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 004 **Supplementary Material to “External Prior Guided Internal Prior Learning for**  
 005 **Real Noisy Image Denoising”**  
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In this supplementary material, we provide:

1. The closed-form solution of the proposed weighted sparse coding model in the main paper.
2. More denoising results on the real noisy images (with no “ground truth”) provided in the dataset [1].
3. More denoising results on the 15 cropped real noisy images (with “ground truth”) used in the dataset [2].
4. More denoising results on the 60 cropped real noisy images (with “ground truth”) from [2].

## 1. Closed-Form Solution of the Weighted Sparse Coding Problem (4)

The weighted sparse coding problem in the main paper is:

$$\min_{\alpha} \|\mathbf{y} - \mathbf{D}\alpha\|_2^2 + \|\mathbf{w}^T \alpha\|_1. \quad (1)$$

Since  $\mathbf{D}$  is an orthonormal matrix, problem (1) is equivalent to

$$\min_{\alpha} \|\mathbf{D}^T \mathbf{y} - \alpha\|_2^2 + \|\mathbf{w}^T \alpha\|_1. \quad (2)$$

For simplicity, we denote  $\mathbf{z} = \mathbf{D}^T \mathbf{y}$ . Since  $\mathbf{w}_i = c * 2\sqrt{2}\sigma^2 / (\Lambda_i + \varepsilon)$  is positive (please refer to Eq. (18) in the main paper), problem (2) can be written as

$$\min_{\alpha} \sum_{i=1}^{p^2} ((\mathbf{z}_i - \alpha_i)^2 + \mathbf{w}_i |\alpha_i|). \quad (3)$$

The problem (3) is separable w.r.t.  $\alpha_i$  and can be simplified to  $p^2$  scalar minimization problems

$$\min_{\alpha_i} (\mathbf{z}_i - \alpha_i)^2 + \mathbf{w}_i |\alpha_i|, \quad (4)$$

where  $i = 1, \dots, p^2$ . Taking derivative of  $\alpha_i$  in problem (4) and setting the derivative to be zero. There are two cases for the solution.

- (a) If  $\alpha_i \geq 0$ , we have

$$2(\alpha_i - \mathbf{z}_i) + \mathbf{w}_i = 0. \quad (5)$$

The solution is

$$\hat{\alpha}_i = \mathbf{z}_i - \frac{\mathbf{w}_i}{2} \geq 0. \quad (6)$$

So  $\mathbf{z}_i \geq \frac{\mathbf{w}_i}{2} > 0$ , and the solution  $\hat{\alpha}_i$  can be written as

$$\hat{\alpha}_i = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}), \quad (7)$$

where  $\text{sgn}(\bullet)$  is the sign function.

- (b) If  $\alpha_i < 0$ , we have

$$2(\alpha_i - \mathbf{z}_i) - \mathbf{w}_i = 0. \quad (8)$$

The solution is

$$\hat{\alpha}_i = \mathbf{z}_i + \frac{\mathbf{w}_i}{2} < 0. \quad (9)$$

So  $\mathbf{z}_i < -\frac{\mathbf{w}_i}{2} < 0$ , and the solution  $\hat{\alpha}_i$  can be written as

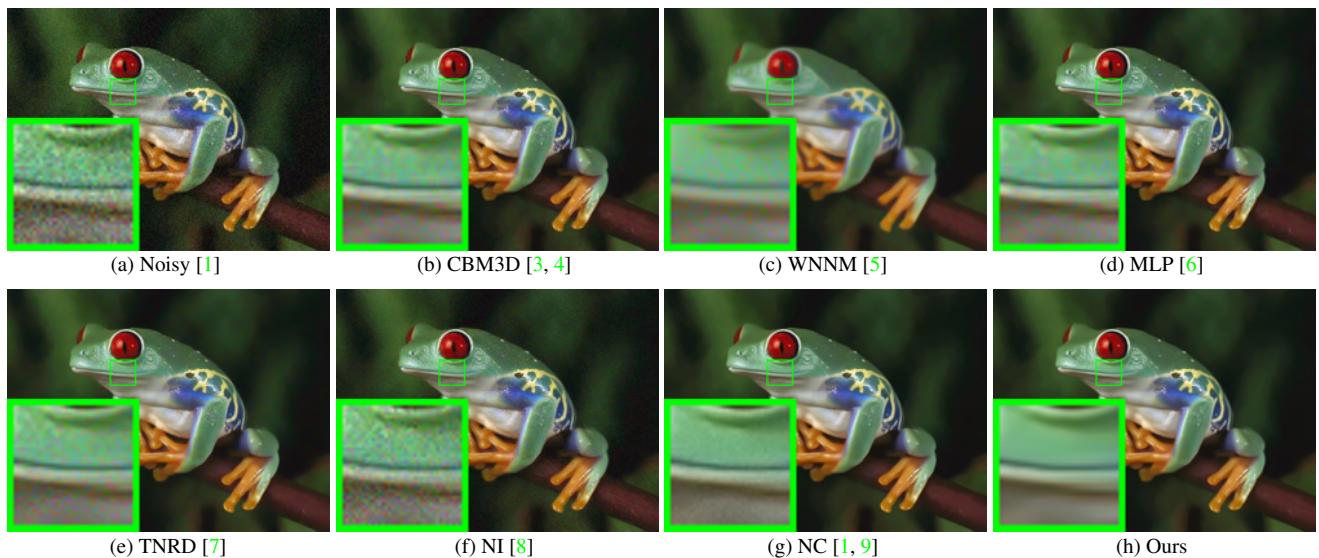
$$\hat{\alpha}_i = \text{sgn}(\mathbf{z}_i) * (-\mathbf{z}_i - \frac{\mathbf{w}_i}{2}) = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}). \quad (10)$$

