Supplementary Material to "External Prior Guided Internal Prior Learning for Real Noisy Image Denoising"

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In this supplementary material, we provide:

1. The closed-form solution of the proposed weighted sparse coding model in the main paper.
2. More denoising results on the real noisy images (with no "ground truth") provided in the dataset [1].
3. More denoising results on the 15 smaller real noisy images (with "ground truth") used in the dataset [2].
4. More denoising results on the 60 real noisy images (with "ground truth") cropped from [2].

1. Closed-Form Solution of the Weighted Sparse Coding Problem

The weighted sparse coding problem in the main paper is:

$$\min_{\boldsymbol{\alpha}} \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\|_2^2 + \|\mathbf{w}^T\boldsymbol{\alpha}\|_1. \tag{1}$$

Since $\mathbf{D}$ is an orthonormal matrix, problem (1) is equivalent to

$$\min_{\boldsymbol{\alpha}} \|\mathbf{D}^T\mathbf{y} - \boldsymbol{\alpha}\|_2^2 + \|\mathbf{w}^T\boldsymbol{\alpha}\|_1. \tag{2}$$

For simplicity, we denote $\mathbf{z} = \mathbf{D}^{\mathbf{T}}\mathbf{y}$. Since $\mathbf{w}_i = c * 2\sqrt{2}\sigma^2/(\boldsymbol{\Lambda}_i + \varepsilon)$ is positive (please refer to Eq. (18) in the main paper), problem (2) can be written as

$$\min_{\boldsymbol{\alpha}} \sum_{i=1}^{p^2} ((\mathbf{z}_i - \boldsymbol{\alpha}_i)^2 + \mathbf{w}_i|\boldsymbol{\alpha}_i|). \tag{3}$$

The problem (3) is separable w.r.t. $\boldsymbol{\alpha}_i$ and can be simplified to p^2 scalar minimization problems

$$\min_{\boldsymbol{\alpha}_i} (\mathbf{z}_i - \boldsymbol{\alpha}_i)^2 + \mathbf{w}_i|\boldsymbol{\alpha}_i|, \tag{4}$$

where $i = 1, ..., p^2$. Taking derivative of $\boldsymbol{\alpha}_i$ in problem (4) and setting the derivative to be zero. There are two cases for the solution.

(a) If $\boldsymbol{\alpha}_i \geq 0$, we have

$$2(\boldsymbol{\alpha}_i - \mathbf{z}_i) + \mathbf{w}_i = 0. \tag{5}$$

The solution is

$$\hat{\boldsymbol{\alpha}}_i = \mathbf{z}_i - \frac{\mathbf{w}_i}{2} \geq 0. \tag{6}$$

So $\mathbf{z}_i \geq \frac{\mathbf{w}_i}{2} > 0$, and the solution $\hat{\boldsymbol{\alpha}}_i$ can be written as

$$\hat{\boldsymbol{\alpha}}_i = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}), \tag{7}$$

where $\text{sgn}(\bullet)$ is the sign function.

(b) If $\boldsymbol{\alpha}_i < 0$, we have

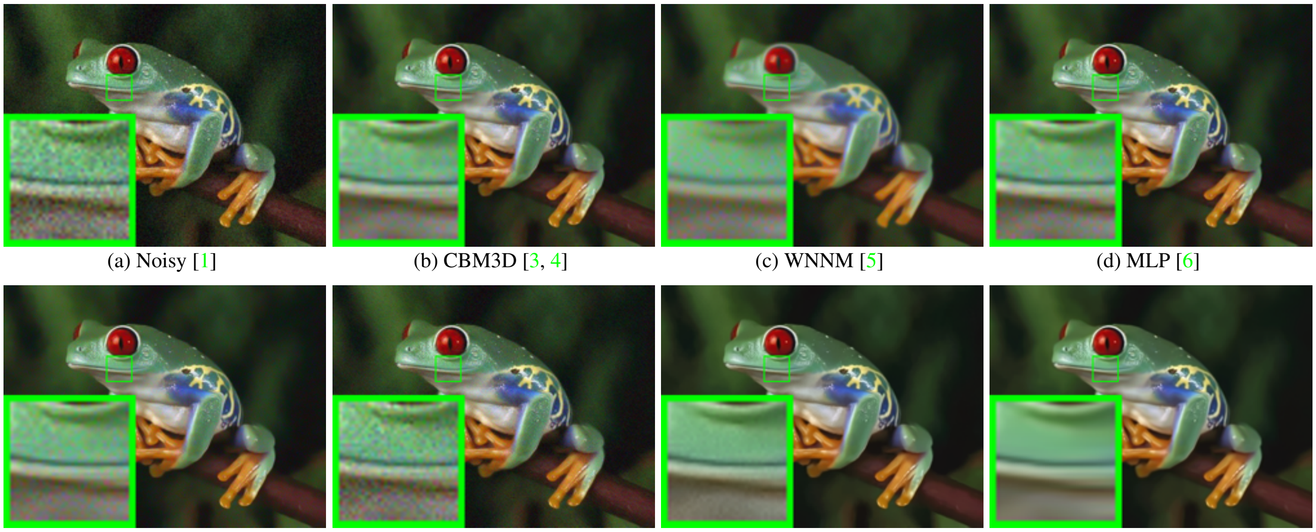
$$2(\boldsymbol{\alpha}_i - \mathbf{z}_i) - \mathbf{w}_i = 0. \tag{8}$$

