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**004 Supplementary Material to “External Prior Guided Internal Prior Learning for**  
 005 **Real Noisy Image Denoising”**

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 008                   Anonymous CVPR submission  
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 011                   Paper ID 1047  
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 015                   In this supplementary material, we provide:  
 016     1. The closed-form solution of the sparse coding problem (6) in the main paper.  
 017     2. More denoising results on the real noisy images (with no “ground truth”) provided in the dataset [1].  
 018     3. More denoising results on the 15 cropped real noisy images (with “ground truth”) used in the dataset [2].  
 019     4. More denoising results on the 60 cropped real noisy images (with “ground truth”) from [2].

020                   **1. Closed-Form Solution of the Weighted Sparse Coding Problem (6)**  
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 022                   For notation simplicity, we ignore the indices  $n, m, t$  in the problem (6) in the main paper. And it turns into the following  
 023 weighted sparse coding problem:  
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$$\min_{\alpha} \|\mathbf{y} - \mathbf{D}\alpha\|_2^2 + \sum_{j=1}^{3p^2} \lambda_j |\alpha_j|. \quad (1)$$

025                   Since  $\mathbf{D}$  is an orthogonal matrix, problem (1) is equivalent to  
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$$\min_{\alpha} \|\mathbf{D}^T \mathbf{y} - \alpha\|_2^2 + \sum_{j=1}^{3p^2} \lambda_j |\alpha_j|. \quad (2)$$

028                   For simplicity, we denote  $\mathbf{z} = \mathbf{D}^T \mathbf{y}$ . Here we have  $\lambda_j > 0, j = 1, \dots, 3p^2$ , then problem (2) can be written as  
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$$\min_{\alpha} \sum_{j=1}^{3p^2} ((\mathbf{z}_j - \alpha_j)^2 + \lambda_j |\alpha_j|). \quad (3)$$

032                   The problem (3) is separable w.r.t. each  $\alpha_j$  and hence can be simplified to  $3p^2$  independent scalar minimization problems  
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$$\min_{\alpha_j} (\mathbf{z}_j - \alpha_j)^2 + \lambda_j |\alpha_j|, \quad (4)$$

035                   where  $j = 1, \dots, 3p^2$ . Taking derivative of  $\alpha_j$  in problem (4) and setting the derivative to be zero. There are two cases for the  
 036 solution.  
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038     (a) If  $\alpha_j \geq 0$ , we have  
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$$2(\alpha_j - \mathbf{z}_j) + \lambda_j = 0. \quad (5)$$

040     The solution is  
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$$\hat{\alpha}_j = \mathbf{z}_j - \frac{\lambda_j}{2} \geq 0. \quad (6)$$

042     So  $\mathbf{z}_j \geq \frac{\lambda_j}{2} > 0$ , and the solution  $\hat{\alpha}_j$  can be written as  
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$$\hat{\alpha}_j = \text{sgn}(\mathbf{z}_j) * (|\mathbf{z}_j| - \frac{\lambda_j}{2}), \quad (7)$$

044     where  $\text{sgn}(\bullet)$  is the sign function.  
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046     (b) If  $\alpha_j < 0$ , we have  
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$$2(\alpha_j - \mathbf{z}_j) - \lambda_j = 0. \quad (8)$$

048     The solution is  
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$$\hat{\alpha}_j = \mathbf{z}_j + \frac{\lambda_j}{2} < 0. \quad (9)$$

108 So  $\mathbf{z}_j < -\frac{\lambda_j}{2} < 0$ , and the solution  $\hat{\alpha}_j$  can be written as  
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$$\hat{\alpha}_j = \text{sgn}(\mathbf{z}_j) * (-\mathbf{z}_j - \frac{\lambda_j}{2}) = \text{sgn}(\mathbf{z}_j) * (|\mathbf{z}_j| - \frac{\lambda_j}{2}). \quad (10)$$

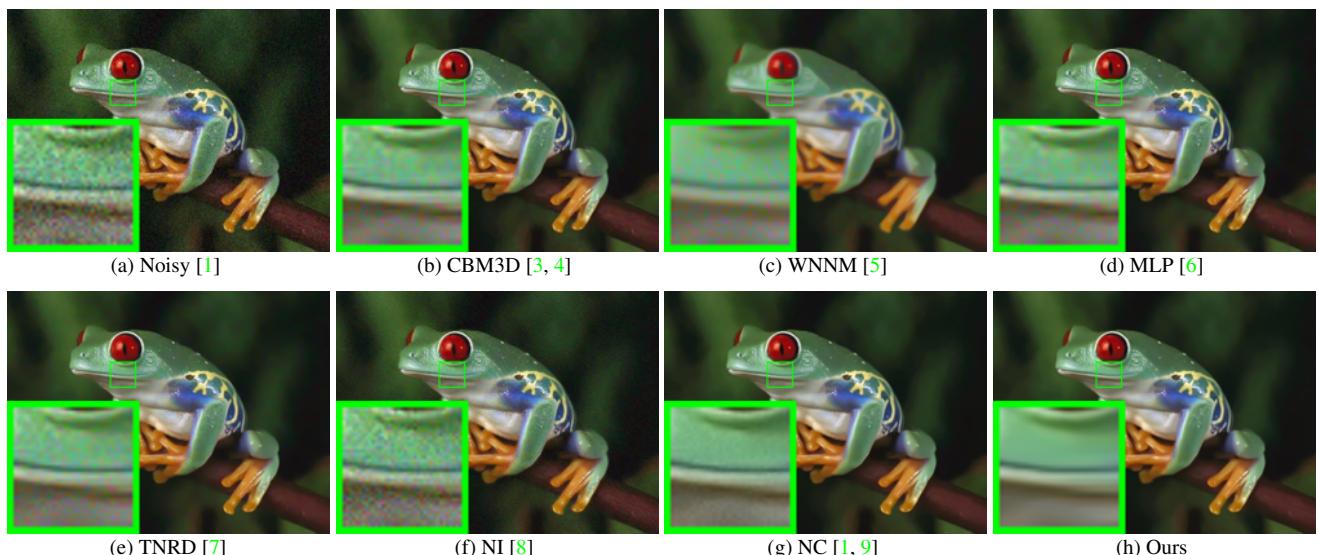
111 In summary, we have the final solution of the weighted sparse coding problem (1) as  
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$$\hat{\alpha} = \text{sgn}(\mathbf{D}^T \mathbf{y}) \odot \max(|\mathbf{D}^T \mathbf{y}| - \lambda, 0), \quad (11)$$