108 In summary, we have the final solution of the weighted sparse coding problem (1) as  
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 110  $\hat{\alpha} = \text{sgn}(\mathbf{D}^T \mathbf{y}) \odot \max(|\mathbf{D}^T \mathbf{y}| - \mathbf{w}/2, 0),$  (11)

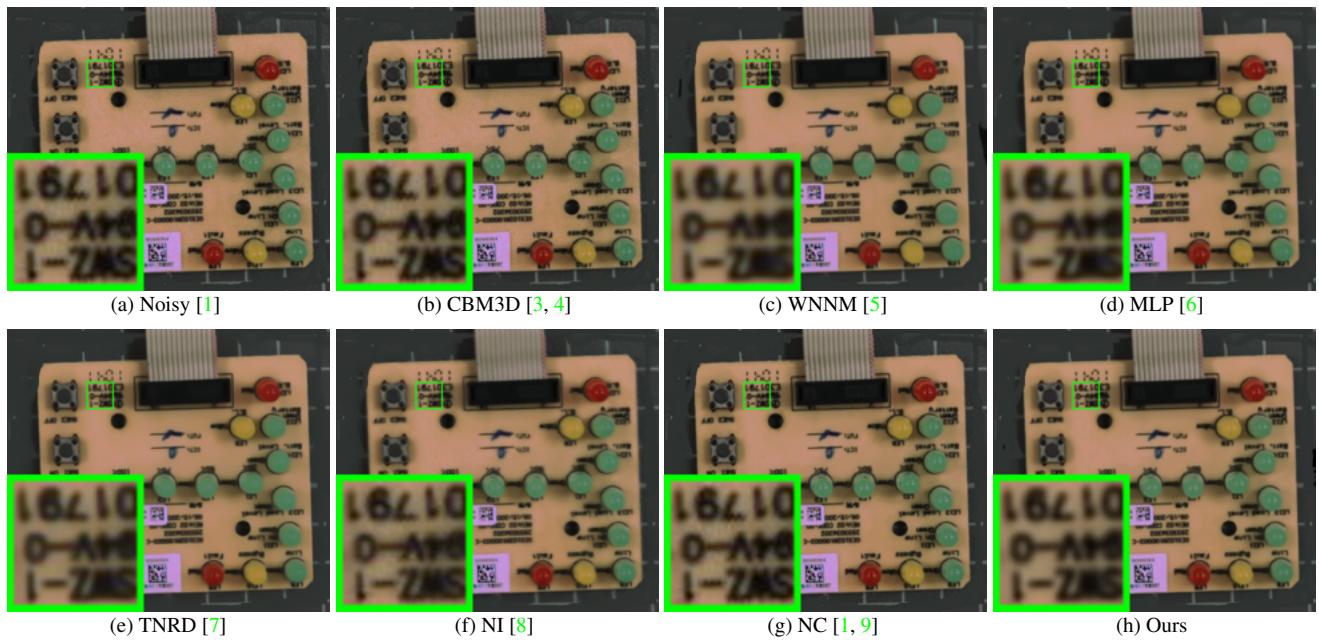
111 where  $\odot$  means element-wise multiplication and  $|\mathbf{D}^T \mathbf{y}|$  is the absolute value of each entry of the vector  $\mathbf{D}^T \mathbf{y}.$

## 112 2. More Results on Real Noisy Images in [1]

114 In this section, we give more visual comparisons of the competing methods on the real noisy images provided in [1]. The  
 115 real noisy images in this dataset [1] have no “ground truth” images and hence we only compare the visual quality of the  
 116 denoised images by different methods. As can be seen from Figures 1-4, our proposed method performs better than the state-  
 117 of-the-art denoising methods. This validates the effectiveness of our proposed external prior guided internal prior learning  
 118 framework for real noisy image denoising.  
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138 Figure 1. Denoised images of the real noisy image “Frog” [1] by different methods. The images are better to be zoomed in on screen.  
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161 Figure 2. Denoised images of the real noisy image “Circuit” [1] by different methods. The images are better to be zoomed in on screen.  
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Figure 3. Denoised images of the real noisy image “Woman” [1] by different methods. The images are better to be zoomed in on screen.

