

# Frequency Response

\* Recap

$$\cos(377t)$$

What is the frequency of this waveform?

$$\cos(\omega t) = \cos(2\pi f t)$$

$f$ : frequency (Hz)

$$\omega = 377$$

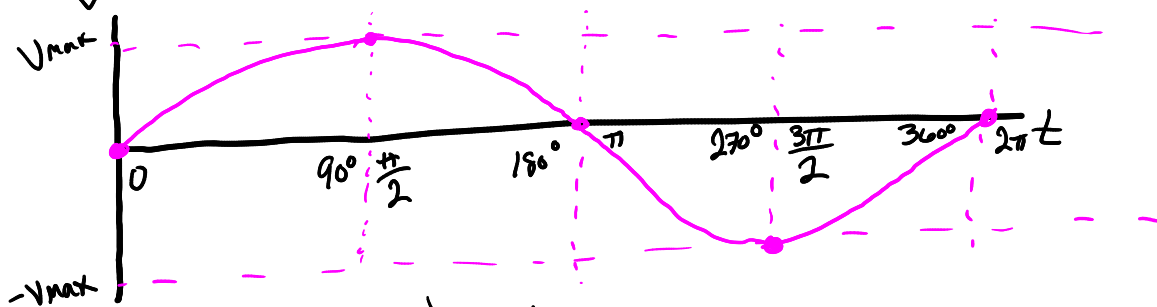
$\omega$ : angular frequency ( $\frac{\text{rad}}{\text{s}}$ )

$$\Rightarrow 2\pi f = 377$$

$$f = 60.03 \text{ Hz}$$

$$v(t) = V_{\text{max}} \sin(\omega t)$$

\* Sinusoidal Waveform:



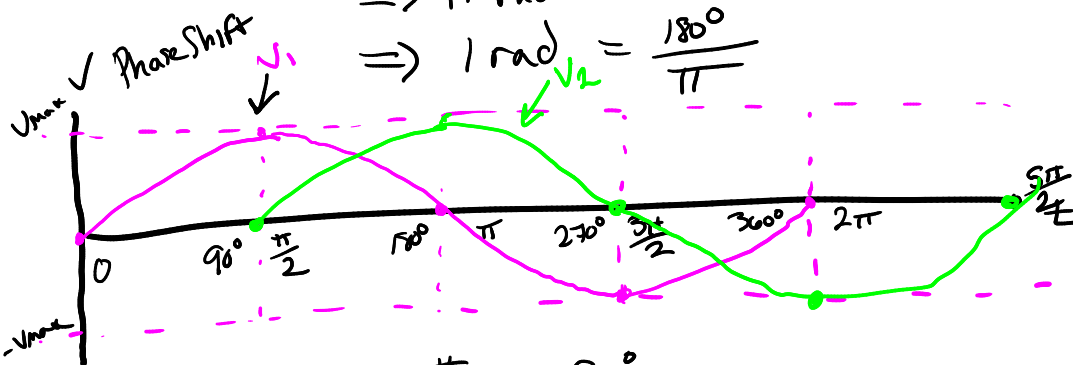
$$V_{\text{RMS}} = \frac{1}{\sqrt{2}} V_{\text{max}}$$

\* How to convert between radians and degrees?

$$2\pi \text{ rad} = 360^\circ$$

$$\Rightarrow \pi \text{ rad} = 180^\circ$$

$$\Rightarrow 1 \text{ rad} = \frac{180^\circ}{\pi}$$



\* phase shift of  $\frac{\pi}{2}$  or  $90^\circ$

\* If  $v(t)$  is written in complex form

$$v(t) = A + jB$$

$$\text{Mag}[v(t)] = \sqrt{A^2 + B^2}$$

$$\text{Phase}[v(t)] = \tan^{-1}\left(\frac{B}{A}\right)$$

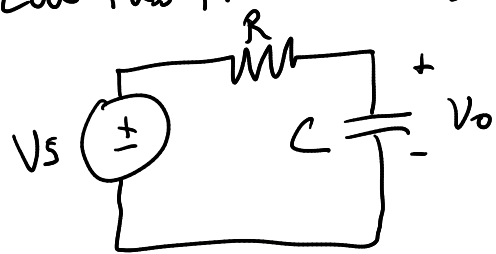
Impedance

$$\text{---} \Rightarrow R \quad R + j0$$

$$\text{---} \Rightarrow j\omega L$$

$$\text{---} \Rightarrow \frac{1}{j\omega C} \text{ or } \frac{-j}{\omega C}$$

\* Low Pass Filter (LPF)



Voltage Divider

$$V_o = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \times V_s$$

$$\Rightarrow V_o = \frac{1}{j\omega CR + 1} \times V_s$$

$$* |V_o| = \frac{1}{\sqrt{(\omega CR)^2 + 1}} |V_s|$$

When  $\omega = 0$ ;  $|V_o| = \frac{1}{\sqrt{0+1}} \times |V_s|$

$$\Rightarrow |V_o| = |V_s|$$

$\omega = \infty$ ;  $|V_o| = \frac{1}{\infty} \times |V_s| \rightarrow 0$

\* Low pass Filters allow low frequencies to pass through

$$V_o = \frac{1 + j0}{j\omega CR + 1} \times V_s$$

\* Phase

$$\angle \frac{x}{y} = \angle x - \angle y$$

$$\phi \Rightarrow \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega CR}{1}\right) = -\tan^{-1}(\omega CR)$$

$$\text{Gain} = \frac{\text{output}}{\text{input}} = \frac{V_o}{V_s} = \frac{1}{\sqrt{(\omega CR)^2 + 1}}$$