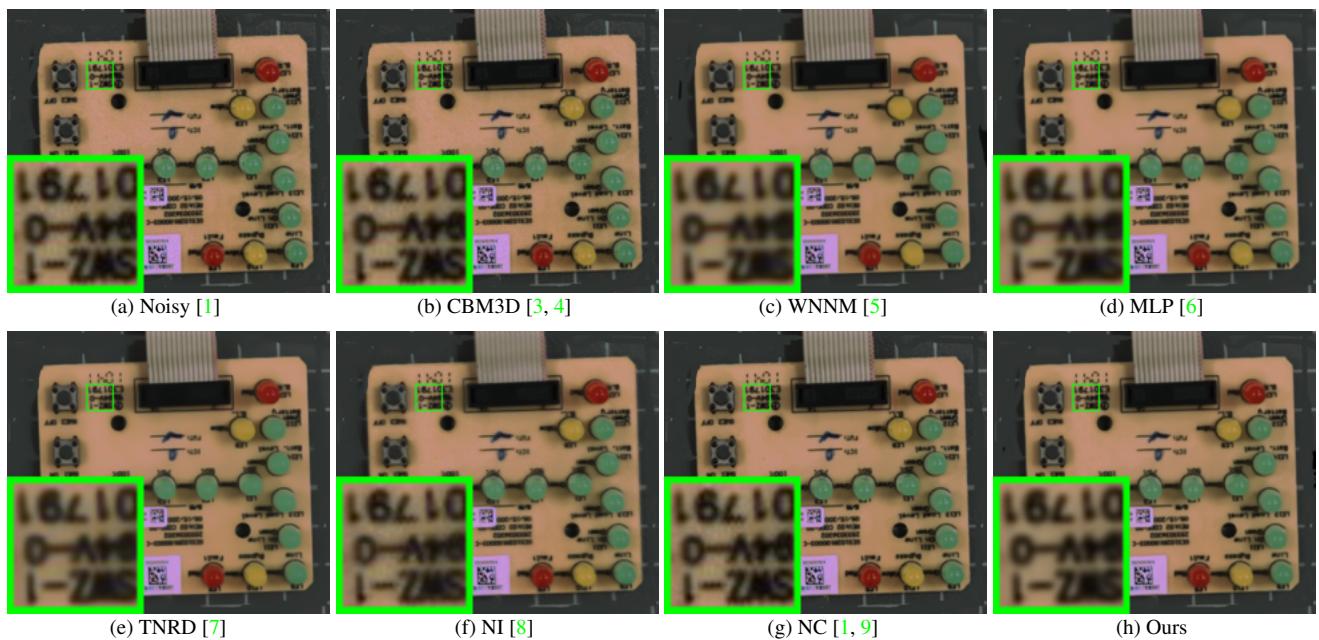
114 where  $\lambda = \frac{1}{2}[\lambda_1, \lambda_2, \dots, \lambda_{3p^2}]$  is the vector of regularization parameter and  $\odot$  means element-wise multiplication.  
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## 2. More Results on Real Noisy Images in [1]

117 In this section, we give more comparisons of the competing methods on the dataset [1]. The real noisy images in dataset  
 118 [1] have no “ground truth” images and hence we only compare the visual quality of the denoised images by different methods.  
 119 One can seen from Figures 1-4 that, our proposed method perfomrs better than the state-of-the-art denoising methods.  
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 138 Figure 1. Denoised images of the real noisy image “Frog” [1] by different methods. The images are better to be zoomed in on screen.  
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 161 Figure 2. Denoised images of the real noisy image “Circuit” [1] by different methods. The images are better to be zoomed in on screen.  
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Figure 3. Denoised images of the real noisy image “Woman” [1] by different methods. The images are better to be zoomed in on screen.

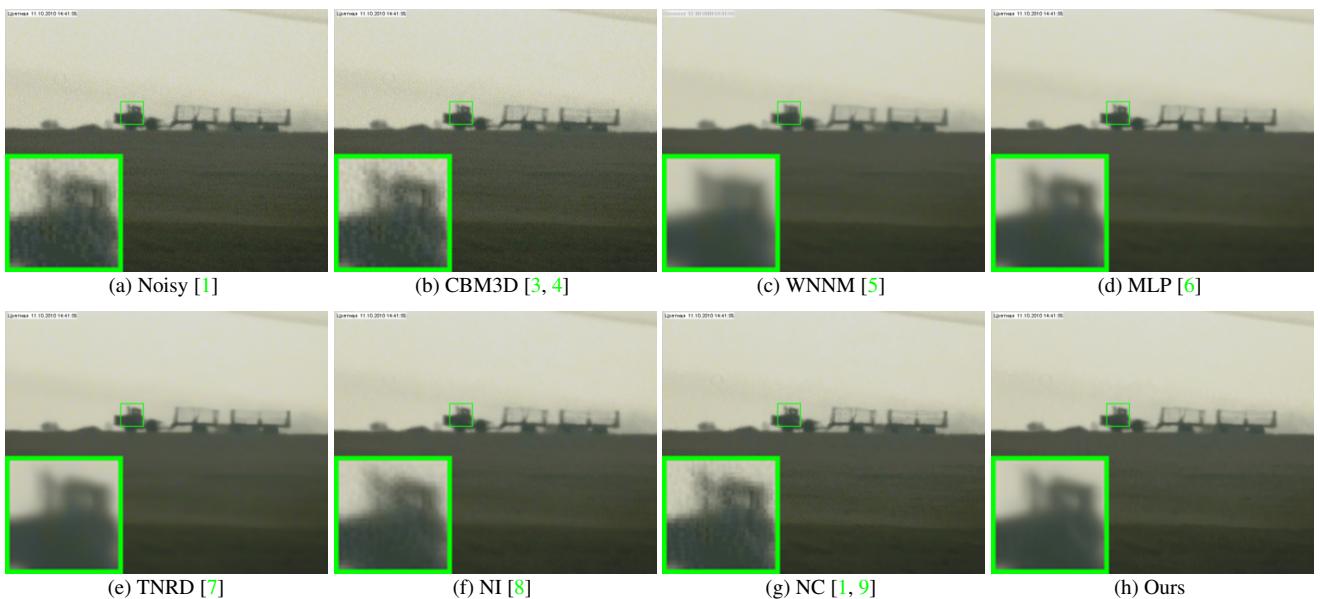


Figure 4. Denoised images of the real noisy image “Vehicle” [1] by different methods. The images are better to be zoomed in on screen.

### 3. More Results on the 15 Cropped Images Used in [2]

In this section, we provide more visual comparisons of the proposed method with the state-of-the-art denoising methods on the 15 cropped real noisy images used in [2]. As can be seen from Figures 5-9, on most cases, our proposed method achieves better performance than the competing methods. This validates the effectiveness of our proposed external prior guided internal prior learning framework for real noisy image denoising.