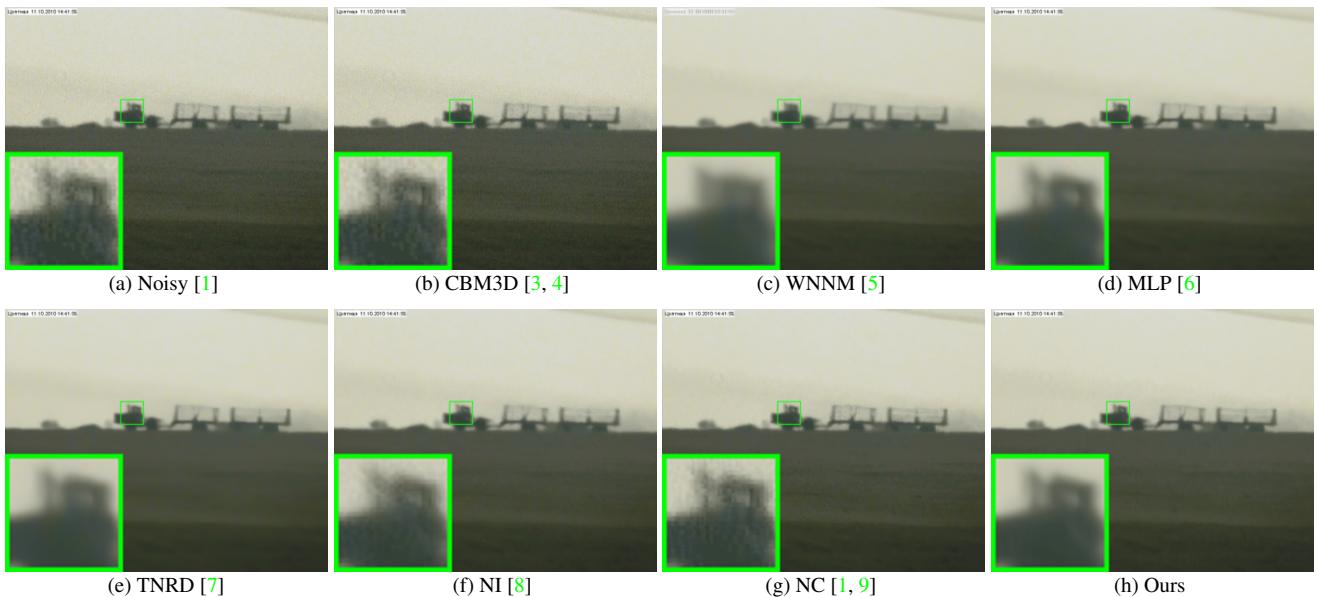


Figure 4. Denoised images of the real noisy image “Vehicle” [1] by different methods. The images are better to be zoomed in on screen.

### 3. More Results on the 15 Cropped Images Used in [2]

In this section, we provide more visual comparisons of the proposed method with the state-of-the-art denoising methods on the 15 cropped real noisy images used in [2]. As can be seen from Figures 5-9, on most cases, our proposed method achieves better performance than the competing methods. This validates the effectiveness of our proposed external prior guided internal prior learning framework for real noisy image denoising.

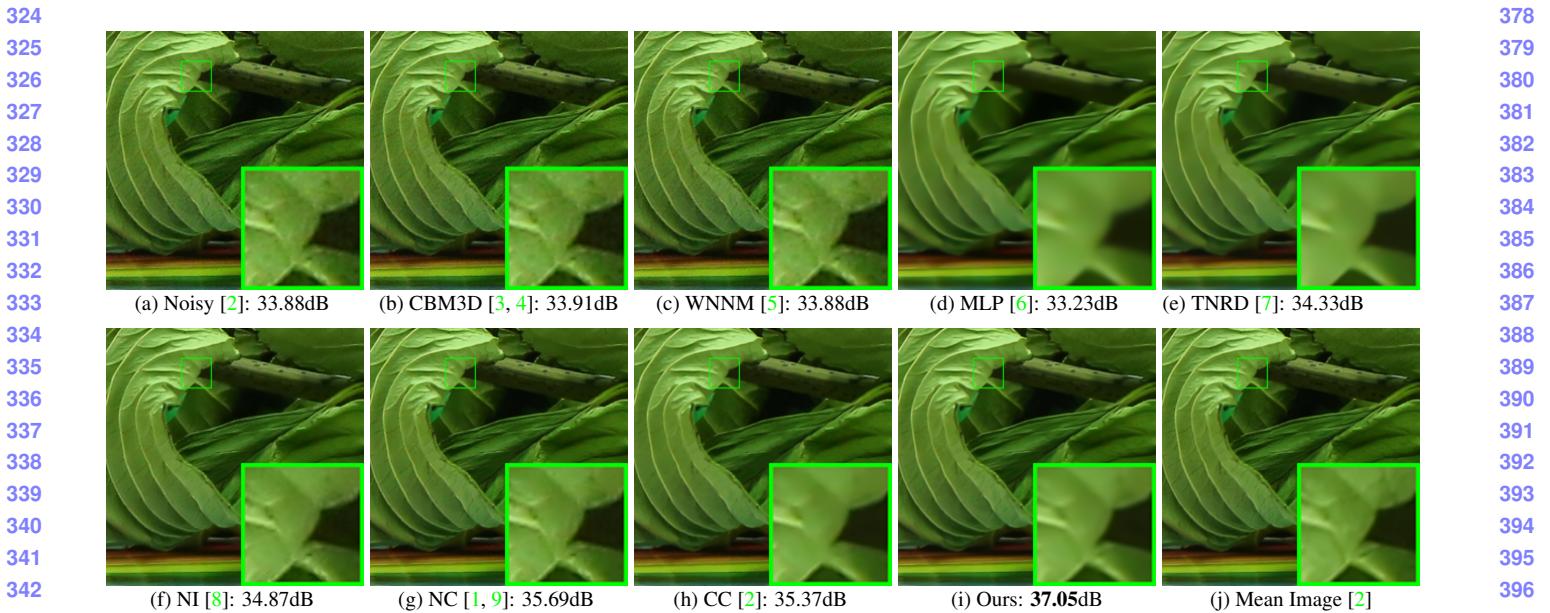


Figure 5. Denoised images of a region cropped from the real noisy image “Canon 5D Mark 3 ISO 3200 2” [2] by different methods. The images are better to be zoomed in on screen.

