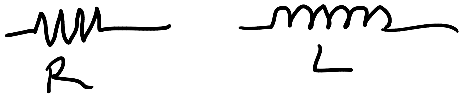
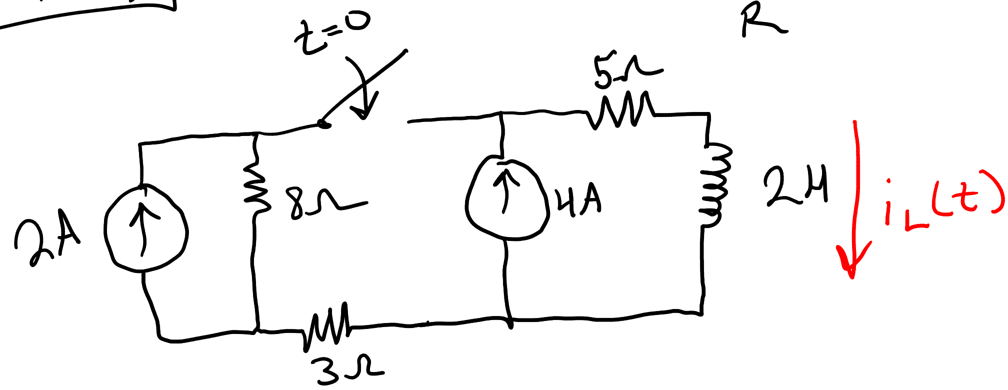
Lecture 21



Ex:



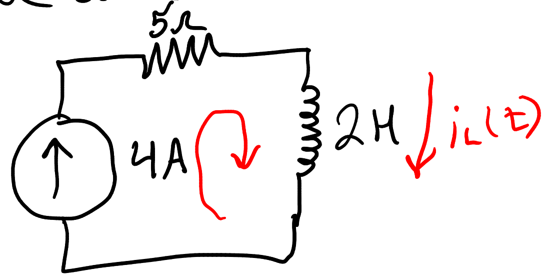
Find $i_L(t)$

$i_L(0^-) = i_L(0^+) = i_L(0)$

* We cannot have instantaneous change in current

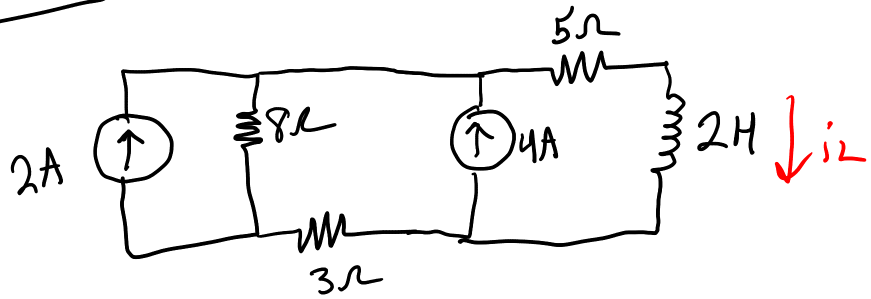
How do we solve this?

t<0)



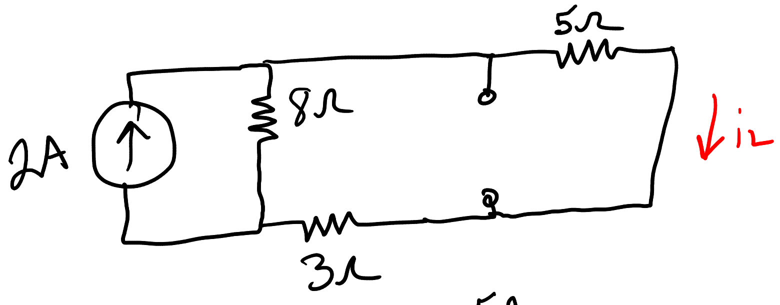
$i_L(0) = 4A$

t>0⁻)