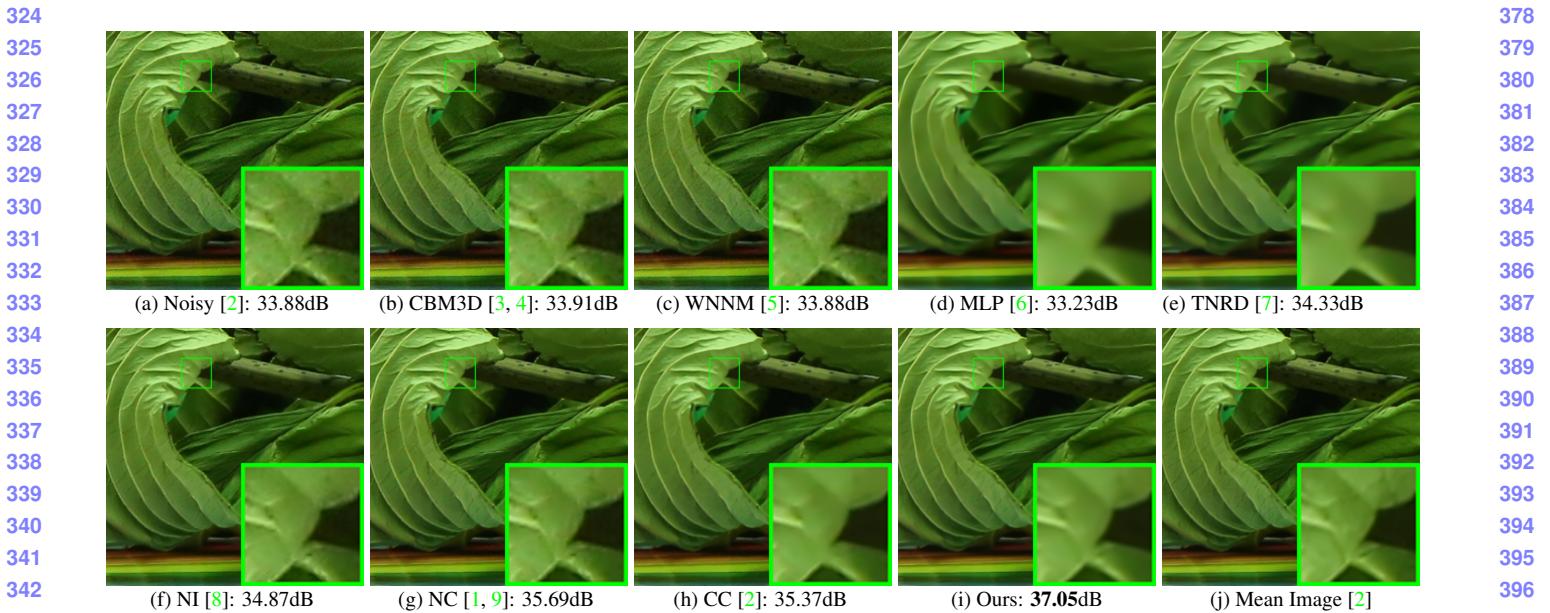


Figure 5. Denoised images of a region cropped from the real noisy image “Canon 5D Mark 3 ISO 3200 2” [2] by different methods. The images are better to be zoomed in on screen.

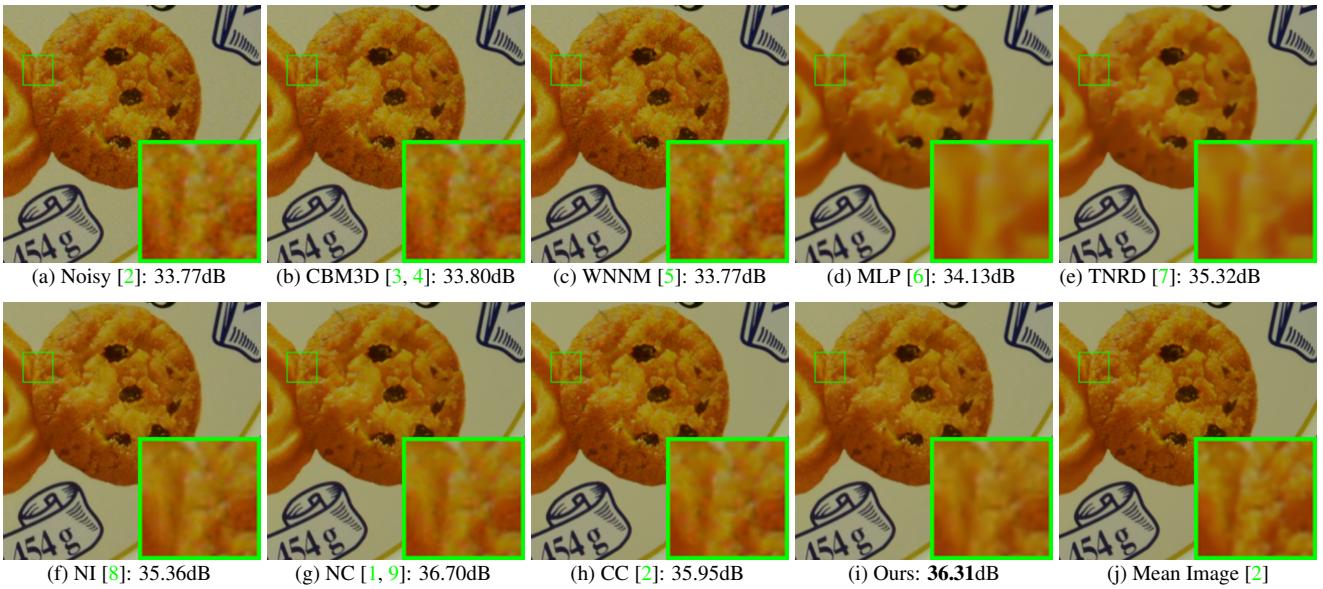


Figure 6. Denoised images of a region cropped from the real noisy image “Canon 5D Mark 3 ISO 3200 2” [2] by different methods. The images are better to be zoomed in on screen.

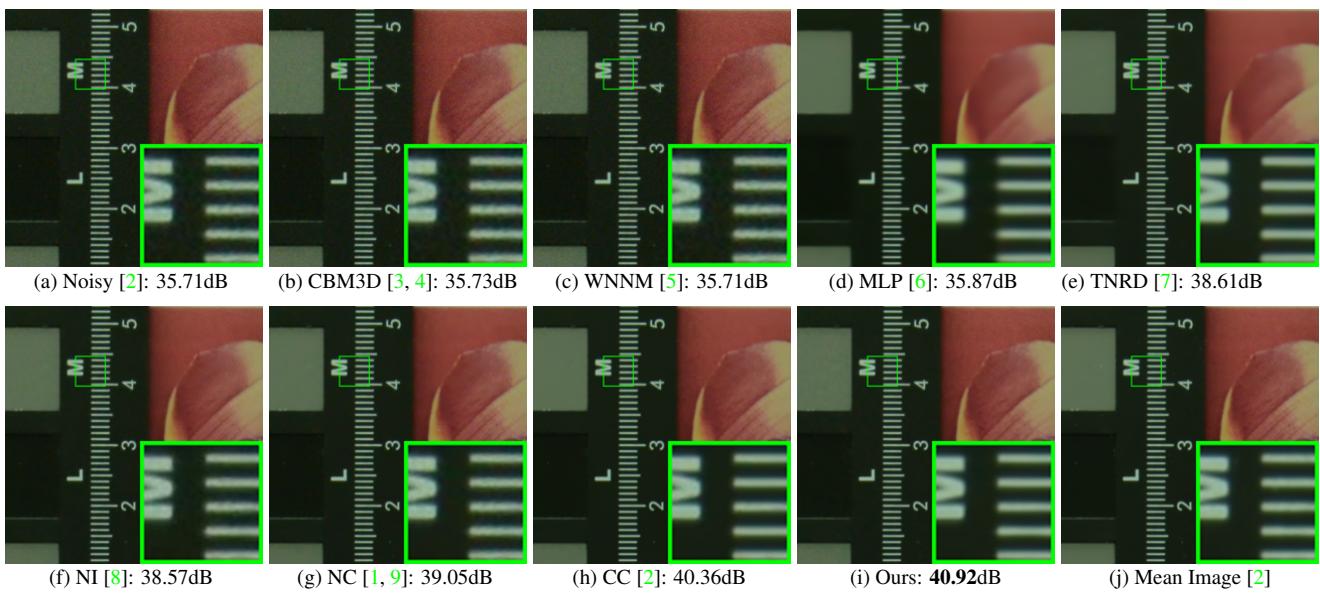


Figure 7. Denoised images of a region cropped from the real noisy image “Nikon D800 ISO 1600 2” [2] by different methods. The images are better to be zoomed in on screen.

