Supplementary Material to "External Prior Guided Internal Prior Learning for Real Noisy Image Denoising"

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In this supplementary material, we provide:

1. The closed-form solution of the proposed weighted sparse coding model in the main paper.
2. More denoising results on the real noisy images (with no "ground truth") provided in the dataset [1].
3. More denoising results on the 15 smaller real noisy images (with "ground truth") used in the dataset [2].
4. More denoising results on the 60 real noisy images (with "ground truth") cropped from [2].

1. Closed-Form Solution of the Weighted Sparse Coding Problem

The weighted sparse coding problem in the main paper is:

$$\min_{\boldsymbol{\alpha}} \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\|_2^2 + \|\mathbf{w}^T\boldsymbol{\alpha}\|_1. \tag{1}$$

Since $\mathbf{D}$ is an orthonormal matrix, problem (1) is equivalent to

$$\min_{\boldsymbol{\alpha}} \|\mathbf{D}^T\mathbf{y} - \boldsymbol{\alpha}\|_2^2 + \|\mathbf{w}^T\boldsymbol{\alpha}\|_1. \tag{2}$$

For simplicity, we denote $\mathbf{z} = \mathbf{D^T y}$. Since $\mathbf{w}_i = c * 2\sqrt{2}\sigma^2/(\mathbf{\Lambda}_i + \varepsilon)$ is positive (please refer to Eq. (18) in the main paper), problem (2) can be written as

$$\min_{\boldsymbol{\alpha}} \sum_{i=1}^{p^2} ((\mathbf{z}_i - \boldsymbol{\alpha}_i)^2 + \mathbf{w}_i|\boldsymbol{\alpha}_i|). \tag{3}$$

The problem (3) is separable w.r.t. $\boldsymbol{\alpha}_i$ and can be simplified to p^2 scalar minimization problems

$$\min_{\boldsymbol{\alpha}_i} (\mathbf{z}_i - \boldsymbol{\alpha}_i)^2 + \mathbf{w}_i|\boldsymbol{\alpha}_i|, \tag{4}$$

where $i = 1, ..., p^2$. Taking derivative of $\boldsymbol{\alpha}_i$ in problem (4) and setting the derivative to be zero. There are two cases for the solution.

(a) If $\boldsymbol{\alpha}_i \geq 0$, we have

$$2(\boldsymbol{\alpha}_i - \mathbf{z}_i) + \mathbf{w}_i = 0. \tag{5}$$

The solution is

$$\hat{\boldsymbol{\alpha}}_i = \mathbf{z}_i - \frac{\mathbf{w}_i}{2} \geq 0. \tag{6}$$

So $\mathbf{z}_i \geq \frac{\mathbf{w}_i}{2} > 0$, and the solution $\hat{\boldsymbol{\alpha}}_i$ can be written as

$$\hat{\boldsymbol{\alpha}}_i = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}), \tag{7}$$

where $\text{sgn}(\bullet)$ is the sign function.

