

Sequence to Sequence Models, Attention and Transformers

CS60010: Deep Learning

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IIT Kharagpur

Mar 23, 24 and 26, 2022

Agenda

- § Understand basic neural language model, structured prediction and conditional language models
- § Using attention to handle information bottleneck problem
- § Self attention and transformer models

Resources

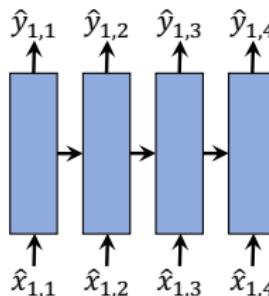
- § CS W182 course by Sergey Levine at UC Berkeley. [[Link](#)] [Lecture 11, 12]
- § “AI Coffee Break with Letitia” youtube channel [[Link](#)]

A Basic Neural Language Model

- § A language model is a model that assigns probabilities to *sequences* representing texts.
- § A language model often is used to generate texts.

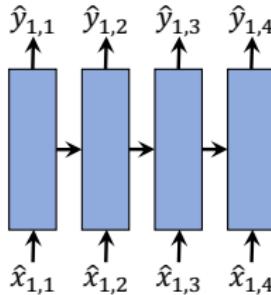
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- § Many language models can be represented as the following general architecture:



- § Why does it need multiple outputs and multiple outputs?
- § Most problems that require multiple outputs have strong *dependencies* between these outputs.
- § This is sometimes referred to as *structured prediction*.

Source: CS W182 course, Sergey Levine, UC Berkeley

A Basic Neural Language Model

- § Lets say we have a text generation model which generates texts given an initial prompt.
- § Let the world of the language model consists of the following three sentences.
 - ▶ I think therefore I am
 - ▶ I like machine learning
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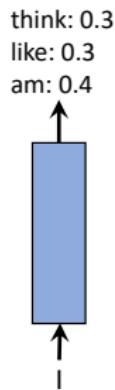


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think: 0.3

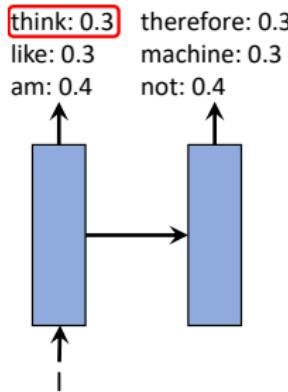
like: 0.3

am: 0.4



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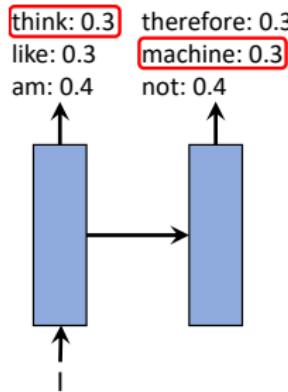
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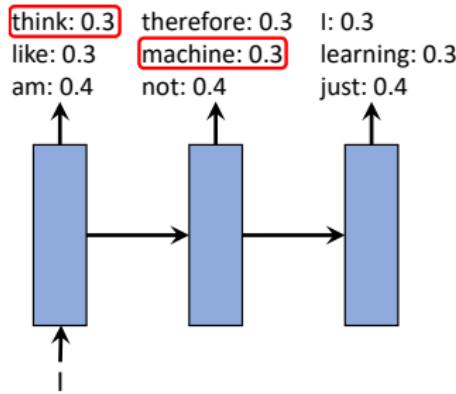
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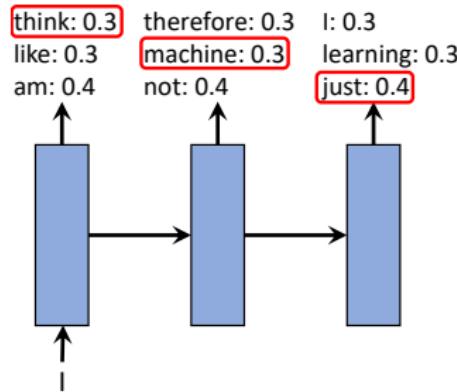


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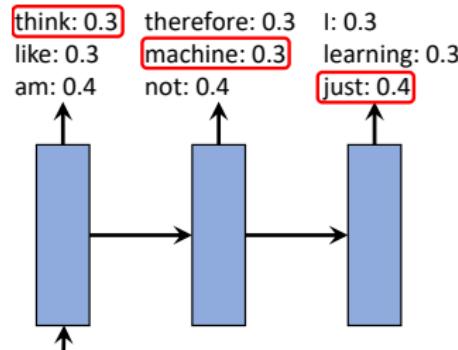
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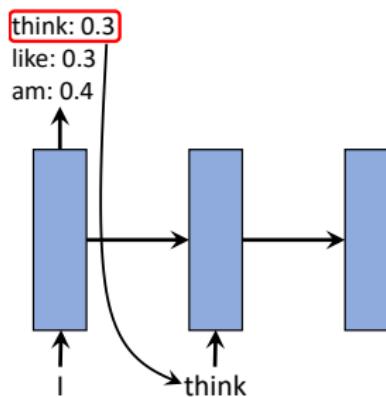


- § Output is nonsensical even though the network did great job individually.

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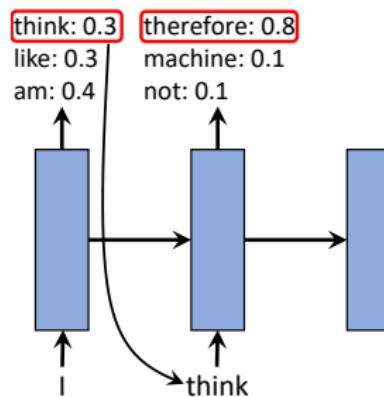
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§ Now the network knows, it is predicting the third word of a sentence where the first two words are 'I think'

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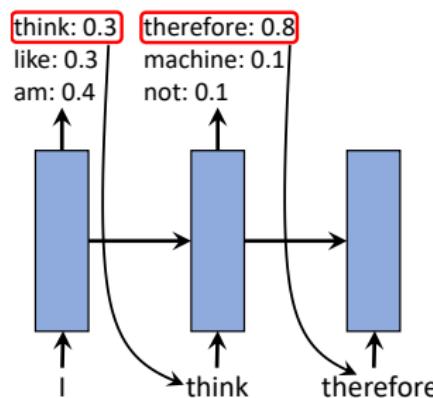
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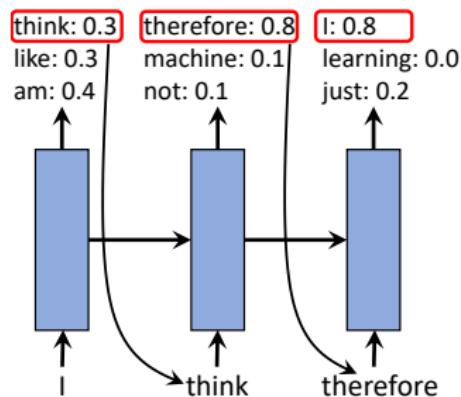
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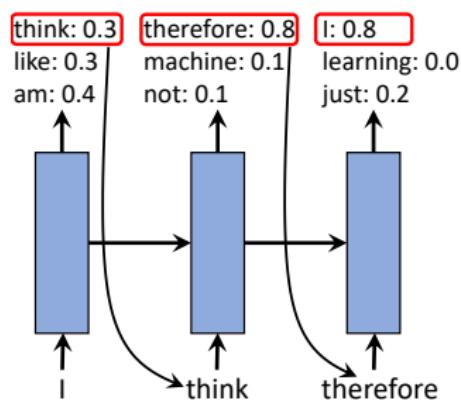
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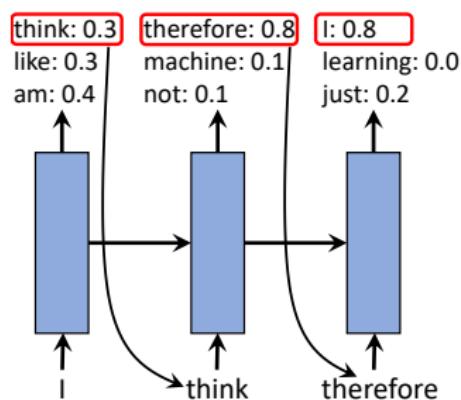
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- § Key idea: Past outputs should influence future outputs.
- § Also known as autoregressive models.

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- § Now the network knows, it is predicting the third word of a sentence where the first two words are 'I think'
- § Key idea: Past outputs should influence future outputs.
- § Also known as autoregressive models.
- § During training: input is the sequence and output is the same sequence offset by 1.

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§ How are the training sequences represented.

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§ Simplest: tokenize the sentence (each word is a token) and use onehot vector representation.

$$x_{1,i} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

§ More complex: word embeddings (we'll cover this later)

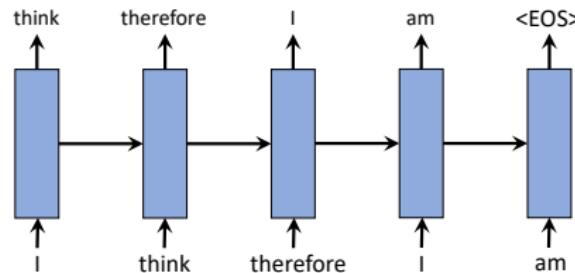
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A Few Details

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A Few Details

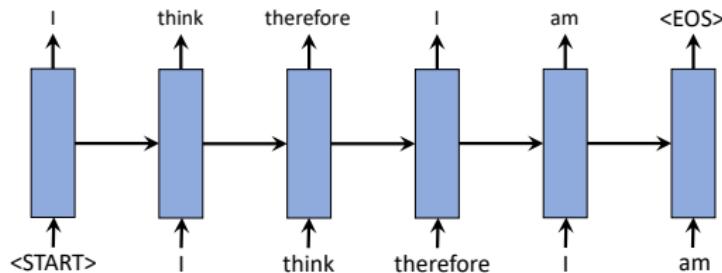
- § How does the model know it has to stop generating words?
- § During training, add a special token $\langle \text{EOS} \rangle$ at the end of the sequence.
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- § Similarly a special `<START>` token is introduced to kick off the start of a sentence.

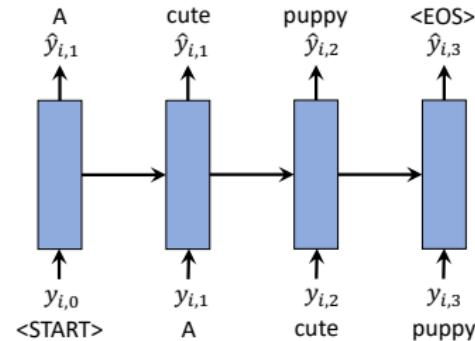
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Conditional Language Models

- § In conditional language models, text is generated conditioned on some input.
- § For example, image captioning conditions text generation on image.

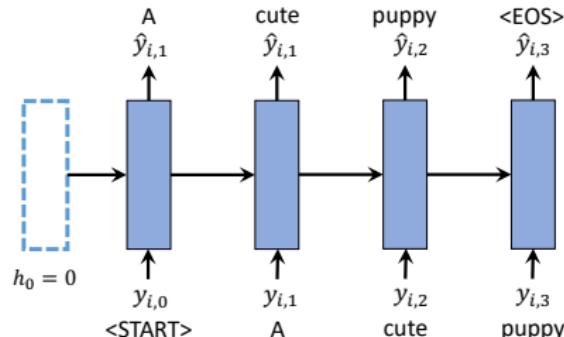
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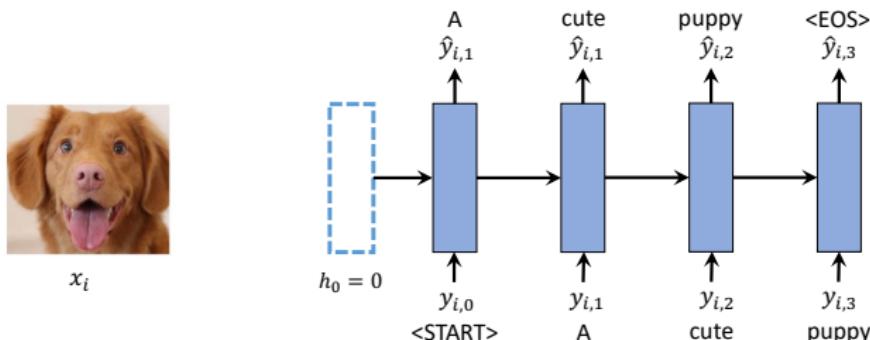
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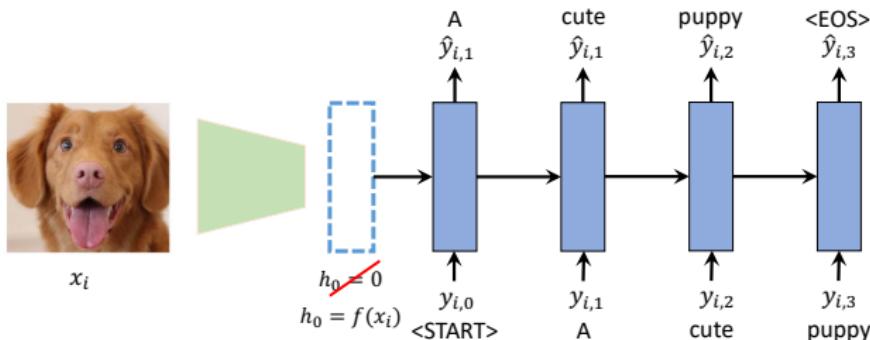
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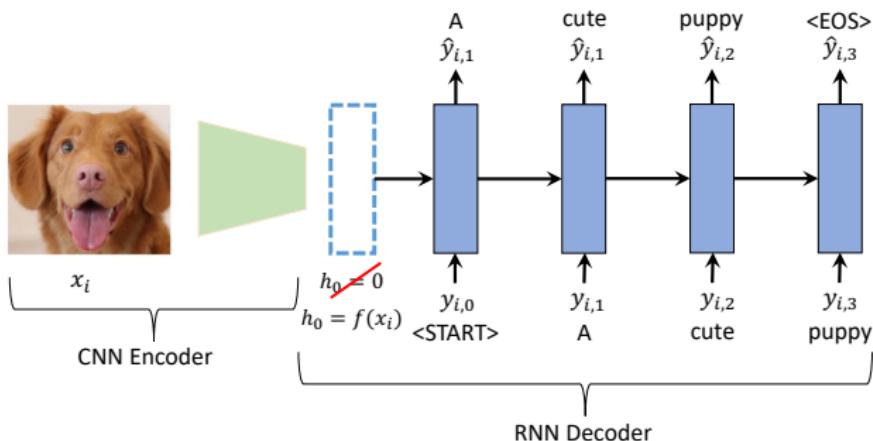
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- § Previously, initial hidden state of RNN was 0.
- § Now, we set the intial state of the RNN as an encoded representation from the image, obtained by passing it thorugh a convnet.
- § Both RNN and ConvNet are trained end-to-end.

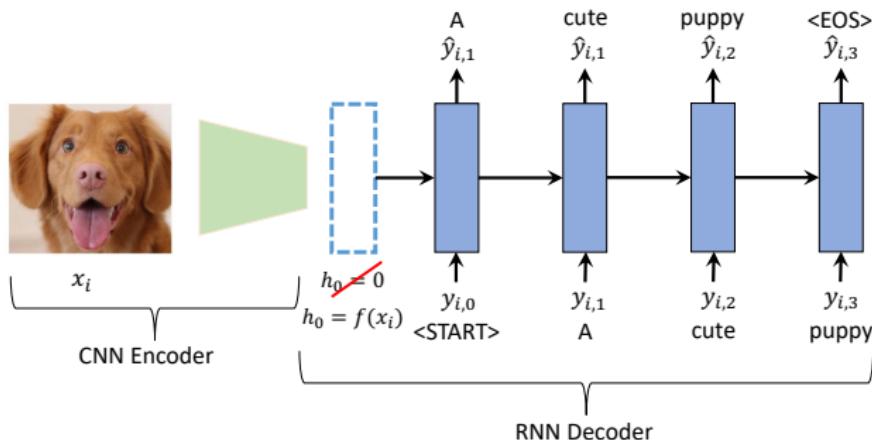
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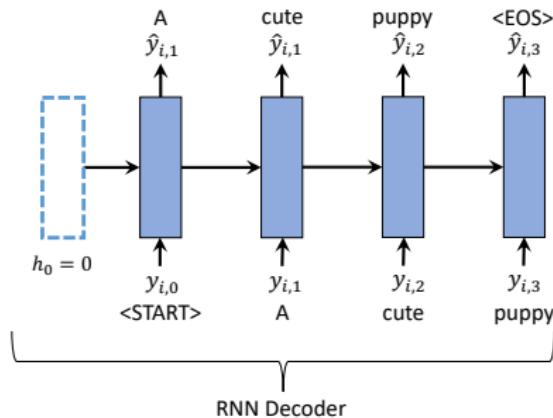
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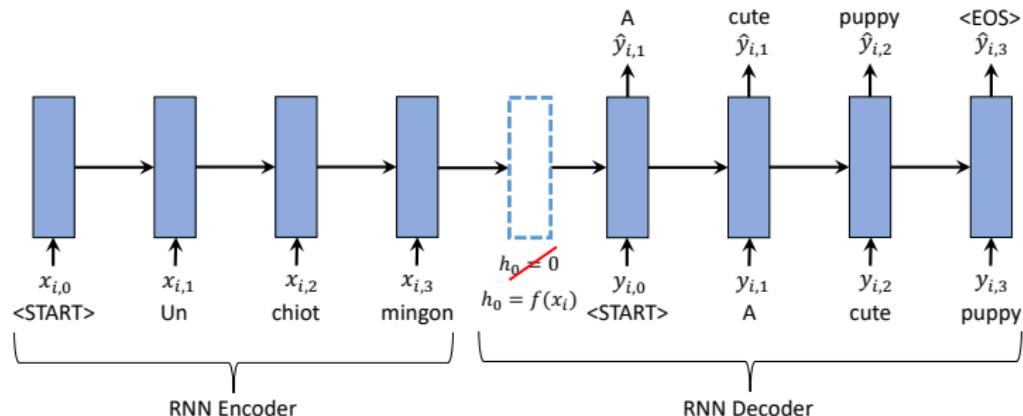
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- § Training data: Paired image-text data.

What if we condition on another sequence?



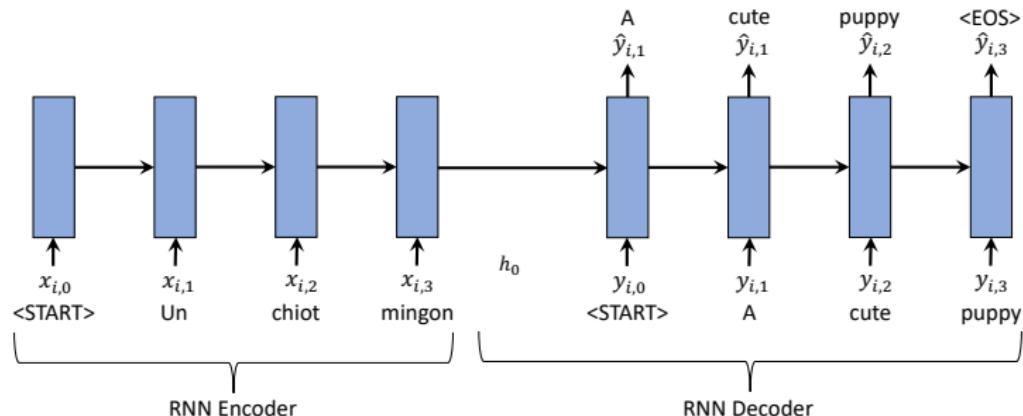
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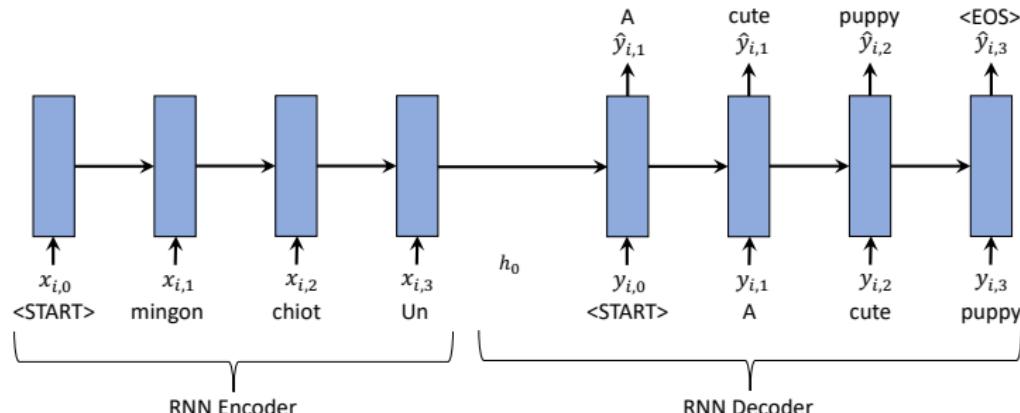


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- § The first RNN reads in French, produces h_0 and the second RNN takes h_0 and produces English text.
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- § h_0 is only ‘virtual’. <EOS> token in French doubles as <START> token in English.