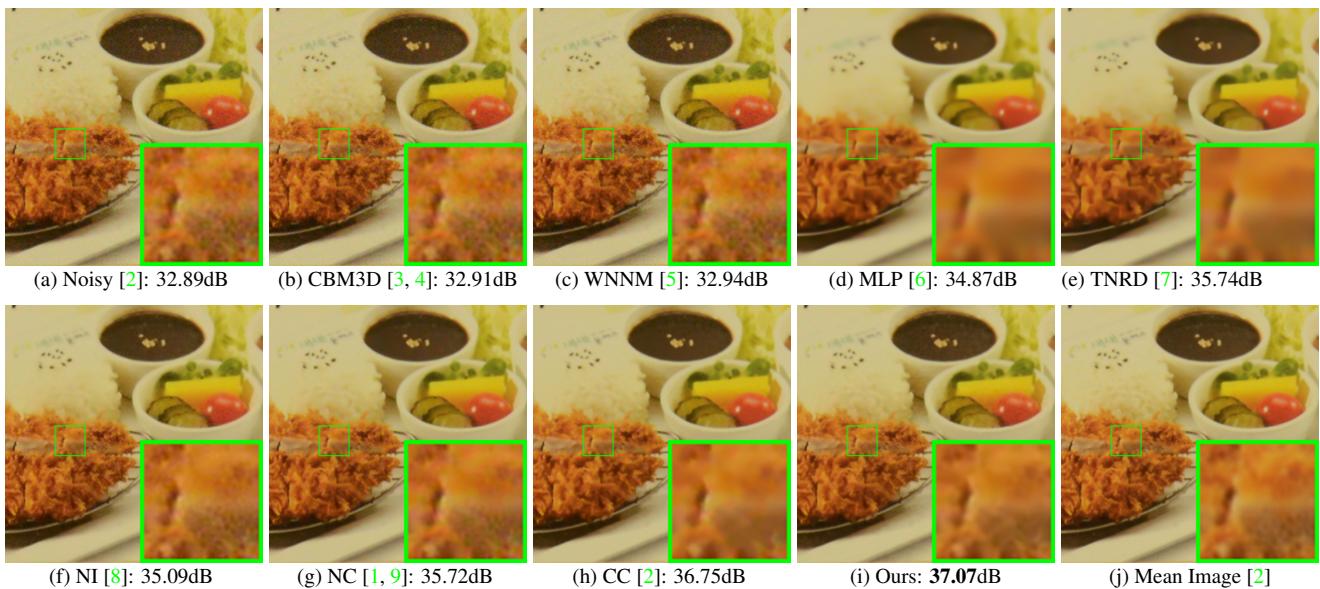


Figure 8. Denoised images of a region cropped from the real noisy image “Nikon D800 ISO 3200 2” [2] by different methods. The images are better to be zoomed in on screen.

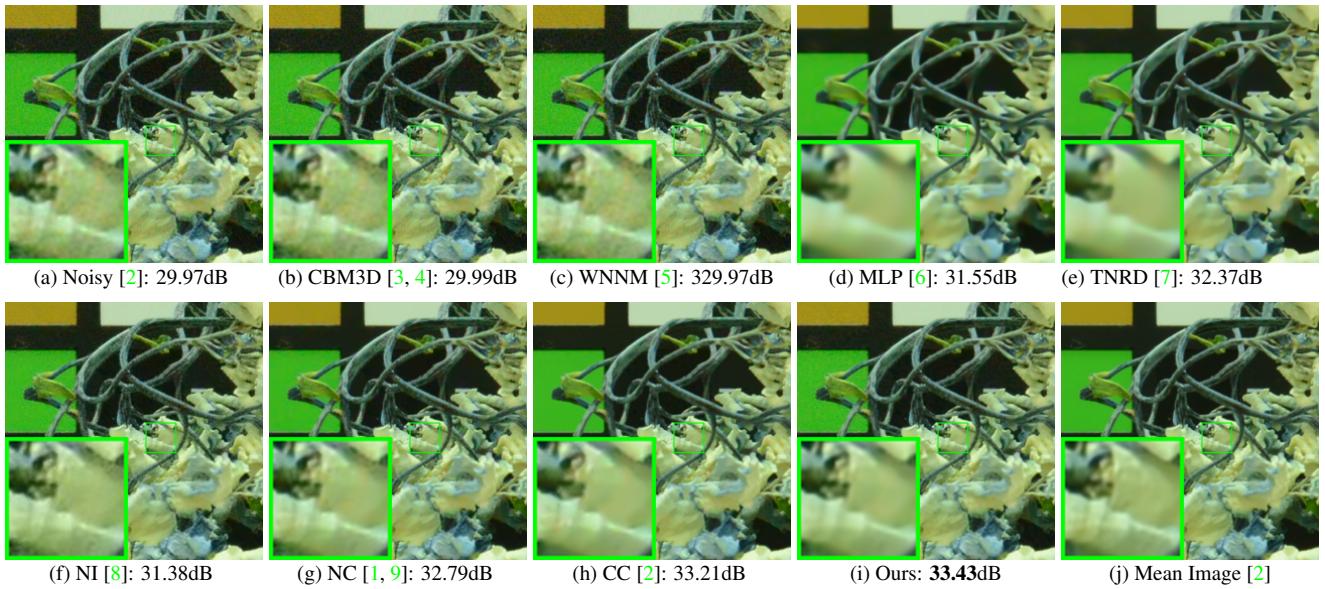


Figure 9. Denoised images of a region cropped from the real noisy image “Nikon D800 ISO 6400 2” [2] by different methods. The images are better to be zoomed in on screen.

#### 4. More Results on the 60 Cropped Images in [2]

In this section, we provide more visual comparisons of the proposed method with the state-of-the-art denoising methods on the 60 cropped real noisy images we cropped from [2]. As can be seen from Figures 10-20, on most cases, our proposed method achieves better performance than the competing methods. This validates the effectiveness of our proposed external prior guided internal prior learning framework for real noisy image denoising.