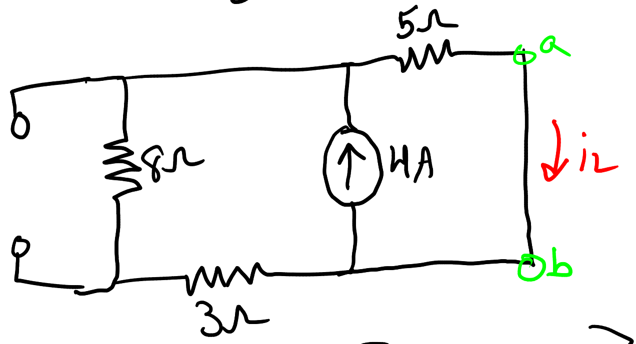


Find $i(\infty)$

Use superposition



$i_2(\infty) = \frac{8 \times 2}{5+3+8} = \frac{16}{16} = 1A$

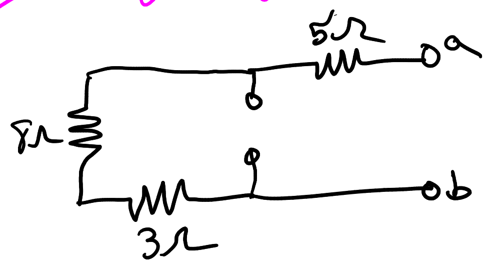


$i_4(\infty) = \frac{11 \times 4}{8+3+5} = 2.75A$

$i_L(\infty) = i_2(\infty) + i_4(\infty) = 3.75A$

$i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-\frac{t}{\tau}}$ $\quad \tau = \frac{L}{R}$; $L = 2H$

Req:

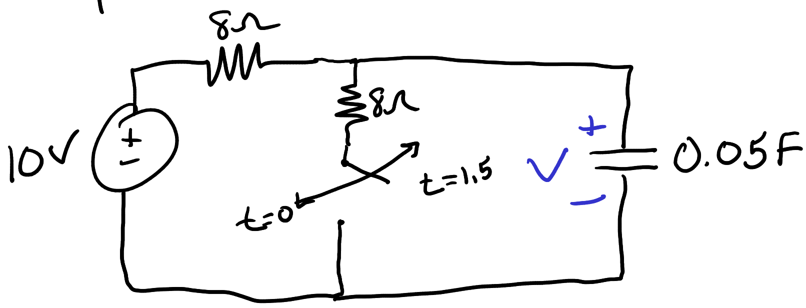


$R_{eq} = ?? = 5+8+3 = 16\Omega$

$\Rightarrow \tau = \frac{2}{16} = \frac{1}{8}$

$$\boxed{i(t) = 3.75 + 0.25e^{-8t}\ A}$$

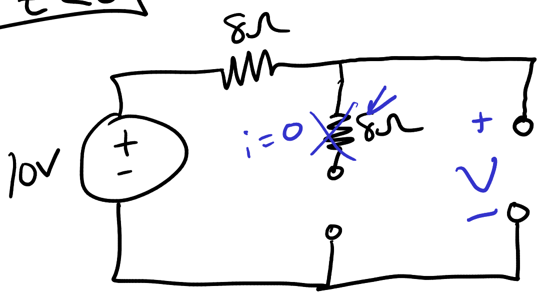
* Sequential Switching



Switch closes @ $t = 0$

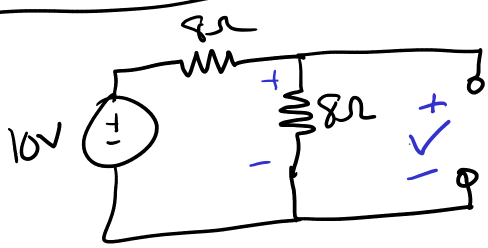
Remains closed till $t = 1.5$ sec, open afterward

for $t < 0^-$



$V(0^-) = 10V$

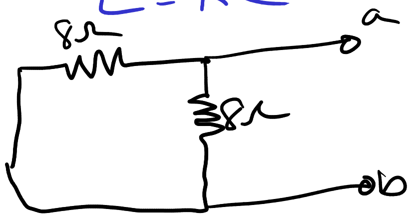
for $0 < t < 1.5$



$$V(\infty) = \frac{8}{8+8} \times 10 = 5V$$

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-\frac{t}{\tau}}$$

$\tau = RC$



$R_{eq} = 8 \| 8 = 4\Omega \Rightarrow \tau = 4 \times 0.05 = 0.20$ sec

$$V(t) = 5 + [10 - 5]e^{-\frac{t}{0.20}}$$

$$\Rightarrow \boxed{5 + 5e^{-5t}\ V} \quad \text{for } 0 < t < 1.5 \text{ sec}$$