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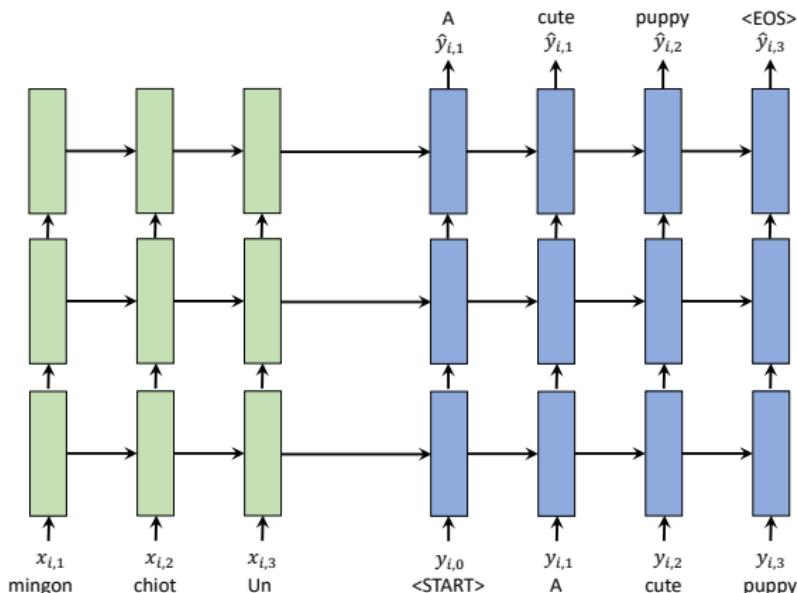


A Few Details



- § Sometimes, the encoder RNN reads the source language sentence in reverse.
- § Typically two separate RNNs (with different weights) are used.
- § Both the RNNs are trained end-to-end on paired data (e.g., pairs of French and English sentences)

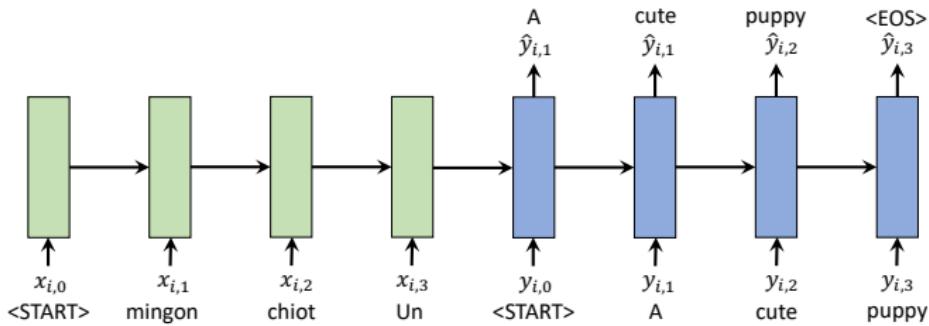
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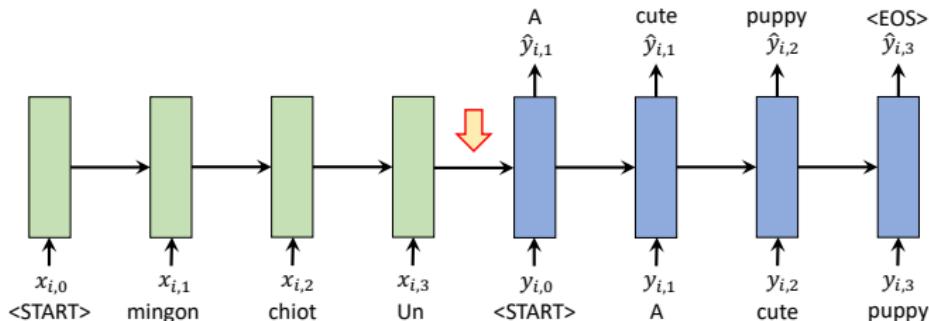
- § RNNs can be stacked.
- § Each RNN layer can use LSTM cells (or GRU)

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The Bottleneck Problem

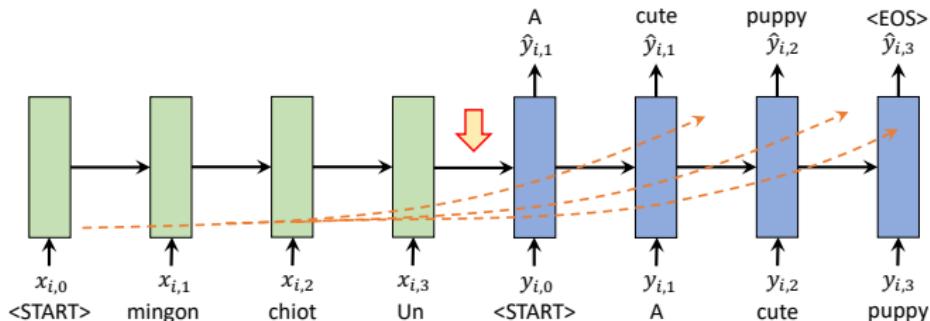


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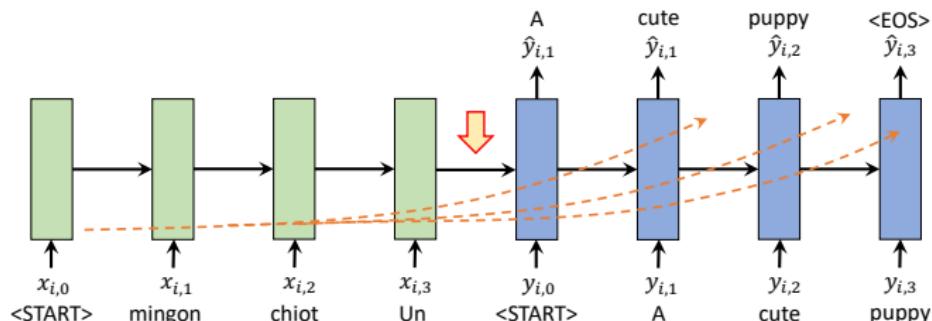
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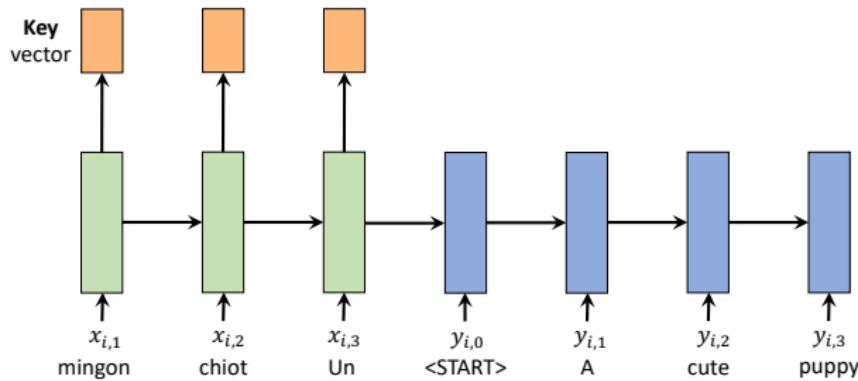
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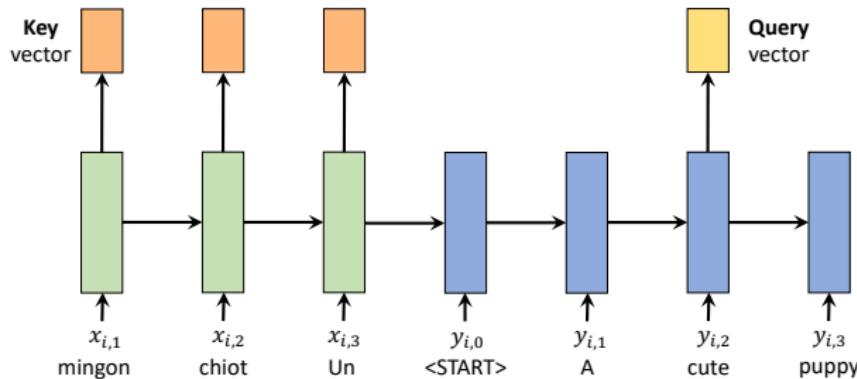
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- § How can we do this?

Can we ‘peek’ at the Input



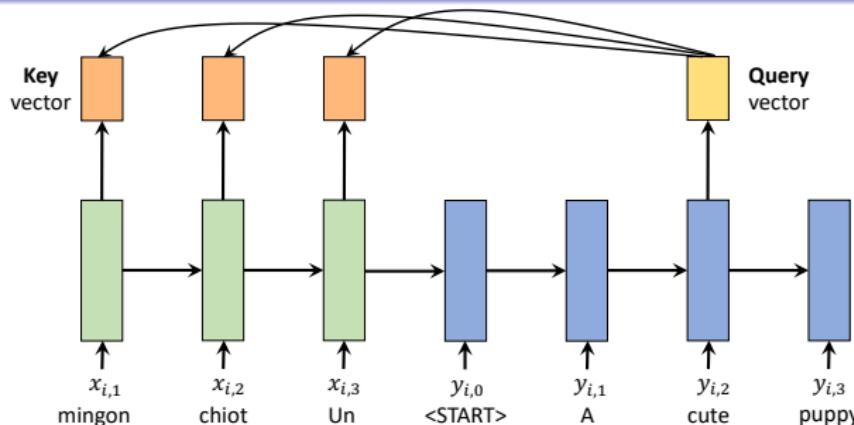
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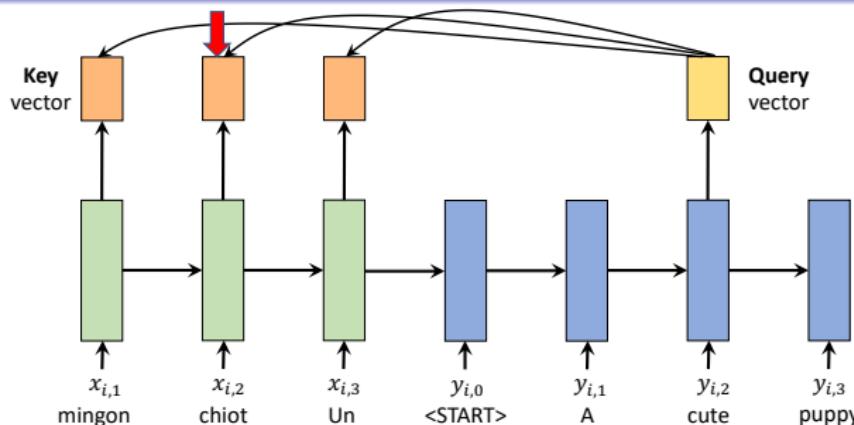
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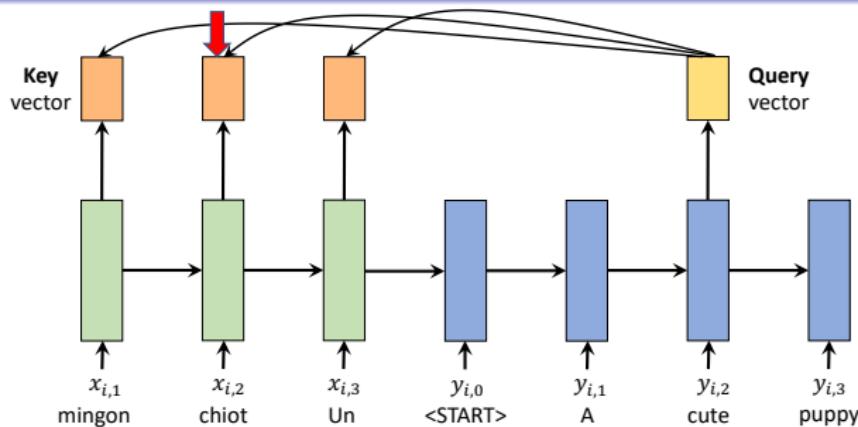
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- § The corresponding hidden state is sent to the decoder

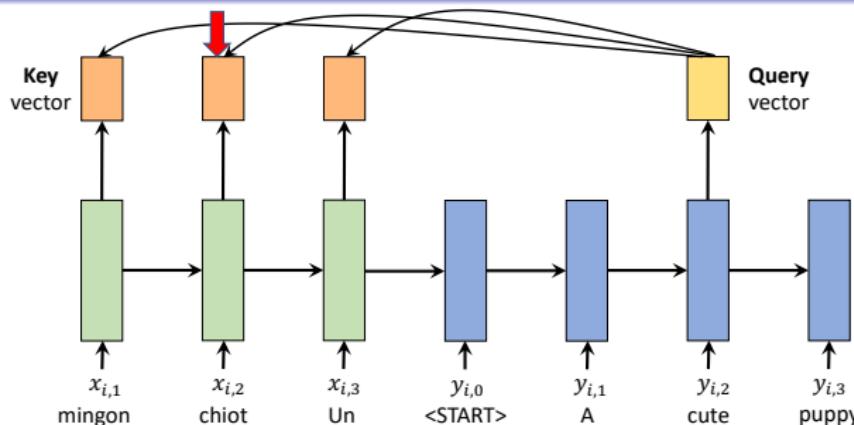
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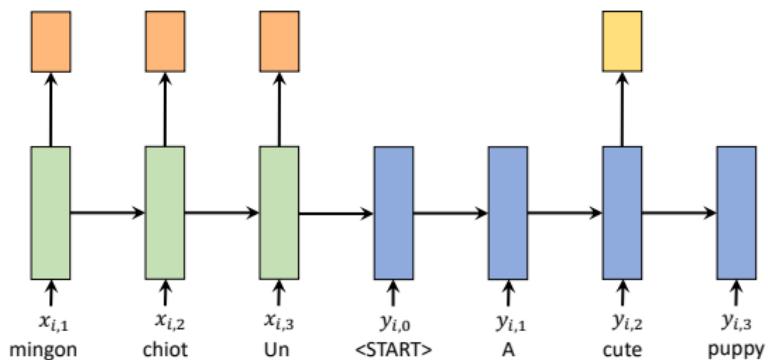
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Can we ‘peek’ at the Input



- § (crude) intuition: key might encode “the subject of the sentence”, and query might ask for “the subject of the sentence”
- § What keys and queries mean is learned as a part of the training process – we do not have to select it manually!

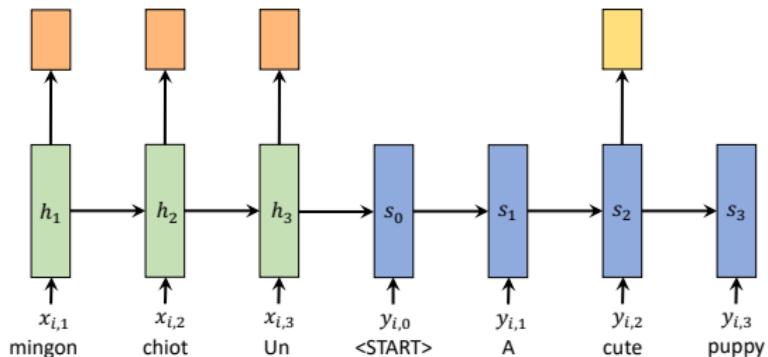
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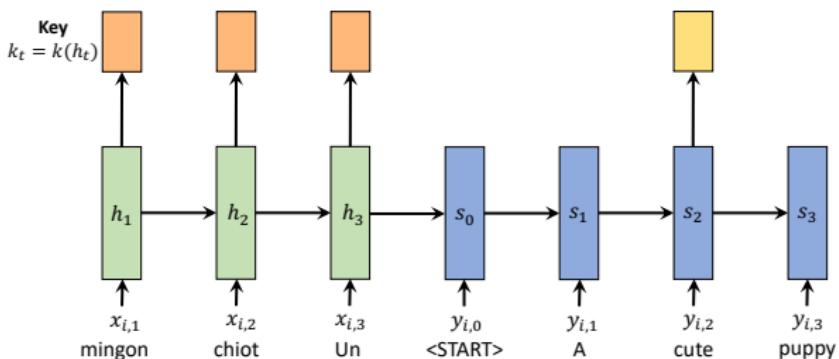


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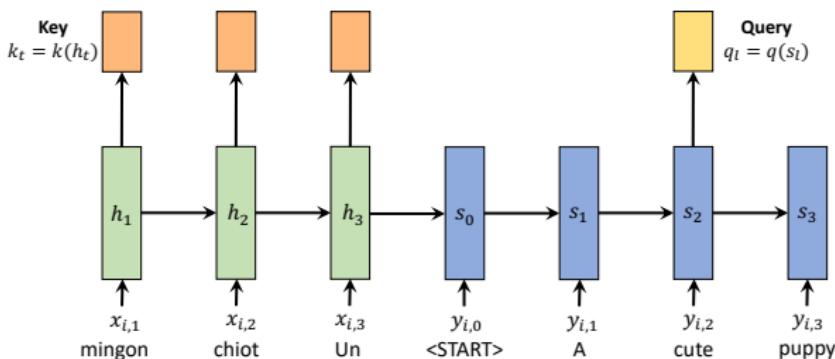
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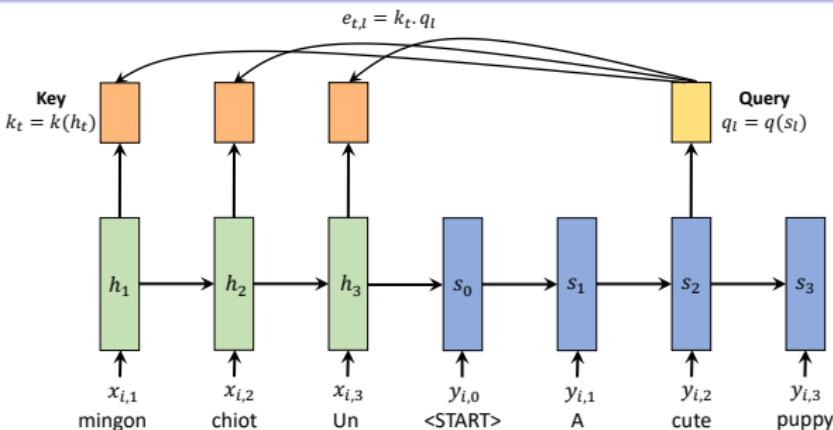
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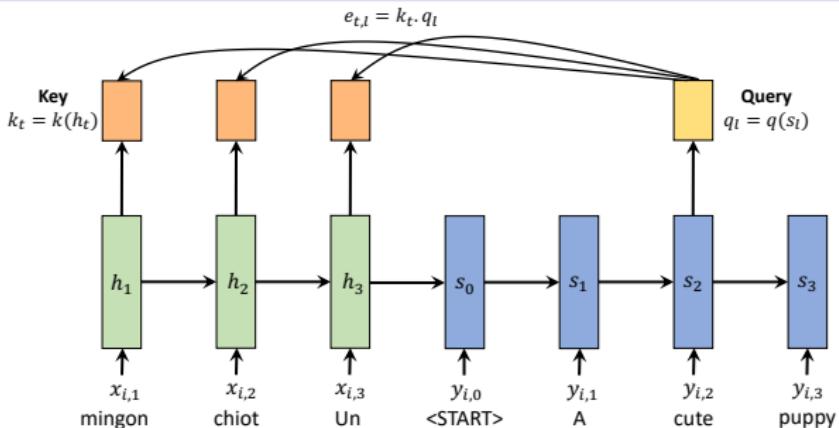


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$$k_t = \sigma(W_k h_t + b_k)$$
- § Similarly query q_l is some learnable function of decoder state s_l
- § Attention $e_{t,l}$ measures the similarity between the key and the query and is given by the dot product between them
- § Intuitively, we want to pull out the hidden state h_t for the timestep t at which $e_{t,l}$ is largest

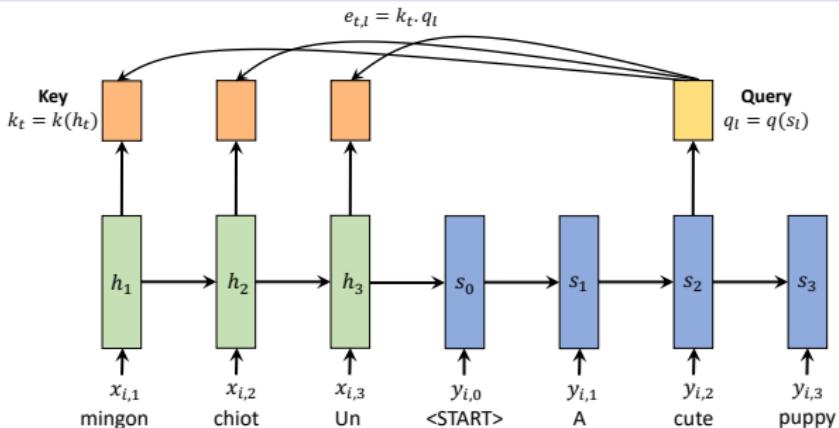
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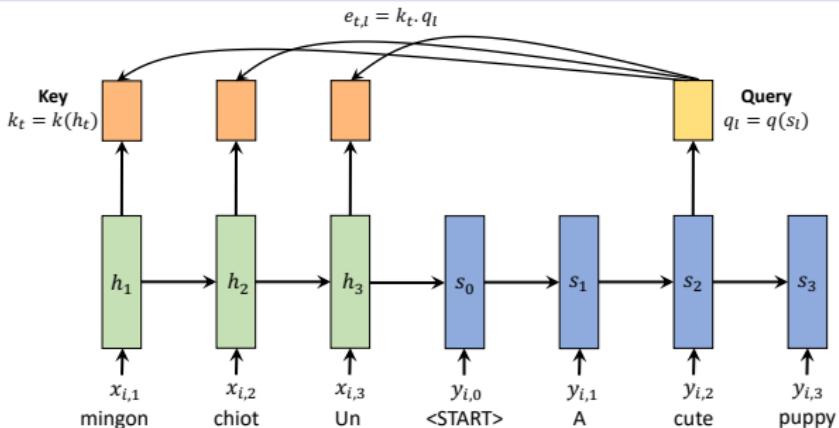
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- § ‘arg max’ is not differentiable, we will not be able to train the network.
- § We will use softmax: $\alpha_{.,l} = \text{softmax}(e_{.,l})$, where $\alpha_{t,l} = \frac{\exp(e_{t,l})}{\sum_{t'} \exp(e_{t',l})}$

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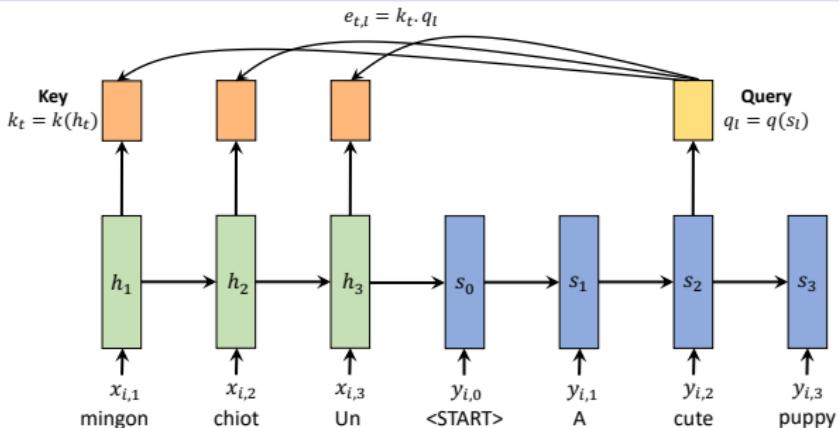
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- § Send $a_l = \sum_t \alpha_{t,l} h_t$. $\alpha_{t,l}$ s are small numbers except for the max $e_{t,l}$

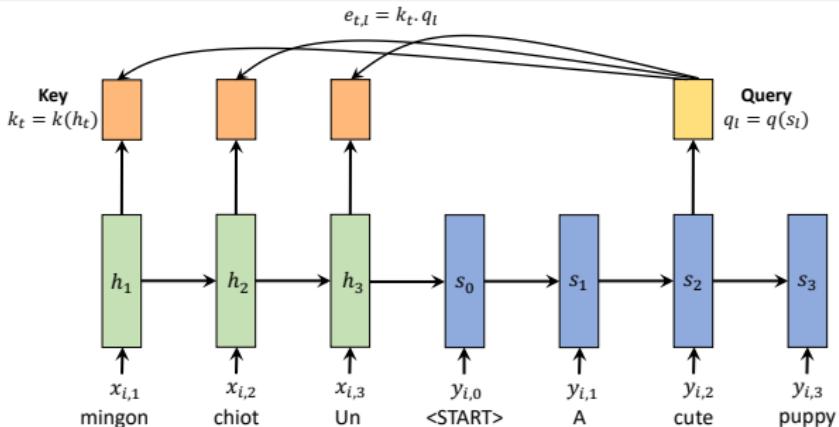
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Attention



§ Send $a_l = \sum_t \alpha_{t,l} h_t$. What does 'sending' mean?

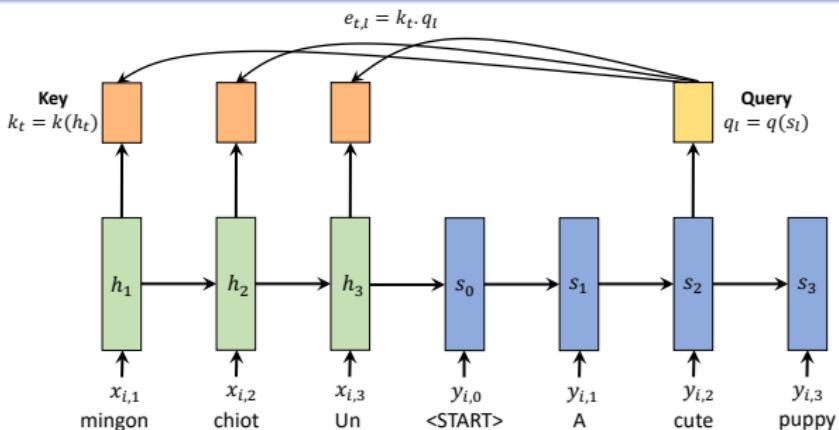
Attention



§ Send $a_l = \sum_t \alpha_{t,l} h_t$. What does 'sending' mean?

► $\hat{y}_l = f(s_l, a_l)$

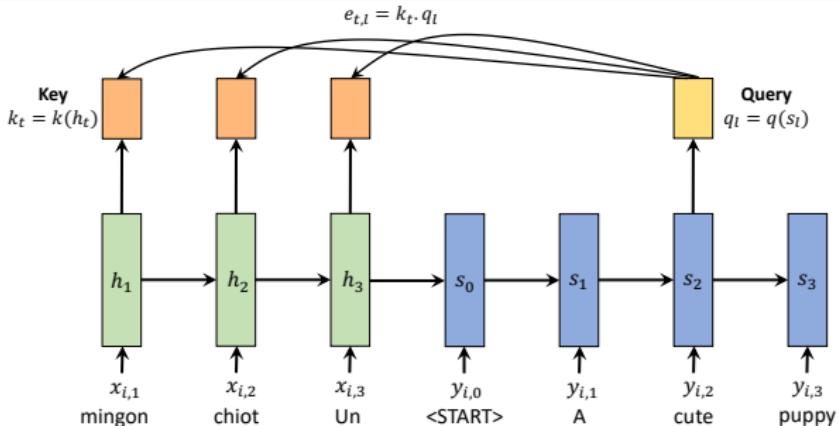
Attention



§ Send $a_l = \sum_t \alpha_{t,l} h_t$. What does 'sending' mean?