(b) If $\boldsymbol{\alpha}_i < 0$, we have

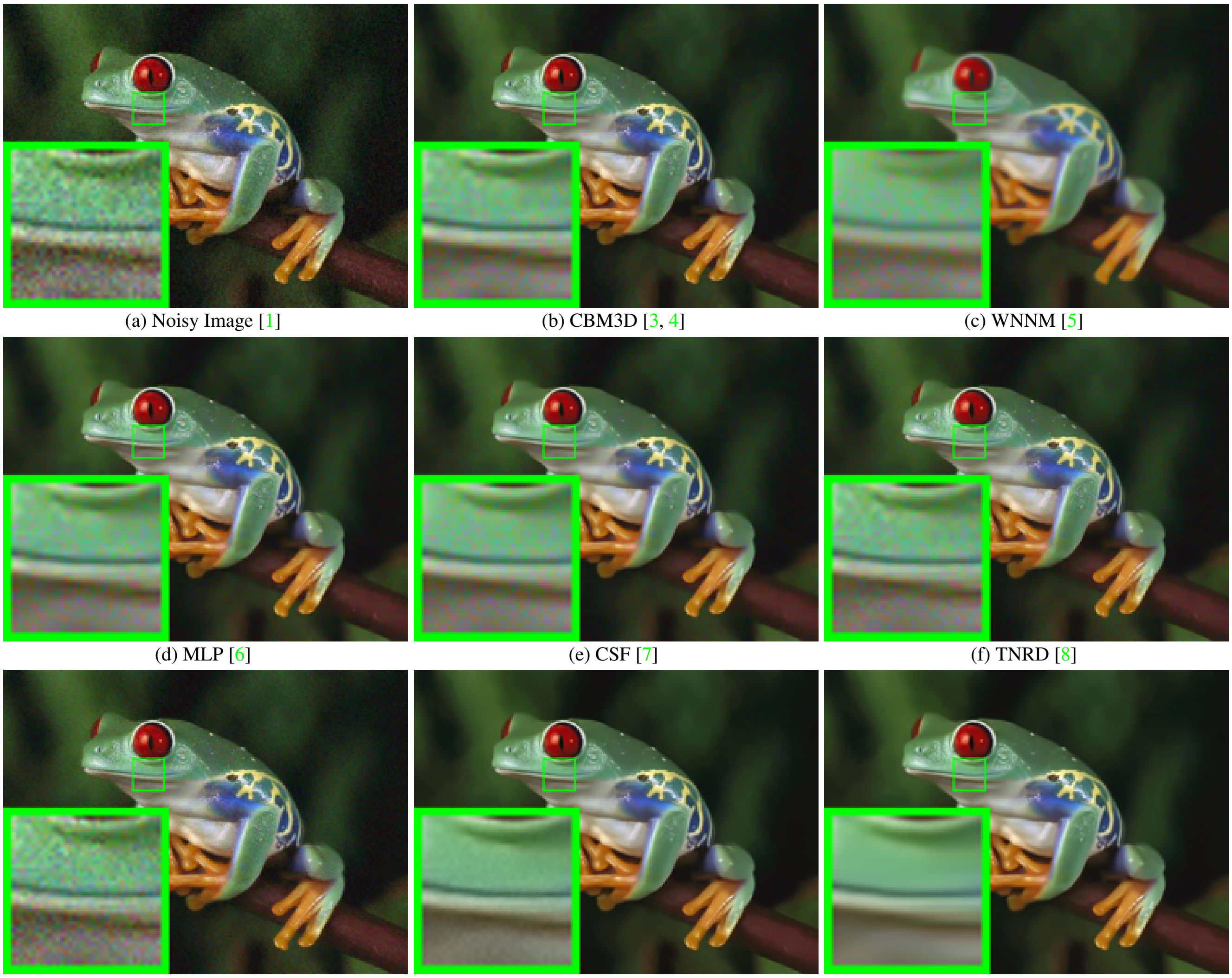
$$2(\boldsymbol{\alpha}_i - \mathbf{z}_i) - \mathbf{w}_i = 0. \tag{8}$$

The solution is

$$\hat{\boldsymbol{\alpha}}_i = \mathbf{z}_i + \frac{\mathbf{w}_i}{2} < 0. \tag{9}$$

So $\mathbf{z}_i < -\frac{\mathbf{w}_i}{2} < 0$, and the solution $\hat{\boldsymbol{\alpha}}_i$ can be written as

$$\hat{\boldsymbol{\alpha}}_i = \text{sgn}(\mathbf{z}_i) * (-\mathbf{z}_i - \frac{\mathbf{w}_i}{2}) = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}). \tag{10}$$



(a) Noisy Image [1] (b) CBM3D [3, 4] (c) WNNM [5]
(d) MLP [6] (e) CSF [7] (f) TNRD [8]
(g) Neat Image [9] (h) Noise Clinic [10, 1] (i) Ours

Figure 1. Denoised images of the real noisy image "Frog" [1] by different methods. The images are better to be zoomed in on screen.

In summary, we have the final solution of the weighted sparse coding problem (1) as

$$\hat{\boldsymbol{\alpha}} = \operatorname{sgn}(\mathbf{D^T y}) \odot \max(|\mathbf{D^T y}| - \mathbf{w}/2, 0), \tag{11}$$

where $\odot$ means element-wise multiplication and $|\mathbf{D^T y}|$ is the absolute value of each entry of the vector $\mathbf{D^T y}$.

2. More Results on Dataset [1]

In this section, we give more visual comparisons of the competing methods on the real noisy images provided in [1]. As can be seen from Figures 1-**??**, our proposed method is much better than the state-of-the-art denoising methods. This validates the effectiveness of our proposed external prior guided internal prior learning framework for real noisy image denoising.

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