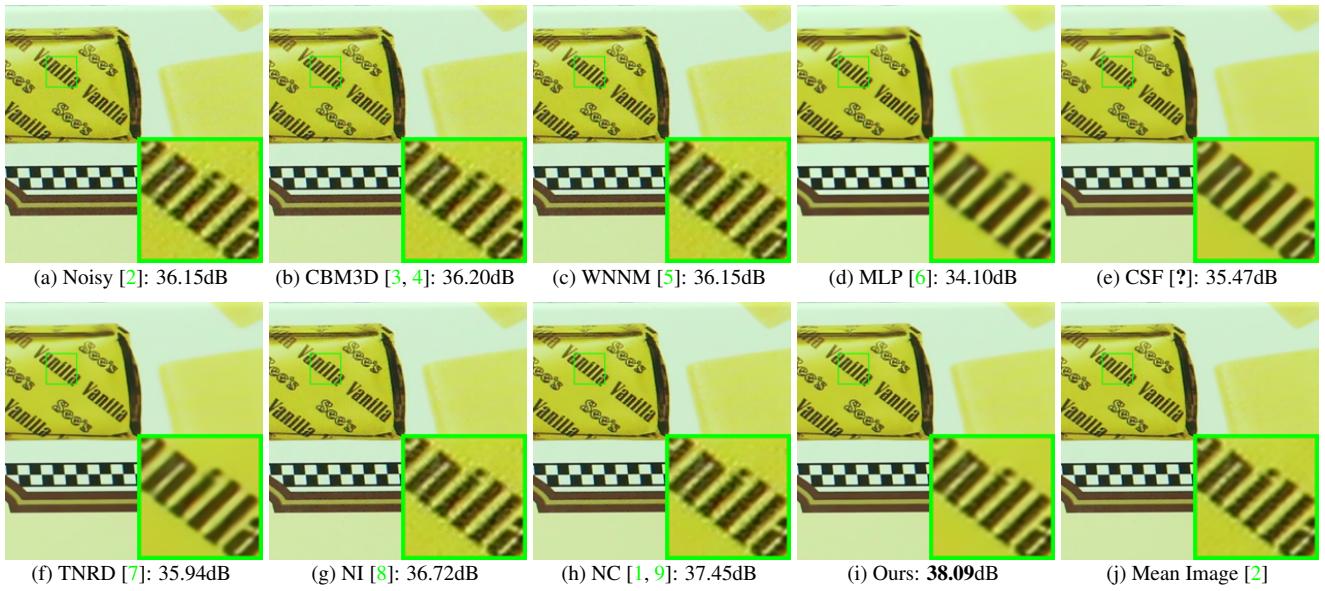


Figure 10. Denoised images of a region cropped from the real noisy image “Canon EOS 5D Mark3 ISO 3200 C1” [2] by different methods. The images are better viewed by zooming in on screen.

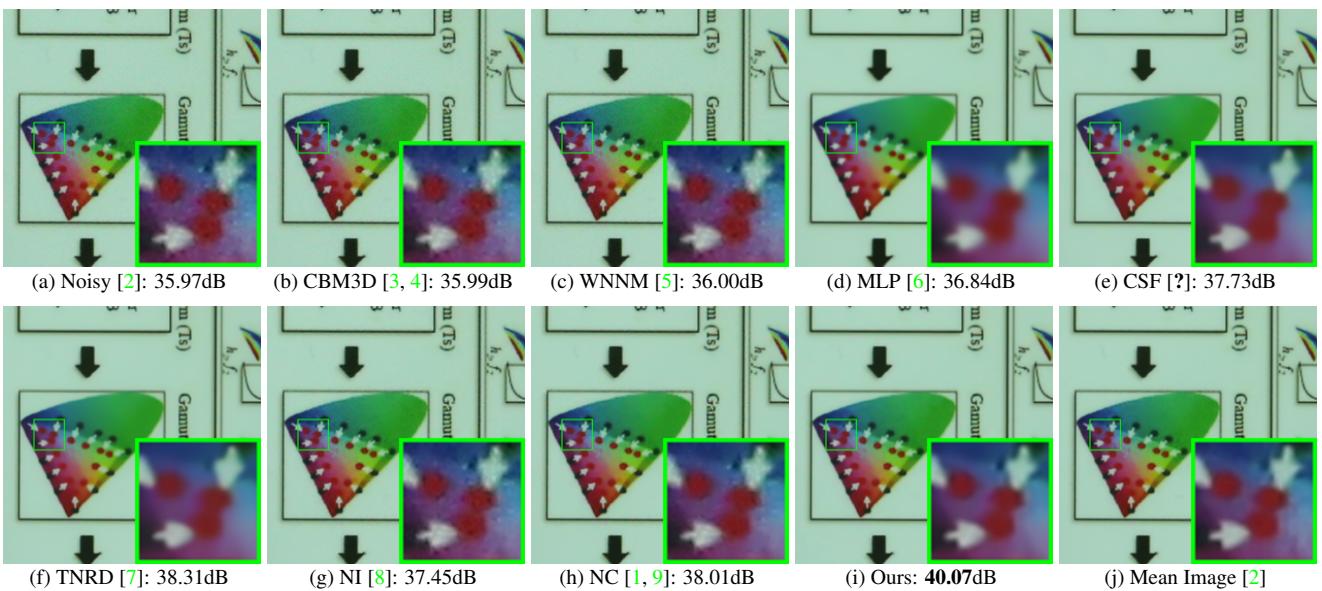


Figure 11. Denoised imagesupp of a region cropped from the real noisy image “Canon EOS 5D Mark3 ISO 3200 C2” [2] by different methods. The images are better viewed by zooming in on screen.