- ▶ $\hat{y}_l = f(s_l, a_l)$
- ▶ Give a_l to next RNN layer if stacked RNN is used

Attention



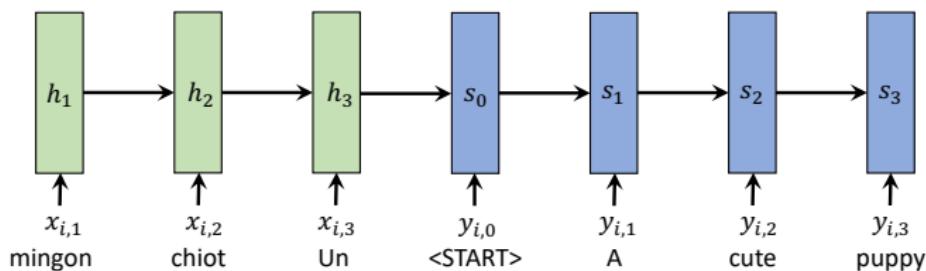
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- ▶ $\hat{y}_l = f(s_l, a_l)$
- ▶ Give a_l to next RNN layer if stacked RNN is used

- ▶ Append a_l to the next decoder step $\bar{s}_l = \begin{bmatrix} s_{l-1} \\ a_{l-1} \\ x_l \end{bmatrix}$

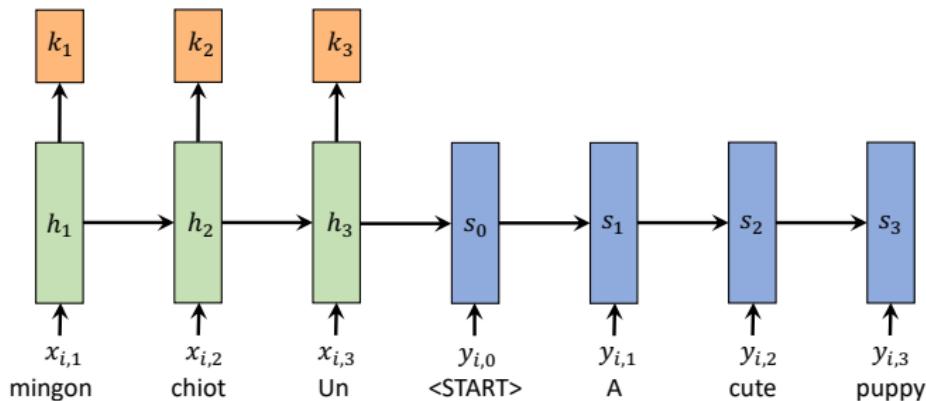
Source: CS W182 course, Sergey Levine, UC Berkeley

Attention Walkthrough (Example)



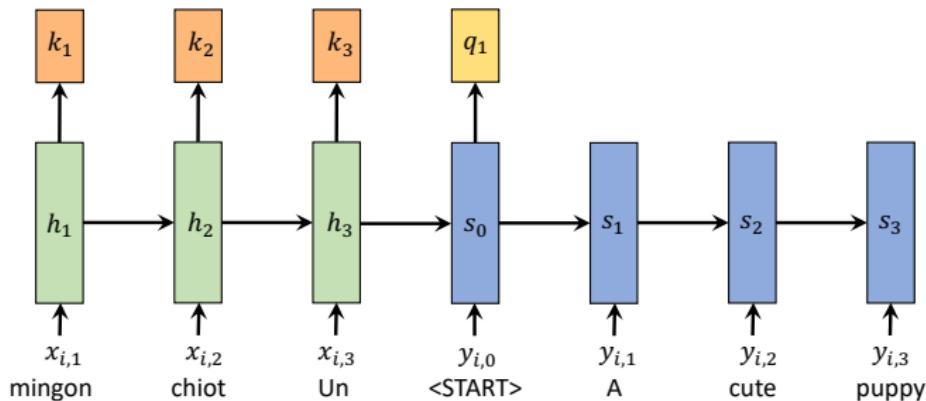
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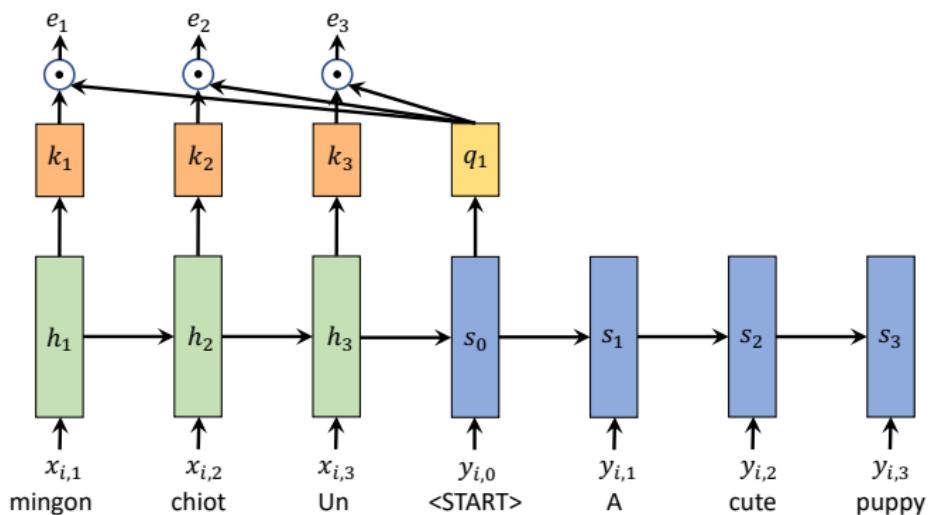
Source: CS W182 course, Sergey Levine, UC Berkeley

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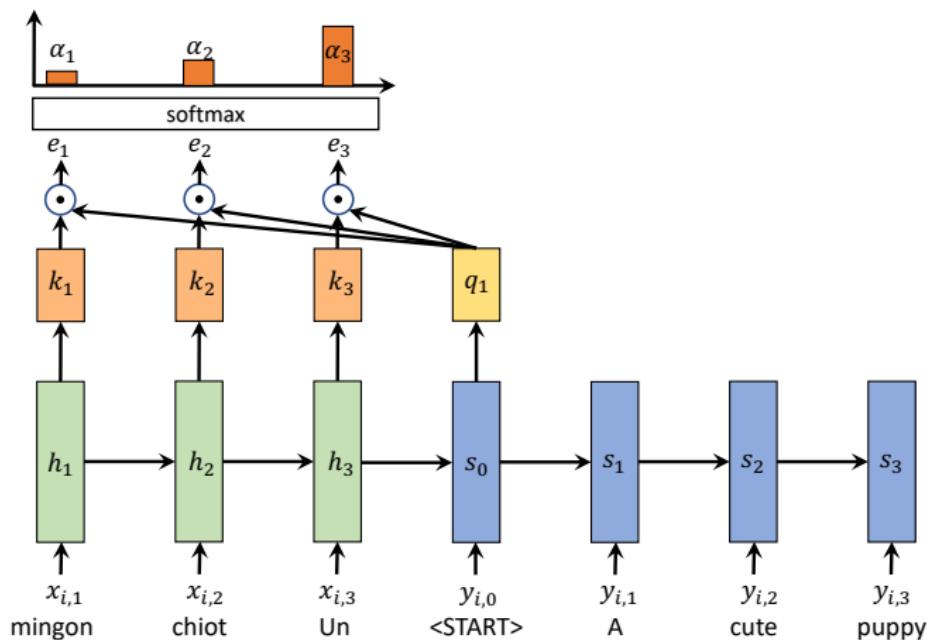
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Attention Walkthrough (Example)



Source: CS W182 course, Sergey Levine, UC Berkeley

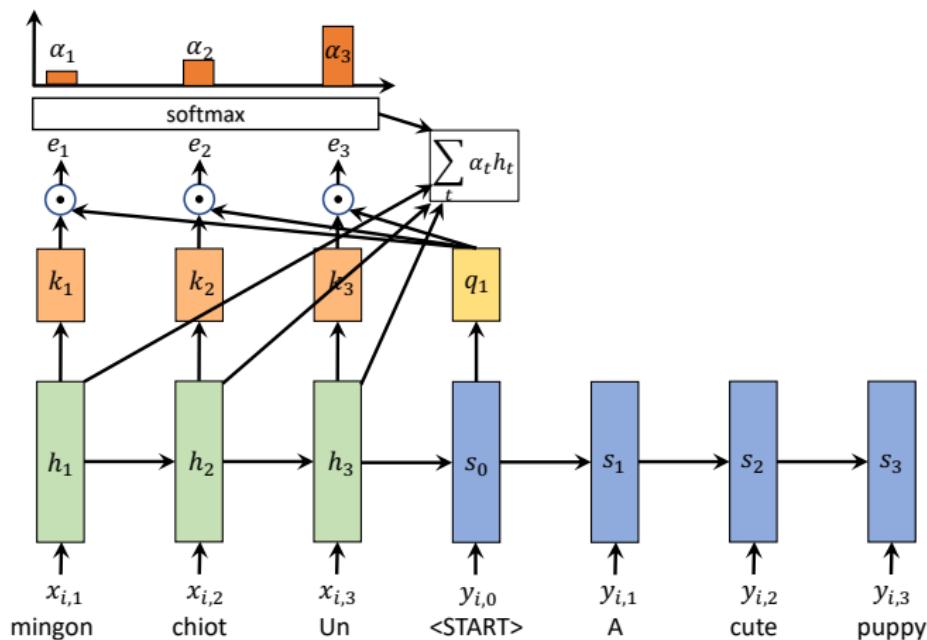
Attention Walkthrough (Example)



Source: CS W182 course, Sergey Levine, UC Berkeley



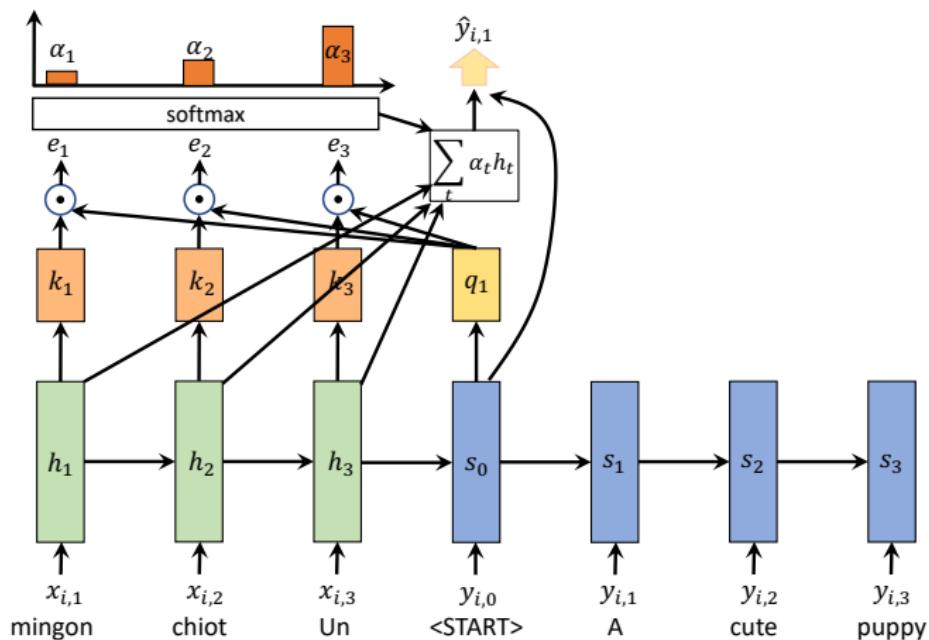
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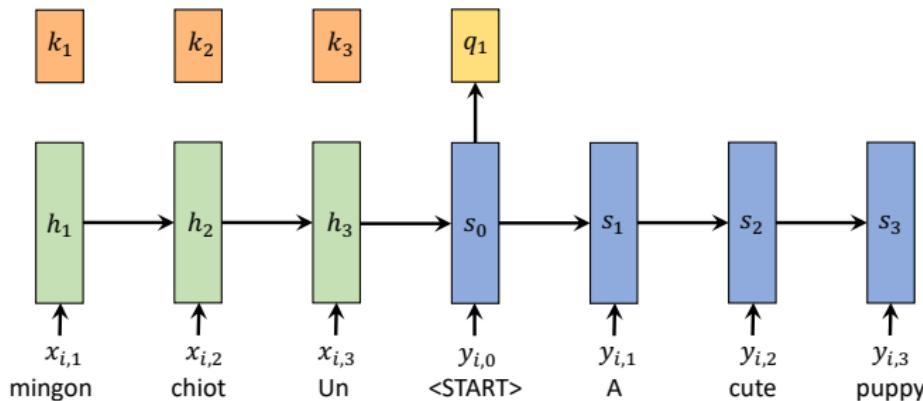
Attention Walkthrough (Example)



Source: CS W182 course, Sergey Levine, UC Berkeley

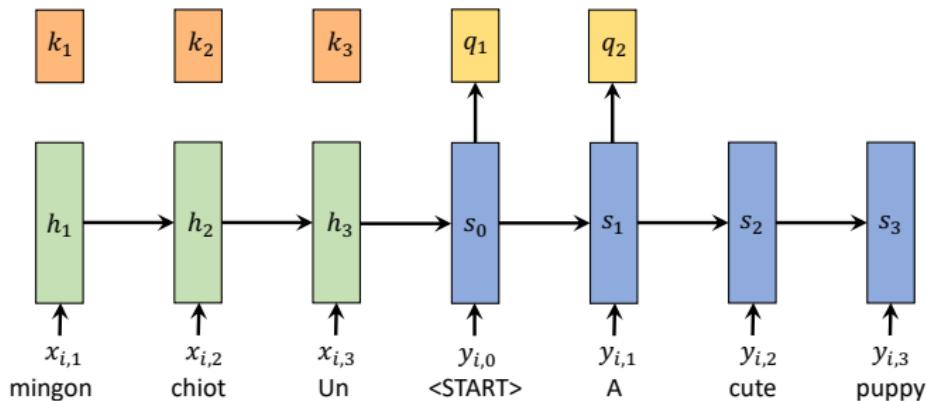


Attention Walkthrough (Example)

 $\hat{y}_{i,1}$ 

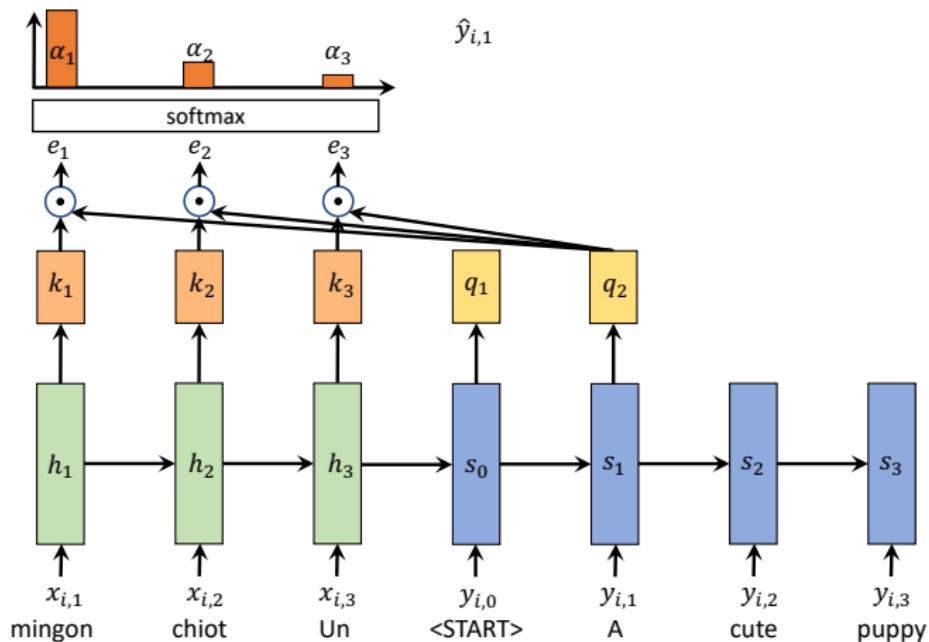
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Attention Walkthrough (Example)

 $\hat{y}_{i,1}$ 

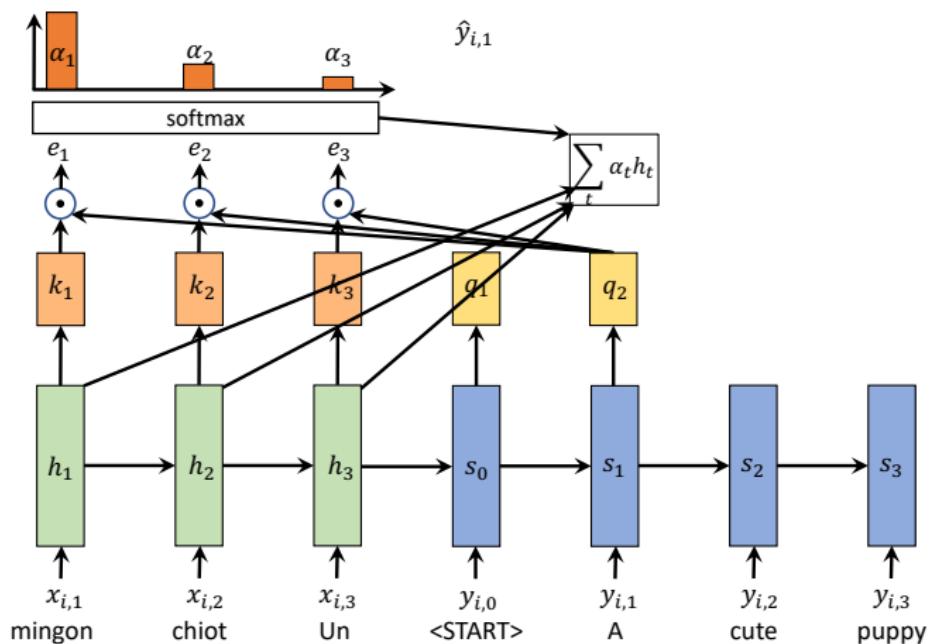
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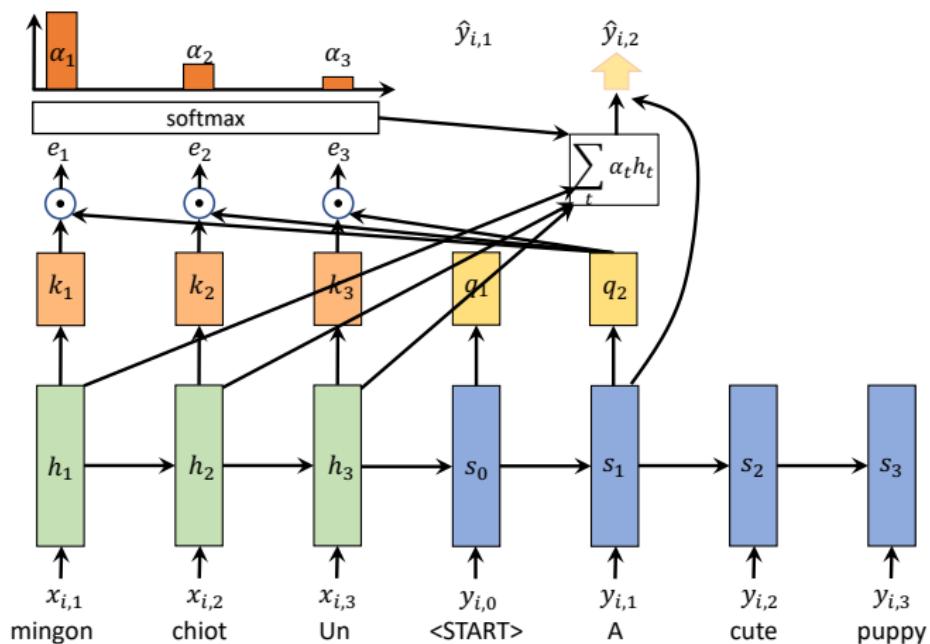
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Attention Walkthrough (Example)



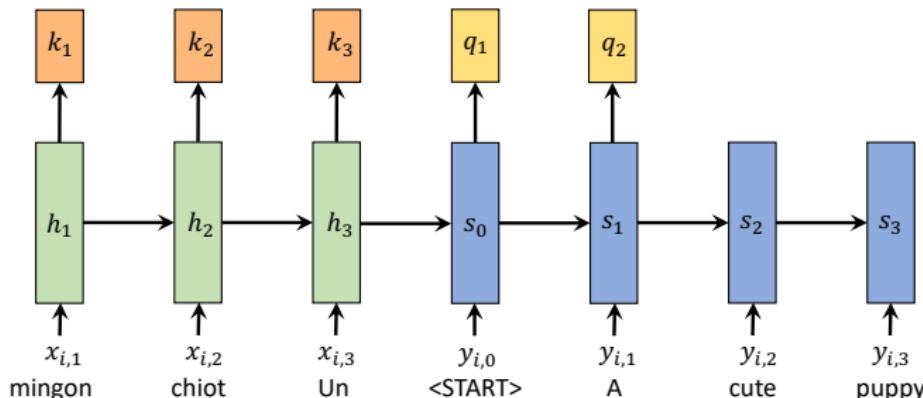
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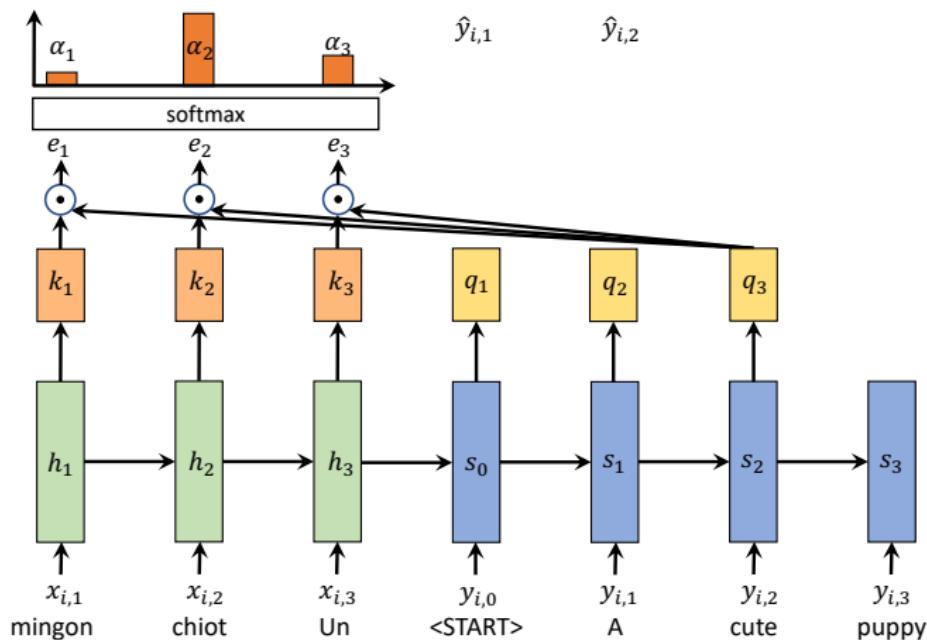
Source: CS W182 course, Sergey Levine, UC Berkeley

Attention Walkthrough (Example)

