Supplementary Material to "External Prior Guided Internal Prior Learning for Real Noisy Image Denoising"

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Paper ID 1047

In this supplementary material, we provide:
1. The closed-form solution of the proposed weighted sparse coding model in the main paper.
2. Comparison of the *Patch Prior based Denoising* (PPD) method and the proposed *Patch Group Prior based Denoising* (PGPD) method.
3. More denoising results on the 20 widely used natural images.
4. The denoising results of the competing methods on the Berkeley Segmentation Data Set [?].

1. Closed-Form Solution of the Weighted Sparse Coding Problem

The weighted sparse coding problem in the main paper is:

$$\min_{\boldsymbol{\alpha}} \|\mathbf{y}-\mathbf{D}\boldsymbol{\alpha}\|_2^2 + \|\mathbf{w}^T\boldsymbol{\alpha}\|_1. \tag{1}$$

Since $\mathbf{D}$ is an orthonormal matrix, problem (1) is equivalent to

$$\min_{\boldsymbol{\alpha}} \|\mathbf{D}^T\mathbf{y}-\boldsymbol{\alpha}\|_2^2 + \|\mathbf{w}^T\boldsymbol{\alpha}\|_1. \tag{2}$$

For simplicity, we denote $\mathbf{z}=\mathbf{D^T y}$. Since $\mathbf{w}_i = c * 2\sqrt{2}\sigma^2/(\boldsymbol{\Lambda}_i+\varepsilon)$ is positive (please refer to Eq. (18) in the main paper), problem (2) can be written as

$$\min_{\boldsymbol{\alpha}} \sum_{i=1}^{p^2}((\mathbf{z}_i-\boldsymbol{\alpha}_i)^2 + \mathbf{w}_i|\boldsymbol{\alpha}_i|). \tag{3}$$

The problem (3) is separable w.r.t. $\boldsymbol{\alpha}_i$ and can be simplified to p^2 scalar minimization problems

$$\min_{\boldsymbol{\alpha}_i}(\mathbf{z}_i-\boldsymbol{\alpha}_i)^2 + \mathbf{w}_i|\boldsymbol{\alpha}_i|, \tag{4}$$

where $i=1,...,p^2$. Taking derivative of $\boldsymbol{\alpha}_i$ in problem (4) and setting the derivative to be zero. There are two cases for the solution.

(a) If $\boldsymbol{\alpha}_i \geq 0$, we have

$$2(\boldsymbol{\alpha}_i-\mathbf{z}_i)+\mathbf{w}_i = 0. \tag{5}$$

The solution is

$$\hat{\boldsymbol{\alpha}}_i = \mathbf{z}_i - \frac{\mathbf{w}_i}{2} \geq 0. \tag{6}$$

So $\mathbf{z}_i \geq \frac{\mathbf{w}_i}{2} > 0$, and the solution $\hat{\boldsymbol{\alpha}}_i$ can be written as

$$\hat{\boldsymbol{\alpha}}_i = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}), \tag{7}$$

where sgn($\bullet$) is the sign function.

(b) If $\boldsymbol{\alpha}_i < 0$, we have

$$2(\boldsymbol{\alpha}_i-\mathbf{z}_i)-\mathbf{w}_i = 0. \tag{8}$$

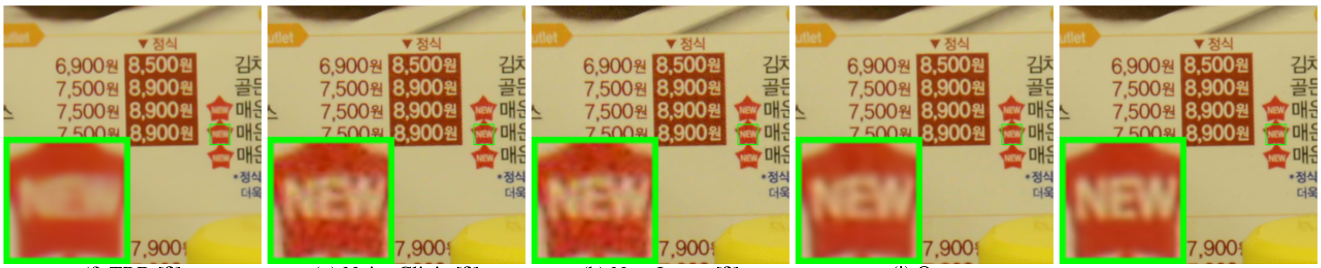
The solution is

$$\hat{\boldsymbol{\alpha}}_i = \mathbf{z}_i + \frac{\mathbf{w}_i}{2} < 0. \tag{9}$$

So $\mathbf{z}_i < -\frac{\mathbf{w}_i}{2} < 0$, and the solution $\hat{\boldsymbol{\alpha}}_i$ can be written as

$$\hat{\boldsymbol{\alpha}}_i = \text{sgn}(\mathbf{z}_i) * (-\mathbf{z}_i - \frac{\mathbf{w}_i}{2}) = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}). \tag{10}$$

(a) Noisy Image (33.30dB/0.8921) | (b) CBM3D [?] (33.33dB/0.8935) | (c) WNNM [?] (33.30dB/0.8922) | (d) MLP [?] (34.22dB/0.9603) | (e) CSF [?] (35.39dB/0.9677)



(f) TRD [?] (35.97dB/0.9705) | (g) Noise Clinic [?] (35.33dB/0.9520) | (h) Neat Image [?] (34.39dB/0.9314) | (i) Ours (**37.28**dB/**0.9729**) | (j) Mean Image

Figure 1. Denoised images of the real noisy image ”$NikonD800ISO3200A3$” by different methods. The images are better to be zoomed in on screen.

In summary, we have the final solution of the weighted sparse coding problem (1) as

$$\hat{\boldsymbol{\alpha}} = \text{sgn}(\mathbf{D}^{\mathbf{T}}\mathbf{y}) \odot \max(|\mathbf{D}^{\mathbf{T}}\mathbf{y}| - \mathbf{w}/2, 0), \tag{11}$$

where $\odot$ means element-wise multiplication and $|\mathbf{D}^{\mathbf{T}}\mathbf{y}|$ is the absolute value of each entry of the vector $\mathbf{D}^{\mathbf{T}}\mathbf{y}$.