# Multi-channel Weighted Nuclear Norm Minimization for Real Color Image Denoising

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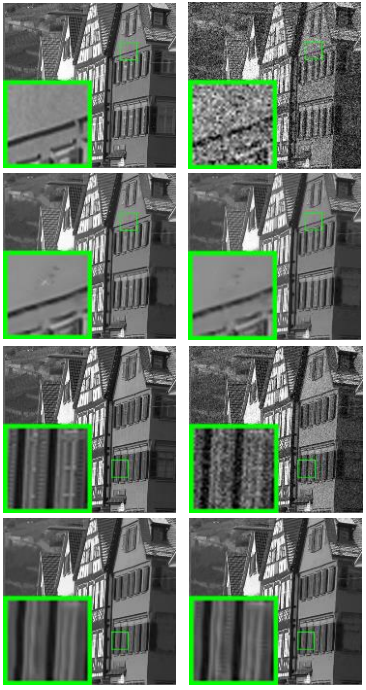
## Insights

**One Fact:** Noise in sRGB space has different variances for different channels.

**Solution:** Introduce weights to balance the noise difference in different channels.



## The MCWNNM Model

**Model:** $\min_{\mathbf{X}} \|\mathbf{W}(\mathbf{Y}-\mathbf{X})\|_F^2 + \|\mathbf{X}\|_{\boldsymbol{w},*}$

**Weights:** $\mathbf{W} = \begin{pmatrix} \sigma_r^{-1}\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_g^{-1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sigma_b^{-1}\mathbf{I} \end{pmatrix}$

**Variable Splitting:**

$$\min_{\mathbf{X},\mathbf{Z}} \|\mathbf{W}(\mathbf{Y}-\mathbf{X})\|_F^2 + \|\mathbf{Z}\|_{\boldsymbol{w},*} \quad \text{s.t.} \quad \mathbf{X}=\mathbf{Z}$$

**Lagrangian:**

$$\mathcal{L}(\mathbf{X},\mathbf{Z},\mathbf{A},\rho) = \|\mathbf{W}(\mathbf{Y}-\mathbf{X})\|_F^2 + \|\mathbf{Z}\|_{\boldsymbol{w},*} + \langle \mathbf{A}, \mathbf{X}-\mathbf{Z}\rangle + \frac{\rho}{2}\|\mathbf{X}-\mathbf{Z}\|_F^2$$

**ADMM:**

(1) **Update X while fixing Z and A**:

$$\mathbf{X}_{k+1} = \arg\min_{\mathbf{X}} \|\mathbf{W}(\mathbf{Y}-\mathbf{X})\|_F^2 + \frac{\rho_k}{2}\|\mathbf{X}-\mathbf{Z}_k+\rho_k^{-1}\mathbf{A}_k\|_F^2$$

(2) **Update Z while fixing X and A**:

$$\mathbf{Z}_{k+1} = \arg\min_{\mathbf{Z}} \frac{\rho_k}{2}\|\mathbf{Z}-(\mathbf{X}_{k+1}+\rho_k^{-1}\mathbf{A}_k)\|_F^2 + \|\mathbf{Z}\|_{\boldsymbol{w},*}$$

(3) **Update A while fixing X and Z**:

$$\mathbf{A}_{k+1} = \mathbf{A}_k + \rho_k(\mathbf{X}_{k+1}-\mathbf{Z}_{k+1})$$

(4) **Update** $\rho_k$: $\rho_{k+1} = \mu * \rho_k$, where $\mu > 1$

**Convergence Guarantee:**

**Theorem 1.** *Assume that the weights in* $\boldsymbol{w}$ *are in a non-descending order, the sequences* $\{\mathbf{X}_k\}$, $\{\mathbf{Z}_k\}$, *and* $\{\mathbf{A}_k\}$ *generated in Algorithm 1 satisfy:*