 $\hat{y}_{i,1}$ $\hat{y}_{i,2}$ 

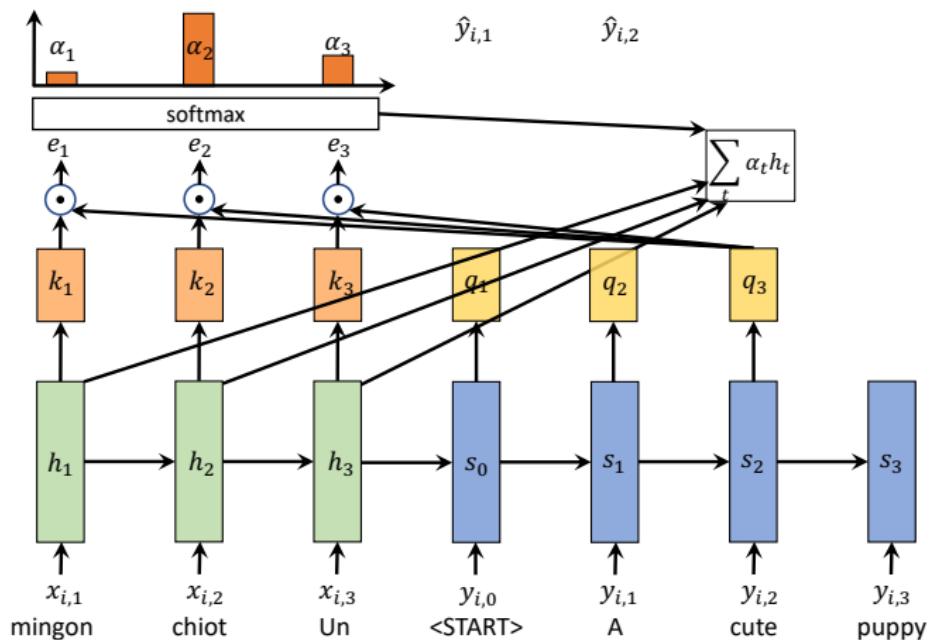
Source: CS W182 course, Sergey Levine, UC Berkeley

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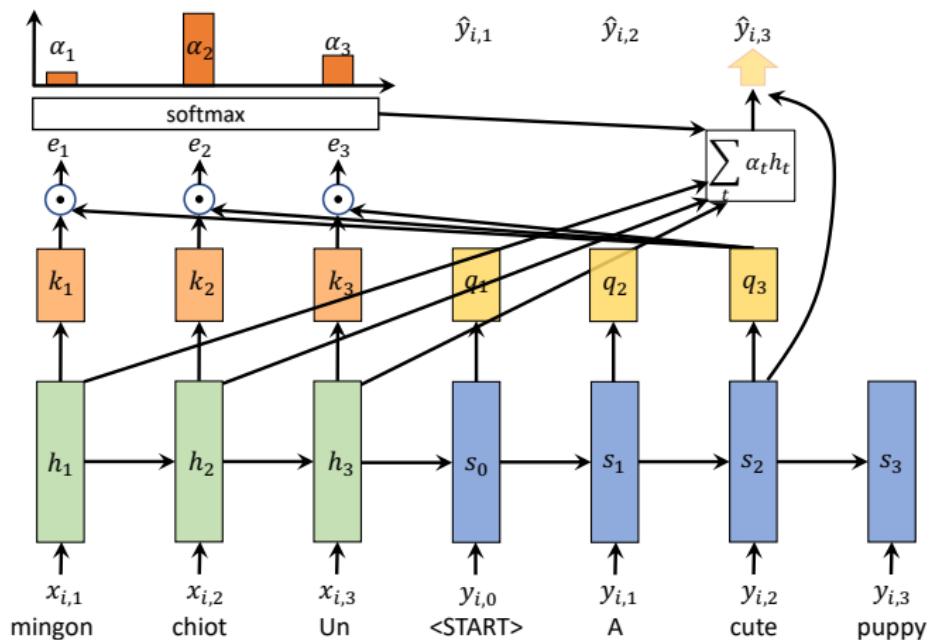
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Attention Walkthrough (Example)



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Attention Walkthrough (Example)



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Attention Equations

 Encoder side:

$$k_t = k(h_t)$$

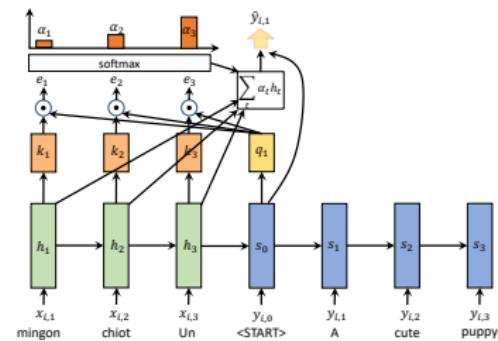
Decoder side:

$$q_l = q(s_l)$$

$$\S \quad e_{t,l} = k_t \cdot q_l$$

$$\alpha_{t,l} = \frac{\exp(e_{t,l})}{\sum \exp(e_{t',l})}$$

$$a_l = \sum_t \alpha_{t,l} h_t$$



Source: CS W182 course, Sergey Levine, UC Berkeley

Attention Equations

- § Encoder side:
 $k_t = k(h_t)$
 - § Decoder side:
 $q_l = q(s_l)$
 - § $e_{t,l} = k_t \cdot q_l$

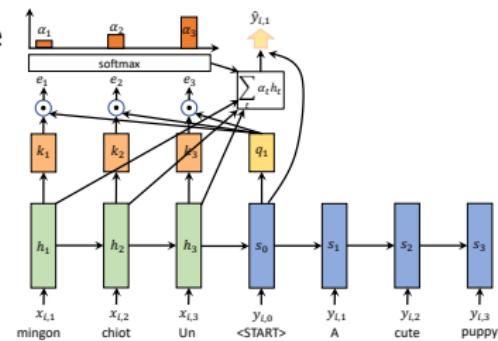
$$\alpha_{t,l} = \frac{\exp(e_{t,l})}{\sum_j \exp(e_{t',l})}$$

$$a_l = \sum_t \alpha_{t,l} h_t$$

- § Can be used in different ways:
 - § Concatenate to hidden state

$$\begin{bmatrix} s_{l-1} \\ a_{l-1} \\ x_l \end{bmatrix}$$
 - § Use for readout:

$$\hat{y}_l = f(s_l, a_l)$$
 - § Concatenate as input to next RNN layer.



Source: CS W182 course, Sergey Levine, UC Berkeley

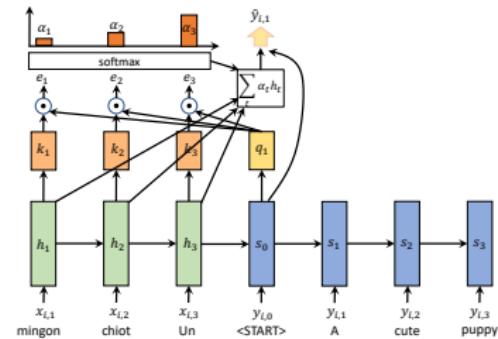
Attention Variants

§ Simple key-query choice: k and q are identity functions: $k_t = h_t$, $q_l = s_l$

$$e_{t,l} = k_t \cdot q_l$$

$$\alpha_{t,l} = \frac{\exp(e_{t,l})}{\sum_{t'} \exp(e_{t',l})}$$

$$a_l = \sum_t \alpha_{t,l} h_t$$



Source: CS W182 course, Sergey Levine, UC Berkeley

Attention Variants

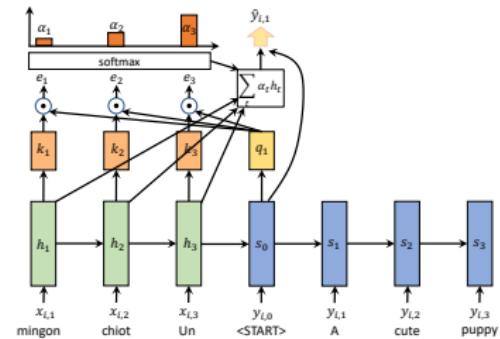
§ Linear multiplicative attention:

$$k_t = W_k h_t, \quad q_l = W_q s_l$$

§ $e_{t,l} = h_t^T W_k^T W_q s_l = h_t^T W_e s_l$

$$\alpha_{t,l} = \frac{\exp(e_{t,l})}{\sum_{t'} \exp(e_{t',l})}$$

$$a_l = \sum_t \alpha_{t,l} h_t$$



Source: CS W182 course, Sergey Levine, UC Berkeley



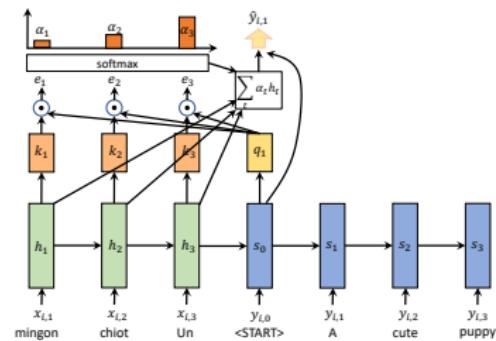
Attention Variants

- § Learned value encoding
 - § Encoder side: $k_t = k(h_t)$
 - § Decoder side: $q_l = q(s_l)$
 - § $e_{t,l} = k_t \cdot q_l$

$$\alpha_{t,l} = \frac{\exp(e_{t,l})}{\sum_{t'} \exp(e_{t',l})}$$

$$a_l = \sum_t \alpha_{t,l} v(h_t)$$

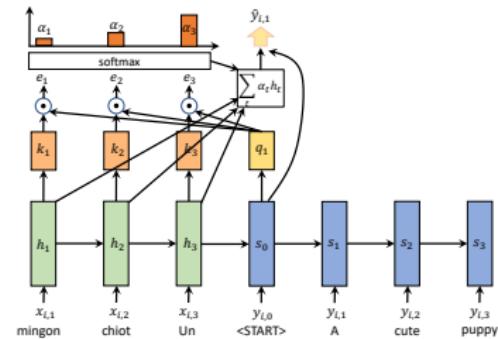
- § $v(\cdot)$ is some learned function and known as the ‘value’.
 - § The interpretation is that now you don’t just compute ‘key’, rather you compute a ‘key-value’ pair of the input hidden states. During decoding, key-query provides the timestep with largest similarity between key and query.
 - § The attention (ideally) collects the value of that timestep from the input. In ‘softmaxed’ version, a weighted combination of the input values are taken.



Source: CS W182 course, Sergey Levine, UC Berkeley

Attention Summary

- § Every encoder step t produces a key k_t
- § Every decoder step l produces a query q_l
- § Decoder gets “sent” encoder activation h_t corresponding to the largest value of $k_t \cdot q_l$
- § Actually gets $a_l = \sum_t \alpha_{t,l} h_t$

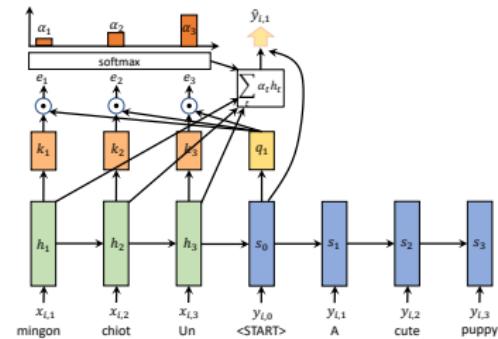


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- § Why is this good?

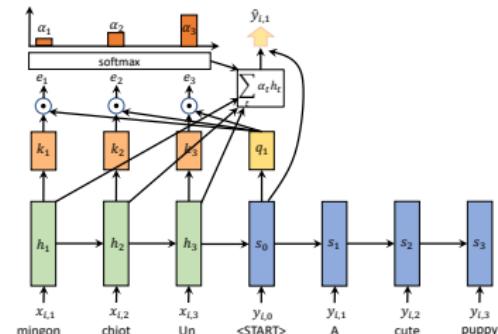


Source: CS W182 course, Sergey Levine, UC Berkeley



Attention Summary

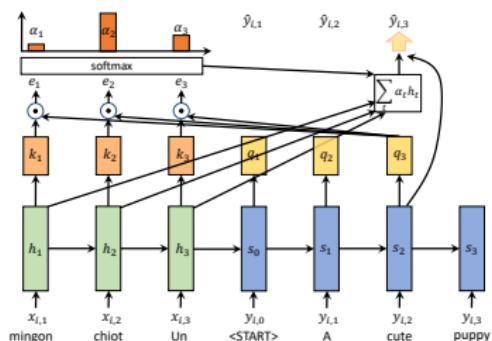
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- § Why is this good?
- § Attention is very powerful, because now **all** decoder steps are connected to **all** encoder steps!
- § Bottleneck is much less important
- § Gradients are much better behaved

Source: CS W182 course, Sergey Levine, UC Berkeley

Attention

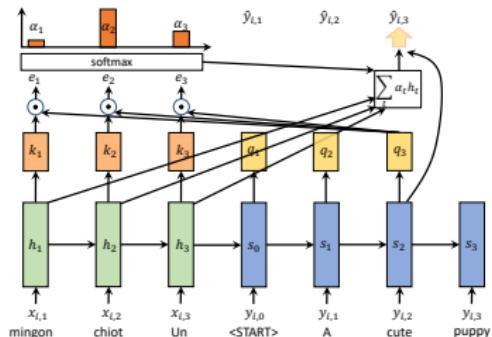


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Attention

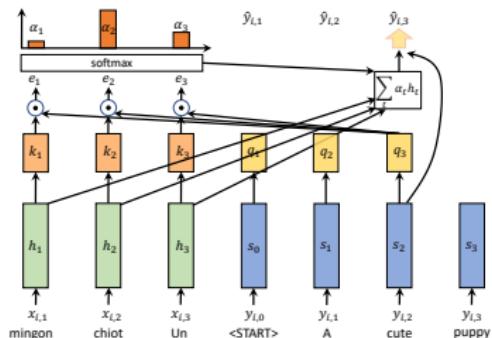
§ If we have **attention**, do we even need recurrent connections?



Source: CS W182 course, Sergey Levine, UC Berkeley

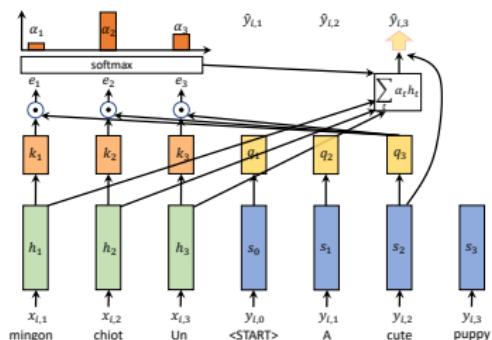
Attention

- § If we have **attention**, do we even need recurrent connections?
- § Can we transform RNN into a **purely attention-based** model?



Source: CS W182 course, Sergey Levine, UC Berkeley

Attention



- § If we have **attention**, do we even need recurrent connections?
- § Can we transform RNN into a **purely attention-based** model?
- § This has a few issues we must overcome:
 - ▶ Now, step $l = 2$ can't access s_0 or s_2
 - ▶ The encoder has no temporal dependences at all.

Source: CS W182 course, Sergey Levine, UC Berkeley

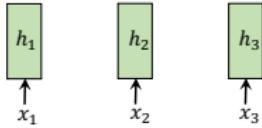
Self-Attention

§ Basic self attention: without making distinction between encoder and decoder

x_1 x_2 x_3

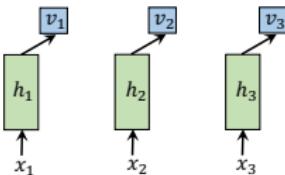
Self-Attention

- § Basic self attention: without making distinction between encoder and decoder
- § Input from each time-step is encoded
 - e.g., $h_t = \sigma(Wx_t + b)$
- § This is not a recurrent model, but still weight sharing



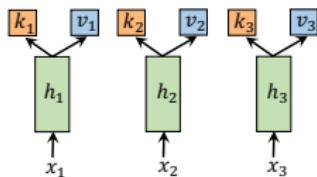
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 - § Every timestep also outputs key $k_t = k(h_t)$, e.g., $k(h_t) = W_k h_t$

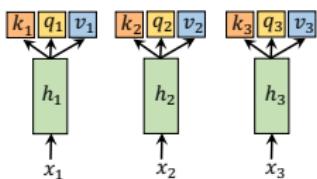


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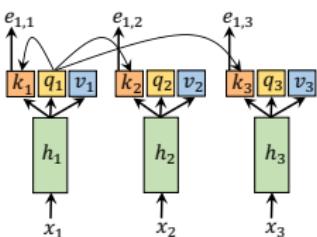
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Self-Attention

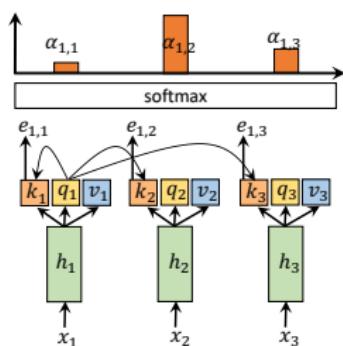
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$$e_{l,t} = q_l \cdot k_t$$



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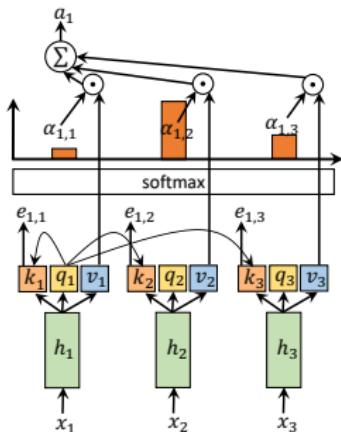
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§ Compute attention scores: $\alpha_{l,t} = \frac{\exp(e_{l,t})}{\sum_{t'} \exp(e_{l,t'})}$

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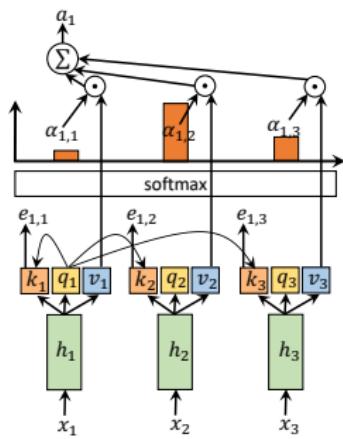
§ Compute attention scores: $\alpha_{l,t} = \frac{\exp(e_{l,t})}{\sum_{t'} \exp(e_{l,t'})}$

§ Compute attention at timestep l : $a_l = \sum_t \alpha_{l,t} v_t$

Source: CS W182 course, Sergey Levine, UC Berkeley

Self-Attention

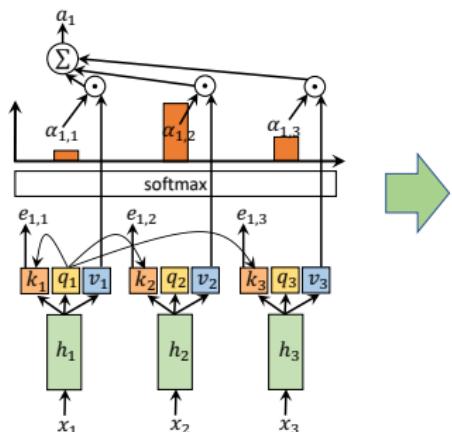
- § At every timestep, self attention takes input and produces an output
- § This can be regarded as a layer



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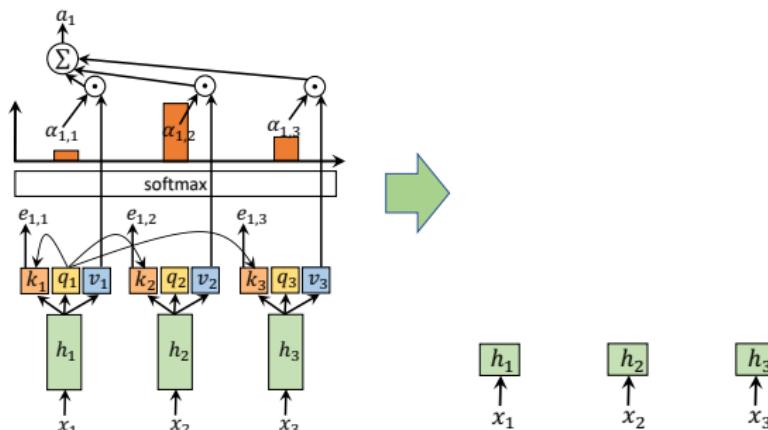


- § We can build an entire network by stacking such layers

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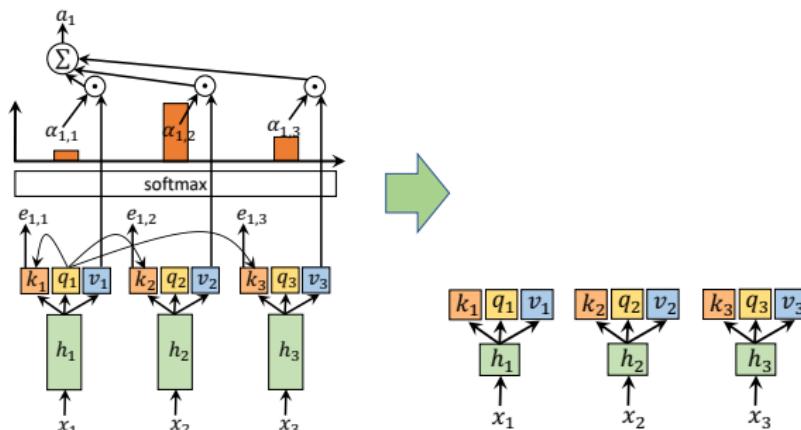


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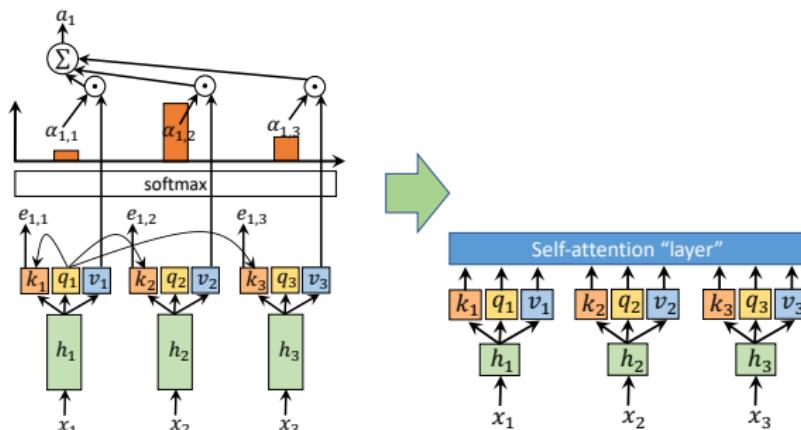


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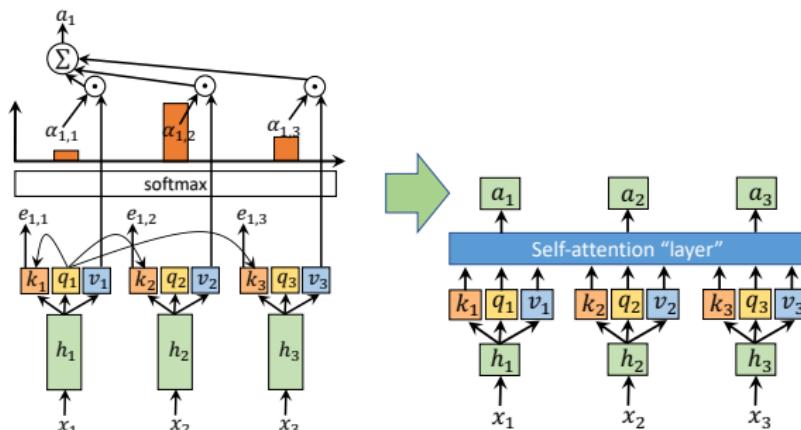


