A Trilateral Weighted Sparse Coding Scheme for Real-World Image Denoising

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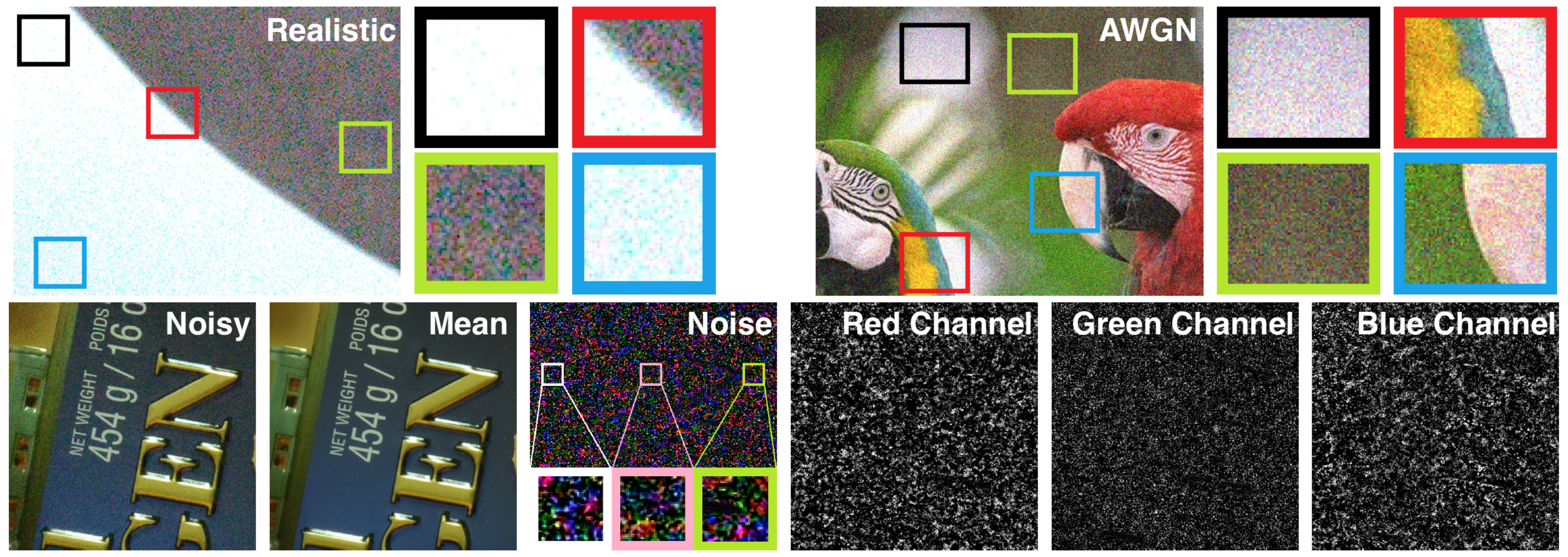
Problem, Motivation, and Contributions

Goal: Estimating the latent clean image from the input real-world noisy image.
Motivation: Realistic noise show channel-wise and locally signal dependent property.



Contributions:

- Propose a trilateral weighted sparse coding (TWSC) scheme for real-world denoising;
- TWSC achieves much better performance than state-of-the-art denoising methods.

The TWSC Scheme

TWSC: Given a color image patch $\mathbf{Y} = \mathbf{X} + \mathbf{N} \in 3p^2 \times N$ and $\mathbf{Y} = \mathbf{DSV}^\top$ is the SVD of $\mathbf{Y}$. The TWSC model can be written as

