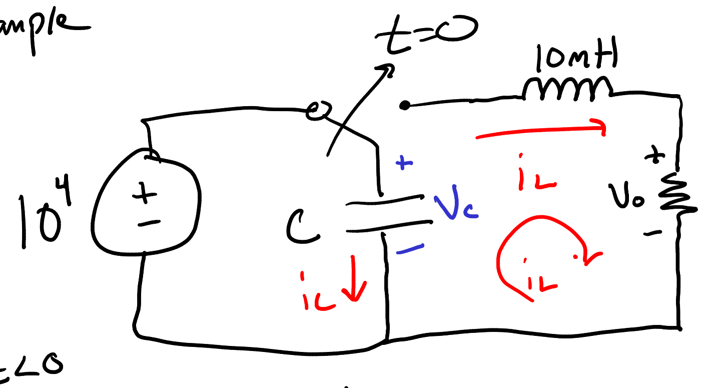
Lecture (6)

Housekeeping: Exam is moved back 1 week

Stun Gun

*Example



$R = 10^6 \Omega$ ← Spark gap

Find $V_o(t)$ and select C so that the response is critically damped

For $t<0$

$$V_c(0) = 10^4 V \qquad i_L(0) = 0 A$$

$t>0$ | Mesh

$$-V_c + V_L + V_o = 0 \Rightarrow V_c = L\frac{di_L}{dt} + i_L R$$

Take the derivative

Remember

$$i_c = C\frac{dV_c}{dt}$$

$$i_c = -i_L$$

$$\frac{dV_c}{dt} = L\frac{d^2 i_L}{dt^2} + R\frac{di_L}{dt}$$

Substitute

$$\Rightarrow \quad i_c = LC\frac{d^2 i_L}{dt^2} + RC\frac{di_L}{dt}$$

Divide by LC

$$\Rightarrow \quad LC\frac{d^2 i_L}{dt^2} + RC\frac{di_L}{dt} + i_L = 0$$

$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = 0$$

*Compare to $\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_o^2 x(t) = 0$

$$\omega_o^2 = \frac{1}{LC}$$

$$2\alpha = \frac{R}{L}$$

$$\therefore \alpha = \frac{R}{2L} = \frac{10^6}{2(10\times10^{-3})} = 50\times10^6$$

* For critical damping $\alpha^2 = \omega_o^2$

$$\alpha^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{L\alpha^2} = \boxed{0.04 pF}$$

$$i_n(t) = (A_1 t + A_2)e^{s_1 t}$$ $S_1 = -\alpha + \cancel{\sqrt{\alpha^2 - \omega_0^2}}$ 0

$S_1 = -\alpha$

*To find A_2,

$$i_n(0) = (A_1(0) + A_2)e^{-50\times10^6(0)}$$

$$\therefore \quad i_n(0) = A_2 \implies A_2 = \emptyset$$

*To obtain A_1,

$$\frac{di_n}{dt} = \frac{d}{dt}(A_1 t + A_2)e^{-50\times10^6 t}$$

$$\implies (A_1 t + A_2)(-50\times10^6)e^{-50\times10^6 t} + e^{-50\times10^6 t}A_1$$

Plug in $t=0$

$$\frac{di_n(0)}{dt} = -50\times10^6 A_2 + A_1 \qquad A_2 = 0$$

$$\therefore \quad \frac{di_n(0)}{dt} = A_1$$

*From Earlier

$$V_c = L\frac{di_L}{dt} + i_L R \implies L\frac{di_L}{dt} = V_c - i_L R$$

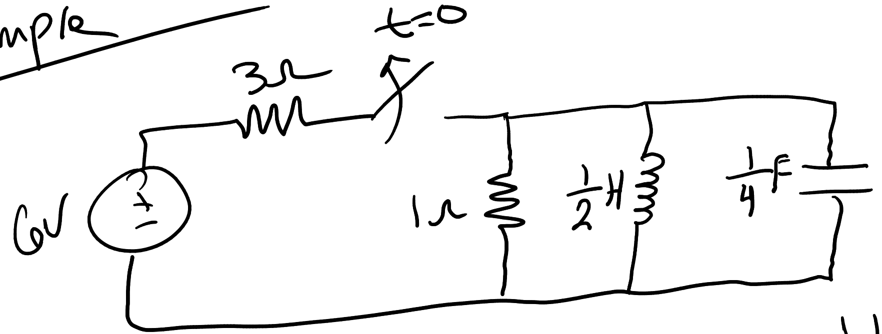
$$\implies \frac{di_L}{dt} = \frac{V_c - i_L R}{L}$$

$$\frac{di_L(0)}{dt} = \frac{V_c(0) - \cancelto{0}{i_L(0)}R}{L} = \frac{10^4}{10\times10^{-3}} = 1\times10^6 = A_1$$

$$i_n(t) = 1\times10^6 t^{-50\times10^6 t} \ A \qquad R = 10^6$$

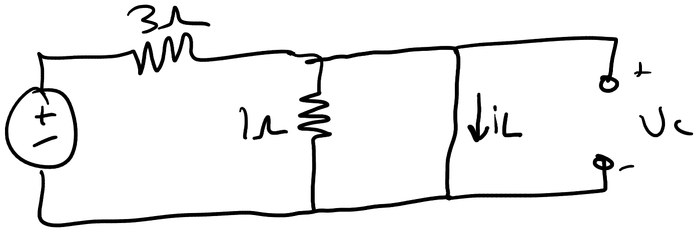
$$V_0 = i_n(t)R \implies \boxed{V_0(t) = (1\times10^{12})te^{-50\times10^6 t} \ V}$$