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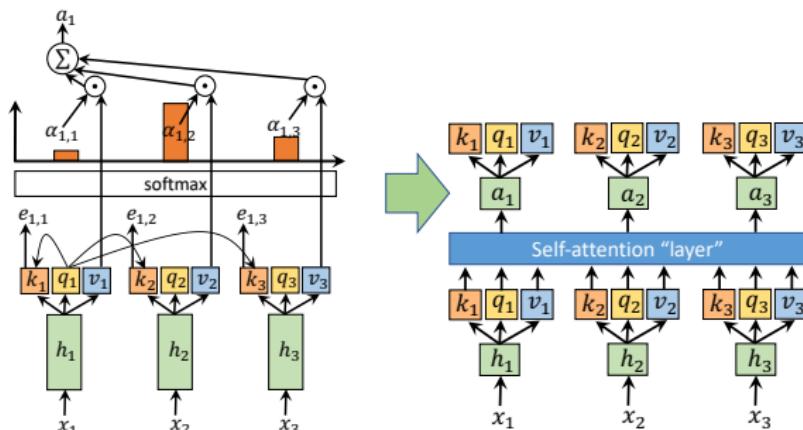


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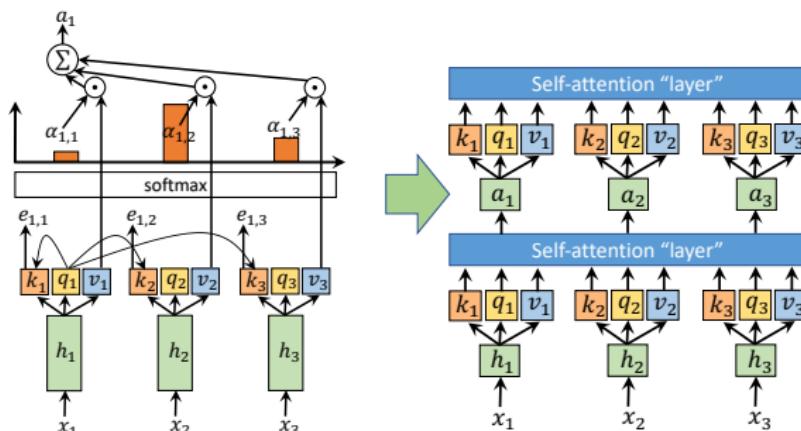


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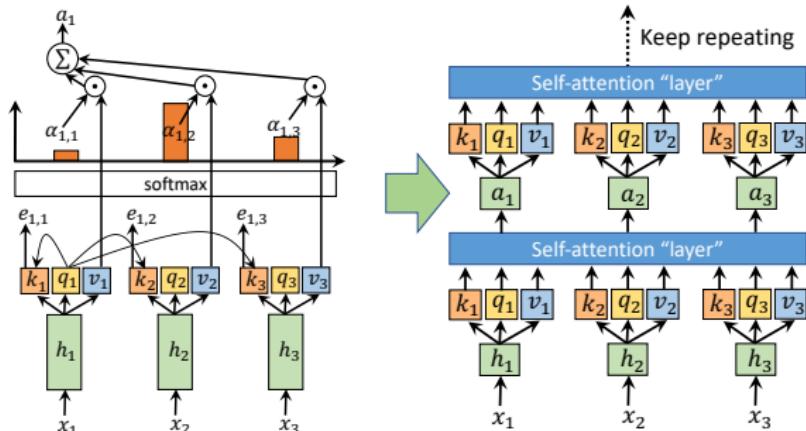


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Source: CS W182 course, Sergey Levine, UC Berkeley

Self-Attention

- § At every timestep, self attention takes input and produces an output
- § This can be regarded as a layer



- § We can build an entire network by stacking such layers
- § This basic idea of getting another sequence from an input sequence is used to get another class of sequence-to-sequence models known as 'Transformers'

Source: CS W182 course, Sergey Levine, UC Berkeley

From Self-Attention to Transformers

- § But to make this actually work, we need to develop a few additional components to address some fundamental limitations

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- ▶ So far, each successive layer is *linear* in the previous one

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From Self-Attention to Transformers

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$$a_l = \sum_t \alpha_{l,t} v_t \text{ where, } v_t = W_v h_t$$
- § Masked decoding
 - ▶ How to prevent attention lookups into the future?

Positional Encoding: What is the Order

§ What we see:

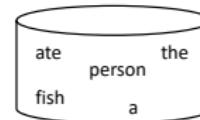
- ▶ the person ate a fish.

Positional Encoding: What is the Order

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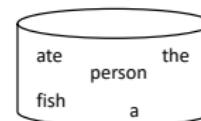


Positional Encoding: What is the Order

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§ Most alternative orderings are nonsense, but some change meaning

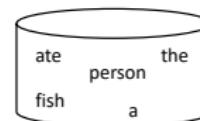
- ▶ the fish ate a person
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Positional Encoding: What is the Order

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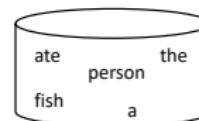
§ In general: the position of words in a sentence carries information!

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§ In general: the position of words in a sentence carries information!

§ Idea: add some information at the beginning that indicates where it is in the sequence!

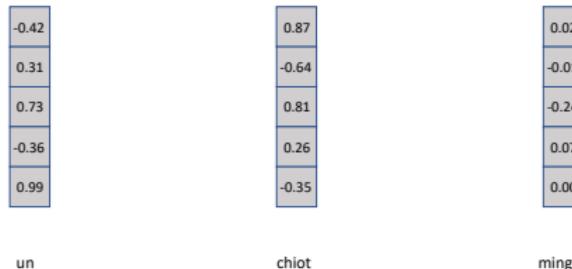
- ▶ $h_t = f(x_t, t)$

Source: CS W182 course, Sergey Levine, UC Berkeley



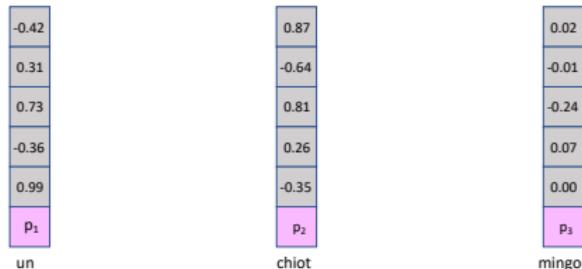
Positional Encoding

- § There are two main ways to provide the model with this information.
- ▶ Concatenating position embedding with word embedding



Positional Encoding

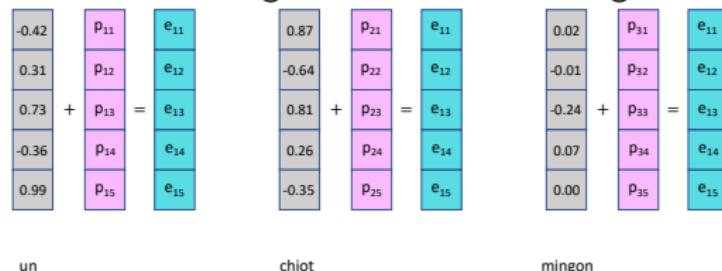
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Positional Encoding

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Positional Encoding

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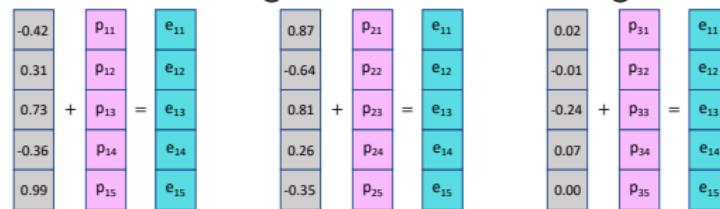
- ▶ Concatenating position embedding with word embedding
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$$\begin{array}{c} \begin{matrix} -0.42 & p_{11} & e_{11} \\ 0.31 & p_{12} & e_{12} \\ 0.73 & + p_{13} = e_{13} \\ -0.36 & p_{14} & e_{14} \\ 0.99 & p_{15} & e_{15} \end{matrix} & \begin{matrix} 0.87 & p_{21} & e_{11} \\ -0.64 & p_{22} & e_{12} \\ 0.81 & + p_{23} = e_{13} \\ 0.26 & p_{24} & e_{14} \\ -0.35 & p_{25} & e_{15} \end{matrix} & \begin{matrix} 0.02 & p_{31} & e_{11} \\ -0.01 & p_{32} & e_{12} \\ -0.24 & + p_{33} = e_{13} \\ 0.07 & p_{34} & e_{14} \\ 0.00 & p_{35} & e_{15} \end{matrix} \end{array}$$

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Positional Encoding

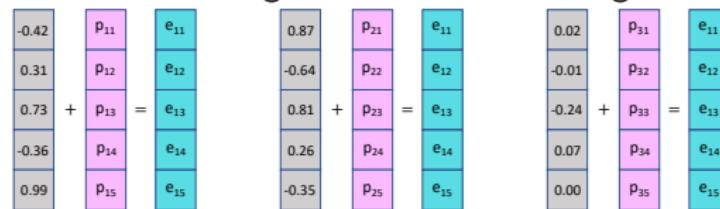
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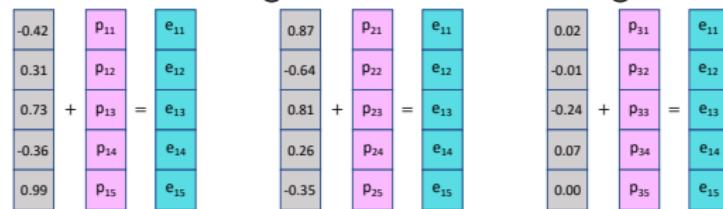
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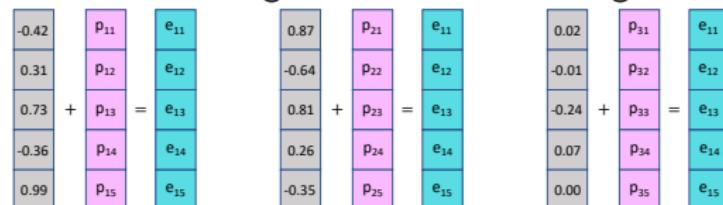
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Positional Encoding

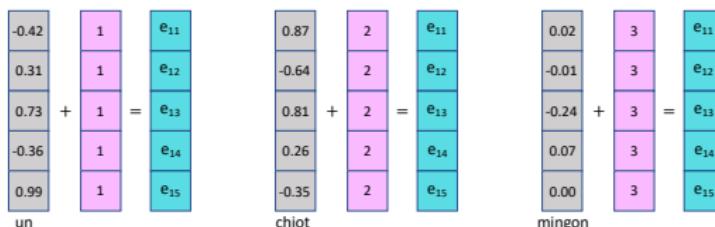
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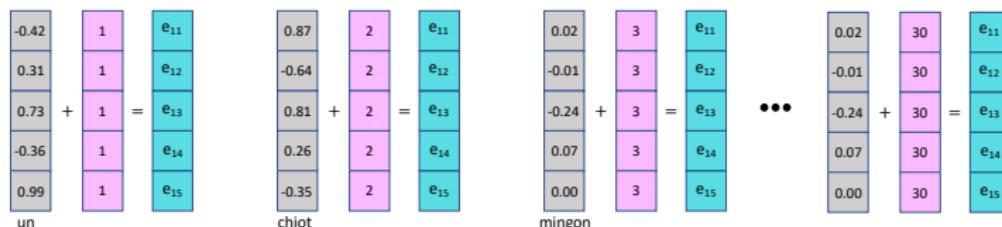
Positional Encoding - Addition

§ Adding the word positions as all dimensions of position embedding



Positional Encoding - Addition

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§ However, adding word positions can significantly distort semantic positions of the words, especially for words in long sentences.

Positional Encoding - Addition

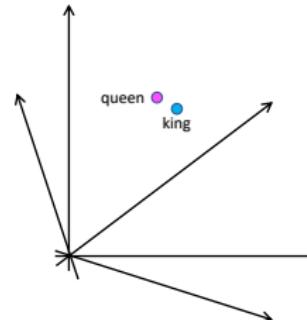
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•••

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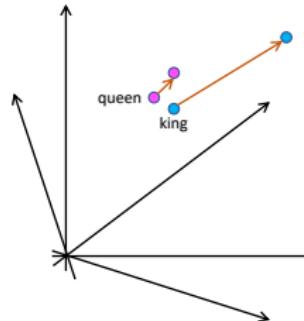
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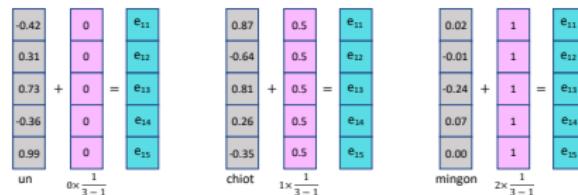
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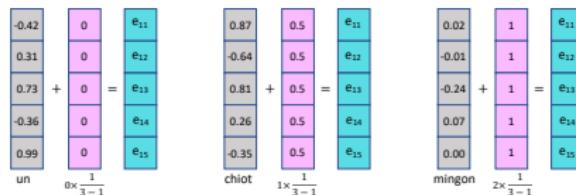
Positional Encoding - Addition

§ How about adding fractions only. The added values will never surpass 1



Positional Encoding - Addition

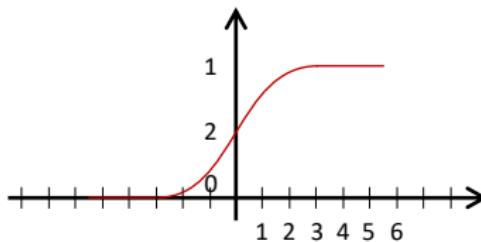
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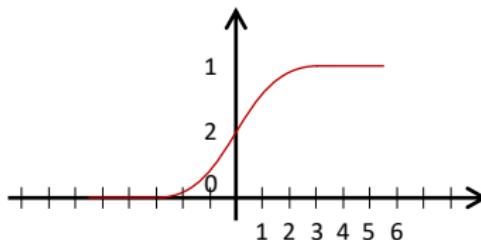
Positional Encoding - Addition

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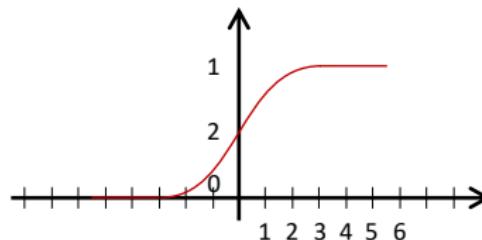
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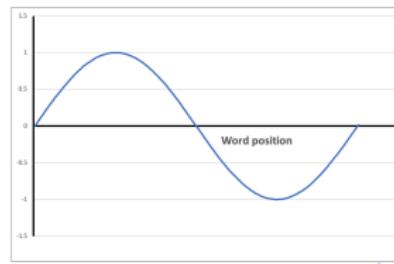
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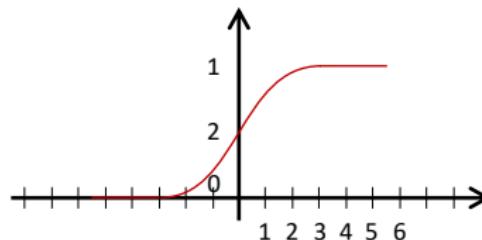


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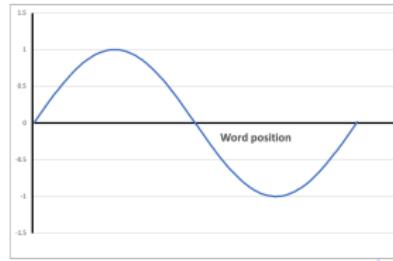
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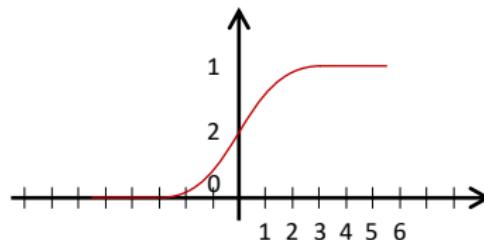
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What is a basic problem?



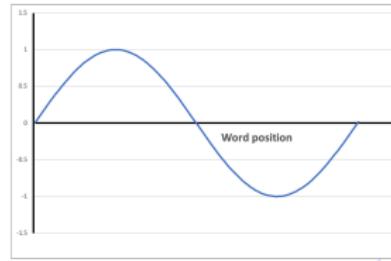
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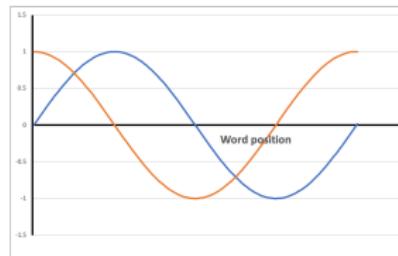
What is a basic problem?



Different positions may have same encoding

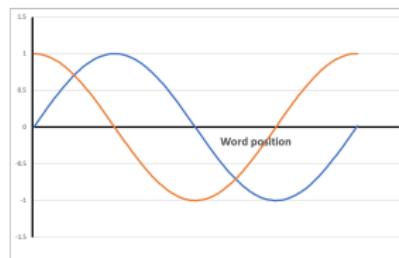
Positional Encoding - Multiple Sinusoids

§ Fix: use a cosine also (with same frequency)



Positional Encoding - Multiple Sinusoids

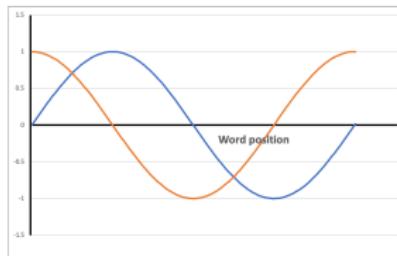
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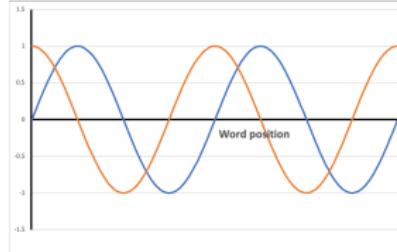
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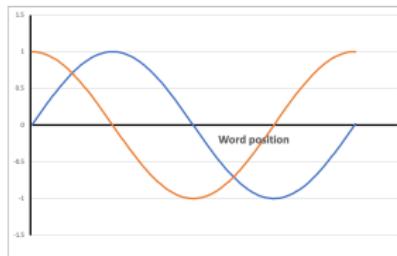


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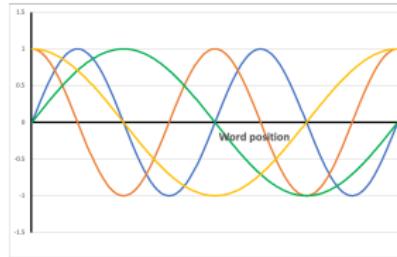


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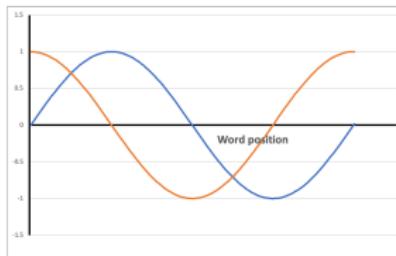


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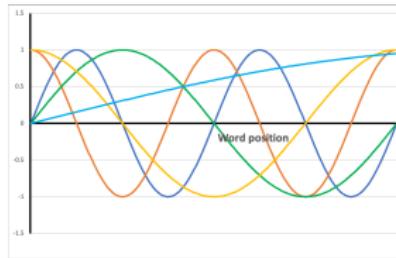


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Positional Encoding - sine-cosine Embedding

$$\mathbf{p}_t = \begin{bmatrix} p_t^{(1)} \\ p_t^{(2)} \\ p_t^{(3)} \\ p_t^{(4)} \\ \vdots \\ p_t^{(d-1)} \\ p_t^{(d)} \end{bmatrix} = \begin{bmatrix} \sin(\omega_1 t) \\ \cos(\omega_1 t) \\ \sin(\omega_2 t) \\ \cos(\omega_2 t) \\ \vdots \\ \sin(\omega_{d/2} t) \\ \cos(\omega_{d/2} t) \end{bmatrix}$$

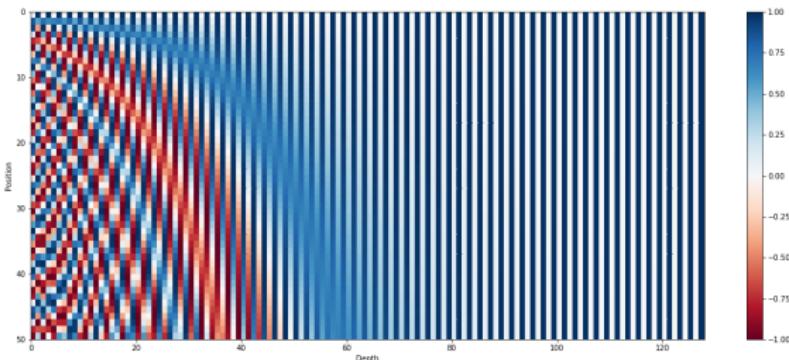
$$p_t^{(i)} = \begin{cases} \sin(\omega_k t), & \text{if } i = 2k - 1 \\ \cos(\omega_k t), & \text{if } i = 2k \end{cases}$$

$$\omega_k = \frac{1}{10000^{2k/d}}$$

k varies from 0 to $\frac{d}{2}$

Positional Encoding - sine-cosine Embedding

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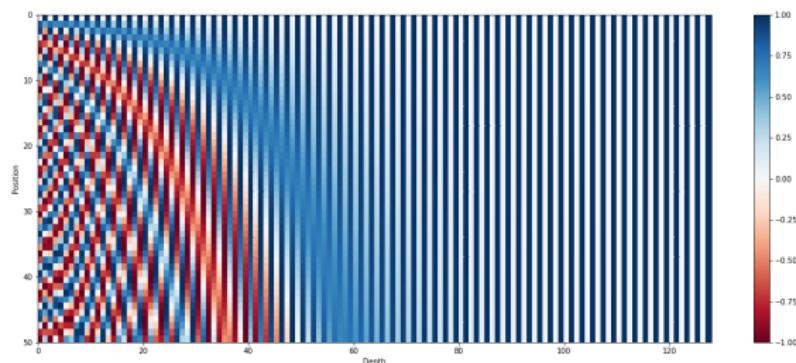
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§ Learnable positional encoding is sometimes also used

From Self-Attention to Transformers

§ But to make this actually work, we need to develop a few additional components to address some fundamental limitations

§ Positional encoding

- ▶ Addresses lack of sequence information

§ Multiheaded attention

- ▶ allows querying multiple positions at each layer

§ Adding nonlinearities

- ▶ So far, each successive layer is *linear* in the previous one

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§ Masked decoding

- ▶ How to prevent attention lookups into the future?

Multi-Head Attention

§ Since we are relying entirely on attention now, we might want to 'query to' different timestep.

Multi-Head Attention

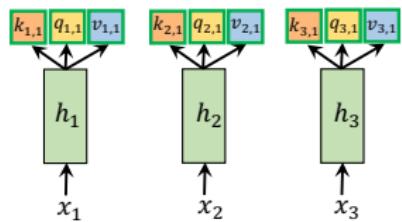
- § Since we are relying entirely on attention now, we might want to ‘query to’ different timestep.
- § A sentence like “The animal didn’t cross the street because it was too tired”, we would want to know
 - ▶ If “animal” refers to “it”
 - ▶ If it is the “animal” who didn’t “cross”

Multi-Head Attention

§ Idea: have multiple keys, queries, and values for every time step!

