Summary. The authors consider the problem of clustering data lying in a union of low-dimensional (linear or affine) subspaces. They propose a new variant in the family of the popular self-expression based subspace clustering methods, which they call simplex representation least-squares regression (SRLSR). All methods in this family seek to build a graph affinity matrix $\boldsymbol{A}$ based on a matrix of self-expressive coefficients $\boldsymbol{C}$, where $\boldsymbol{X} \approx \boldsymbol{X}\boldsymbol{C}$. The key difference between SRLSR and prior methods is how the self-expressive coefficients $\boldsymbol{C}$ are regularized to promote subspace-preserving and well-connected affinities. The formulation constrains the columns $\boldsymbol{c}_j$ to lie on a scaled standard simplex, while also including squared ℓ_2 regularization. In contrast, other well-known methods such as SSC, LSR, and LRR impose ℓ_1, squared ℓ_2, and nuclear norm regularization respectively. To solve the associated convex optimization problem, the authors implement a standard ADMM based algorithm. A comprehensive evaluation of the method on standard benchmarks and against previous state-of-the-art methods demonstrates significantly improved performance, both in terms of clustering accuracy and runtime.

Discussion of novelty and significance of contribution. The literature on subspace clustering is nearly saturated with variants of self-expression based methods. The proposed SRLSR offers seemingly incremental changes relative to prior work. I argue that SRLSR appears closely related to elastic-net subspace clustering (EnSC) [1]–a previous state-of-the-art method which the authors do not compare against.

In principle, this relative lack of novelty could be overcome by providing rigorous theoretical or empirical insight into why the proposed innovations are essential. However the authors provide neither. In general, their justifications for the method formulation are vague and informal.

The authors also fail to provide novel algorithmic contributions. The implemented algorithm is a standard ADMM-based approach.

In addition, while the experimental evaluation is comprehensive and the results convincing, the applications considered are all standard benchmarks that have been reported in the literature for up to a decade. While it is important to compare to previous methods in a common setting, I worry that these tasks are becoming stale and losing relevance to the community.

In summary, the authors have produced a new variant method that performs remarkably well in benchmark experiments. However, without significant contributions to the analysis, algorithms, or applications for this method, the contribution is insufficient for publication in its current form.

Major comments.

1. **Novelty of the formulation and similarity to EnSC.** The proposed SRLSR method solves the optimization problem

$$\underset{\boldsymbol{C}}{\text{minimize}} \ \frac{1}{2}\|\boldsymbol{X} - \boldsymbol{X}\boldsymbol{C}\|_F^2 + \frac{\lambda}{2}\|\boldsymbol{C}\|_F^2 \quad \text{s.t.} \quad \boldsymbol{C} \geq \boldsymbol{0},\ \boldsymbol{1}^\top \boldsymbol{C} = s\boldsymbol{1}^\top. \tag{1}$$

   Under the assumption that the data $\boldsymbol{X}$ are symmetrized, i.e. $-\boldsymbol{x} \in \boldsymbol{X}$ whenever $\boldsymbol{x} \in \boldsymbol{X}$[1], I

[1] A reasonable assumption since one can always symmetrize the data during preprocessing without changing the union of subspace structure.

claim that this problem is equivalent to

$$\underset{\boldsymbol{C}}{\text{minimize}}\ \frac{1}{2}\|\boldsymbol{X}-\boldsymbol{X}\boldsymbol{C}\|_F^2+\frac{\lambda}{2}\|\boldsymbol{C}\|_F^2 \quad \text{s.t.} \quad \|\boldsymbol{c}_j\|_1 = s,\ j=1,\dots,N, \tag{2}$$

since one can easily convert between the $\boldsymbol{C}$ variables of the two problems without increasing the objective. Relaxing the ℓ_1 equality constraint in (2), and then replacing the constraint with a regularization penalty, you obtain

$$\underset{\boldsymbol{C}}{\text{minimize}}\ \frac{1}{2}\|\boldsymbol{X}-\boldsymbol{X}\boldsymbol{C}\|_F^2+\frac{\lambda}{2}\|\boldsymbol{C}\|_F^2+\gamma\|\boldsymbol{C}\|_{1,\infty}. \tag{3}$$

This relaxed formulation then bears a strong resemblance to elastic-net subspace clustering

$$\underset{\boldsymbol{C}}{\text{minimize}}\ \frac{1}{2}\|\boldsymbol{X}-\boldsymbol{X}\boldsymbol{C}\|_F^2+\frac{\lambda}{2}\|\boldsymbol{C}\|_F^2+\gamma\|\boldsymbol{C}\|_1, \tag{4}$$

as both include least-squares and ℓ_1 regularization on the columns of $\boldsymbol{C}$. To address this apparent similarity, the authors should explain, through theory or experiments, why satisfying the scaled affine constraint in 1, or equivalently the ℓ_1 equality constraint in 2 is crucial to the method's success.

2. **Vague and informal justifications.** The authors claim repeatedly that by enforcing non-negative self-expressive coefficients, this helps preserve the "inherent correlations among the data points". They do not justify this claim however, or explain what they mean more fully. More generally, I do not see the problem with allowing negative coefficients. I argued above that after symmetrizing the data (a reasonable pre-processing step), enforcing non-negativity makes no difference.

   Similarly, the authors claim without justification that the scaled affine constraint makes the method more flexible and robust to corruptions relative to previous methods.

   To address these issues, the authors should provide clear explanations as to how SRLSR promotes affinity matrices with better structure, and back these explanations up with theoretical or empirical evidence.

3. **Misleading synthetic experiment.** The authors do provide one simple synthetic experiment intended to give intuition into the advantages of SRLSR (Figure 1). However this experiment is contrived to advantage the proposed method. The construction $\boldsymbol{x}_3+\boldsymbol{x}_5=\boldsymbol{x}_1$ is a degenerate case where SSC and non-negative SSC can fail. Moreover, had $\boldsymbol{x}_2$ been the target rather than $\boldsymbol{x}_1$, SRLSR (with the same s parameter) would have failed while the affine constraint method would have succeeded.

4. **Lack of algorithmic contribution.** New subspace clustering methods often improve on previous state-of-the-art by developing more efficient scalable algorithms. The authors here however implement a standard ADMM based algorithm lacking original contribution.

5. **Runtime analysis.** Confusingly, the authors nonetheless claim improved computational efficiency on the basis of runtime experiments, despite using an algorithm with the same cost per iteration as the original SSC implementation. This may be due to SRLSR converging in fewer iterations than other ADMM based algorithms–the authors state SRLSR is limited to only 5 iterations–however this argument would need to be developed more fully.

6. **Missing EnSC evaluation.** The experiments convincingly demonstrate that SRLSR significantly improves over a large set of comparison methods, and across three image clustering tasks. Nonetheless, the evaluation is incomplete since the authors do not compare with the seemingly most closely related method: elastic-net subspace clustering (EnSC) [1](see Comment 1).

7. **Experiment novelty.** The experiments also lack novelty. Yale-B face image clustering and Hopkins-155 motion segmentation are both standard benchmarks that have been reported in the subspace clustering literature for nearly a decade. Clustering MNIST images on the basis of scattering transform features was introduced more recently. However, the MNIST dataset is nonetheless viewed as a "toy" dataset with limited relevance to the community. The contribution of the experiments would be improved if the authors evaluated on a clustering task with more application potential to the larger machine learning & computer vision communities.

References

[1] C. You, C.-G. Li, D. P. Robinson, and R. Vidal, "Oracle based active set algorithm for scalable elastic net subspace clustering," in *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 3928–3937, 2016. 1, 3