Supplementary Material to "External Prior Guided Internal Prior Learning for Real Noisy Image Denoising"

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In this supplementary material, we provide:
1. The closed-form solution of the proposed weighted sparse coding model in the main paper.
2. More denoising results on the 20 real noisy imagessupp (with no "ground truth") provided in the dataset [1].
3. More denoising results on the 15 smaller real noisy imagessupp (with "ground truth") used in the dataset [2].
4. More denoising results on the 60 real noisy imagessupp (with "ground truth") cropped from [2].

1. Closed-Form Solution of the Weighted Sparse Coding Problem

The weighted sparse coding problem in the main paper is:

$$\min_{\boldsymbol{\alpha}} \|\mathbf{y}-\mathbf{D}\boldsymbol{\alpha}\|_2^2 + \|\mathbf{w}^T\boldsymbol{\alpha}\|_1. \tag{1}$$

Since $\mathbf{D}$ is an orthonormal matrix, problem (1) is equivalent to

$$\min_{\boldsymbol{\alpha}} \|\mathbf{D}^T\mathbf{y}-\boldsymbol{\alpha}\|_2^2 + \|\mathbf{w}^T\boldsymbol{\alpha}\|_1. \tag{2}$$

For simplicity, we denote $\mathbf{z} = \mathbf{D}^{\mathbf{T}}\mathbf{y}$. Since $\mathbf{w}_i = c * 2\sqrt{2}\sigma^2/(\boldsymbol{\Lambda}_i + \varepsilon)$ is positive (please refer to Eq. (18) in the main paper), problem (2) can be written as

$$\min_{\boldsymbol{\alpha}} \sum_{i=1}^{p^2}((\mathbf{z}_i - \boldsymbol{\alpha}_i)^2 + \mathbf{w}_i|\boldsymbol{\alpha}_i|). \tag{3}$$

The problem (3) is separable w.r.t. $\boldsymbol{\alpha}_i$ and can be simplified to p^2 scalar minimization problems

$$\min_{\boldsymbol{\alpha}_i}(\mathbf{z}_i - \boldsymbol{\alpha}_i)^2 + \mathbf{w}_i|\boldsymbol{\alpha}_i|, \tag{4}$$

where $i = 1, ..., p^2$. Taking derivative of $\boldsymbol{\alpha}_i$ in problem (4) and setting the derivative to be zero. There are two cases for the solution.

(a) If $\boldsymbol{\alpha}_i \geq 0$, we have

$$2(\boldsymbol{\alpha}_i - \mathbf{z}_i) + \mathbf{w}_i = 0. \tag{5}$$

The solution is

$$\hat{\boldsymbol{\alpha}}_i = \mathbf{z}_i - \frac{\mathbf{w}_i}{2} \geq 0. \tag{6}$$

So $\mathbf{z}_i \geq \frac{\mathbf{w}_i}{2} > 0$, and the solution $\hat{\boldsymbol{\alpha}}_i$ can be written as

$$\hat{\boldsymbol{\alpha}}_i = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}), \tag{7}$$

where sgn($\bullet$) is the sign function.