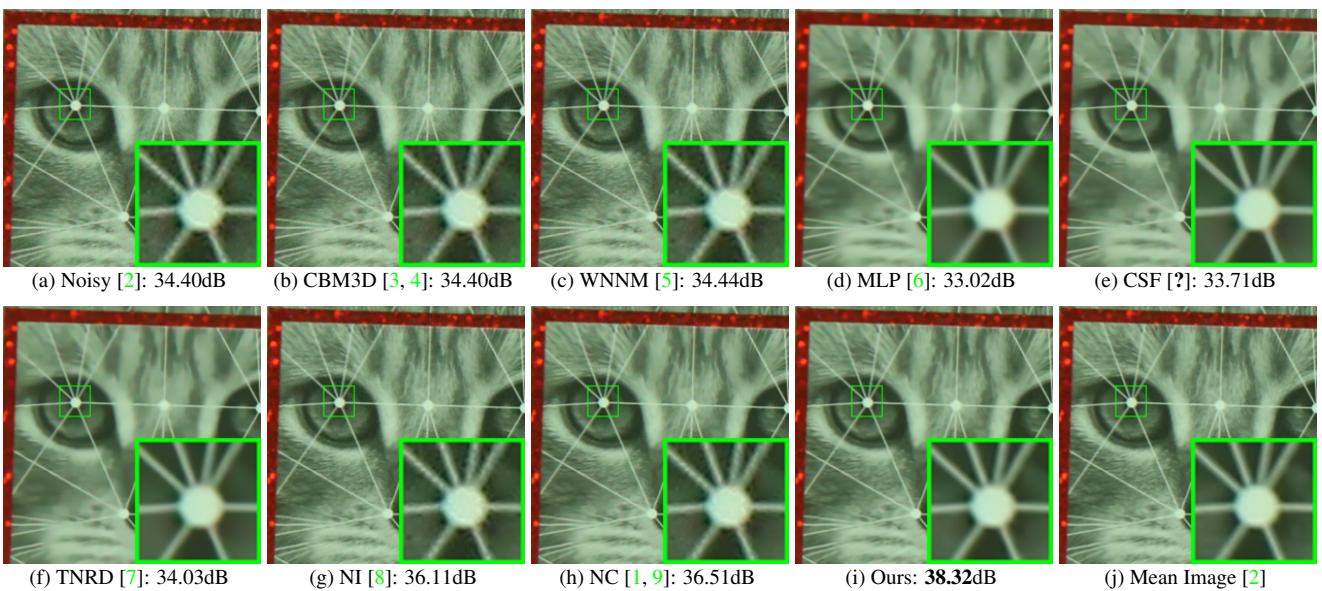


Figure 12. Denoised imagesupp of a region cropped from the real noisy image “Canon EOS 5D Mark3 ISO 3200 C3” [2] by different methods. The images are better viewed by zooming in on screen.



Figure 13. Denoised imagesupp of a region cropped from the real noisy image “Nikon D600 ISO 3200 C1” [2] by different methods. The images are better viewed by zooming in on screen.

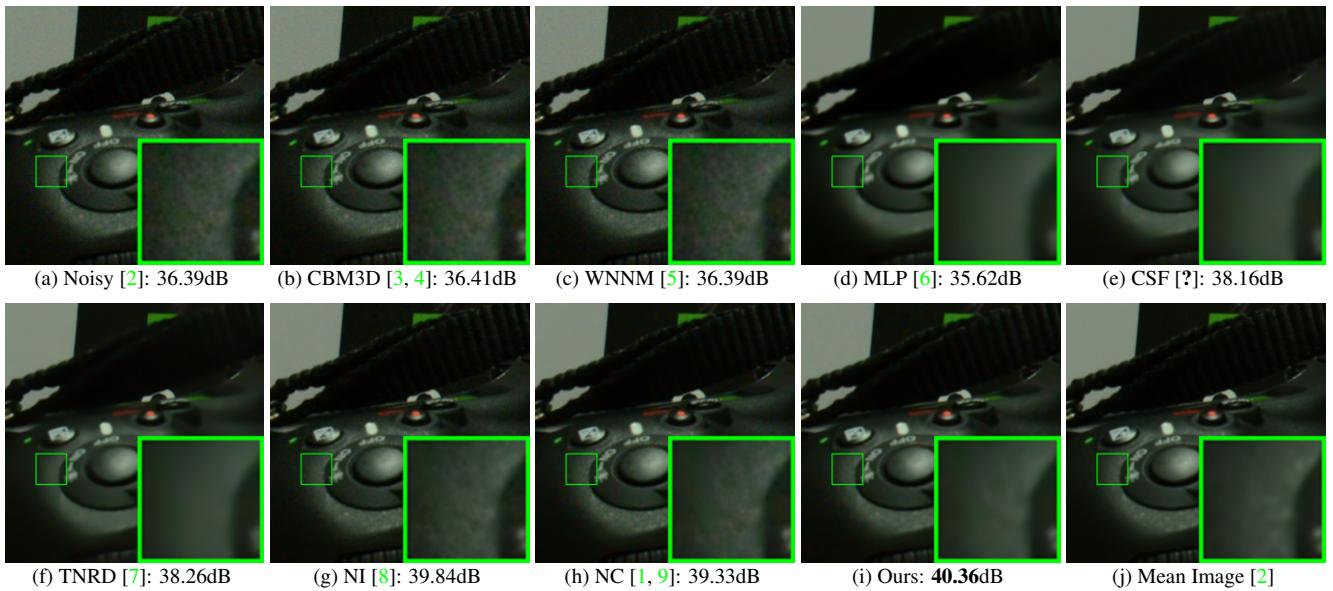


Figure 14. Denoised imagesupp of a region cropped from the real noisy image “Nikon D600 ISO 3200 C2” [2] by different methods. The images are better viewed by zooming in on screen.

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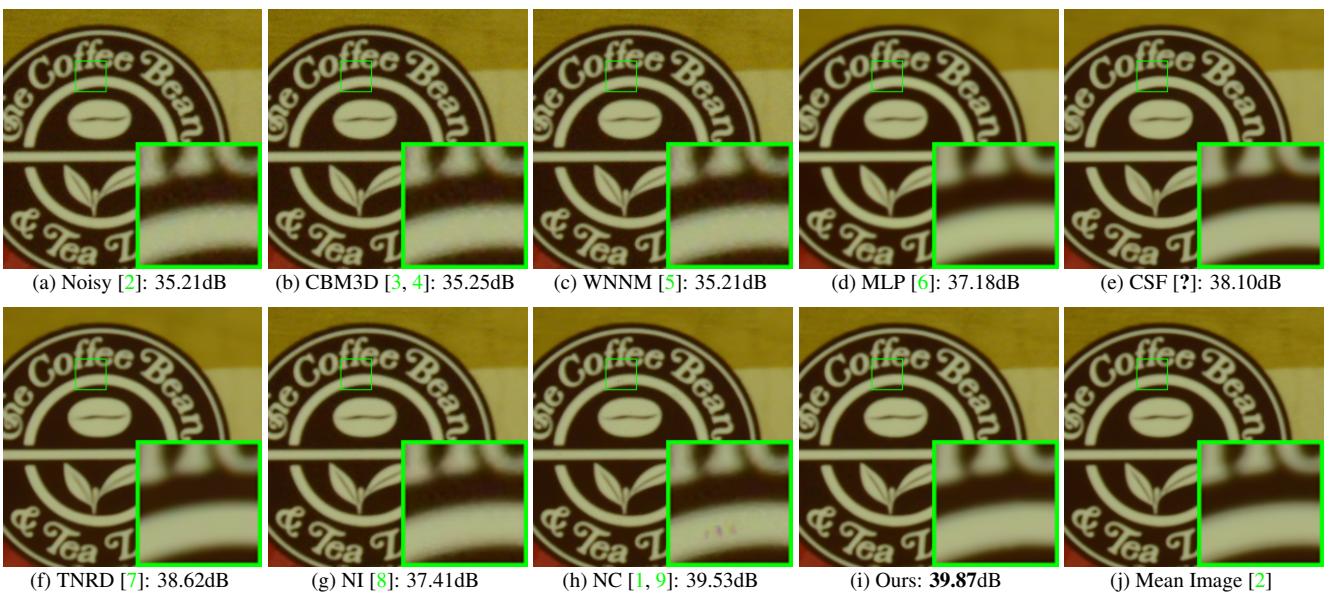


Figure 15. Denoised imagesupp of a region cropped from the real noisy image “Nikon D800 ISO 1600 B2” [2] by different methods. The images are better viewed by zooming in on screen.