for $t > 1.5$

$V(1.5) = V(1.5^-) = V(1.5^+)$

$$V(1.5) = 5 + 5e^{-5(1.5)} = 5.002V$$

$V(\infty) = 10V$ * from earlier

$\tau = RC$

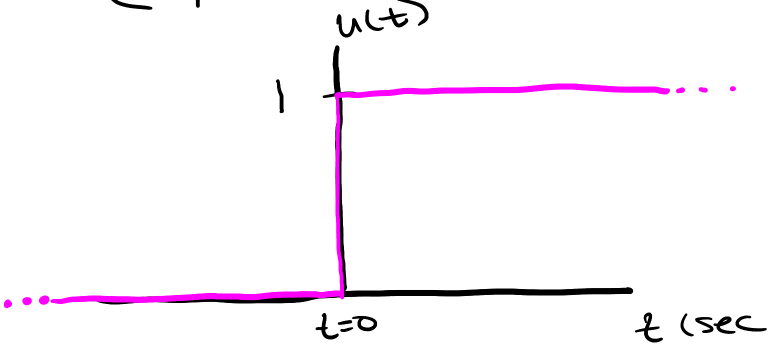
$\tau = 8(0.05)$

$\tau = 0.40$ sec

$$\boxed{V(t) = 10 + [-4.998]e^{-\frac{(t-1.5)}{0.40}}}$$

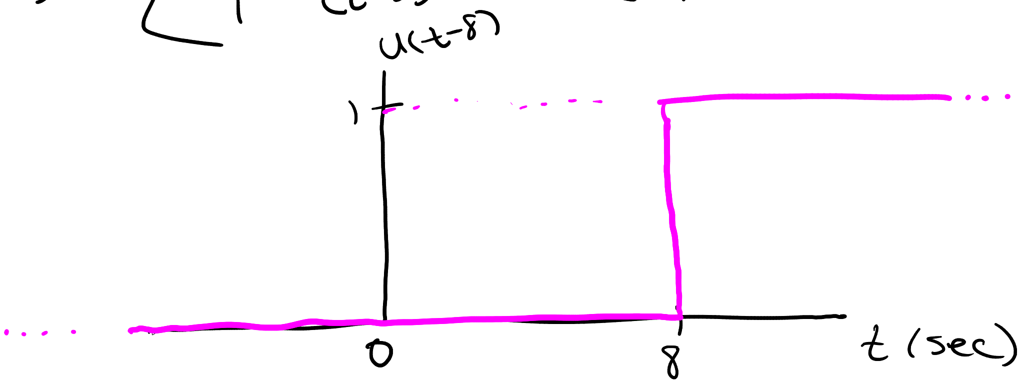
* Unit Step

$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$



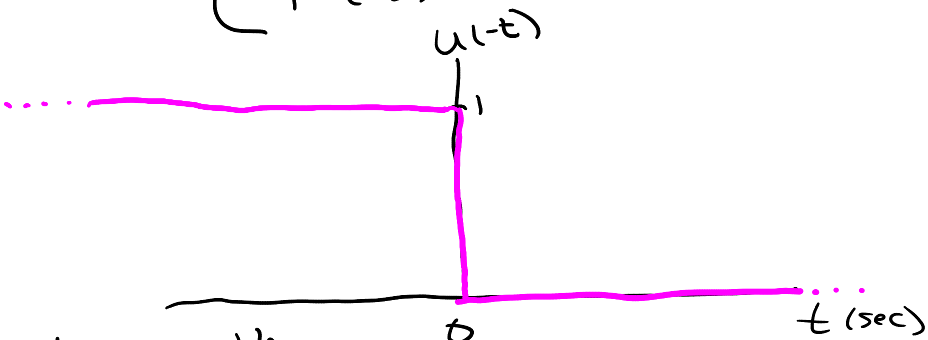
* Time Delay

$$u(t-8) = \begin{cases} 0 & (t-8)<0 \\ 1 & (t-8)>0 \end{cases} = \begin{cases} 0 & t<8 \\ 1 & t>8 \end{cases}$$

