

Frequency Response

*Recap

$$\cos(377t)$$

What is the frequency of this waveform?

$$\cos(\omega t) = \cos(2\pi f t)$$

$$\omega = 377$$

$$\Rightarrow 2\pi f = 377$$

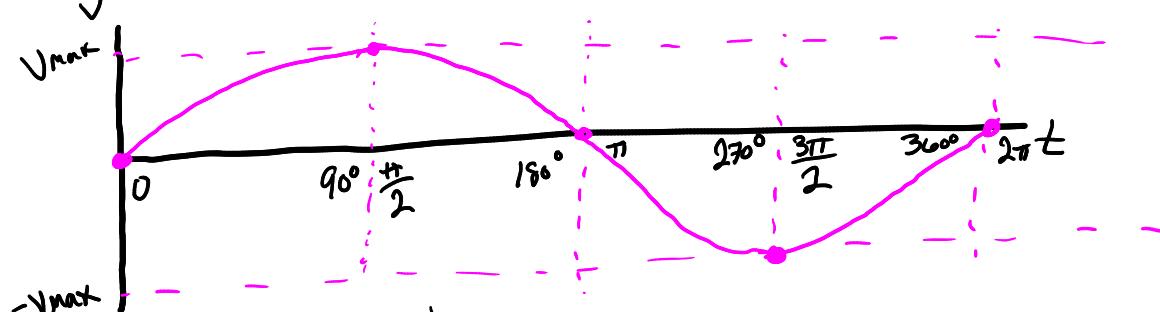
$$f = 60.03 \text{ Hz}$$

f: frequency (Hz)

ω : angular frequency ($\frac{\text{rad}}{\text{s}}$)

$$v(t) = V_{\max} \sin(\omega t)$$

* Sinusoidal Waveform:



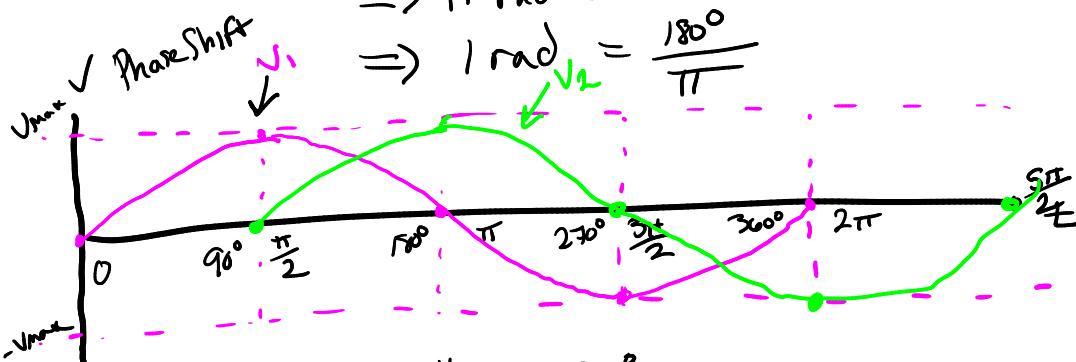
$$V_{\text{rms}} = \frac{1}{\sqrt{2}} V_{\max}$$

* How to convert between radians and degrees?

$$2\pi \text{ rad} = 360^\circ$$

$$\Rightarrow \pi \text{ rad} = 180^\circ$$

$$\Rightarrow 1 \text{ rad} = \frac{180^\circ}{\pi}$$



* Phase Shift of $\frac{\pi}{2}$ or 90°

* If $v(t)$ is written in complex form

$$v(t) = A + jB$$

$$\text{Mag}[v(t)] = \sqrt{A^2 + B^2}$$

$$\text{Phase}[v(t)] = \tan^{-1}\left(\frac{B}{A}\right)$$

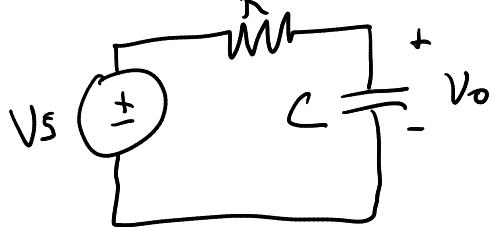
Impedance

$$\text{---} \Rightarrow R \quad R + j0$$

$$\text{---} \Rightarrow j\omega L$$

$$\text{---} \Rightarrow \frac{1}{j\omega C} \text{ or } \frac{j}{\omega C}$$

* Low Pass Filter (LPF)



Voltage Divider

$$V_o = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \times V_s$$

$$\Rightarrow V_o = \frac{1}{j\omega CR + 1} \times V_s$$

$$|V_o| = \frac{1}{\sqrt{(j\omega CR)^2 + 1^2}} |V_s|$$

$$\text{When } \omega=0; |V_o| = \frac{1}{j\omega CR + 1} \times |V_s|$$

$$\Rightarrow |V_o| = |V_s|$$

$$\omega=\infty; |V_o| = \frac{1}{\infty} \times |V_s| \rightarrow 0$$

* Low pass Filters allow low frequencies to pass through

$$V_o = \frac{1 + j\omega}{j\omega CR + 1} \times V_s$$

* Phase

$$\angle \frac{V_o}{V_s} = \angle x - \angle y$$

$$\phi \Rightarrow \tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{wCR}{x}\right) = -\tan^{-1}(wCR)$$

$$\text{Gain} = \frac{\text{Output}}{\text{Input}} = \frac{V_o}{V_s} = \frac{1}{\sqrt{(wCR)^2 + 1}}$$