$$(a)\ \lim_{k\to\infty} \|\mathbf{X}_{k+1}-\mathbf{Z}_{k+1}\|_F = 0;$$

$$(b)\ \lim_{k\to\infty} \|\mathbf{X}_{k+1}-\mathbf{X}_k\|_F = 0;$$

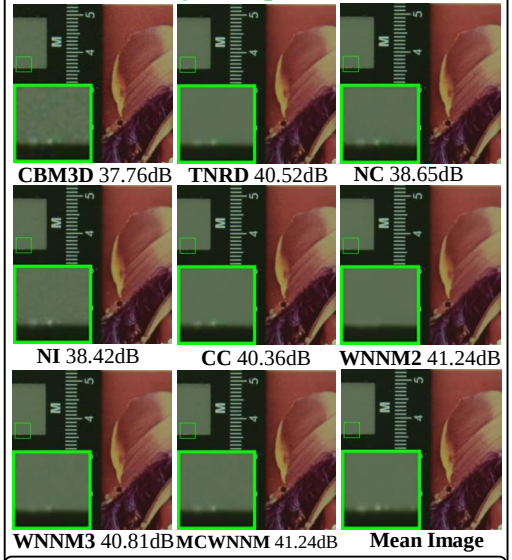
$$(c)\ \lim_{k\to\infty} \|\mathbf{Z}_{k+1}-\mathbf{Z}_k\|_F = 0.$$

## Experiments

### PSNR Results on Real Color Images

| Camera Settings | CBM3D | MLP | TNRD | DnCNN | NI | NC | CC | WNNM-1 | WNNM-2 | WNNM-3 | MC-WNNM |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Canon 5D ISO = 3200 | 39.76 | 39.00 | 39.51 | 37.26 | 35.68 | 36.20 | 38.37 | 37.51 | 39.74 | 39.98 | **41.13** |
| | 36.40 | 36.34 | 36.47 | 34.13 | 34.03 | 34.35 | 35.37 | 33.86 | 35.12 | 36.65 | **37.28** |
| | 36.37 | 36.33 | 36.45 | 34.09 | 32.63 | 33.10 | 34.91 | 31.43 | 33.14 | 34.63 | **36.52** |
| Nikon D600 ISO = 3200 | 34.18 | 34.70 | 34.79 | 33.62 | 31.78 | 32.28 | 34.98 | 33.46 | 35.08 | 35.08 | **35.53** |
| | 35.07 | 36.20 | 36.37 | 34.48 | 35.16 | 35.34 | 35.95 | 36.09 | 36.42 | 36.84 | **37.02** |
| | 37.13 | 39.33 | 39.49 | 35.41 | 39.98 | 40.51 | **41.15** | 39.86 | 40.78 | 39.24 | 39.56 |
| Nikon D800 ISO = 1600 | 36.81 | 37.95 | 38.11 | 35.79 | 34.84 | 35.09 | 37.99 | 36.35 | 38.28 | 38.61 | **39.26** |
| | 37.76 | 40.23 | 40.52 | 36.08 | 38.42 | 38.65 | 40.36 | 39.99 | 41.24 | 40.81 | **41.43** |
| | 37.51 | 37.94 | 38.17 | 35.48 | 35.79 | 35.85 | 38.30 | 37.15 | 38.04 | 38.96 | **39.55** |
| Nikon D800 ISO = 3200 | 35.05 | 37.55 | 37.69 | 34.08 | 38.36 | 38.56 | 39.01 | 38.60 | **39.93** | 37.97 | 38.91 |
| | 34.07 | 35.91 | 35.90 | 33.70 | 35.53 | 35.76 | 36.75 | 36.04 | 37.32 | 37.30 | **37.41** |
| | 34.42 | 38.15 | 38.21 | 33.31 | 40.05 | 40.59 | 39.06 | 39.75 | **41.52** | 38.68 | 39.39 |
| Nikon D800 ISO = 6400 | 31.13 | 32.69 | 32.81 | 29.83 | 34.08 | 34.25 | 34.61 | 33.29 | **35.20** | 34.57 | 34.80 |
| | 31.22 | 32.33 | 32.33 | 30.55 | 32.13 | 32.38 | 33.21 | 31.16 | 33.61 | 33.43 | **33.95** |
| | 30.97 | 32.29 | 32.29 | 30.09 | 31.52 | 31.76 | 33.22 | 31.98 | 33.62 | **34.02** | 33.94 |
| **Average** | 35.19 | 36.46 | 36.61 | 33.86 | 35.33 | 35.65 | 36.88 | 35.77 | 37.27 | 37.12 | **37.71** |
| **Time** | 7.8 | 20.4 | 6.7 | 180.3 | **0.9** | 18.2 | NA | 689.1 | 465.3 | 198.6 | 202.9 |

### Visual Quality Comparison




**Matlab Code & More Details Available @ Github**