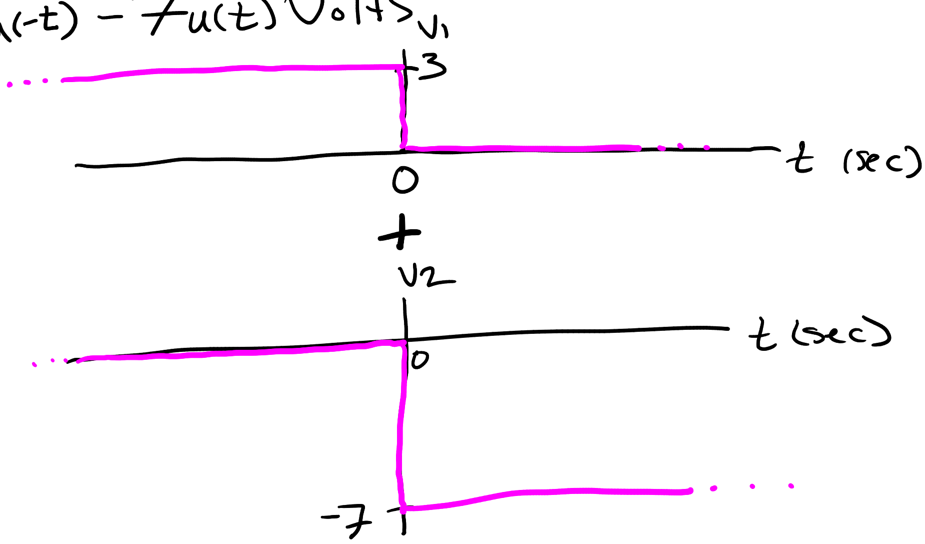


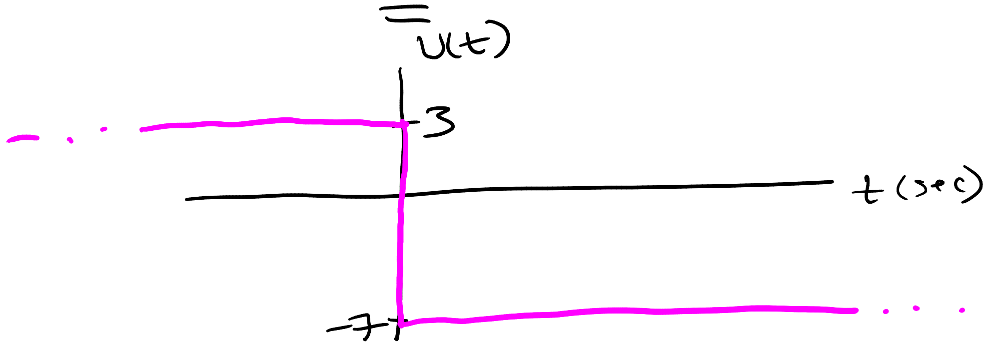
* Time Reversal

$$u(-t) = \begin{cases} 0 & (-t)<0 \\ 1 & (-t)>0 \end{cases} = \begin{cases} 0 & t>0 \\ 1 & t<0 \end{cases}$$



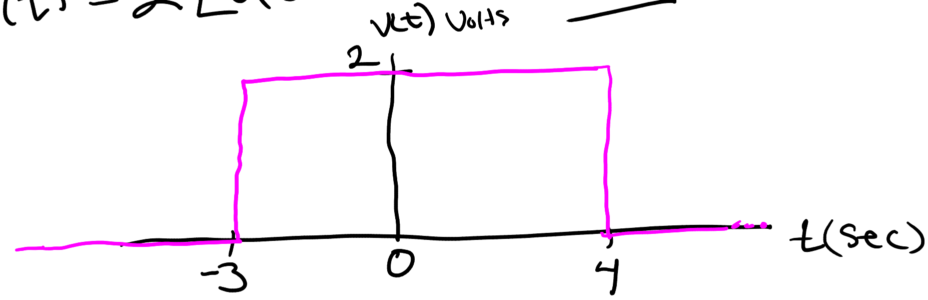
* $v(t) = \overset{V_1}{3u(-t)} - \overset{V_2}{7u(t)}$ Volts