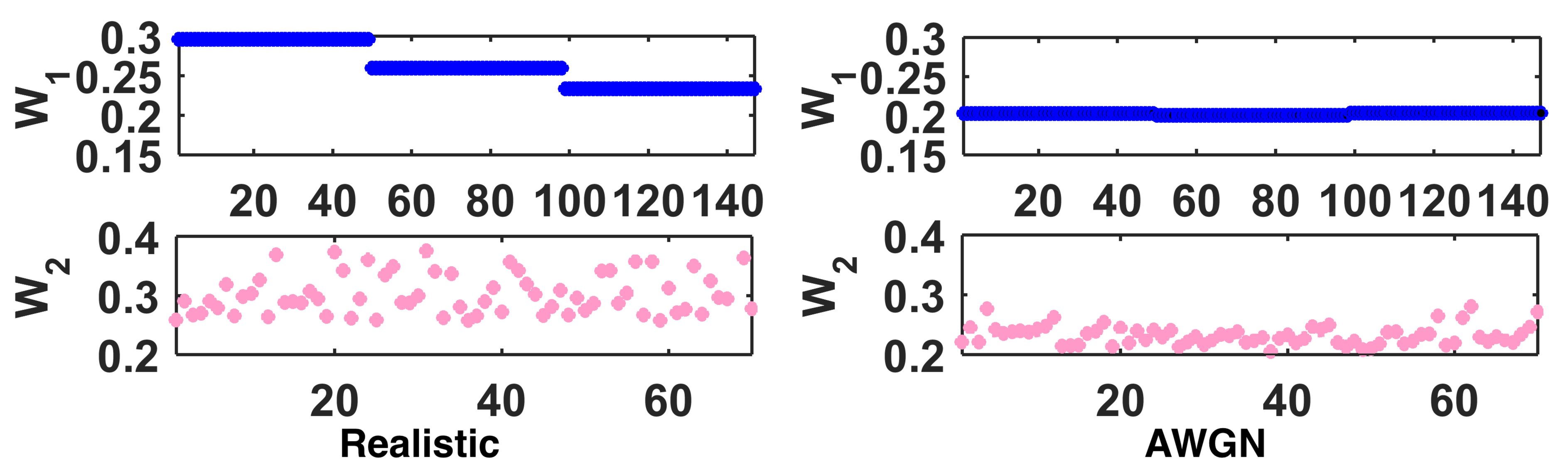
$$\hat{\mathbf{C}} = \arg\min_{\mathbf{C}} \|\mathbf{W}_1(\mathbf{Y} - \mathbf{DC})\mathbf{W}_2\|_F^2 + \|\mathbf{W}_3^{-1}\mathbf{C}\|_1. \quad (1)$$

The estimation of $\mathbf{X}$ can be $\hat{\mathbf{X}} = \mathbf{D}\hat{\mathbf{C}}$.

Formulation of weight matrices:

$$\begin{aligned} \mathbf{W}_1 &= \mathrm{diag}(\sigma_r^{-1/2}\mathbf{I}_{p^2}, \sigma_g^{-1/2}\mathbf{I}_{p^2}, \sigma_b^{-1/2}\mathbf{I}_{p^2}), \\ \mathbf{W}_2 &= \mathrm{diag}(\sigma_1^{-1/2}, ..., \sigma_M^{-1/2}), \mathbf{W}_3 = \mathbf{S}, \end{aligned} \quad (2)$$

Visualization of weight matrices:



Optimization

Variable Splitting: $\min_{\mathbf{C},\mathbf{Z}} \|\mathbf{W}_1(\mathbf{Y} - \mathbf{DW}_3\mathbf{C})\mathbf{W}_2\|_F^2 + \|\mathbf{Z}\|_1$ s.t. $\mathbf{C} = \mathbf{Z}$.

ADMM:

(a) $\mathbf{C}_{k+1} = \arg\min_{\mathbf{C}} \|\mathbf{W}_1(\mathbf{Y} - \mathbf{DW}_3\mathbf{C})\mathbf{W}_2\|_F^2 + \frac{\rho_k}{2}\|\mathbf{C} - \mathbf{Z}_k + \rho_k^{-1}\mathbf{\Delta}_k\|_F^2$.
The solution $\mathbf{C}_{k+1}$ satisfies $\mathbf{AC}_{k+1} + \mathbf{C}_{k+1}\mathbf{B}_k = \mathbf{E}_k$, where
$\mathbf{A} = \mathbf{W}_3^\top\mathbf{D}^\top\mathbf{W}_1^\top\mathbf{W}_1\mathbf{DW}_3, \mathbf{B}_k = \frac{\rho_k}{2}(\mathbf{W}_2\mathbf{W}_2^\top)^{-1}$,
$\mathbf{E}_k = \mathbf{W}_3^\top\mathbf{D}^\top\mathbf{W}_1^\top\mathbf{W}_1\mathbf{Y} + (\frac{\rho_k}{2}\mathbf{Z}_k - \frac{1}{2}\mathbf{\Delta}_k)(\mathbf{W}_2\mathbf{W}_2^\top)^{-1}$.
(Solution) $\mathbf{C}_{k+1} = \mathrm{vec}^{-1}((\mathbf{I}_M \otimes \mathbf{A} + \mathbf{B}_k^\top \otimes \mathbf{I}_{3p^2})^{-1}\mathrm{vec}(\mathbf{E}_k))$.
BUT! Is $(\mathbf{I}_M \otimes \mathbf{A} + \mathbf{B}_k^\top \otimes \mathbf{I}_{3p^2})^{-1}$ exist?