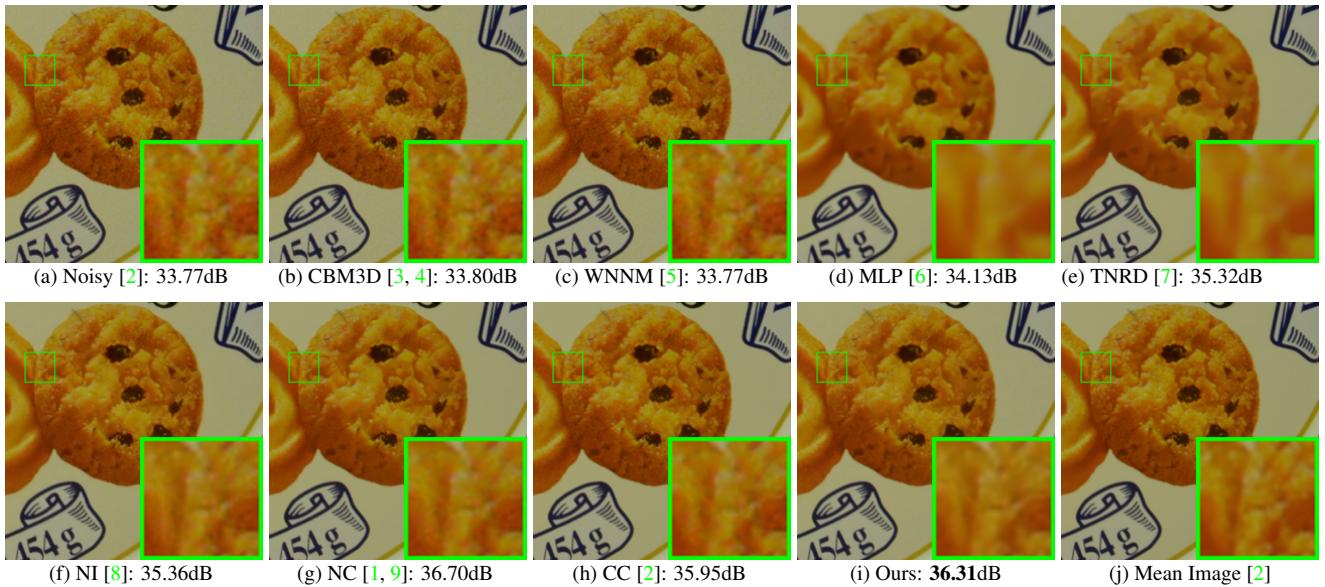


Figure 6. Denoised images of a region cropped from the real noisy image “Canon 5D Mark 3 ISO 3200 2” [2] by different methods. The images are better to be zoomed in on screen.

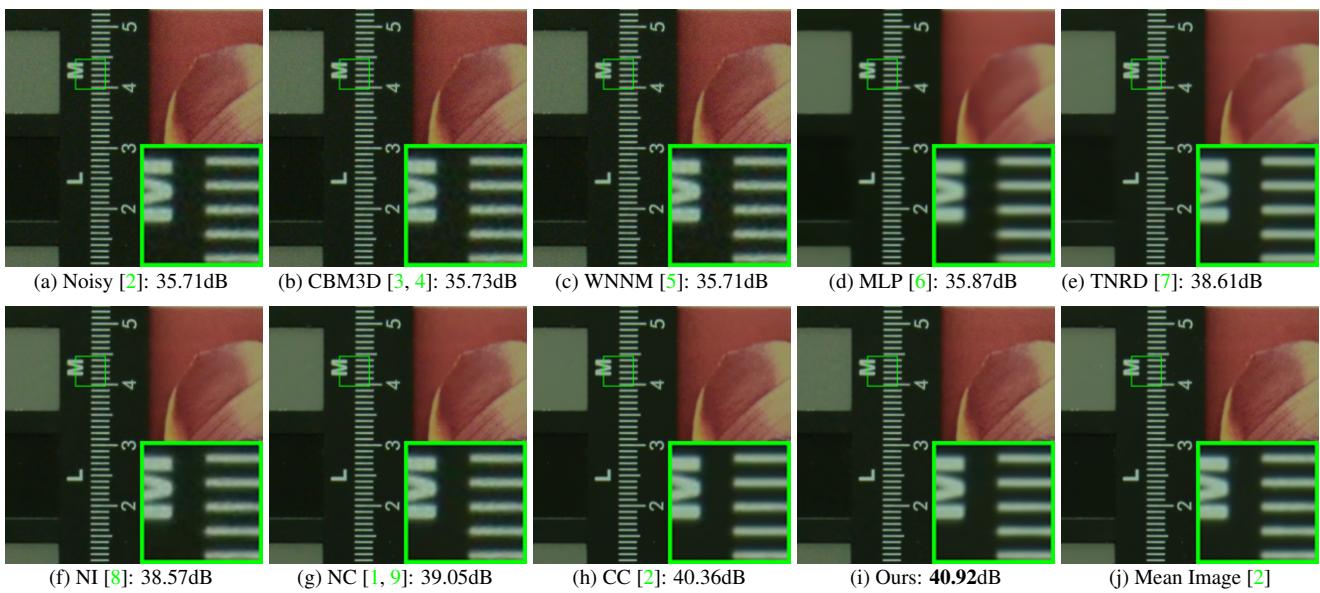


Figure 7. Denoised images of a region cropped from the real noisy image “Nikon D800 ISO 1600 2” [2] by different methods. The images are better to be zoomed in on screen.