* Example, $R = 2k\Omega$, $C = 0.1\mu F$ find $|V_o|$ at $f = 2 \text{ kHz}$

$$V_s = 4V$$

$$\frac{V_o}{V_s} = \frac{1}{\sqrt{[2\pi(2000)(0.1 \times 10^{-6})(2000)]^2 + 1}} = \frac{1}{2705} = 0.3696$$

$$V_o = 4 \times 0.3696 = \boxed{1.478V}$$

On a log scale

$$\frac{V_o}{V_s} = 20 \log_{10}(0.3696) = -8.64 \text{ dB}$$

$$\text{Account for } V_s = 4 \Rightarrow 20 \log_{10}(1.478) = 3.393 \text{ dB}$$

* Cut off frequency : The frequency where the output power is $\frac{1}{2}$ the input power

$$\Rightarrow P_o = \frac{1}{2} P_{in} \quad P = \frac{V^2}{R} \quad P \approx V^2$$

$$|V_o|^2 = \frac{1}{2} |V_s|^2 \quad \frac{1}{T_2} = 0.707$$

$$\Rightarrow |V_o| = \frac{1}{\sqrt{2}} |V_s| \quad \text{In dB, } 20 \log_{10}(\frac{1}{\sqrt{2}}) \approx -3 \text{ dB}$$

* To find the cutoff frequency (assume unity $V_s=1$)

$$\frac{1}{\sqrt{(wCR)^2 + 1}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sqrt{(wCR)^2 + 1} = \sqrt{2} \Rightarrow (wCR)^2 + 1 = 2$$

$$\Rightarrow (wCR)^2 = 1 \Rightarrow wCR = 1 \Rightarrow w = \frac{1}{RC}$$

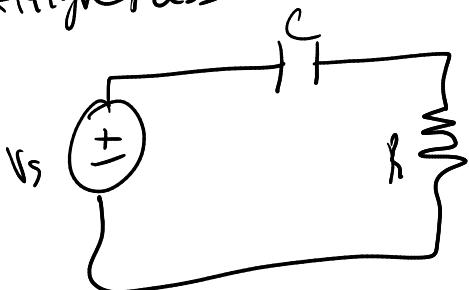
from corner $f_c = \frac{1}{2\pi RC}$

$$f_c = 795.78 \text{ Hz} \quad \text{when } V_s = 4V \Rightarrow 20 \log_{10}(4) \approx 12 \text{ dB}$$

$$- 3 \text{ dB}$$

$$9 \text{ dB}$$

* High Pass



$$\frac{|V_o|}{|V_s|} = \frac{wRC}{\sqrt{(wCR)^2 + 1}}$$

$$V_o = \frac{R}{R + \frac{1}{jwC}} V_s$$

$$V_o = \frac{jwRC}{jwRC + 1} V_s$$

$$\omega = 0 \Rightarrow \frac{0}{T_0 + 1} = 0$$

$$\omega = \infty \Rightarrow \omega \gg 1 \Rightarrow \frac{wRC}{\sqrt{(wRC)^2}} \Rightarrow |V_o| = |V_s|$$

* High Pass : pass through high frequency, block low frequencies.

* RL Lowpass



$$\frac{|V_o|}{|V_s|} = \frac{R}{R + j\omega L}$$

$$\omega = 0 \rightarrow |V_o| = |V_s|$$

$$\omega = \infty \rightarrow \frac{R}{\infty} = 0$$

* RL High Pass

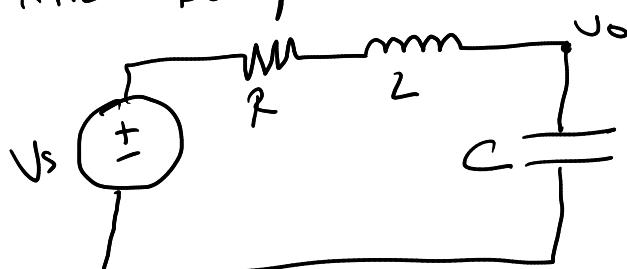


$$\frac{|V_o|}{|Vs|} = \frac{j\omega L}{R + j\omega L}$$

$$\omega = 0 \rightarrow \frac{0}{R+0} = 0$$

$$\omega = \infty \rightarrow |V_o| = |Vs|$$

* RLC Bandpass Filter



$$V_o = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} * Vs$$

Multiply by $\frac{1}{j\omega C}$

$$\frac{V_o}{Vs} = \frac{1}{j\omega CR - \omega^2 LC + 1}$$

$$\left| \frac{V_o}{Vs} \right| = \frac{1}{\sqrt{(j\omega CR)^2 + (1-\omega^2 LC)^2}}$$

$$\phi = \frac{\tan^{-1}\left(\frac{0}{1}\right)}{\tan^{-1}\left(\frac{j\omega CR}{1-\omega^2 LC}\right)}$$

* Resonant Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