(b) If $\boldsymbol{\alpha}_i < 0$, we have

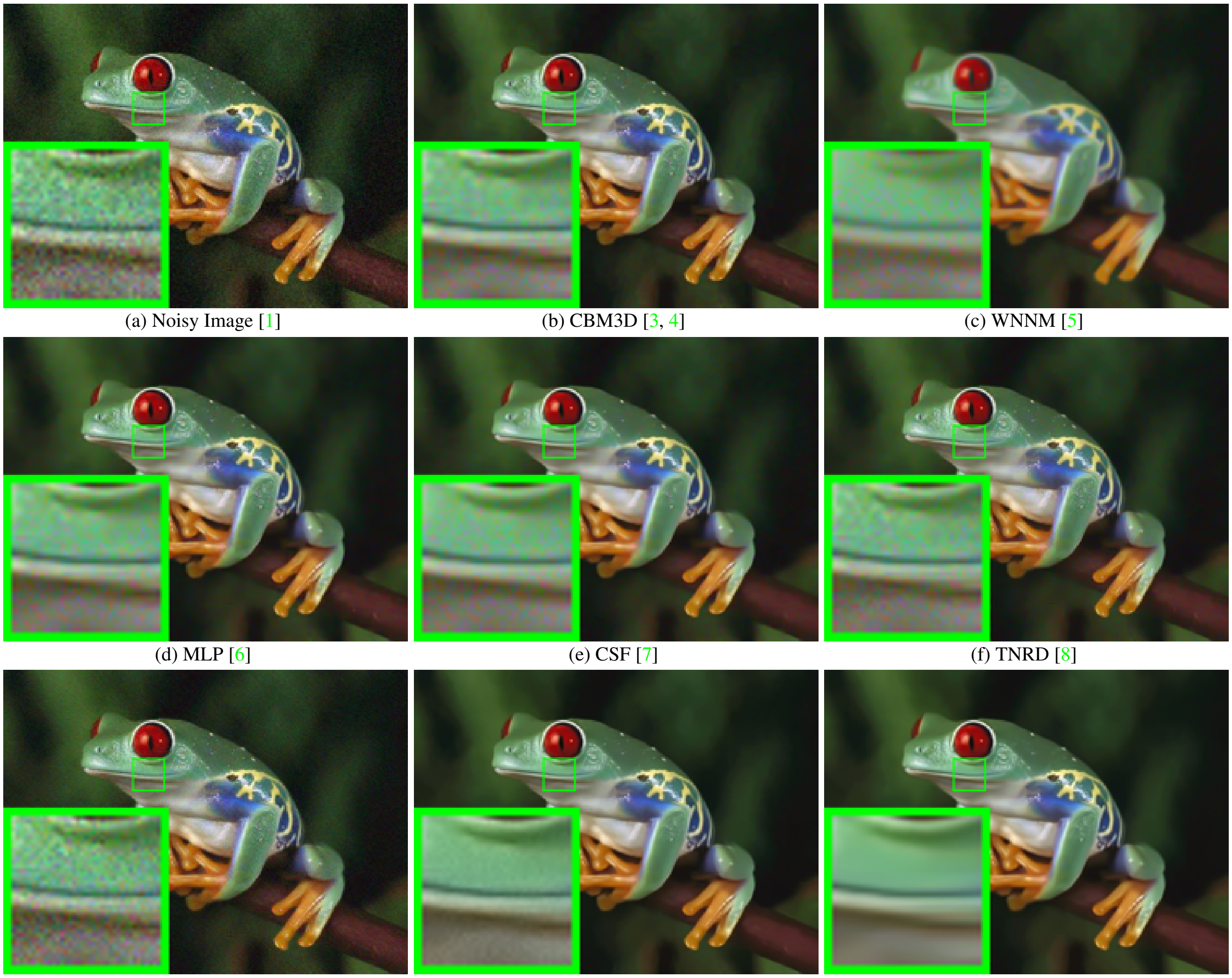
$$2(\boldsymbol{\alpha}_i - \mathbf{z}_i) - \mathbf{w}_i = 0. \tag{8}$$

The solution is

$$\hat{\boldsymbol{\alpha}}_i = \mathbf{z}_i + \frac{\mathbf{w}_i}{2} < 0. \tag{9}$$

So $\mathbf{z}_i < -\frac{\mathbf{w}_i}{2} < 0$, and the solution $\hat{\boldsymbol{\alpha}}_i$ can be written as

$$\hat{\boldsymbol{\alpha}}_i = \text{sgn}(\mathbf{z}_i) * (-\mathbf{z}_i - \frac{\mathbf{w}_i}{2}) = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}). \tag{10}$$



(a) Noisy Image [1] (b) CBM3D [3, 4] (c) WNNM [5]
(d) MLP [6] (e) CSF [7] (f) TNRD [8]
(g) Neat Image [9] (h) Noise Clinic [10, 1] (i) Ours

Figure 1. (a) A real noisy image captured by a digital camera [1]. (b)~(i) Denoised imagessupp by different methods. Among them, CBM3D [3, 4] and WNNM [5] are prior model based methods; MLP [6], CSF [7] and TNRD [8] are discriminative learning based methods; and Neat Image [9] and Noise Clinic [10, 1] are methods designed for real noisy imagessupp. Clearly, our method produces a cleaner image.

In summary, we have the final solution of the weighted sparse coding problem (1) as

$$\hat{\boldsymbol{\alpha}} = \mathrm{sgn}(\mathbf{D}^{\mathbf{T}}\mathbf{y}) \odot \max(|\mathbf{D}^{\mathbf{T}}\mathbf{y}| - \mathbf{w}/2, 0), \tag{11}$$

where $\odot$ means element-wise multiplication and $|\mathbf{D}^{\mathbf{T}}\mathbf{y}|$ is the absolute value of each entry of the vector $\mathbf{D}^{\mathbf{T}}\mathbf{y}$.

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