Chapter 9
Second Order Circuits   RLC

$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$
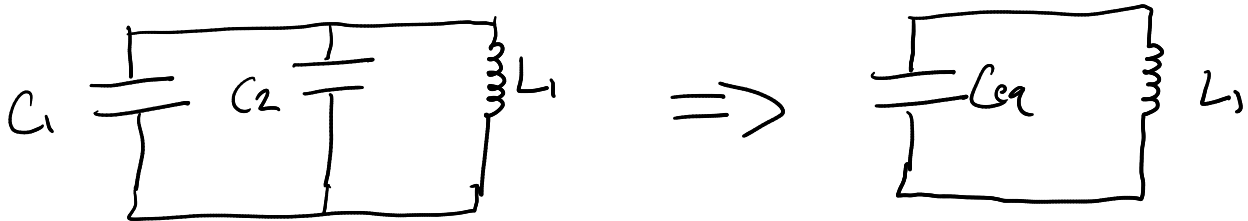
Where $f(t)$ = input to system

$x(t)$ = output of system

α = damping coefficient

ω_0 = resonant frequency

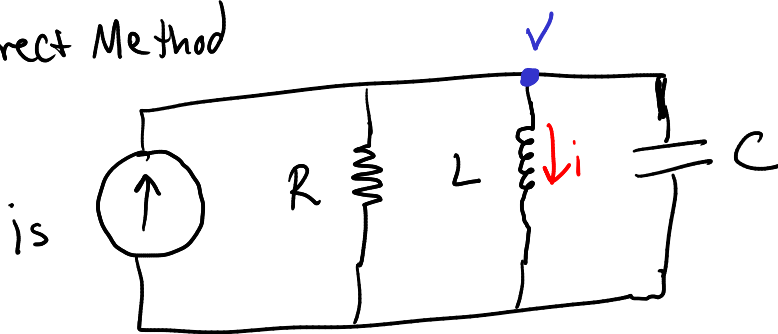$*$ Second order implies two irreducible energy storage elements

$*$ Two methods for obtaining second order equations

1) Direct Method   ← Start here

2) Operator Method

$*$ Direct Method

Given

$$i_c = C\frac{dv}{dt} \quad \Leftarrow$$

$$V = L\frac{di}{dt}$$

Nodal eq for V:)

$$i_s = i_R + i_L + i_c \Rightarrow i_s = \frac{V}{R} + i + C\frac{dv}{dt}$$

Substitute for V

$$\Rightarrow i_s = \frac{L}{R}\frac{di}{dt} + i + LC\frac{d^2i}{dt^2}$$

Divide by LC and rearrange

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i = \frac{i_s}{LC}$$

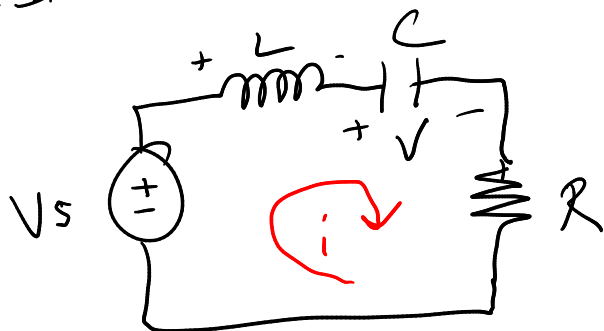$$\frac{d^2 x(t)}{dt^2} + 2\alpha \frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$

$$2\alpha = \frac{1}{RC}$$

$$\therefore \boxed{\alpha = \frac{1}{2RC} \frac{rad}{sec}}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \frac{rad}{sec}}$$

* In terms of V

Given

$$V = L\frac{di}{dt}$$

$$i = C\frac{dV}{dt}$$

KVL

$$V_S = V_L + V_C + V_R \implies V_S = L\frac{di}{dt} + V_C + iR$$

$$\implies \frac{di}{dt} + \frac{V}{L} + \frac{R}{L}i = \frac{V_S}{L} \qquad \text{Divide by } L$$

Plug in for i

$$\implies C\frac{d^2V}{dt^2} + \frac{V}{L} + \frac{RC}{L}\frac{dV}{dt} = \frac{V_S}{L}$$

$$\implies \frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \frac{1}{LC}V = \frac{1}{LC}V_S$$

$$2\alpha = \frac{R}{L}$$

$$\therefore \boxed{\alpha = \frac{R}{2L}}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

*Steps for the Direct Method

1) Identify the first and second variables X_1 and X_2. These variables are capacitor voltages or inductor currents

2) Write one first order differential equation obtaining

$$\frac{dx_1}{dt} = f(x_1, x_2)$$

3) Obtain an additional first-order differential equation in terms of the second variable so that $\frac{dx_2}{dt} = Kx_1$

or $x_1 = \frac{1}{K}\frac{dx_2}{dt}$

4) Substitute the equation of Step 3 into the equation of Step 2 thus obtaining a second order diff. eq.