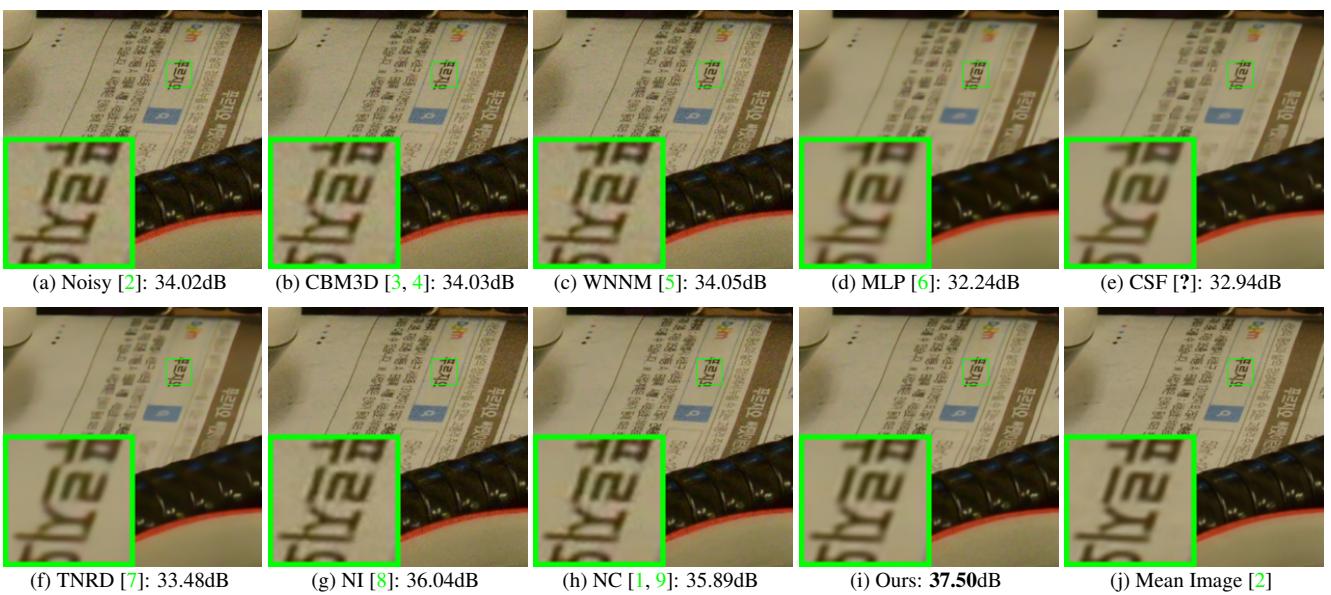


Figure 16. Denoised imagesupp of a region cropped from the real noisy image “Nikon D800 ISO 3200 A1” [2] by different methods. The images are better viewed by zooming in on screen.

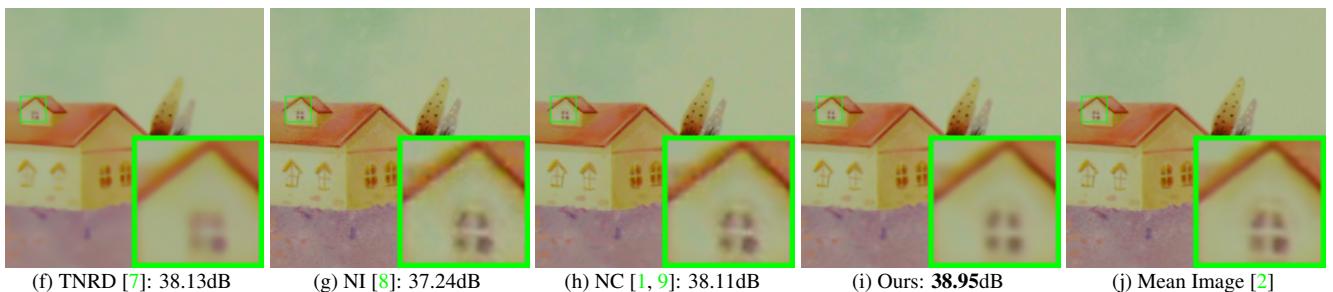
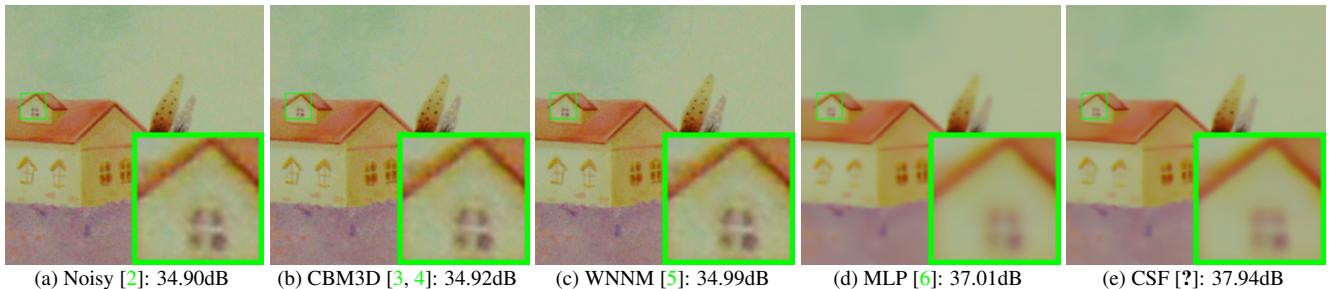


Figure 17. Denoised imagesupp of a region cropped from the real noisy image “Nikon D800 ISO 3200 A2” [2] by different methods. The images are better viewed by zooming in on screen.

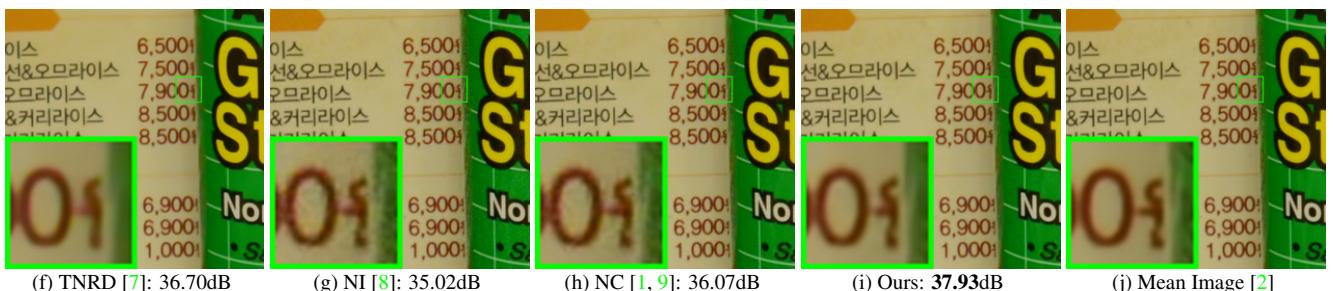
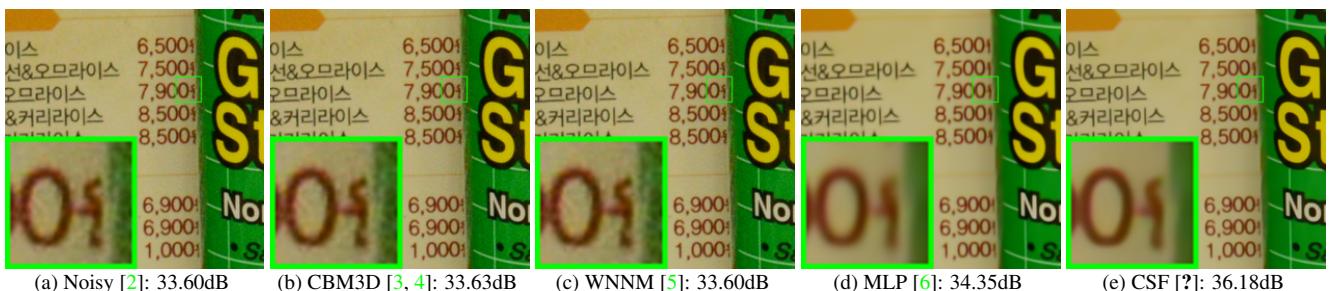


