# Multi-channel weighted nuclear norm minimization for color image denoising

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## Abstract

*Motivated by the weighted Orthogonal Procrustes Problem, we propose a noval weighted Frobenious norm based weighted sparse coding model for non-Gaussian error modeling. We solve this model in an alternative manner. Updating of each variable has closed-form solutions and the overall model converges to a stationary point. The proposed model is applied in real image denoising problem and extensive experiments demonstrate that the proposed model can much better performance (over 1.0dB improvement on PSNR) than state-of-the-art image denoising methods, including some excellant commercial software. The noval weighted Frobeniius norm can perfectly fit the non-Gaussian property of real noise.*

## 1. Introduction

Image denoising is an important step in enhance the quality of images in computer vision systems. It aims to recover the latent clean image $\mathbf{x}$ from the observed noisy version $\mathbf{y} = \mathbf{x} + \mathbf{n}$, where $\mathbf{n}$ is often assumed to be additive white Gaussian noise. Most denoising methods [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] are designed for grayscale images, and other color image denoising methods [**?** ] treat equally the R, G, B channels in color images. However, in many computer vision tasks, the multiple channels in natural images being processed often exhibit distinct properties, e.g., contain different noise levels. For example, the noise levels among the R, G, B channels are different in real noisy images due to the different sampling frequency in color demosaicking. The This is caused by the color demosaicking during the transformation from raw data to RGB images in the standard in-camera imaging pipeline. Usually, the G channel contains the least noise levels among the three channels. Hence, in order to deal with each channel more effectively, different noise levels should be plugged into different channels for color image denoising.

The non-local self similarity property of images has been extensively employed in image denoising algorithms []. Among these methods, the weighted nuclear norm minimization (WNNM) achieved the state-of-the-art performance in grayscale image denoising with additive white Gaussian noise (AWGN). Though among the most effective methods, how WNNM can be extended to deal with color image denoising and real-world image denoising is still an open problem. In this paper, we proposed a multi-channel weighted nuclear norm minimization model to deal with the color image denoising problem as well as the real noisy image denoising problem.

## 2. Related Work

The WNNM model

$$\min_{\mathbf{X}} \|\mathbf{Y} - \mathbf{X}\|_F^2 + \|\mathbf{X}\|_{*,\mathbf{w}} \tag{1}$$

is firstly proposed for grayscale image denoising problem. How to extend it to deal with color images or hyperspectral images is still an open problem. This model treat each

去噪过程如下：

1. 初始化：

我们从原始噪声图得到相似块矩阵$\mathbf{Y}$，我们采用[**?** ]的方法估计彩色带噪图的噪声水平$\sigma_0$。初始化权重矩

阵$\mathbf{W}^{(0)} = \frac{1}{\sigma_0}\mathbf{I}$, 初始化字典$\mathbf{D}^{(0)} = \mathbf{I}$。

2. 进入内部迭代优化：

对于每一次迭代，模型都需要反复迭代求解$\mathbf{D}$,$\mathbf{C}$直到收敛。For $k = 0, 1, 2, ...$:

a. update $\mathbf{C}$

$$\min_{\mathbf{C}} \frac{1}{2}\|(\mathbf{Y} - \mathbf{D}^{(k)}\mathbf{C})\mathbf{W}^{(k)}\|_F^2 + \lambda\|\mathbf{C}\|_1. \quad (2)$$

有闭合解，每一列单独求解：

$$(\hat{\mathbf{c}}_i)^{(k+1)} = \arg\min_{\mathbf{c}_i} \frac{1}{2}\|(\mathbf{y}_i - \mathbf{D}^{(k)}\mathbf{c}_i)\mathbf{W}_{ii}\|_2^2 + \lambda\|\mathbf{c}_i\|_1. \quad (3)$$