The solution is

$$\hat{\boldsymbol{\alpha}}_i = \mathbf{z}_i + \frac{\mathbf{w}_i}{2} < 0. \tag{9}$$

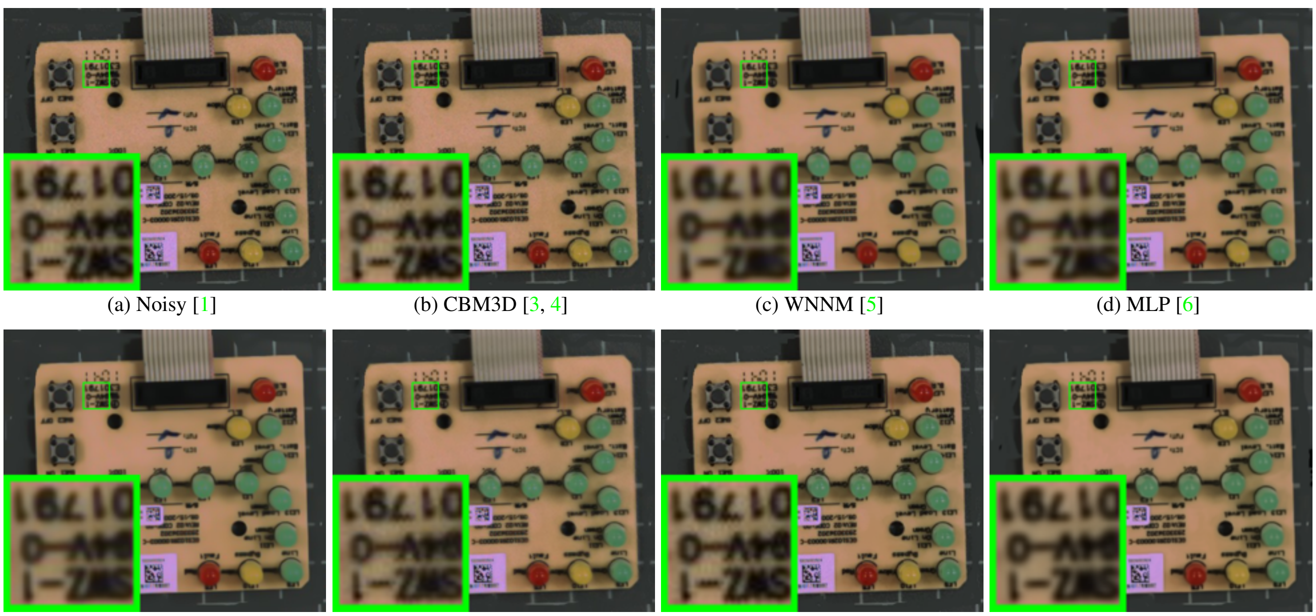
So $\mathbf{z}_i < -\frac{\mathbf{w}_i}{2} < 0$, and the solution $\hat{\boldsymbol{\alpha}}_i$ can be written as

$$\hat{\boldsymbol{\alpha}}_i = \text{sgn}(\mathbf{z}_i) * (-\mathbf{z}_i - \frac{\mathbf{w}_i}{2}) = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}). \tag{10}$$



(a) Noisy [1] (b) CBM3D [3, 4] (c) WNNM [5] (d) MLP [6]
(e) TNRD [8] (f) NI [9] (g) NC [10, 1] (h) Ours

Figure 1. Denoised images of the real noisy image "Frog" [1] by different methods. The images are better to be zoomed in on screen.



(a) Noisy [1] (b) CBM3D [3, 4] (c) WNNM [5] (d) MLP [6]
(e) TNRD [8] (f) NI [9] (g) NC [10, 1] (h) Ours

Figure 2. Denoised images of the real noisy image "Circuit" [1] by different methods. The images are better to be zoomed in on screen.