*Example



Determine and plot $i_L(t)$ assuming Steady State

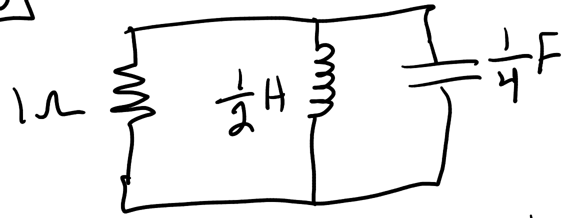
For $t<0$,



$$V_c(0) = 0V \checkmark$$

$$i_L(0) = \frac{6V}{3\Omega} = 2A$$

t > 0)



Parallel RLC Solution

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{1}{LC} i_L = 0$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(1)(\frac{1}{4})} = 2 \qquad \omega_0^2 = \frac{1}{LC} = \frac{1}{(\frac{1}{2})(\frac{1}{4})} = 8$$

$\alpha^2 < \omega_0^2$ must underdamped circuit

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{8-4} = 2\ \frac{\text{rad}}{\text{sec}}$$

$$S_1 = -\alpha + j\omega_d = -2 + j2$$

$$S_2 = -\alpha - j\omega_d = -2 - j2$$

$$i_L(t) = e^{-\alpha t}(B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$$

$$= e^{-2t}(B_1 \cos(2t) + B_2 \sin(2t))$$

$i_L(0) = B_1 = 2$ *

To find $\frac{di_L}{dt}$, use inductor voltage

$$\frac{di_L(0)}{dt} = 2B_2 - 2B_1$$

$$V_L = L\frac{di_L}{dt} \Rightarrow \frac{di_L}{dt} = \frac{V}{2}$$

$$\frac{di_L(0)}{dt} = 0 \qquad V_L(0) = 0V$$

$$\Rightarrow 2B_2 = 2B_1 \Rightarrow B_2 = 2$$

$$\boxed{i_L(t) = e^{-2t}[2\cos(2t) + 2\sin(2t)]\ A}$$

* Forced Response

Total Response $x(t)$ = natural + forced

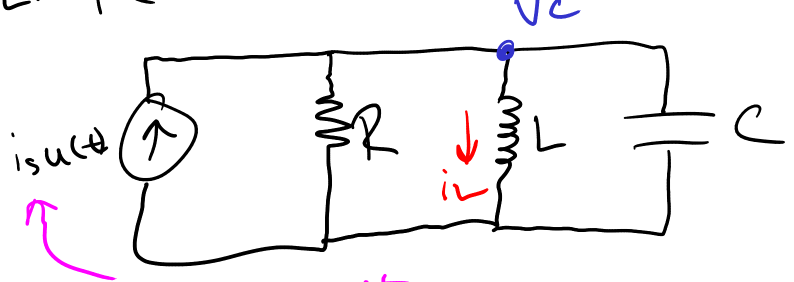
$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$ forcing function

* The response to a forcing function will often be of the same form as the forcing function

Forced Responses

Forcing Function	Assumed Response
K	A
Kt	$At + B$
Kt^2	$At^2 + Bt + C$
$K\sin(\omega t)$	$A\sin(\omega t) + B\cos(\omega t)$
Ke^{-at}	Ae^{-at}

* Example



Source current applied at $t=0$

Given
$i_s = 8e^{-2t}\,A$
$R = 6\,\Omega \quad C = \frac{1}{42}\,F$
$L = 7\,H$

Find the forced Response

$$V_c = V_L = L\frac{di_L}{dt}$$

$$\frac{dV_c}{dt} = L\frac{d^2 i_L}{dt^2}$$

KCL: $$i_L + \frac{V_c}{R} + C\frac{dV_c}{dt} = i_s$$

$$\Rightarrow i_L + \frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = i_s$$

$$\Rightarrow \frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{1}{LC}i_L = \frac{i_s}{LC}$$

Plug in numbers :) $i_s = 8e^{-2t}$

$$\Rightarrow \frac{d^2 i_L}{dt^2} + 7\frac{di}{dt} + 6i_L = 48e^{-2t}$$

* Assumed Response

* plug derivatives in!!

$$i_{Lf} = Be^{-2t} \qquad \frac{di_{Lf}}{dt} = -2Be^{-2t} \qquad \frac{d^2 i_{Lf}}{dt} = 4Be^{-2t}$$

$$\Rightarrow 4Be^{-2t} - 14Be^{-2t} + 6Be^{-2t} = 48e^{-2t}$$

$$-4Be^{-2t} = 48e^{-2t}$$

$$B = -12 \quad \text{and} \quad \boxed{i_{Lf} = -12e^{-2t}\,A}$$

*Example

A circuit is described by

$$\frac{d^2 i}{dt^2} + 9\frac{di}{dt} + 20i = 6 i_s$$

where $i_s = 6 + 2t$ A

Find the forced response i_F for $t > 0$

$$\frac{d^2 i}{dt^2} + 9\frac{di}{dt} + 20i = 36 + 12t$$

Assumed Response

$$i_F = At + B$$

$$\frac{di_F}{dt} = A \quad \frac{d^2 i_F}{dt^2} = 0$$

$$9A + 20At + 20B = 36 + 12t$$

t: $20A = 12 \Rightarrow A = \frac{12}{20} = 0.6$

con: $9A + 20B = 36 \Rightarrow 20B = 30.6 \Rightarrow B = 1.53$

$$\boxed{i_F = 0.6t + 1.53 \text{ A}}$$