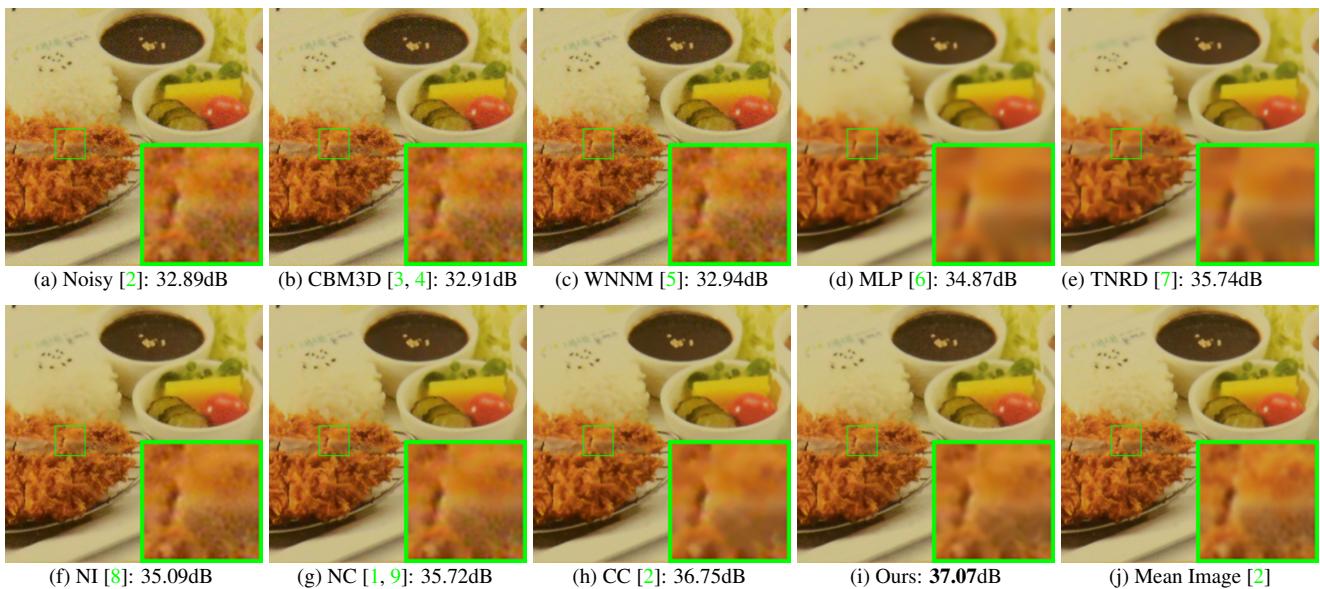


Figure 8. Denoised images of a region cropped from the real noisy image “Nikon D800 ISO 3200 2” [2] by different methods. The images are better to be zoomed in on screen.

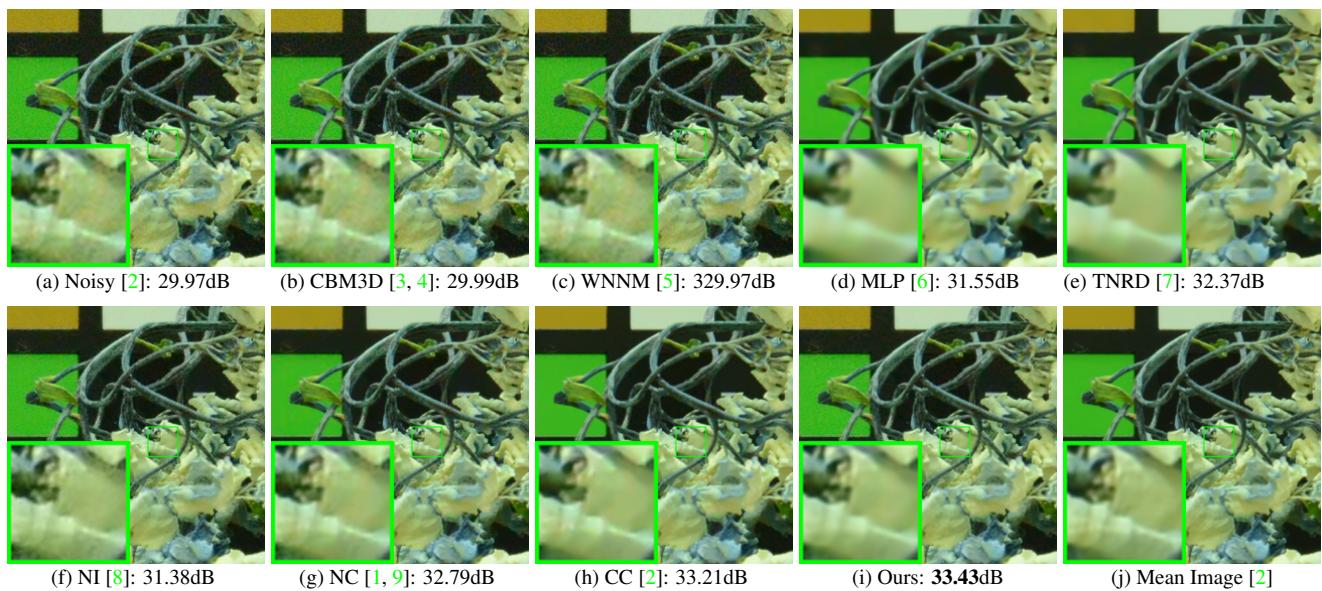


Figure 9. Denoised images of a region cropped from the real noisy image “Nikon D800 ISO 6400 2” [2] by different methods. The images are better to be zoomed in on screen.

#### 4. More Results on the 60 Cropped Images in [2]

In this section, we provide more visual comparisons of the proposed method with the state-of-the-art denoising methods on the 60 cropped real noisy images we cropped from [2]. As can be seen from Figures 10-15, on most cases, our proposed method achieves better performance than the competing methods. This validates the effectiveness of our proposed external prior guided internal prior learning framework for real noisy image denoising.