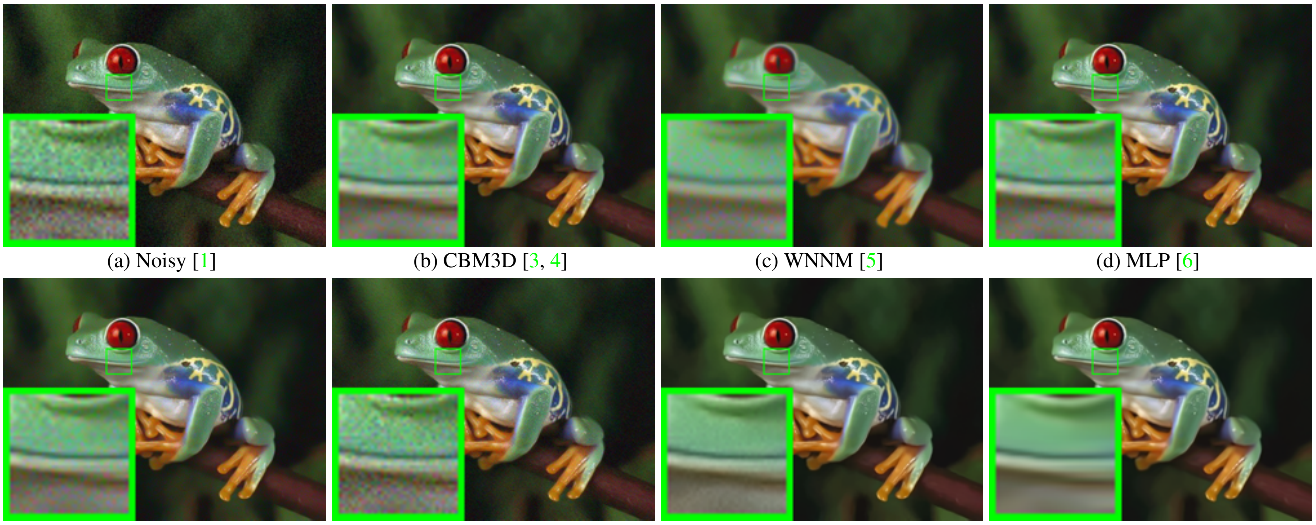
In summary, we have the final solution of the weighted sparse coding problem (1) as

$$\hat{\boldsymbol{\alpha}} = \text{sgn}(\mathbf{D}^{\mathbf{T}}\mathbf{y}) \odot \max(|\mathbf{D}^{\mathbf{T}}\mathbf{y}| - \mathbf{w}/2, 0), \tag{11}$$

where $\odot$ means element-wise multiplication and $|\mathbf{D}^{\mathbf{T}}\mathbf{y}|$ is the absolute value of each entry of the vector $\mathbf{D}^{\mathbf{T}}\mathbf{y}$.

2. More Results on Dataset [1]

In this section, we give more visual comparisons of the competing methods on the real noisy images provided in [1]. As can be seen from Figures 1-**??**, our proposed method is much better than the state-of-the-art denoising methods. This validates the effectiveness of our proposed external prior guided internal prior learning framework for real noisy image denoising.



(a) Noisy [1] (b) CBM3D [3, 4] (c) WNNM [5] (d) MLP [6]

(e) TNRD [8] (f) NI [9] (g) NC [10, 1] (h) Ours

Figure 3. Denoised images of the real noisy image "Frog" [1] by different methods. The images are better to be zoomed in on screen.

References

[1] M. Lebrun, M. Colom, and J. M. Morel. The noise clinic: a blind image denoising algorithm. http://www.ipol.im/pub/art/2015/125/. Accessed 01 28, 2015. 1, 2, 3

[2] S. Nam, Y. Hwang, Y. Matsushita, and S. J. Kim. A holistic approach to cross-channel image noise modeling and its application to image denoising. *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 1683–1691, 2016. 1

[3] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian. Image denoising by sparse 3-D transform-domain collaborative filtering. *IEEE Transactions on Image Processing*, 16(8):2080–2095, 2007. 2, 3

[4] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian. Color image denoising via sparse 3D collaborative filtering with grouping constraint in luminance-chrominance space. *IEEE International Conference on Image Processing (ICIP)*, pages 313–316, 2007. 2, 3

[5] S. Gu, L. Zhang, W. Zuo, and X. Feng. Weighted nuclear norm minimization with application to image denoising. *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 2862–2869, 2014. 2, 3

[6] H. C. Burger, C. J. Schuler, and S. Harmeling. Image denoising: Can plain neural networks compete with BM3D? *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 2392–2399, 2012. 2, 3

[7] U. Schmidt and S. Roth. Shrinkage fields for effective image restoration. *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 2774–2781, June 2014.

[8] Y. Chen, W. Yu, and T. Pock. On learning optimized reaction diffusion processes for effective image restoration. *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 5261–5269, 2015. 2, 3

[9] Neatlab ABSoft. Neat Image. https://ni.neatvideo.com/home. 2, 3

[10] M. Lebrun, M. Colom, and J.-M. Morel. Multiscale image blind denoising. *IEEE Transactions on Image Processing*, 24(10):3149–3161, 2015. 2, 3