Figure 18. Denoised imagesupp of a region cropped from the real noisy image “Nikon D800 ISO 3200 A3” [2] by different methods. The images are better viewed by zooming in on screen.

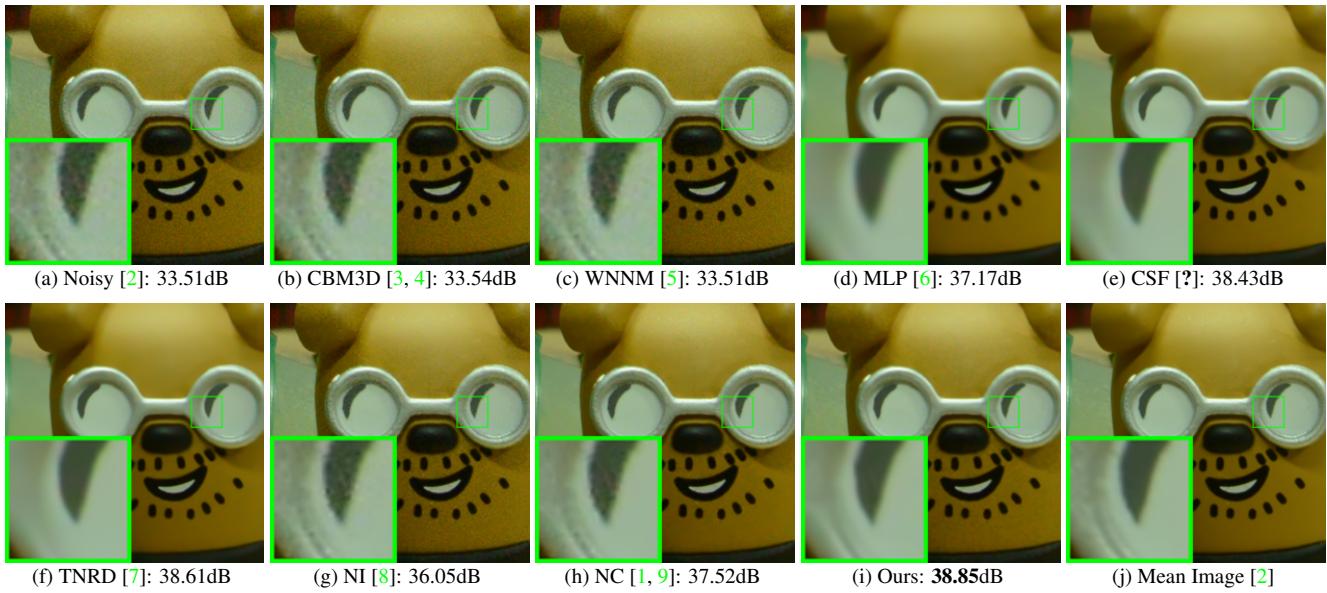


Figure 19. Denoised imagesupp of a region cropped from the real noisy image “Nikon D800 ISO 3200 A4” [2] by different methods. The images are better viewed by zooming in on screen.

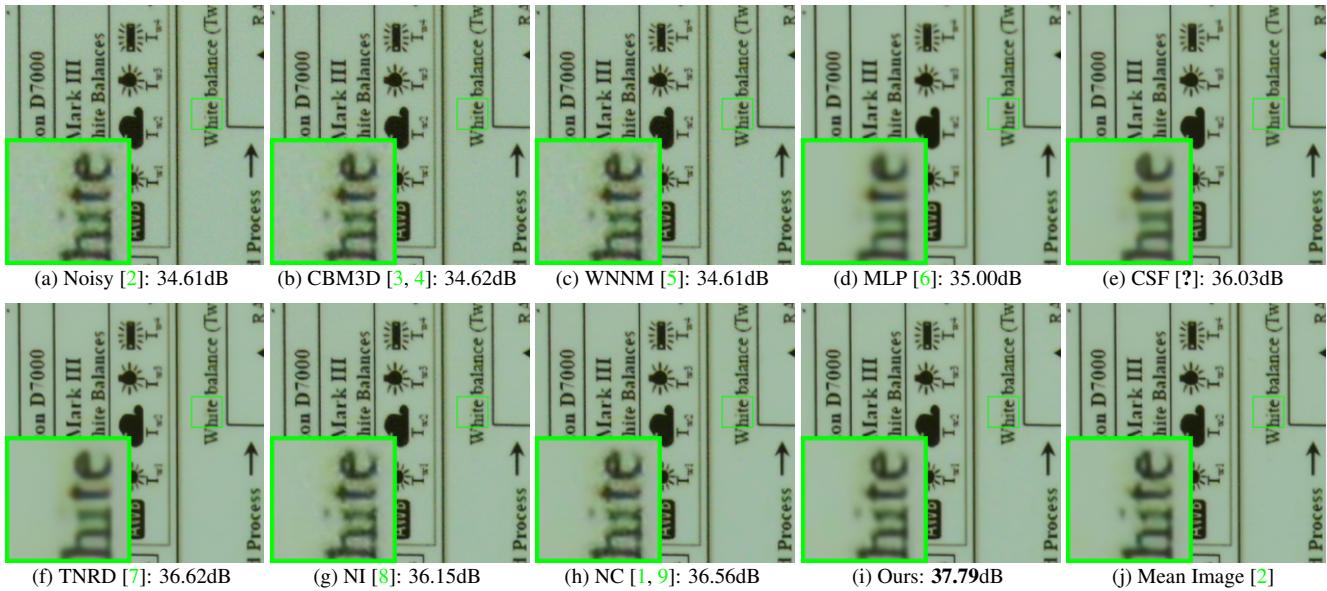


Figure 20. Denoised imagesupp of a region cropped from the real noisy image “Nikon D800 ISO 3200 A5” [2] by different methods. The images are better viewed by zooming in on screen.

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