Multi-Head Attention

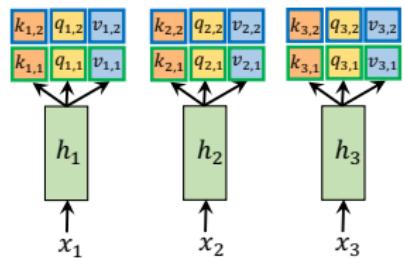
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Source: CS W182 course, Sergey Levine, UC Berkeley

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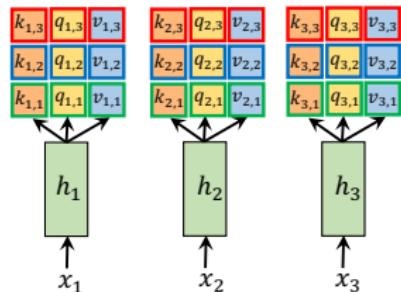
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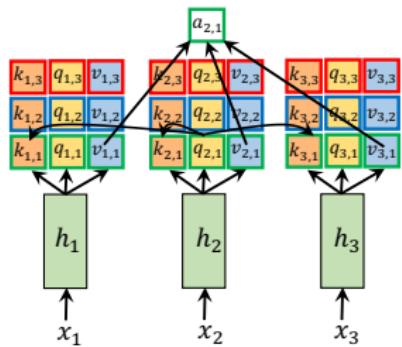
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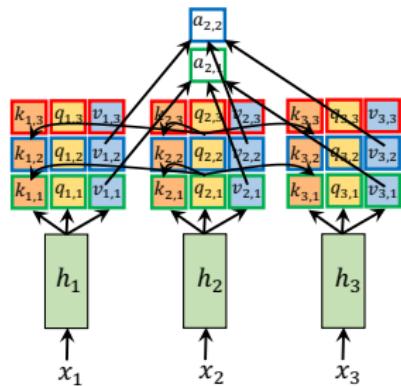
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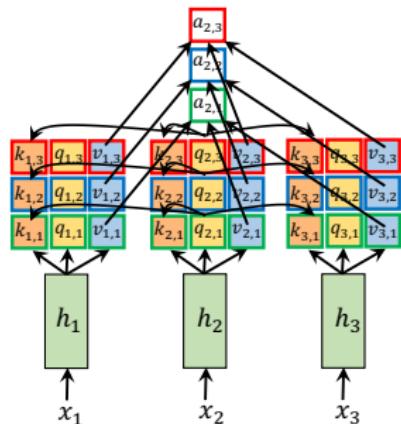


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Multi-Head Attention

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§ Compute weights *independently* for each head

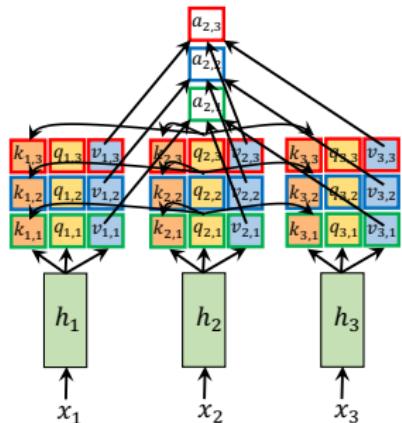


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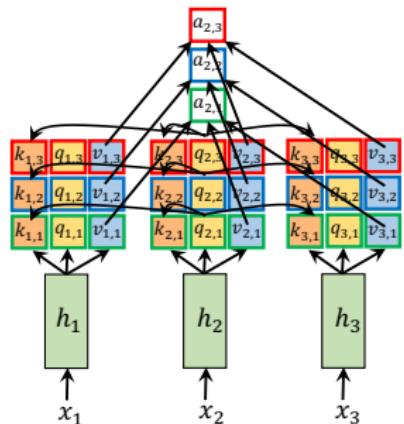
§ Compute weights *independently* for each head
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Multi-Head Attention

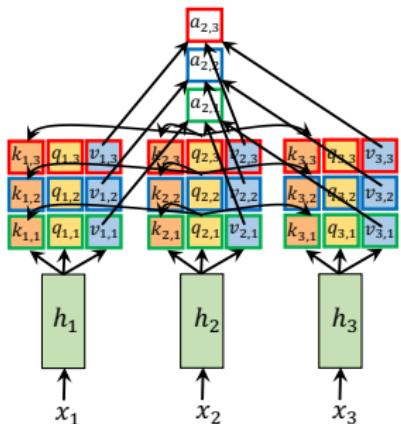
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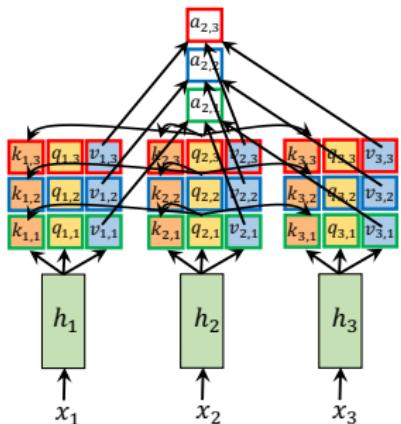


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$$a_2 = \begin{bmatrix} a_{2,1} \\ a_{2,2} \\ a_{2,3} \end{bmatrix}$$

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- § Around 8 heads per layer tend to work well.

From Self-Attention to Transformers

§ But to make this actually work, we need to develop a few additional components to address some fundamental limitations

§ Positional encoding

- ▶ Addresses lack of sequence information

§ Multiheaded attention

- ▶ allows querying multiple positions at each layer

§ Adding nonlinearities

- ▶ So far, each successive layer is *linear* in the previous one

$$a_l = \sum_t \alpha_{l,t} v_t \text{ where, } v_t = W_v h_t$$

§ Masked decoding

- ▶ How to prevent attention lookups into the future?

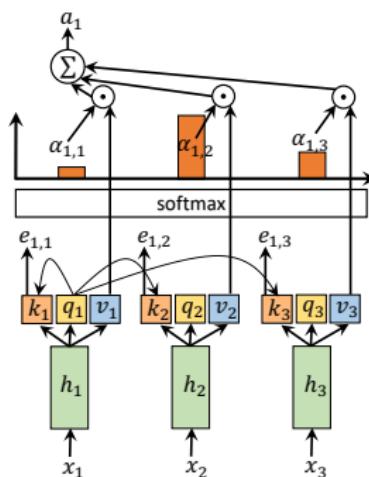
Self-Attention is Linear

$$\S \quad k(h_t) = W_k h_t, \quad q(h_t) = W_q h_t, \quad v(h_t) = W_v h_t$$

$$e_{l,t} = q_l \cdot k_t$$

$$\alpha_{l,t} = \frac{\exp(e_{l,t})}{\sum_{t'} \exp(e_{l,t'})}$$

$$a_l = \sum_t \alpha_{l,t} v_t = \sum_t \alpha_{l,t} W_v h_t = W_v \sum_t \alpha_{l,t} h_t$$

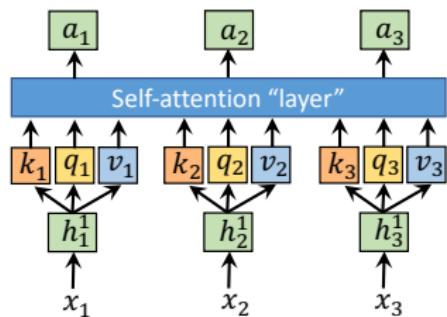


§ We make a non-linear choice of the timesteps we need to attend to, but we combine the linear transformations of those timesteps

§ Every self-attention “layer” is a linear transformation of the previous layer (with nonlinear weights)

S In many situations, This is not very expressive

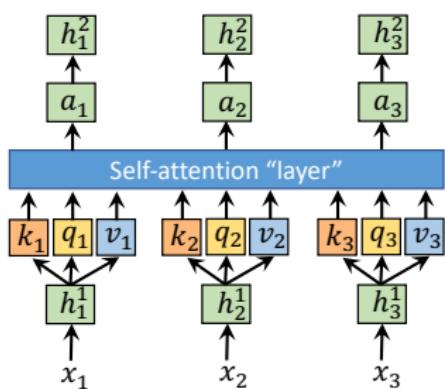
Alternating Self-Attention and Non-Linearity



Source: CS W182 course, Sergey Levine, UC Berkeley

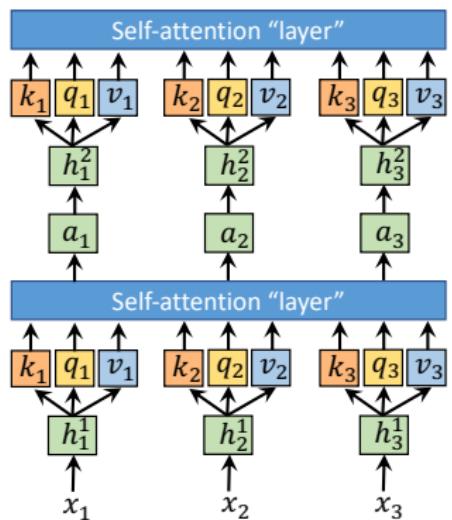
Alternating Self-Attention and Non-Linearity

- § Some *learnable* non-linear function e.g.,
$$h_t^l = \sigma(W^l a_t + b^l)$$
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- § Referred to as “position-wise feedforward network”

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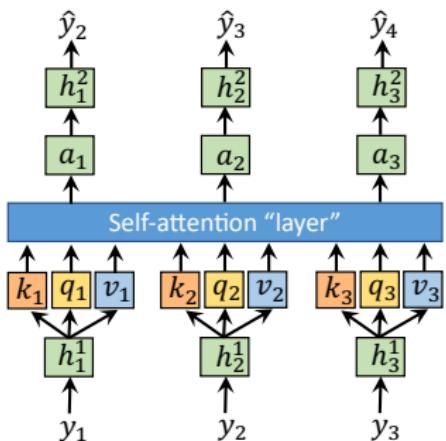
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Self-Attention Can See the Future

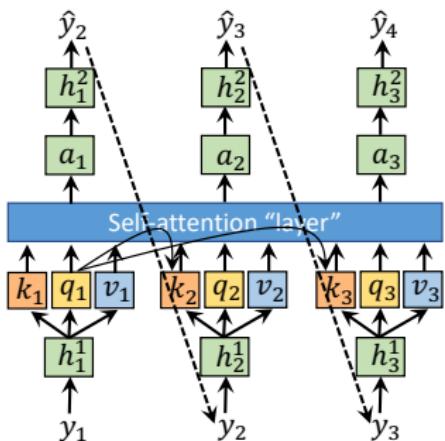
- § Nothing is preventing a crude self-attention ‘language model’ to look into the future
- § (In reality, we have many alternating self-attention layers and position-wise feedforward networks, not just one)



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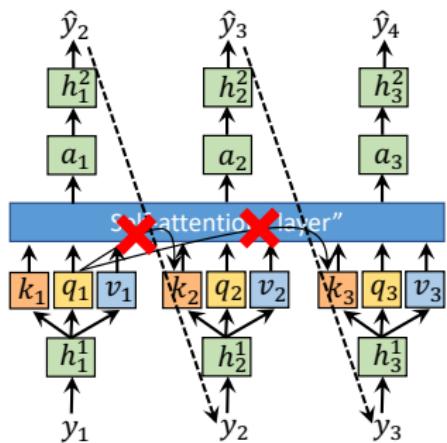
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- § Easy solution of this circular dependency:

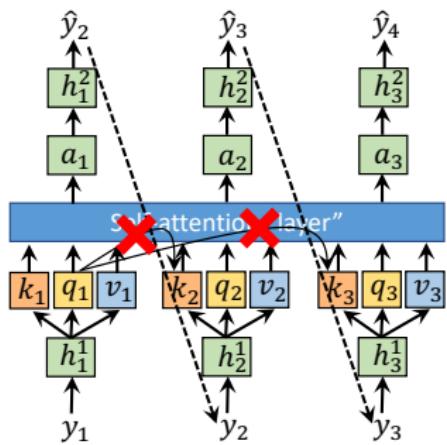
$$\underline{e_{l,t} = q_l \cdot k_t}$$

$$e_{l,t} = \begin{cases} q_l \cdot k_t, & \text{if } l \geq t \\ -\infty, & \text{otherwise} \end{cases}$$

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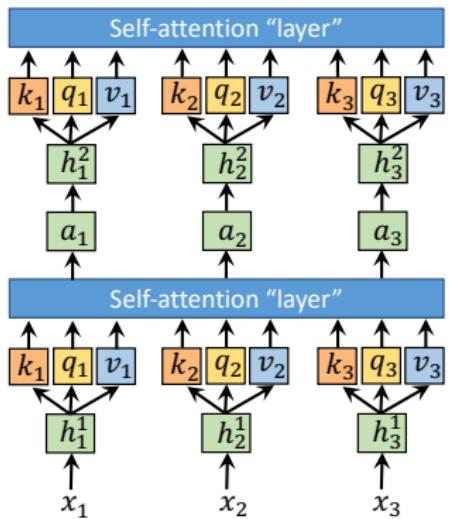
$$e_{l,t} = \begin{cases} q_l \cdot k_t, & \text{if } l \geq t \\ -\infty, & \text{otherwise} \end{cases}$$

- § In practice: Just replace $\exp(e_{l,t})$ with 0 if $l < t$ inside the softmax

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Transformer

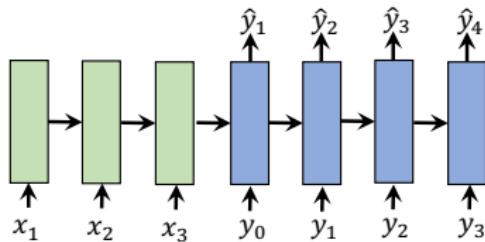
- § We will combine the pieces that we learnt to get the classic Transformer model
- § There are a number of model designs that use successive self-attention and position-wise nonlinear layers to process sequences
- § These are generally called “Transformers” because they transform one sequence into another at each layer
 - See Vaswani *et al.* **Attention Is All You Need**, NeurIPS 2017
- § The “classic” transformer (Vaswani *et al.* 2017) is a sequence to sequence model.
- § A number of well-known follow works also use transformers for language modeling (BERT, GPT, etc.)



Source: CS W182 course, Sergey Levine, UC Berkeley

The “Classic” Transformer

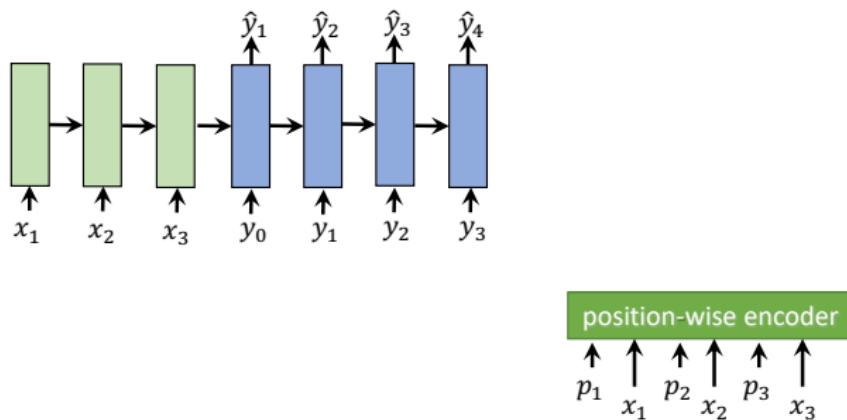
§ As compared to a sequence to sequence RNN model