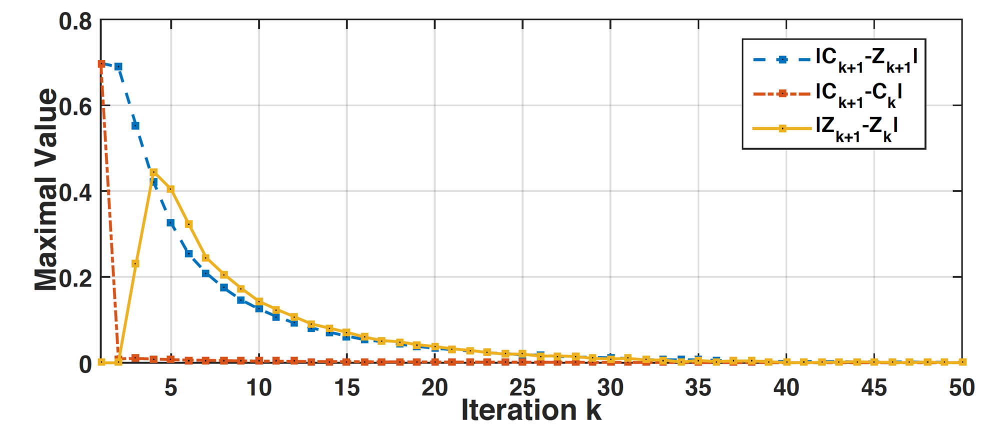
(b) $\mathbf{Z}_{k+1} = \arg\min_{\mathbf{Z}} \frac{\rho_k}{2}\|\mathbf{Z} - (\mathbf{C}_{k+1} + \rho_k^{-1}\mathbf{\Delta}_k)\|_F^2 + \|\mathbf{Z}\|_1$.

(c) $\mathbf{\Delta}_{k+1} = \mathbf{\Delta}_k + \rho_k(\mathbf{C}_{k+1} - \mathbf{Z}_{k+1})$.

(d) $\rho_{k+1} = \mu\rho_k$, where $\mu \geq 1$.

Theoretical Analysis

Convergence:



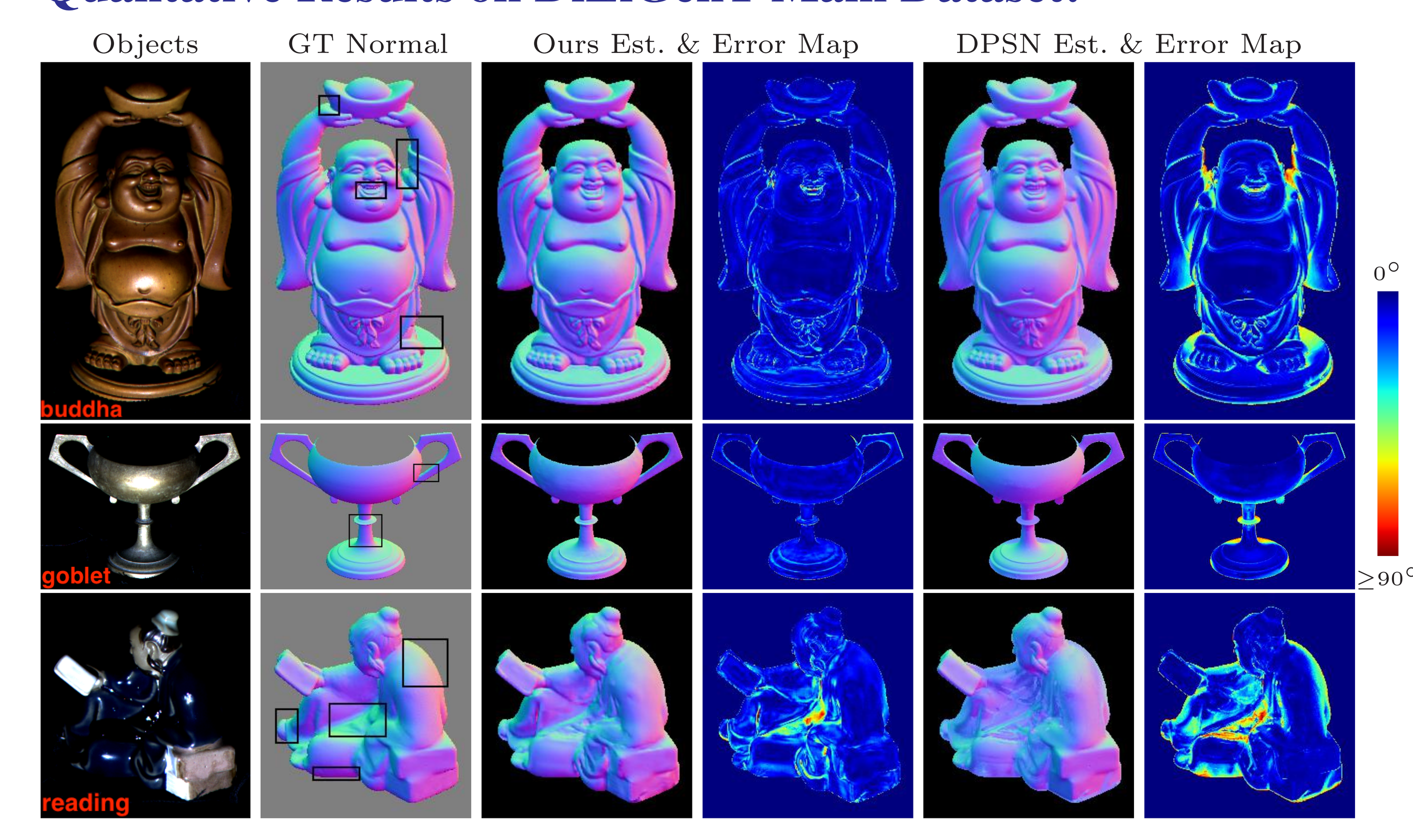
Existence of the Solution to ADMM (a):

Theorem 1. *Assume that $\mathbf{A} \in \mathbb{R}^{3p^2 \times 3p^2}$, $\mathbf{B} \in \mathbb{R}^{M \times M}$ are both symmetric and positive semi-definite matrices. If at least one of $\mathbf{A}, \mathbf{B}$ is positive definite, the Sylvester equation $\mathbf{AC} + \mathbf{CB} = \mathbf{E}$ has a unique solution for $\mathbf{C} \in \mathbb{R}^{3p^2 \times M}$.*

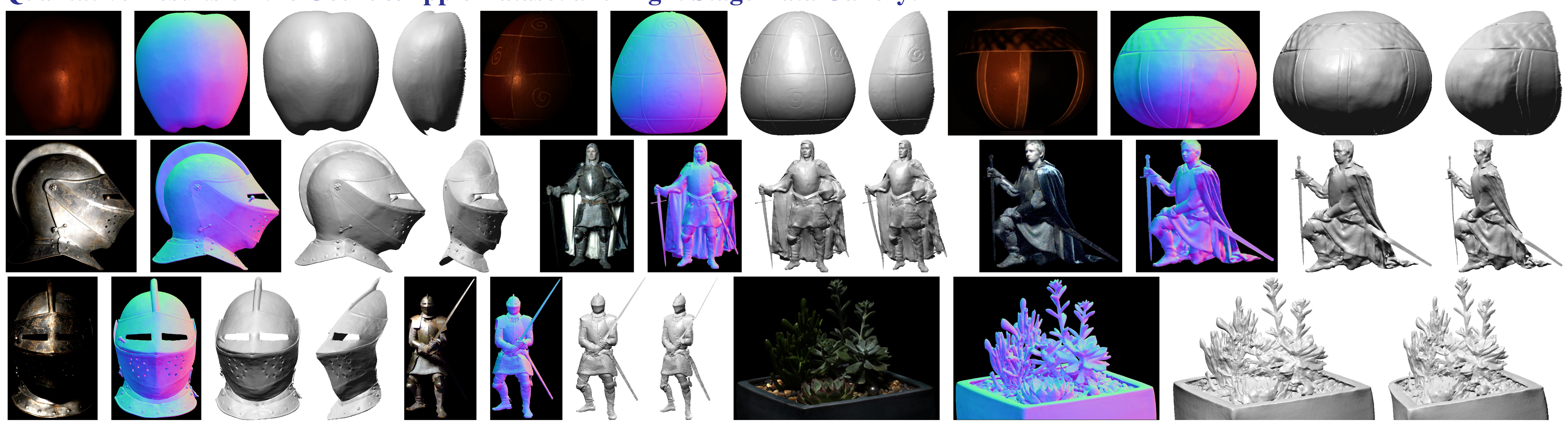
Corollary 1. *The Solution to ADMM (a) exists and is unique.*