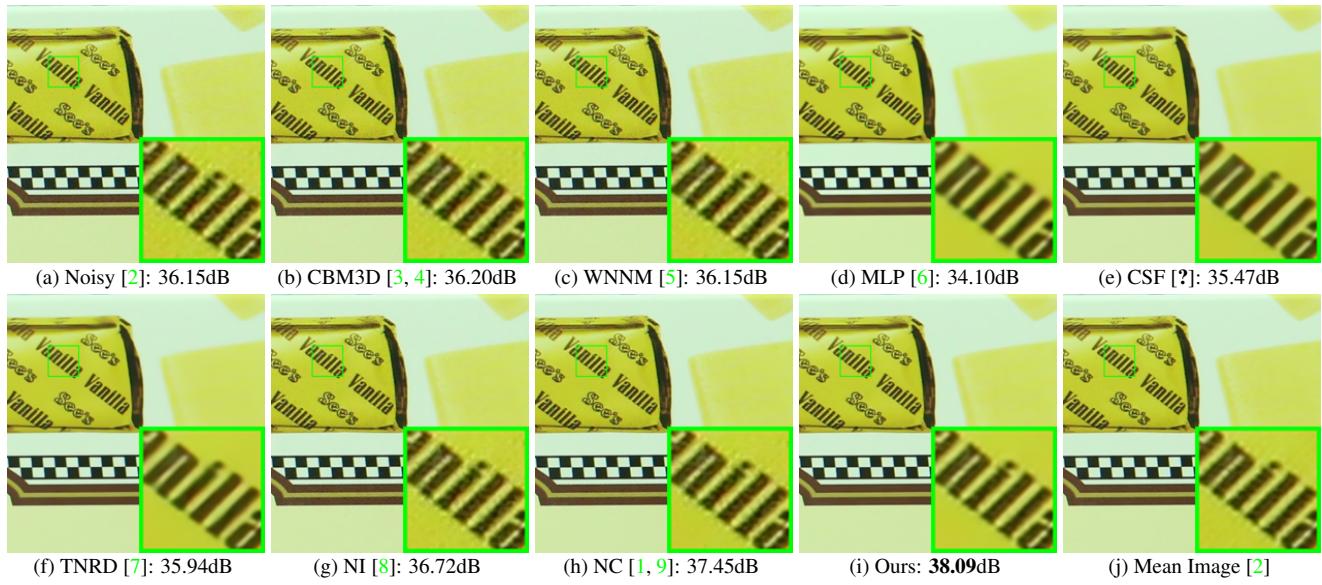


Figure 10. Denoised images of a region cropped from the real noisy image “Canon EOS 5D Mark3 ISO 3200 C1” [2] by different methods. The images are better viewed by zooming in on screen.

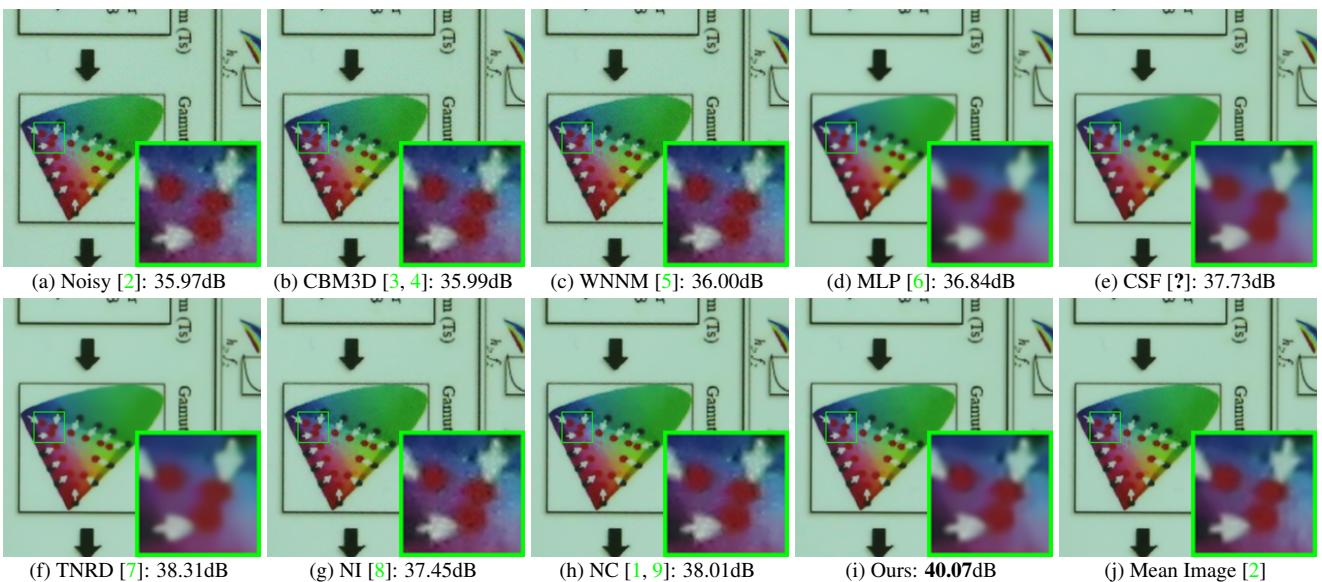


Figure 11. Denoised imagesupp of a region cropped from the real noisy image “Canon EOS 5D Mark3 ISO 3200 C2” [2] by different methods. The images are better viewed by zooming in on screen.