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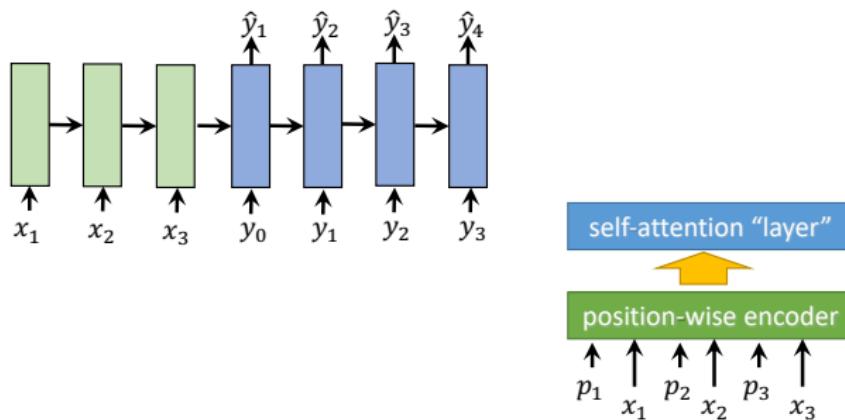
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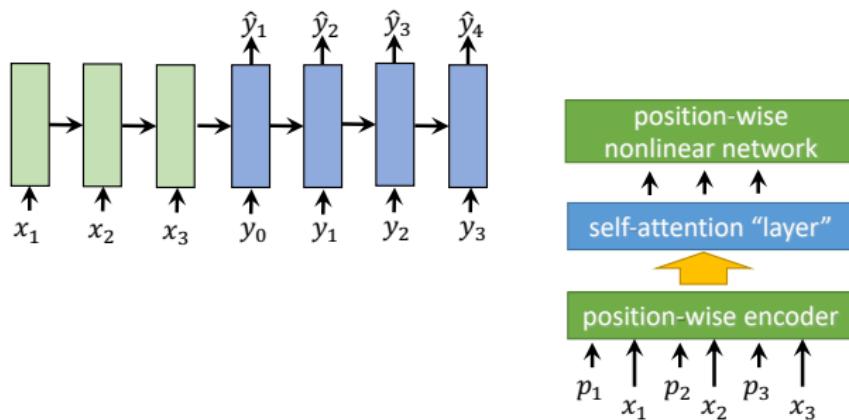
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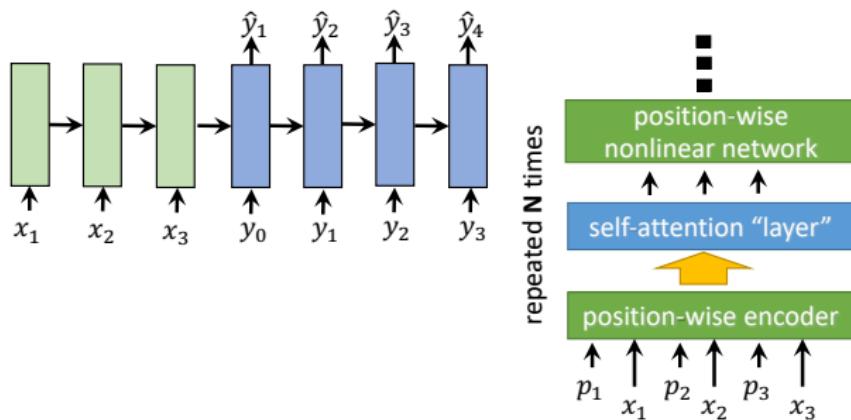
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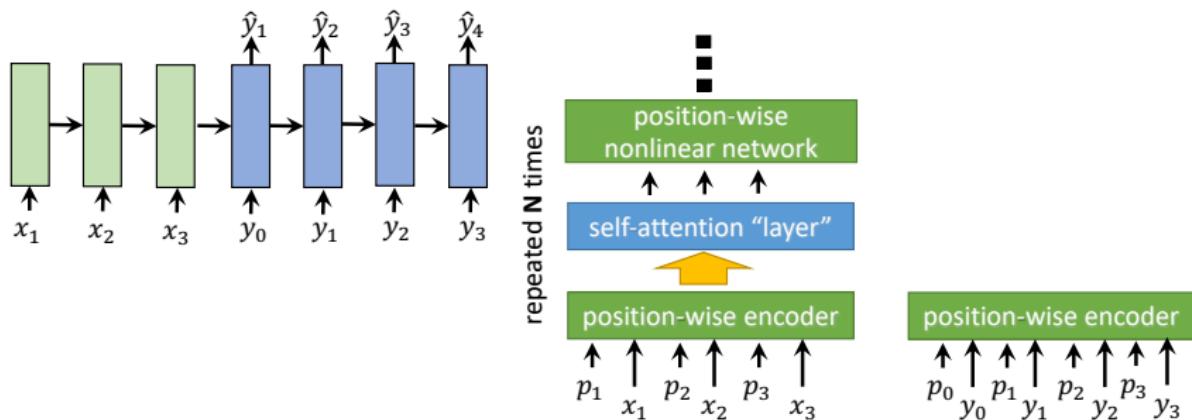
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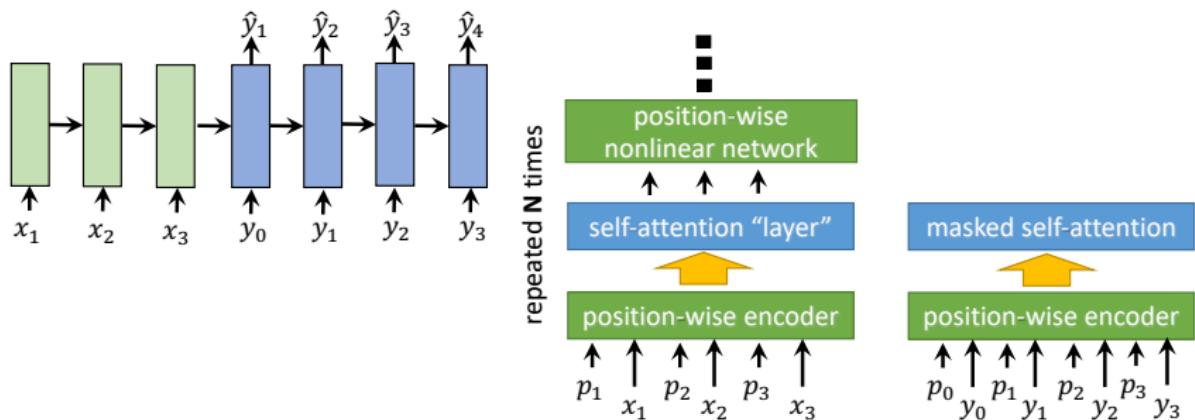
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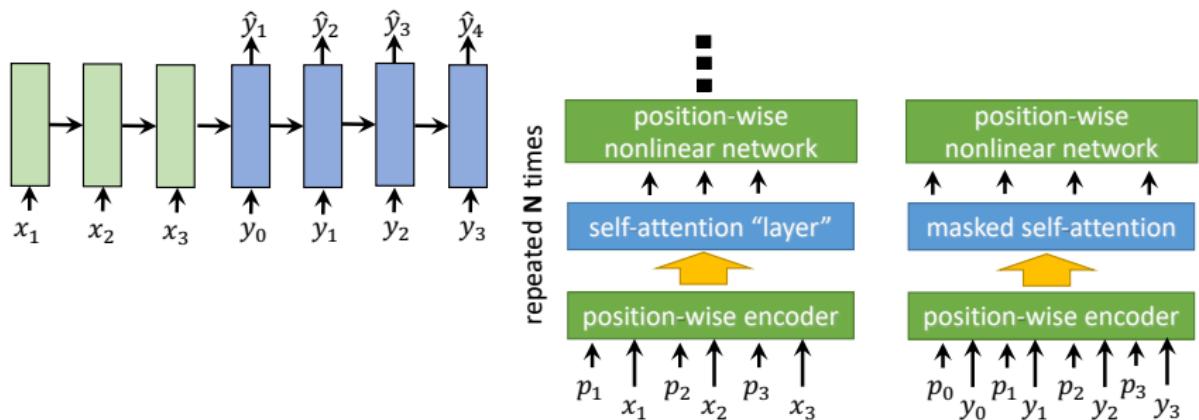
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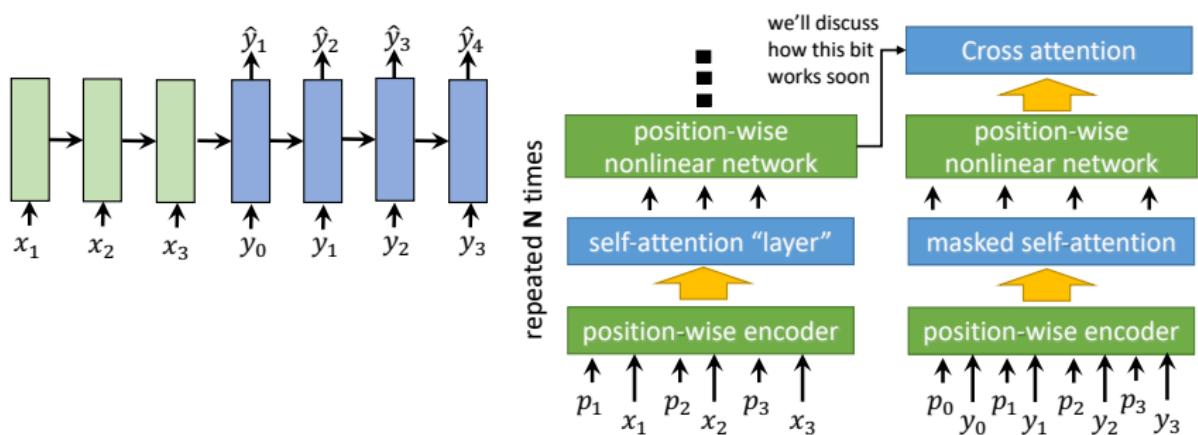
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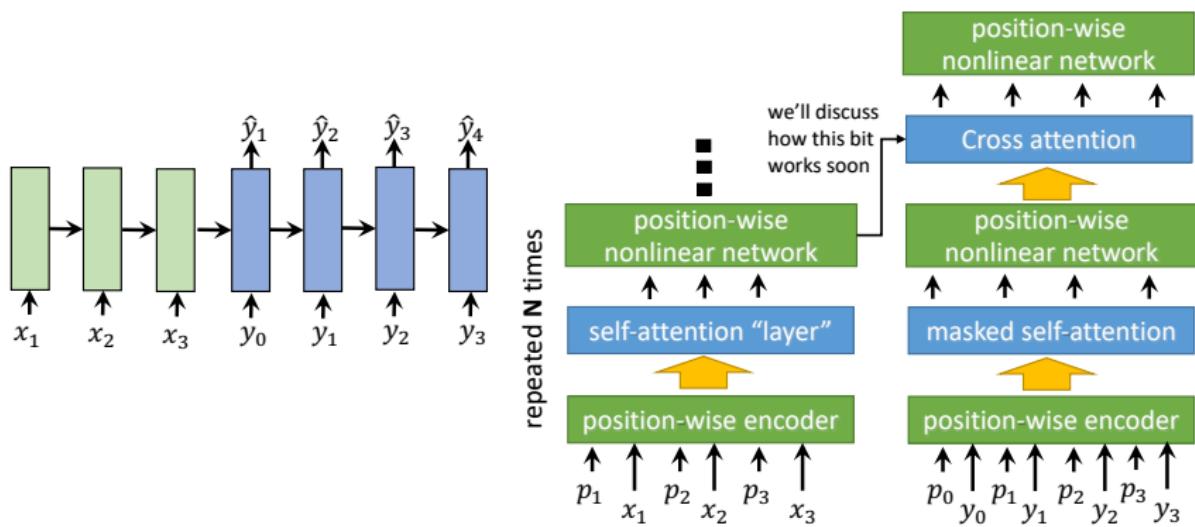
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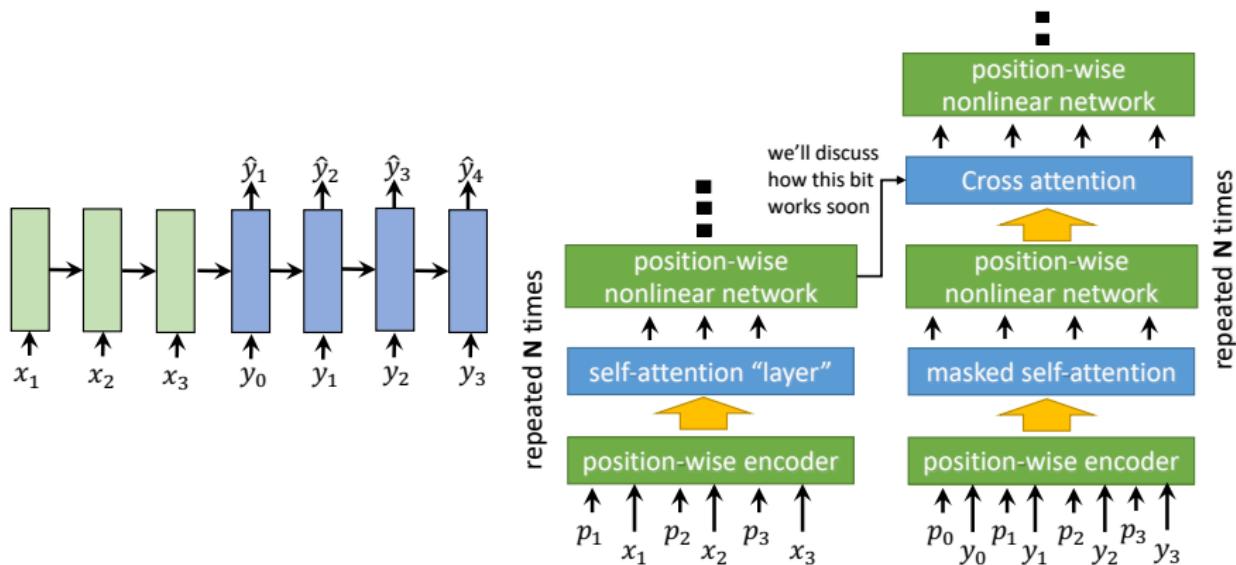
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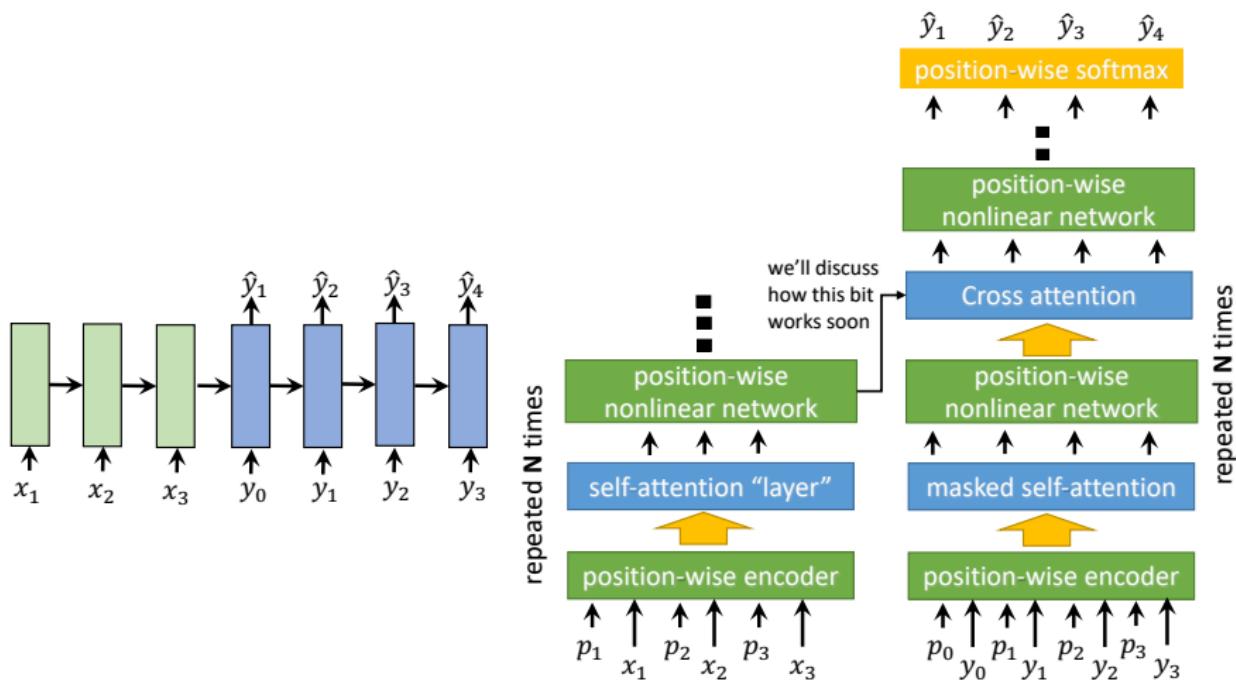
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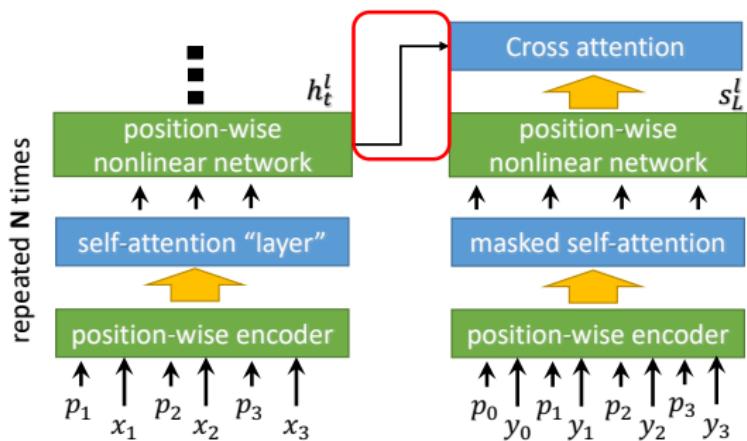
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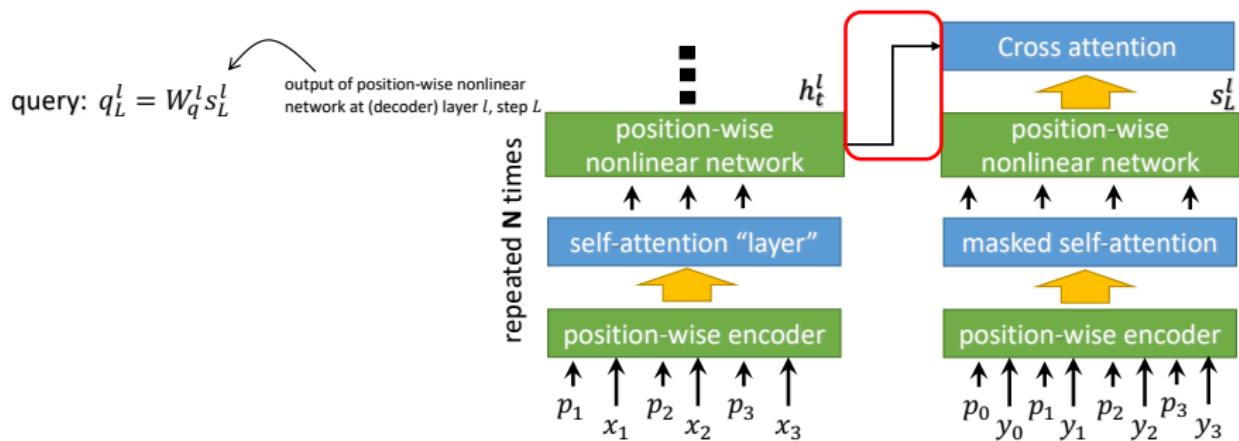
Cross-Attention: Combining Encoder and Decoder Values

§ Much like the standard attention



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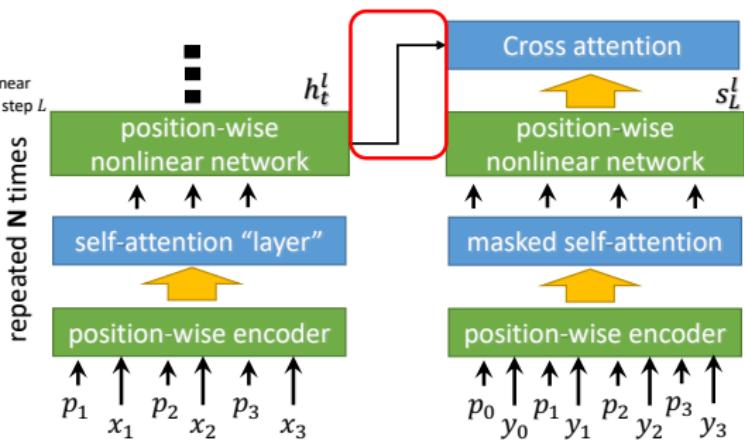
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Cross-Attention: Combining Encoder and Decoder Values

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$$\begin{aligned} \text{query: } q_L^l &= W_q^l s_L^l && \text{output of position-wise nonlinear network at (decoder) layer } l, \text{ step } L \\ \text{key: } k_t^l &= W_k^l h_t^l && \text{output of position-wise nonlinear network at (encoder) layer } l, \text{ step } t \\ \text{value: } v_t^l &= W_v^l h_t^l && \end{aligned}$$



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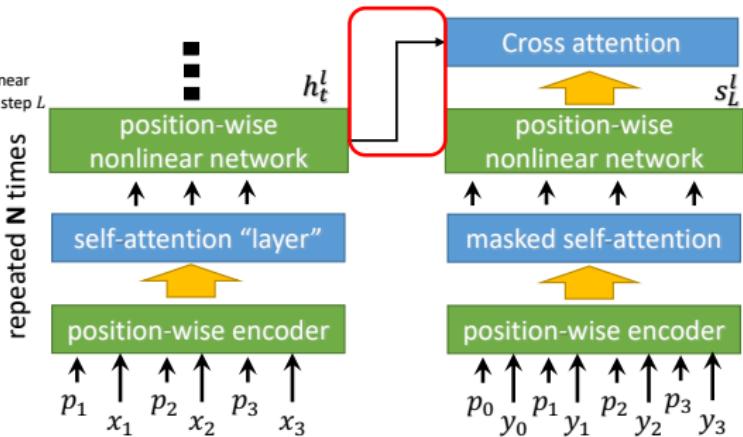
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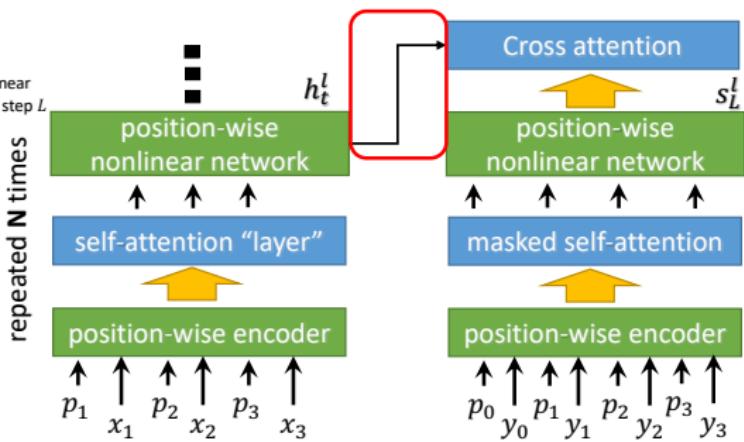
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 c_L^l &= \sum_t \alpha_{L,t}^l v_t^l && \text{cross attention output}
 \end{aligned}$$



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Cross-Attention: Combining Encoder and Decoder Values

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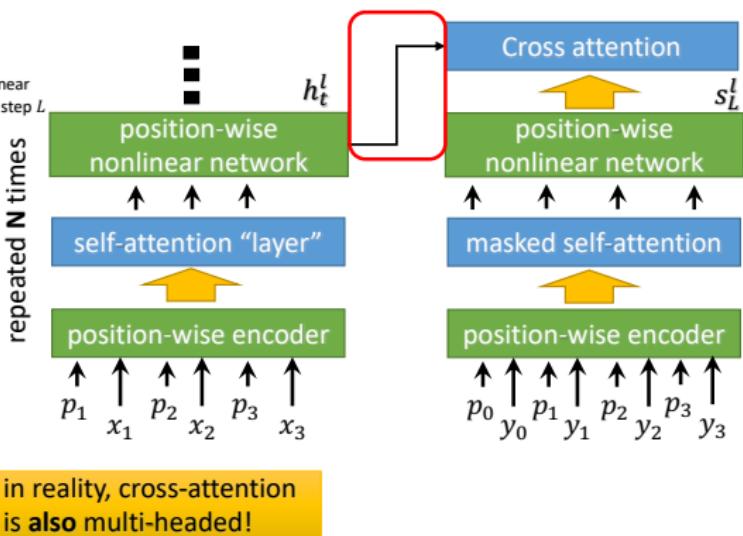
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 repeated N times



One Last Detail: Layer Normalization

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$$\sigma = \sqrt{\frac{1}{B} \sum_{i=1}^B (a_i - \mu)^2}$$

$$\bar{a}_i = \frac{a_i - \mu}{\sigma} \gamma + \beta$$

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One d dimensional vector a

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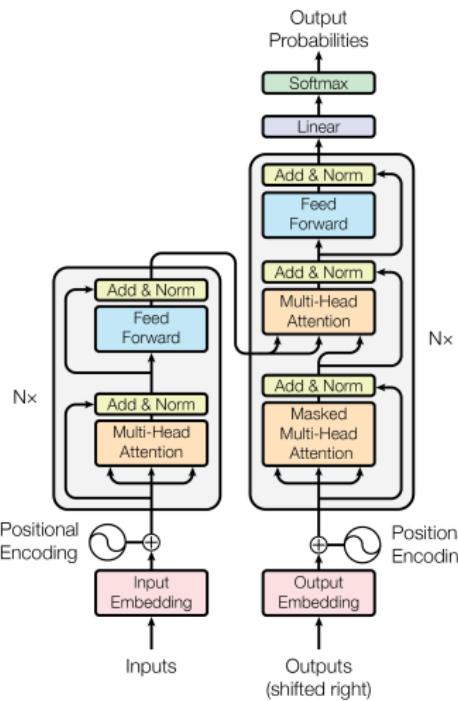
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Putting it all together

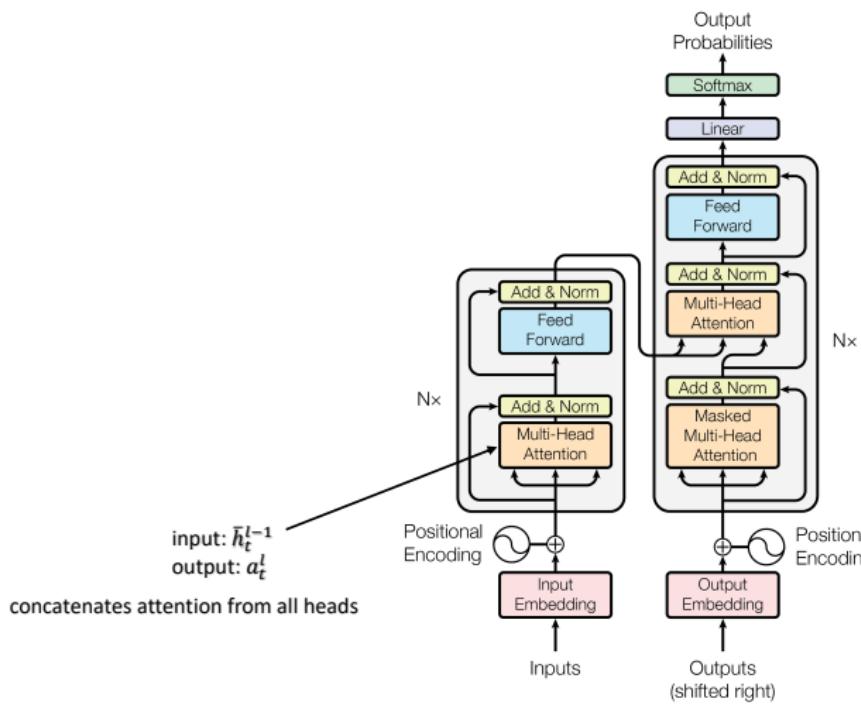
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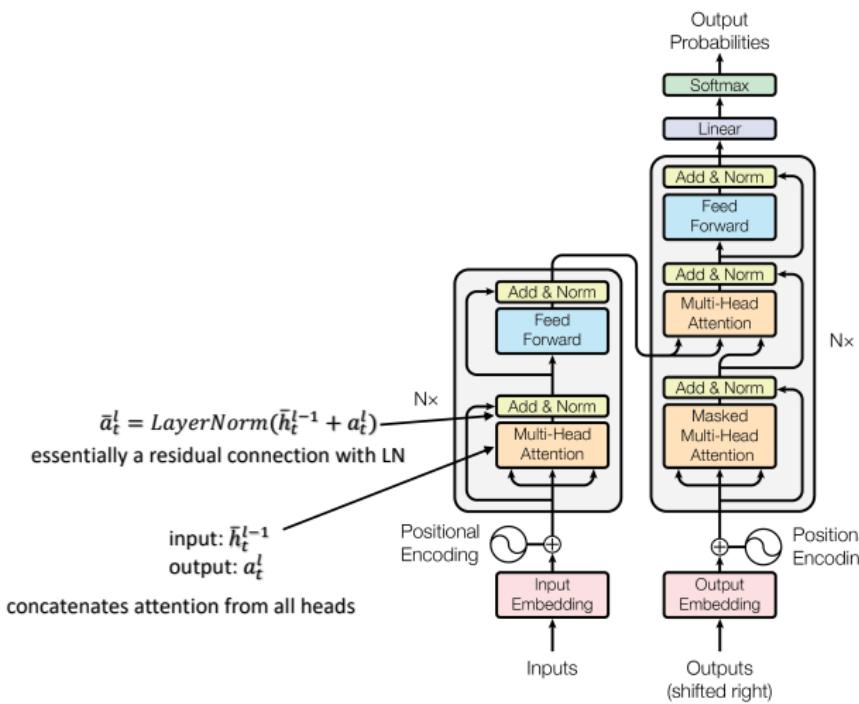
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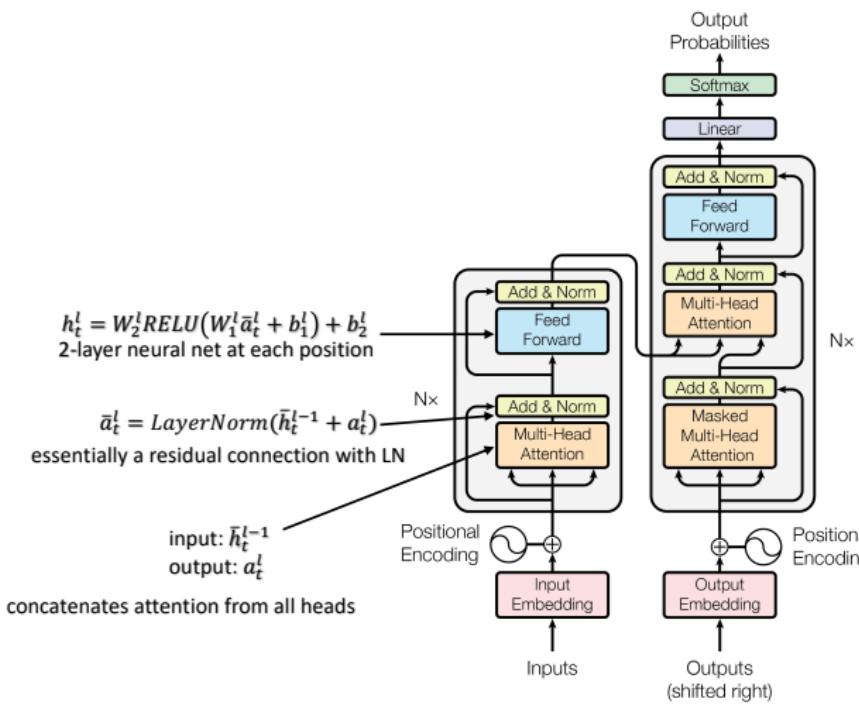
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Source: CS W182 course, Sergey Levine, UC Berkeley

Putting it all together

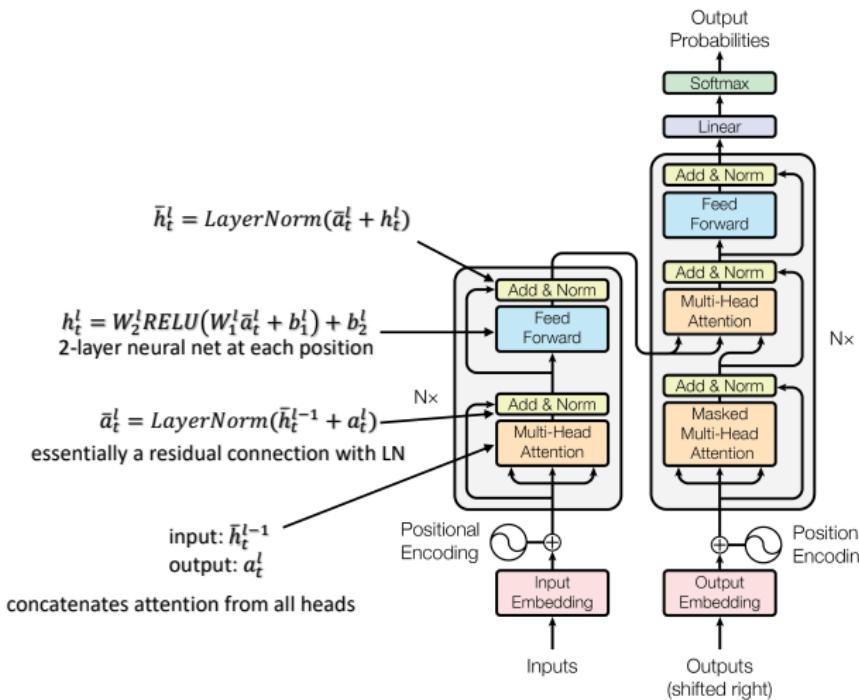
§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017