Experimental Results

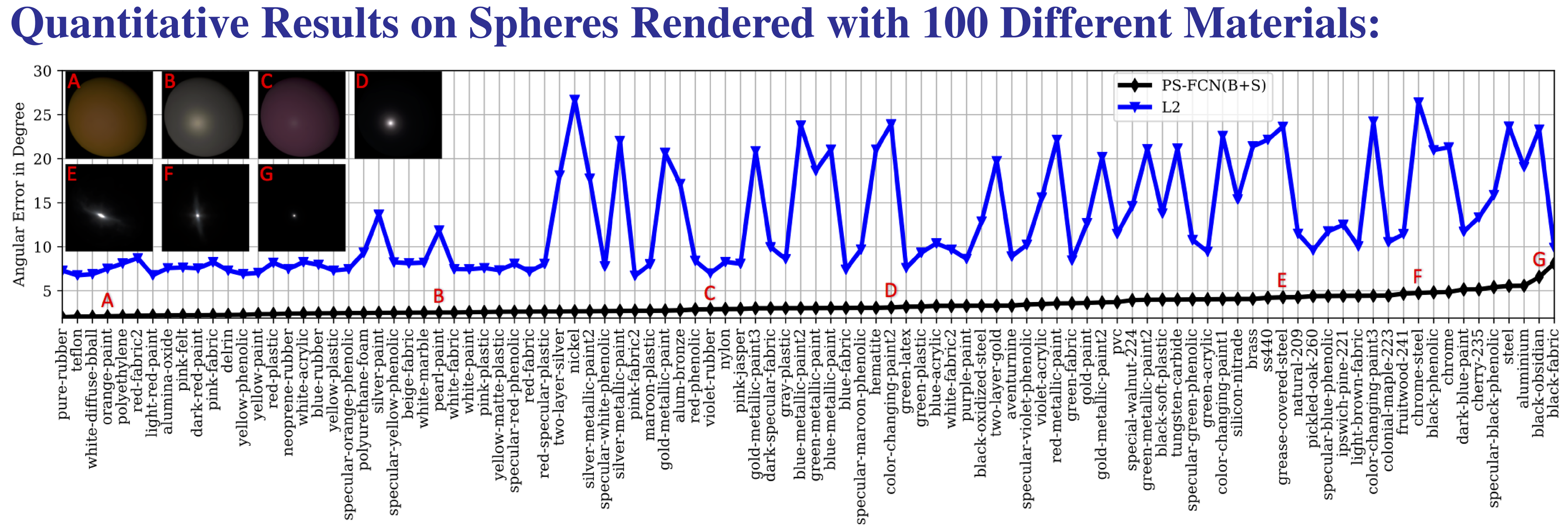
Qualitative Results on DiLiGenT Main Dataset:



Qualitative Results on the Gourd&Apple Dataset and Light Stage Data Gallery:



Quantitative Results on Spheres Rendered with 100 Different Materials:



Quantitative Results of Uncalibrated PS-FCN on DiLiGenT Main Dataset:

Method	ball	cat	pot1	bear	pot2	buddha	goblet	reading	cow	harvest	Avg.
AM07	7.27	31.45	18.37	16.81	49.16	32.81	46.54	53.65	54.72	61.70	37.25
SM10	8.90	19.84	16.68	11.98	50.68	15.54	48.79	26.93	22.73	73.86	29.59
WT13	**4.39**	36.55	**9.39**	**6.42**	14.52	**13.19**	20.57	58.96	19.75	55.51	23.93
PF14	4.77	**9.54**	9.51	9.07	15.90	14.92	29.93	24.18	19.53	29.21	16.66
LC18	9.30	12.60	12.40	10.90	15.70	19.00	**18.30**	**22.30**	15.00	28.00	16.30
UPS-FCN	6.62	14.68	13.98	11.23	**14.19**	15.87	20.72	23.26	**11.91**	**27.79**	**16.02**

Project Webpage:
Code & Dataset & Model