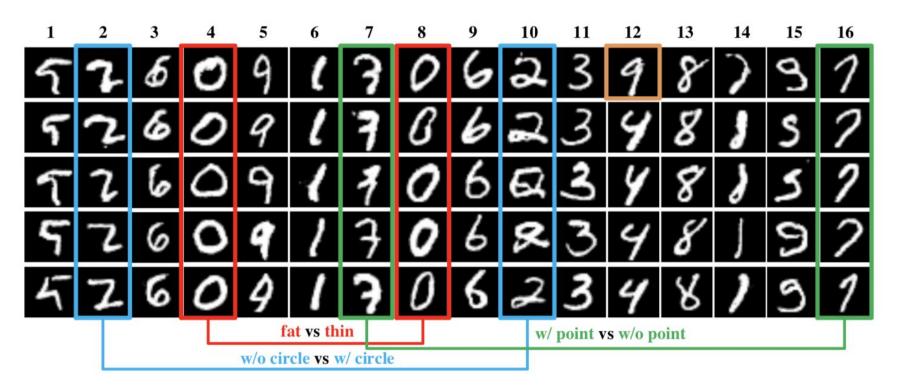
neration:	Image Quality	Modeling Subspaces?
0003236669 5170285392	Not Satisfied	Yes
7447956811 13909111987 6649013996 2683097141 7304576709	Satisfied	No
7510325202 65879301R4 6836408389 65879301R4 7867120165 65879301R4	favorable	Yes

Generation Performance of the contrastive methods.





#### Generation:



Samples generated from joint unsupervised training on the MNIST dataset using the proposed Sub-GAN by setting K=16. The Sub-GAN can discover multiple hidden attributes of the data. The diversity of generation can be controlled by the given number of clusters.





#### Generation:

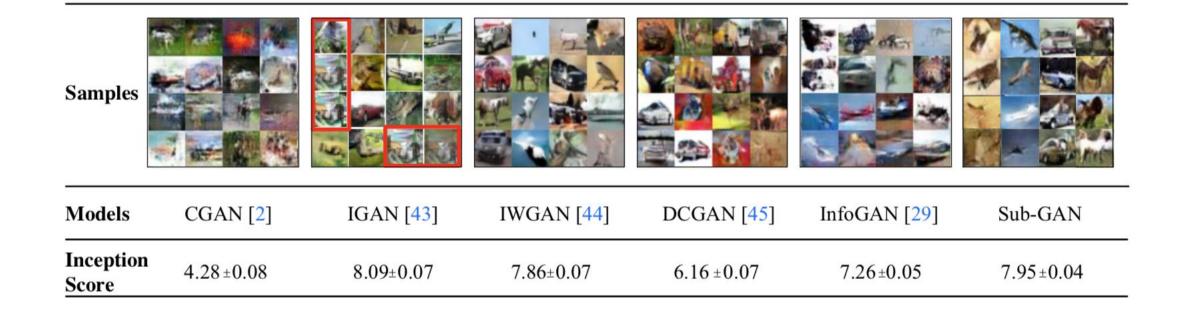
Datasets	Real	CGAN [2]	IGAN [43]	IWGAN [44]	DCGAN [45]	InfoGAN [29	] Sub-GAN
MNIST CIFAR	$2.96 \\ 3.21$	$0.92 \\ 1.02$	$1.81 \\ 2.20$	$1.78 \\ 2.03$	$1.63 \\ 1.95$	$2.11 \\ 2.48$	$2.36 \\ 2.72$

Comparison of the diversity scores on both MNIST and CIFAR datasets with K = 10. The proposed Sub-GAN achieves best performance against contrastive methods, which alleviates the mode collapse problem in training GANs.





#### Generation:



Inception scores for samples derived from various generative models on the CIFAR-10 dataset.





#### Clustering:

Some samples might be wrongly grouped based on the global similarity to all others.



Refine the assignment in D based on the similarity of samples in local batches.

Refinement in $D$	$1^{\mathrm{st}}$ Epoch	$20^{\mathrm{th}}$ Epoch	$40^{ m th}$ Epoch	Last Epoch
W/o	75.23	82.96	83.11	83.87
W/	$\boldsymbol{77.12}$	$\bf 83.45$	$\bf 84.24$	$\bf 85.32$

Clustering accuracy (%) on the MNIST dataset under K = 10 with/without the refinement operation in the discriminator.





### Clustering Performance:

Methods	MNIST			CIFAR			
	K = 10	K = 16	K = 20	K = 10	K = 16	K = 20	
K-means	53.49	60.36	62.55	42.62	46.81	51.02	
SSC[4]	62.71	66.82	70.19	50.31	52.77	53.98	
LSR [46]	66.85	70.21	73.83	53.97	55.80	59.24	
SMR[47]	73.39	81.27	83.63	56.24	59.02	62.73	
NSN [48]	68.75	71.04	73.67	52.29	56.55	59.03	
SSC-OMP [35]	76.33	79.25	82.52	51.21	53.02	57.84	
ORGEN $[31]$	71.04	74.07	78.65	52.29	55.61	58.08	
iPursuit [49]	61.35	64.28	68.84	59.21	62.53	65.66	
DEC [39]	84.30	83.28	83.02	61.03	65.29	67.31	
CatGAN [50]	80.21	84.92	90.30	67.42	67.85	68.76	
InfoGAN [29]	70.63	73.77	78.69	71.02	73.64	74.07	
Sub-GAN	$\bf 85.32$	90.36	90.81	78.95	81.35	82.44	

Unsupervised clustering performance (adjusted clustering accuracy) of the contrasted methods on the MNIST and CIFAR datasets with different K's.

## Conclusion





#### The proposed unsupervised Sub-GAN model:

- Jointly learning the latent subspaces of the ambient space and generating instances correspondingly
- Clusterer: aims to discover distinctive subspaces of high-dimensional data in an unsupervised fashion, which is updated on each epoch based on the feedback from the discriminator
- **Generator**: produces samples conditioned on a one-hot vector indicating the belonged cluster and a base vector of subspace derived from the clusterer
- **Discriminator**: not only needs to distinguish between real and fake samples, but also requires to classify them to belonged subspaces. It also provides distinctive representations of data samples for updating the clusterer