Source: CS W182 course, Sergey Levine, UC Berkeley

Putting it all together

§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017

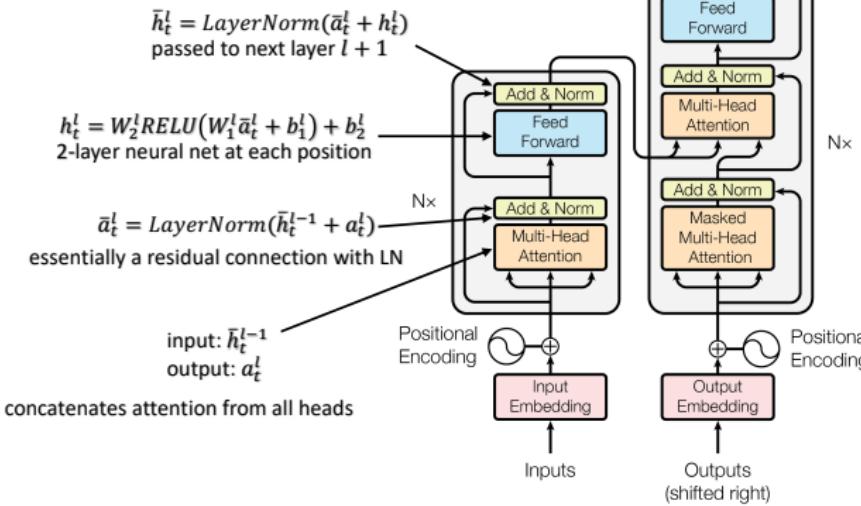


Source: CS W182 course, Sergey Levine, UC Berkeley

Putting it all together

§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017

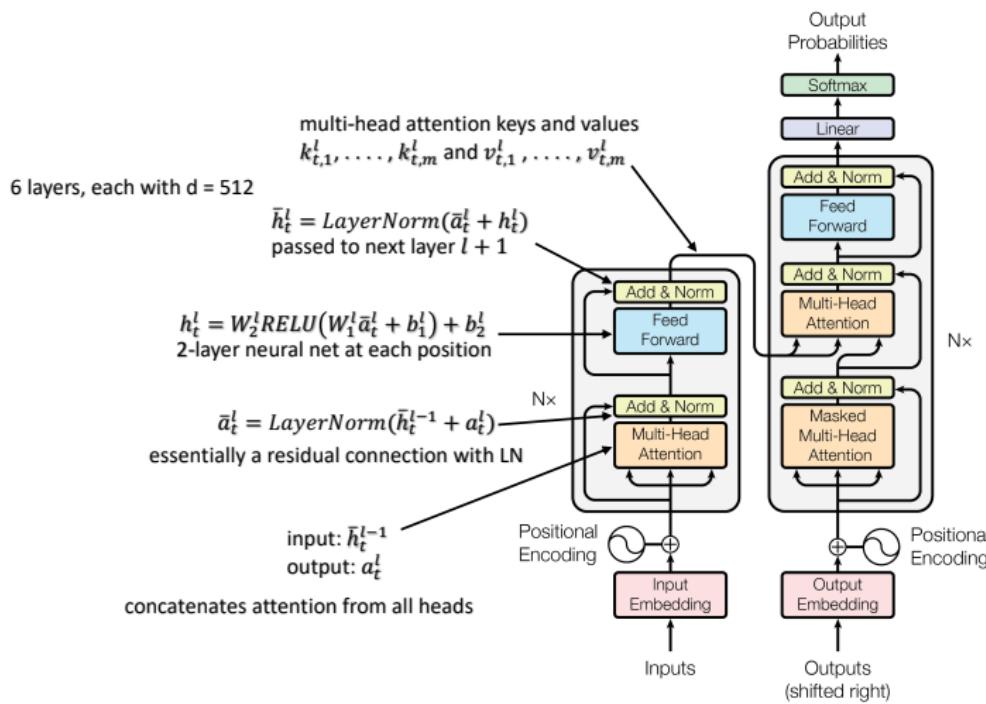
6 layers, each with $d = 512$



Source: CS W182 course, Sergey Levine, UC Berkeley

Putting it all together

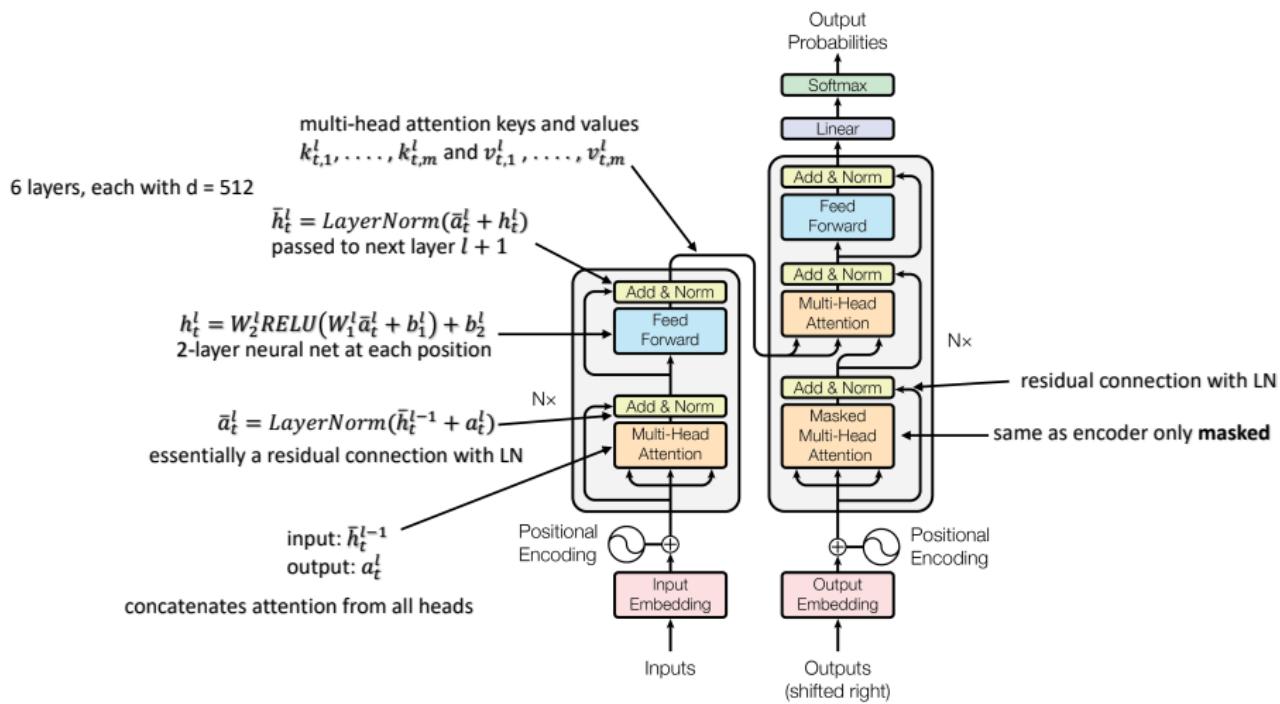
§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017



Source: CS W182 course, Sergey Levine, UC Berkeley

Putting it all together

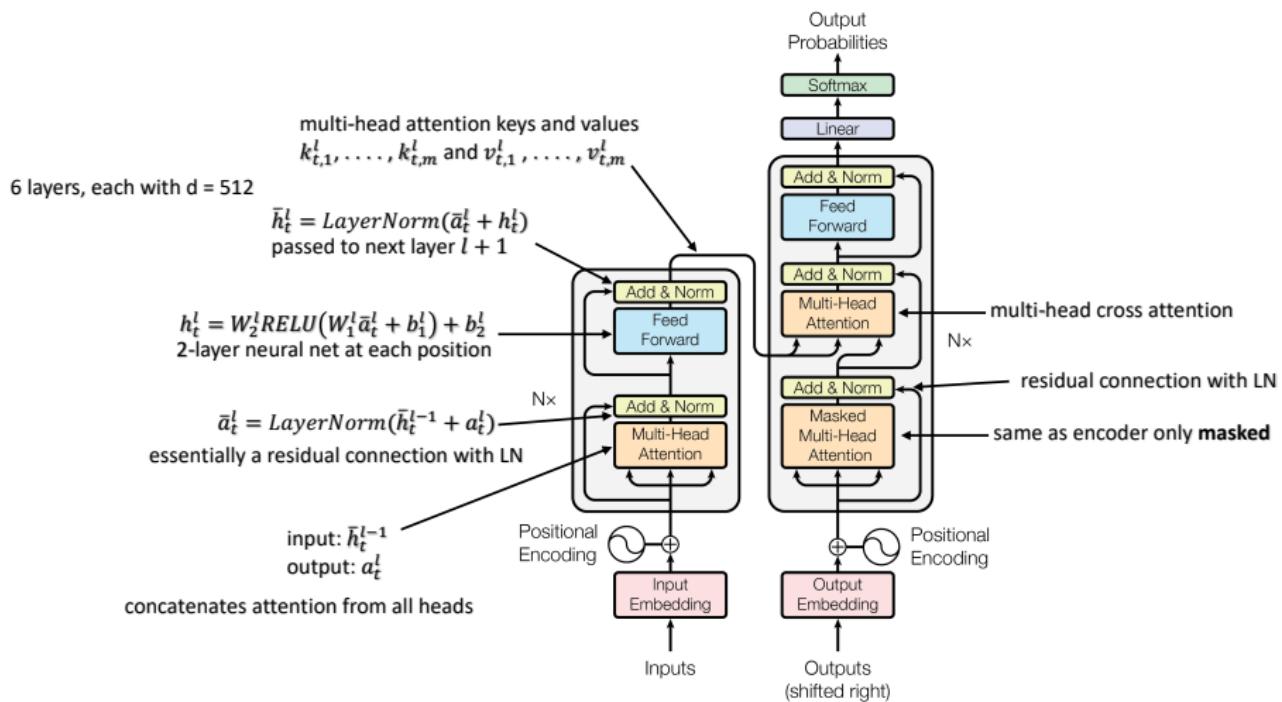
§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017



Source: CS W182 course, Sergey Levine, UC Berkeley

Putting it all together

§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017

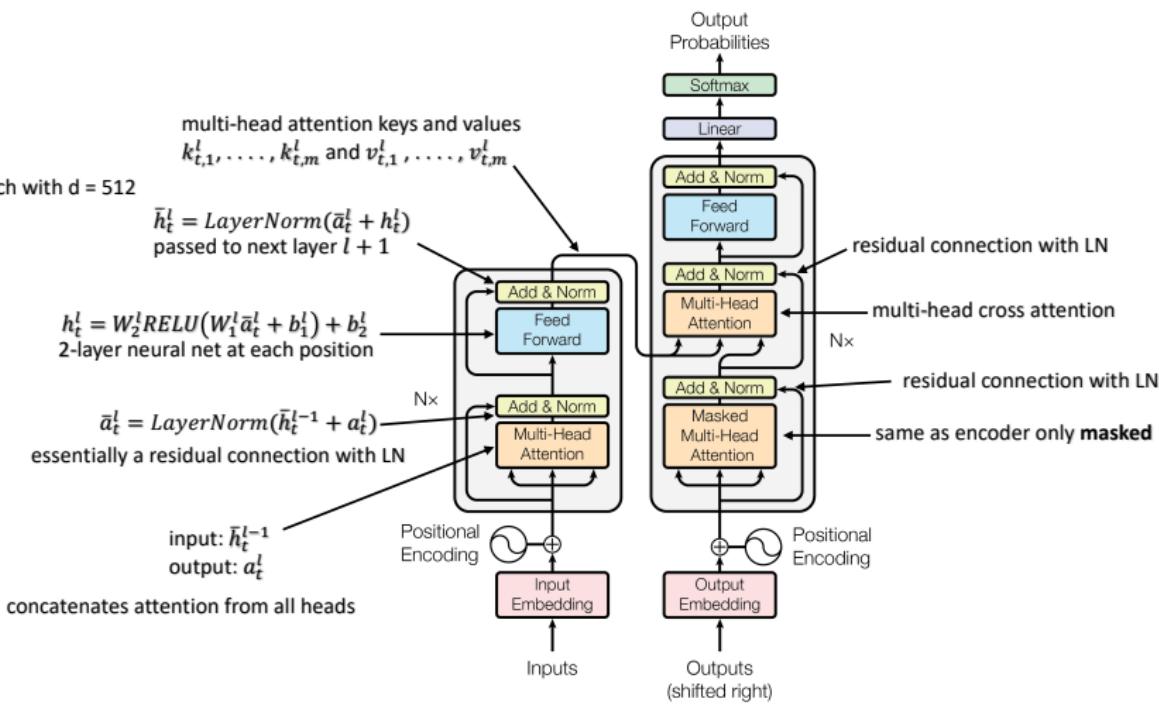


Source: CS W182 course, Sergey Levine, UC Berkeley

Putting it all together

§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017

6 layers, each with $d = 512$

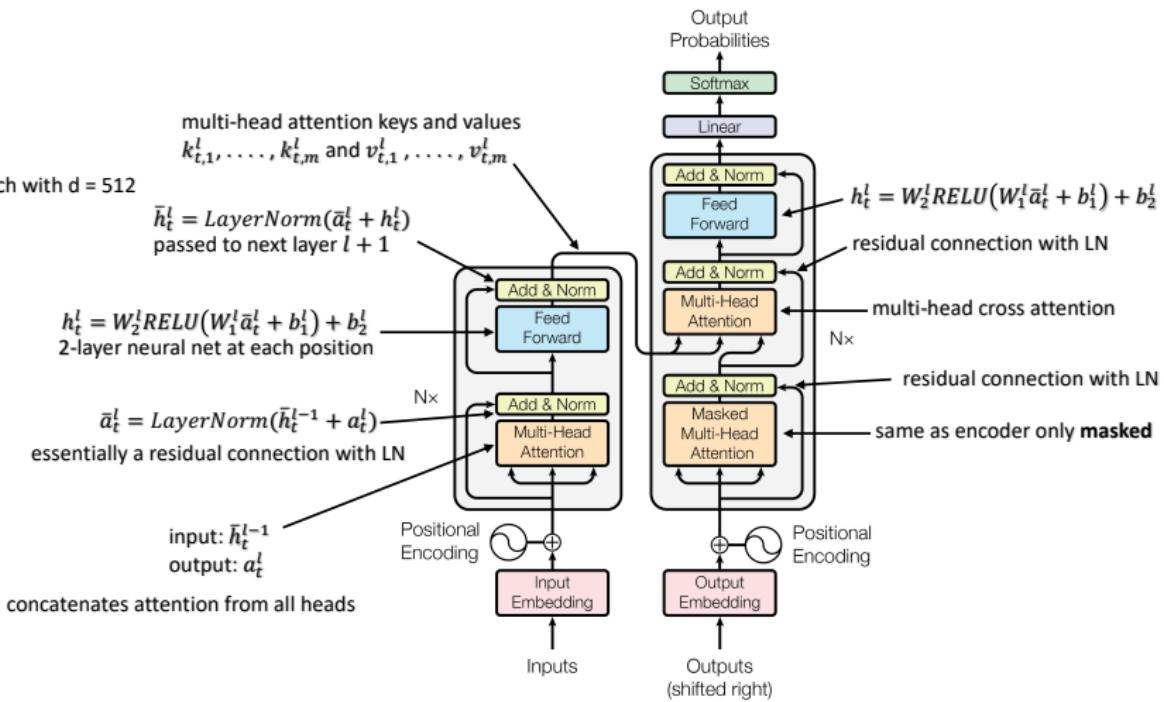


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Putting it all together

§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017

6 layers, each with $d = 512$

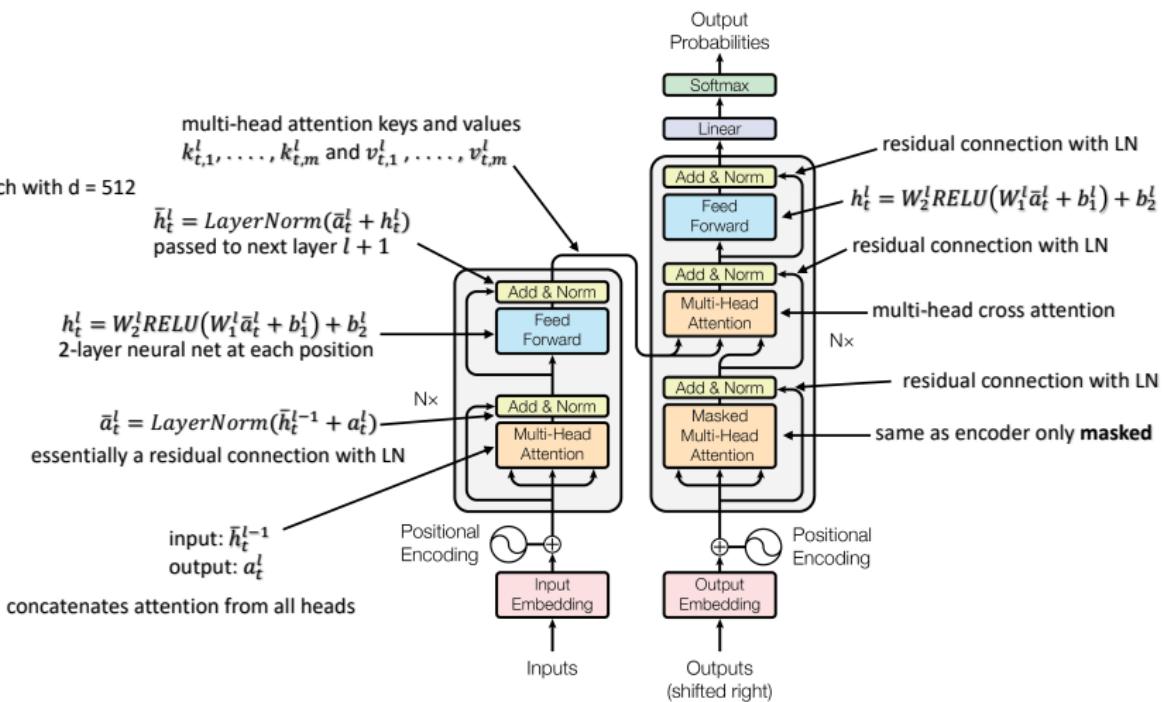


Source: CS W182 course, Sergey Levine, UC Berkeley

Putting it all together

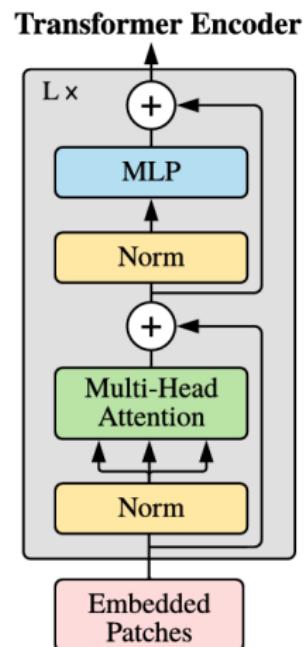
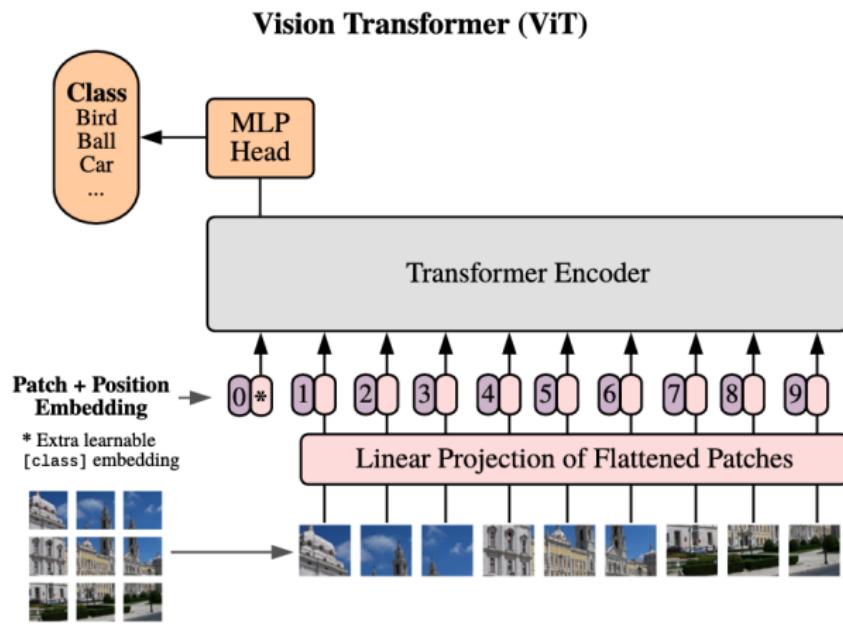
§ The Transformer from Vaswani et al. 'Attention Is All You Need', NeurIPS, 2017

6 layers, each with $d = 512$



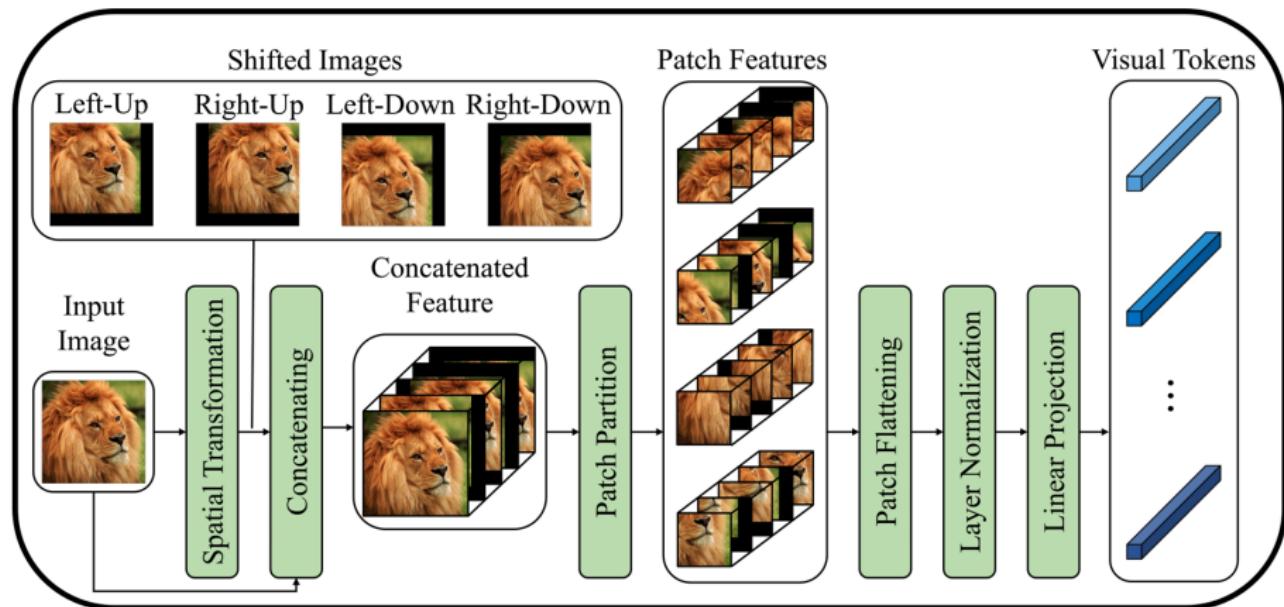
Source: CS W182 course, Sergey Levine, UC Berkeley

Vision Transformer: ViT



Source: An Image is Worth 16×16 Words: Transformers for Image Recognition at Scale

ViT for Small-Size Datasets



(a) Shifted Patch Tokenization

Source: Vision Transformer for Small-Size Datasets