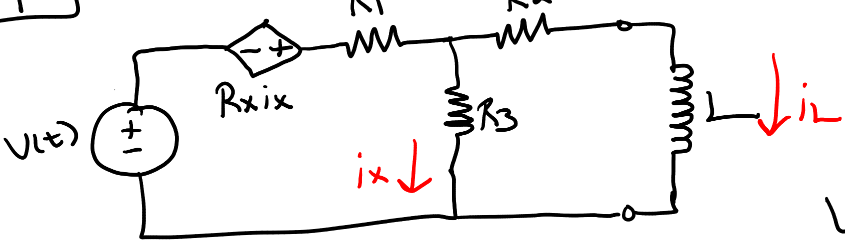


* Pulse

$$v(t) = 2\left[u(t+3) - u(t-4)\right]$$



*Example



Given

$R_1 = 8\Omega$, $R_2 = 1.6\Omega$

$R_3 = 4\Omega$, $R_x = 2\Omega$

$L = 3H$

$V(t) = -9u(-t) + 6u(t)$ ✓

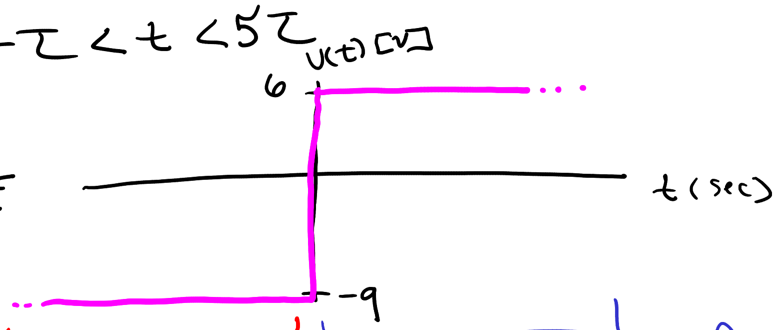
a) Find $i_L(t)$ over $-\infty < t < \infty$

b) Plot $i_L(t)$ over $-\tau < t < 5\tau$

* Take a good look at $v(t)$

$$i_L(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R_{eq}}$$

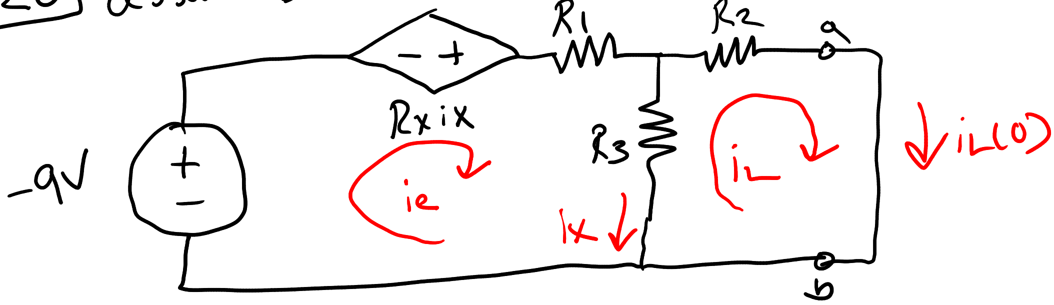


up to $t = 0^-$

establishes $i_L(0)$, $ix(0)$

serves as a forcing function

$t<0$) assume DCSS



Note $ix(0) = ie - i_L$

using mesh analysis

ie) $9 - R_x ix + R_1 ie + R_3(ie - i_L)$

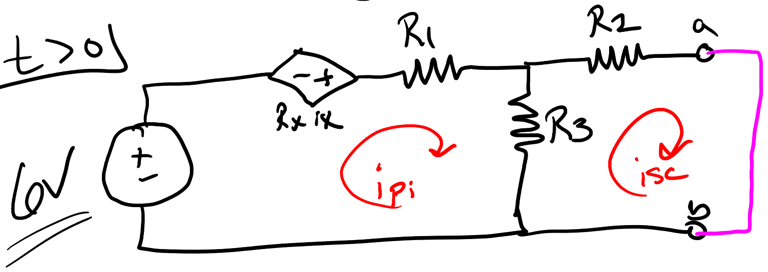
$$\Rightarrow -R_x(i_e - i_L) + R_1(i_e) + R_3(i_e - i_L) = -9 \quad ①$$