闭合解为：

$$(\hat{\mathbf{c}}_i)^{(k+1)} = \text{sgn}(\mathbf{D}^\top\mathbf{y}) \odot \max(|\mathbf{D}^\top\mathbf{y}| - \frac{\lambda}{(\mathbf{W}_{ii})^2}, 0), \quad (4)$$

b. update $\mathbf{D}$

$$\min_{\mathbf{D}} \frac{1}{2}\|(\mathbf{Y} - \mathbf{D}\mathbf{C}^{(k+1)})\mathbf{W}\|_F^2 \quad \text{s.t.} \quad \mathbf{D}^\top\mathbf{D} = \mathbf{I}. \quad (5)$$

等价于

$$\min_{\mathbf{D}} \|(\mathbf{Y}\mathbf{W}) - \mathbf{D}(\mathbf{C}^{(k+1)}\mathbf{W})\|_F^2 \quad \text{s.t.} \quad \mathbf{D}^\top\mathbf{D} = \mathbf{I}, \quad (6)$$

闭合解为：$\hat{\mathbf{D}}^{(k+1)} = \mathbf{V}\mathbf{U}^\top$, $\mathbf{C}\mathbf{W}(\mathbf{Y}\mathbf{W})^\top = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top$.

c. update $\mathbf{W}$ 根据贝叶斯法则，权重矩阵的第$i$项为

$$\mathbf{W}_{ii} = \frac{\frac{1}{N}\sum_{i=1}^{N}\|\mathbf{y}_i - \mathbf{D}\mathbf{c}_i\|_2}{\sigma_{\mathbf{y}_i}\|\mathbf{y}_i - \mathbf{D}\mathbf{c}_i\|_2} \quad (7)$$

3. 外部迭代优化：

更新每个块的噪声水平：

$$\sigma_{\mathbf{y}_i} = \sqrt{\sigma_0^2 - \|\mathbf{y}_i - \mathbf{D}\mathbf{c}_i\|_2^2} \quad (8)$$

然后重复步骤2.

## References

[1] M. Elad and M. Aharon. Image denoising via sparse and redundant representations over learned dictionaries. *IEEE Transactions on Image Processing*, 15(12):3736–3745, 2006. 1

[2] J. Mairal, F. Bach, J. Ponce, G. Sapiro, and A. Zisserman. Non-local sparse models for image restoration. *IEEE International Conference on Computer Vision (ICCV)*, pages 2272–2279, 2009. 1

[3] W. Dong, L. Zhang, G. Shi, and X. Li. Nonlocally centralized sparse representation for image restoration. *IEEE Transactions on Image Processing*, 22(4):1620–1630, 2013. 1

[4] A. Buades, B. Coll, and J. M. Morel. A non-local algorithm for image denoising. *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 60–65, 2005. 1

[5] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian. Image denoising by sparse 3-D transform-domain collaborative filtering. *IEEE Transactions on Image Processing*, 16(8):2080–2095, 2007. 1

[6] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian. Color image denoising via sparse 3D collaborative filtering with grouping constraint in luminance-chrominance space. *IEEE International Conference on Image Processing (ICIP)*, pages 313–316, 2007. 1

[7] J. Xu, L. Zhang, W. Zuo, D. Zhang, and X. Feng. Patch group based nonlocal self-similarity prior learning for image denoising. *IEEE International Conference on Computer Vision (ICCV)*, pages 244–252, 2015. 1

[8] S. Gu, L. Zhang, W. Zuo, and X. Feng. Weighted nuclear norm minimization with application to image denoising. *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 2862–2869, 2014. 1

[9] H. C. Burger, C. J. Schuler, and S. Harmeling. Image denoising: Can plain neural networks compete with BM3D? *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 2392–2399, 2012. 1

[10] U. Schmidt and S. Roth. Shrinkage fields for effective image restoration. *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 2774–2781, June 2014. 1

[11] Y. Chen, W. Yu, and T. Pock. On learning optimized reaction diffusion processes for effective image restoration. *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 5261–5269, 2015. 1

[12] S. Roth and M. J. Black. Fields of experts. *International Journal of Computer Vision*, 82(2):205–229, 2009. 1

[13] D. Zoran and Y. Weiss. From learning models of natural image patches to whole image restoration. *IEEE Interna-*

*tional Conference on Computer Vision (ICCV)*, pages 479–486, 2011. 1