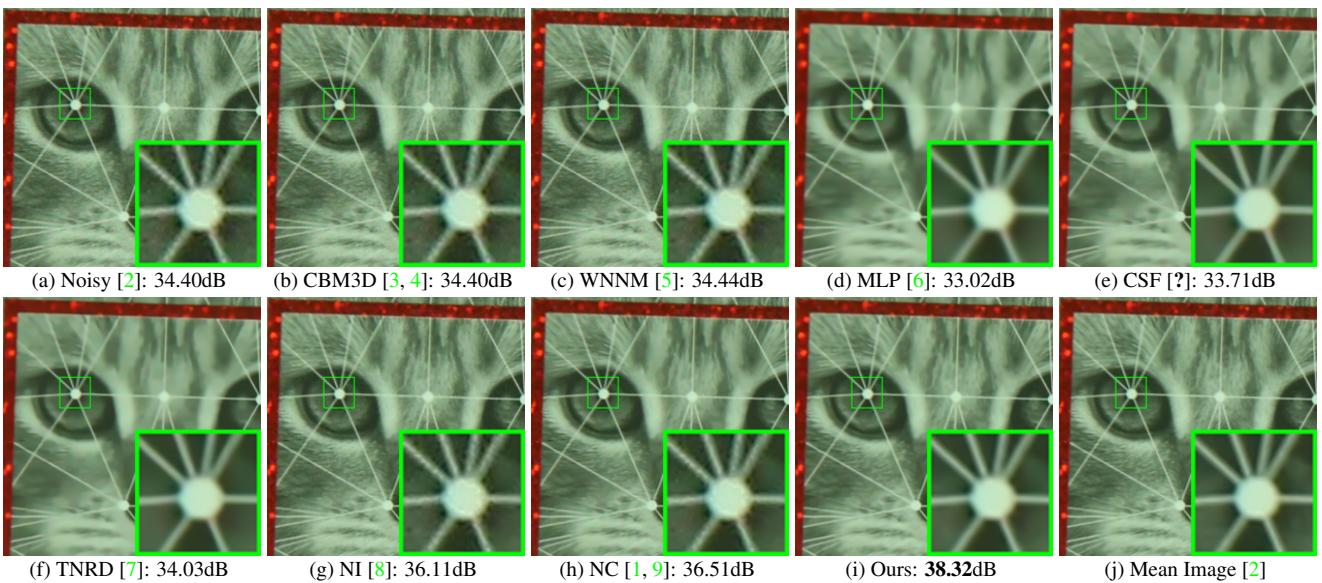


Figure 12. Denoised imagesupp of a region cropped from the real noisy image “Canon EOS 5D Mark3 ISO 3200 C3” [2] by different methods. The images are better viewed by zooming in on screen.

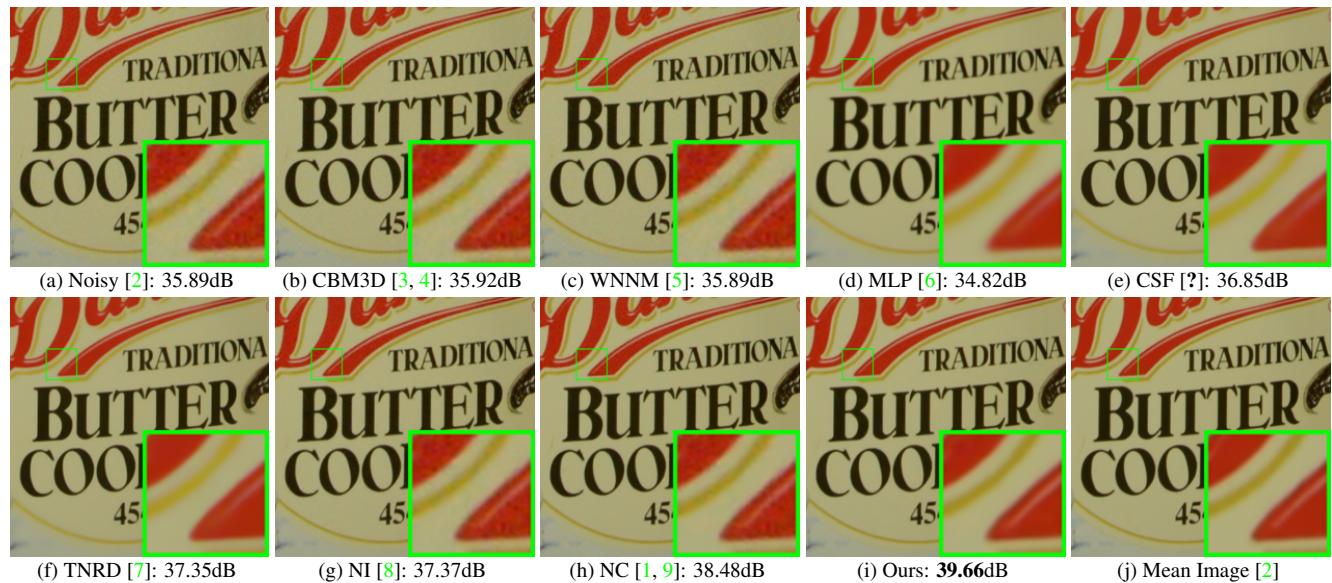


Figure 13. Denoised images of a region cropped from the real noisy image “Nikon D600 ISO 3200 C1” [2] by different methods. The images are better viewed by zooming in on screen.

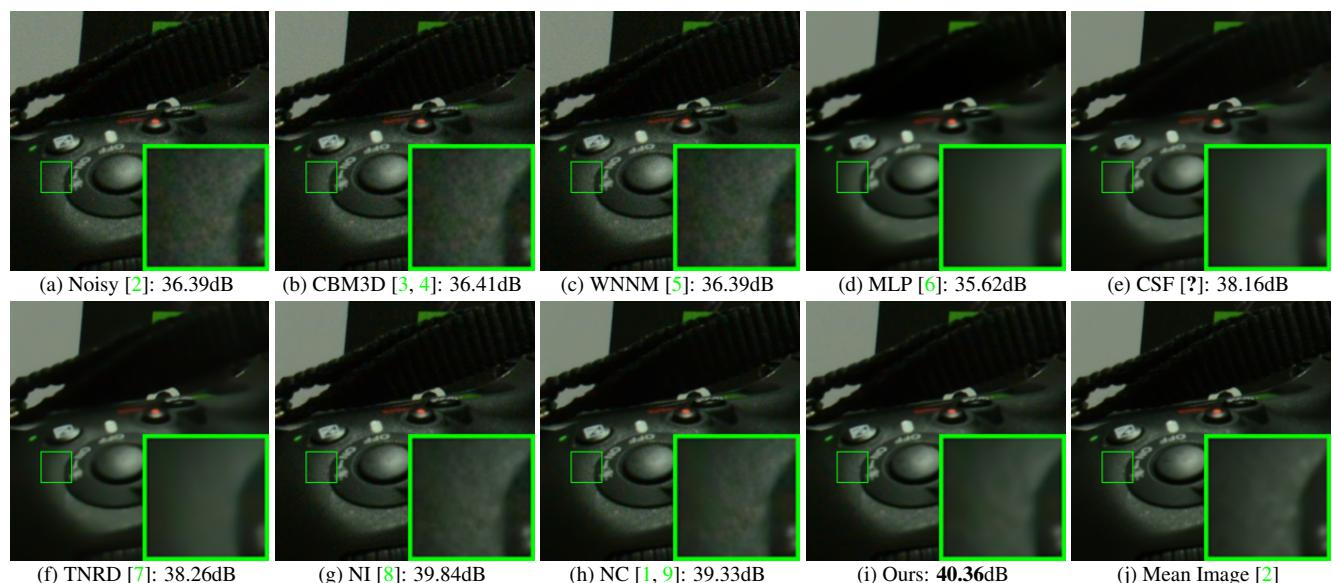


Figure 14. Denoised images of a region cropped from the real noisy image “Nikon D600 ISO 3200 C2” [2] by different methods. The images are better viewed by zooming in on screen.