\* Example,  $R = 2k\Omega$ ,  $C = 0.1\mu F$  Find  $|V_o|$  at  $f = 2kHz$   
 $V_s = 4V$

$$\frac{V_o}{V_s} = \frac{1}{\sqrt{[2\pi(2000)(0.1 \times 10^{-6})(2000)]^2 + 1}} = \frac{1}{2.705} = 0.3696$$

$$V_o = 4 \times 0.3696 = \boxed{1.478V}$$

On a log scale

$$\frac{V_o}{V_s} = 20 \log_{10}(0.3696) = -8.64 \text{ dB}$$

Account for  $V_s = 4 \Rightarrow 20 \log_{10}(1.478) = 3.393 \text{ dB}$

\* Cutoff frequency: The frequency where the output power is  $\frac{1}{2}$  the input power

$$\Rightarrow P_o = \frac{1}{2} P_{in}$$

$$P = \frac{V^2}{R} \quad P \propto V^2$$

$$|V_o|^2 = \frac{1}{2} |V_s|^2$$

$$\frac{1}{\sqrt{2}} = 0.707$$

$$\Rightarrow |V_o| = \frac{1}{\sqrt{2}} |V_s|$$

$$\text{In dB, } 20 \log_{10} \left( \frac{1}{\sqrt{2}} \right) \approx -3 \text{ dB}$$

\* To find the cutoff frequency (assume unity  $V_s = 1$ )

$$\frac{1}{\sqrt{(wCR)^2 + 1}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sqrt{(wCR)^2 + 1} = \sqrt{2} \Rightarrow (wCR)^2 + 1 = 2$$

$$\Rightarrow (wCR)^2 = 1 \Rightarrow wCR = 1 \Rightarrow w = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC}$$

from earlier

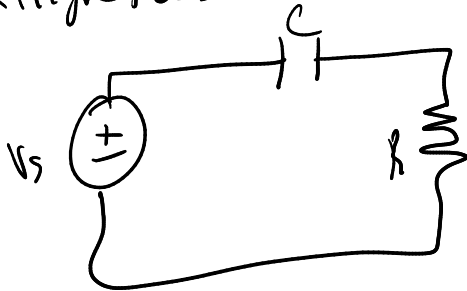
$$f_c = 795.78 \text{ Hz}$$

$$\text{When } V_s = 4V \Rightarrow 20 \log_{10}(4) \approx 12 \text{ dB}$$

$$- 3 \text{ dB}$$

$$\hline 9 \text{ dB}$$

\* High Pass



$$V_o = \frac{R}{R + \frac{1}{j\omega C}} V_s$$

$$V_o = \frac{j\omega RC}{j\omega RC + 1} V_s$$

$$\frac{|V_o|}{|V_s|} = \frac{\omega RC}{\sqrt{(wCR)^2 + 1}}$$

$$\omega = 0 \Rightarrow \frac{0}{\sqrt{0+1}} = 0$$

$$\omega = \infty \Rightarrow \omega \gg 1 \Rightarrow \frac{\omega RC}{\sqrt{(wRC)^2}}$$

$$\Rightarrow |V_o| = |V_s|$$

\* High Pass : pass through high frequency, block low frequencies.

\* RL Low pass

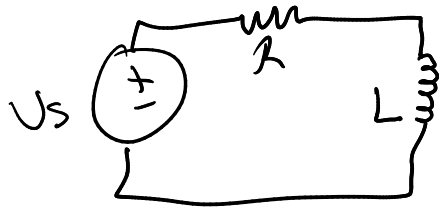


$$\frac{|V_o|}{|V_s|} = \frac{R}{R + j\omega L}$$

$$\omega = 0 \rightarrow |V_o| = |V_s|$$

$$\omega = \infty \rightarrow \frac{R}{\infty} = 0$$

\* RL High Pass

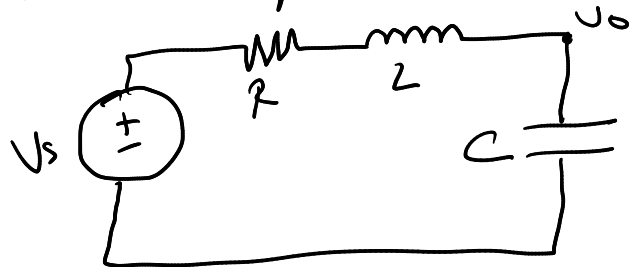


$$\frac{|V_o|}{|V_s|} = \frac{j\omega L}{R + j\omega L}$$

$$\omega = 0 \rightarrow \frac{0}{R + 0} = 0$$

$$\omega = \infty \rightarrow |V_o| = |V_s|$$

\* RLC Bandpass Filter



$$V_o = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} \times V_s$$

Multiply by  $\frac{1}{j\omega C}$

$$\frac{V_o}{V_s} = \frac{1}{j\omega CR - \omega^2 LC + 1}$$

$$\left| \frac{V_o}{V_s} \right| = \frac{1}{\sqrt{(\omega CR)^2 + (1 - \omega^2 LC)^2}}$$

$$\phi = \frac{\tan^{-1}\left(\frac{0}{1}\right)}{\tan^{-1}\left(\frac{\omega CR}{1 - \omega^2 LC}\right)}$$

\* Resonant Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