$i_L)$ $$R_3(i_L - i_e) + R_2 i_L = 0 \quad ②$$

* Solving these equations yields $i_L(0) = -0.75A$, $i_x(0) = -0.3A$

* Find isc and Req seen at port a,b for $t>0$

$t>0)$



$i_{sc} = I_{no}$

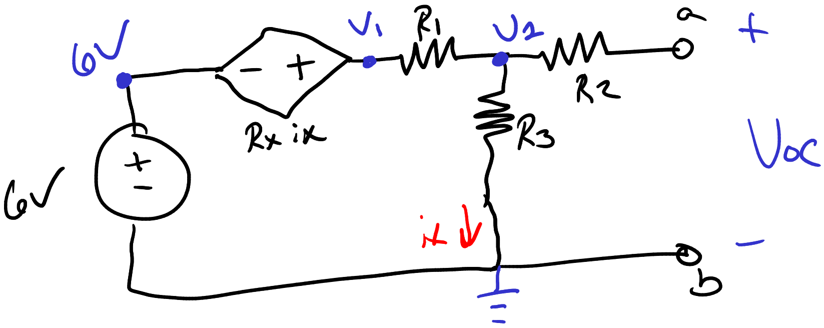
Mesh yields $i_{sc} = 0.5A$

Need Req, we have 2 approaches

* Can use $R_{eq} = \frac{V_{oc}}{I_{sc}} = \frac{V_{th}}{I_{No}}$

* can turn off independent sources then apply either 1V or 1A to a,b

I will seek V_{oc}



$$i_x = \frac{V_2}{R_3}$$

Using Nodal $V_1 - 6 = R_x i_x \Rightarrow V_1 - 6 = \frac{R_x}{R_3} V_2$

$V_2)$ $$\frac{V_2 - V_1}{R_1} + \frac{V_2 - 0}{R_3} = 0 \Rightarrow \left(-\frac{1}{R_1}\right)V_1 + \left(\frac{1}{R_1} + \frac{1}{R_3}\right)V_2 = 0 \quad ①$$

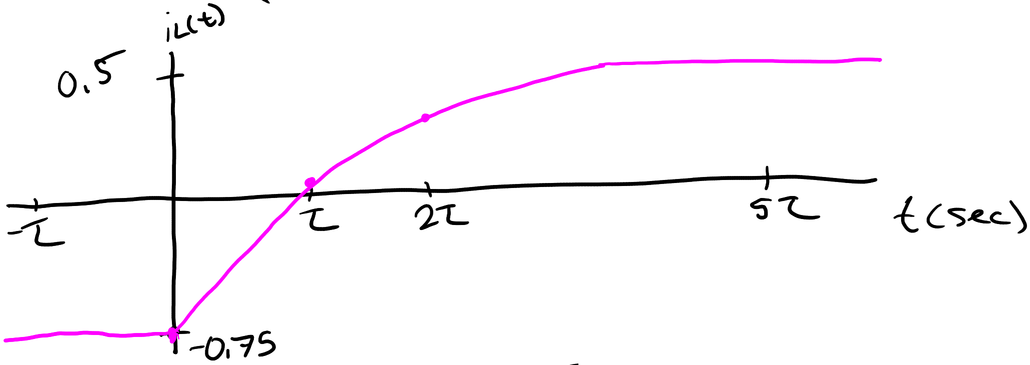
$$(1)V_1 + \left(\frac{-R_x}{R_3}\right)V_2 = 6 \quad ②$$

Solving yields

$V_1 = 7.2V$ $V_2 = 2.4V = V_{oc}$

$$R_{eq} = \frac{2.4V}{0.5A} = 4.8\Omega$$

$$i_L(t) = \begin{cases} -0.75\ A & t < 0 \\ 0.5 - 1.25e^{\frac{-t}{0.625}} & t > 0 \end{cases}$$



$\tau = 0.040\ A$ $\quad 2\tau = 0.331\ A$