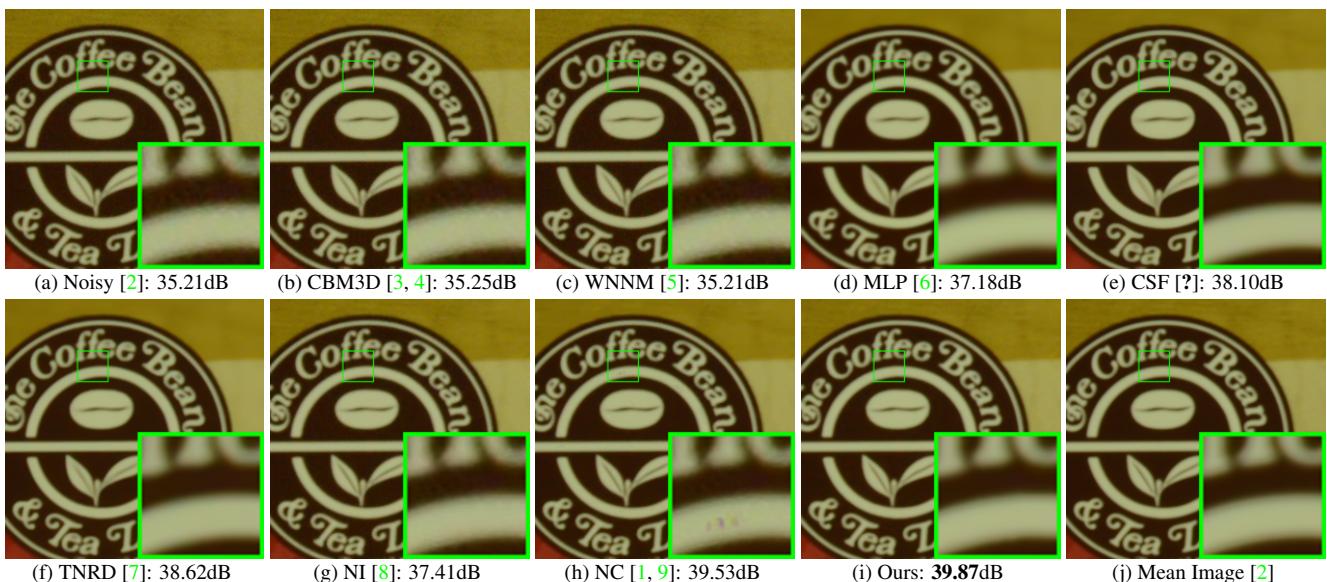


Figure 15. Denoised images of a region cropped from the real noisy image “Nikon D800 ISO 1600 B2” [2] by different methods. The images are better viewed by zooming in on screen.

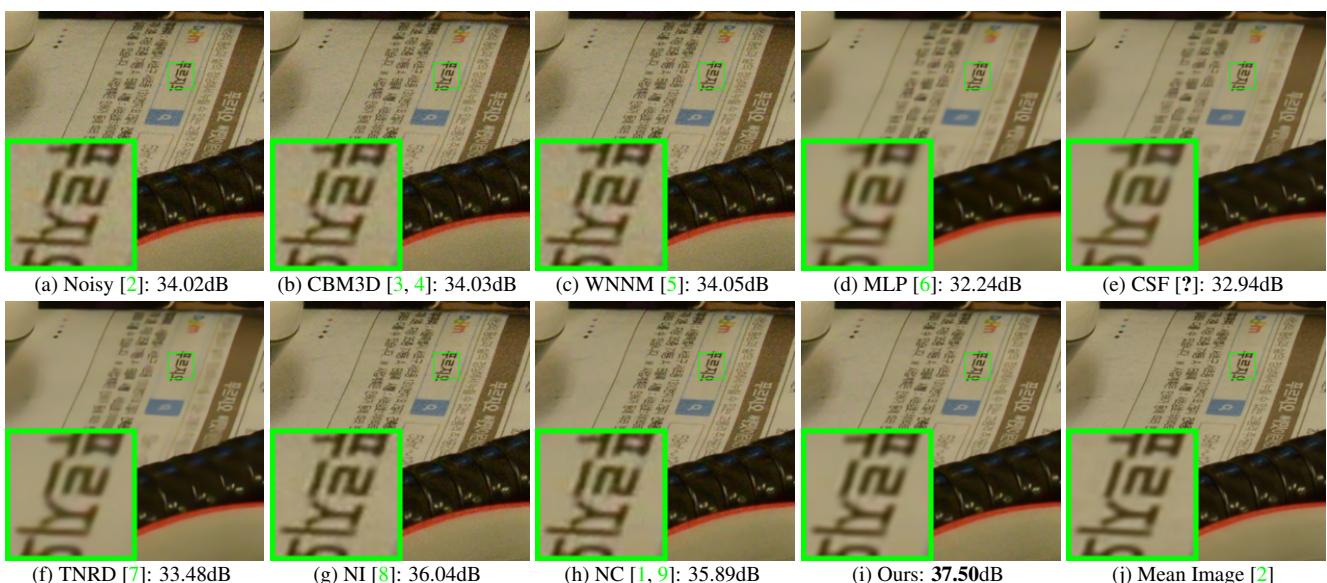


Figure 16. Denoised images of a region cropped from the real noisy image “Nikon D800 ISO 3200 A1” [2] by different methods. The images are better viewed by zooming in on screen.

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