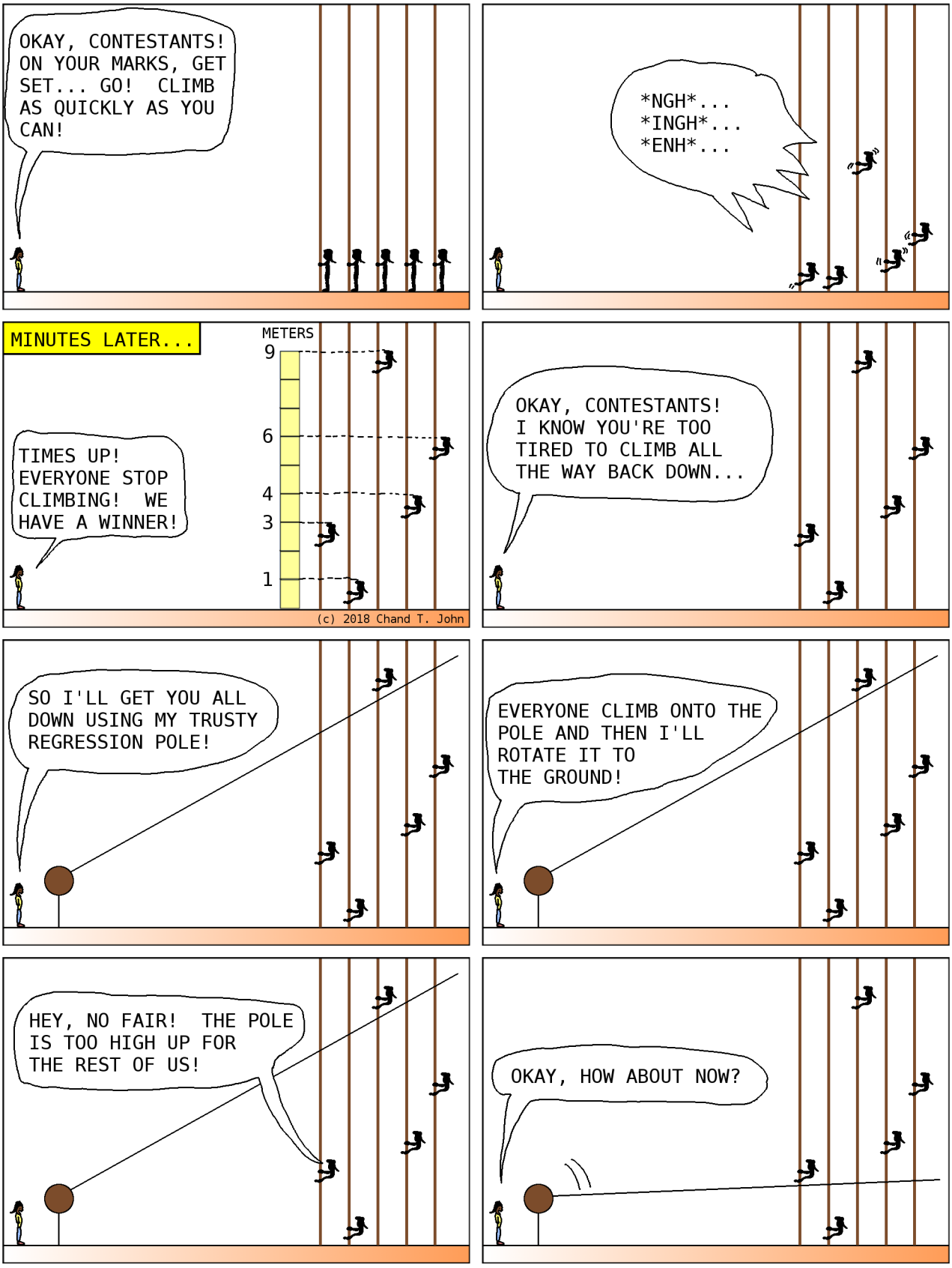
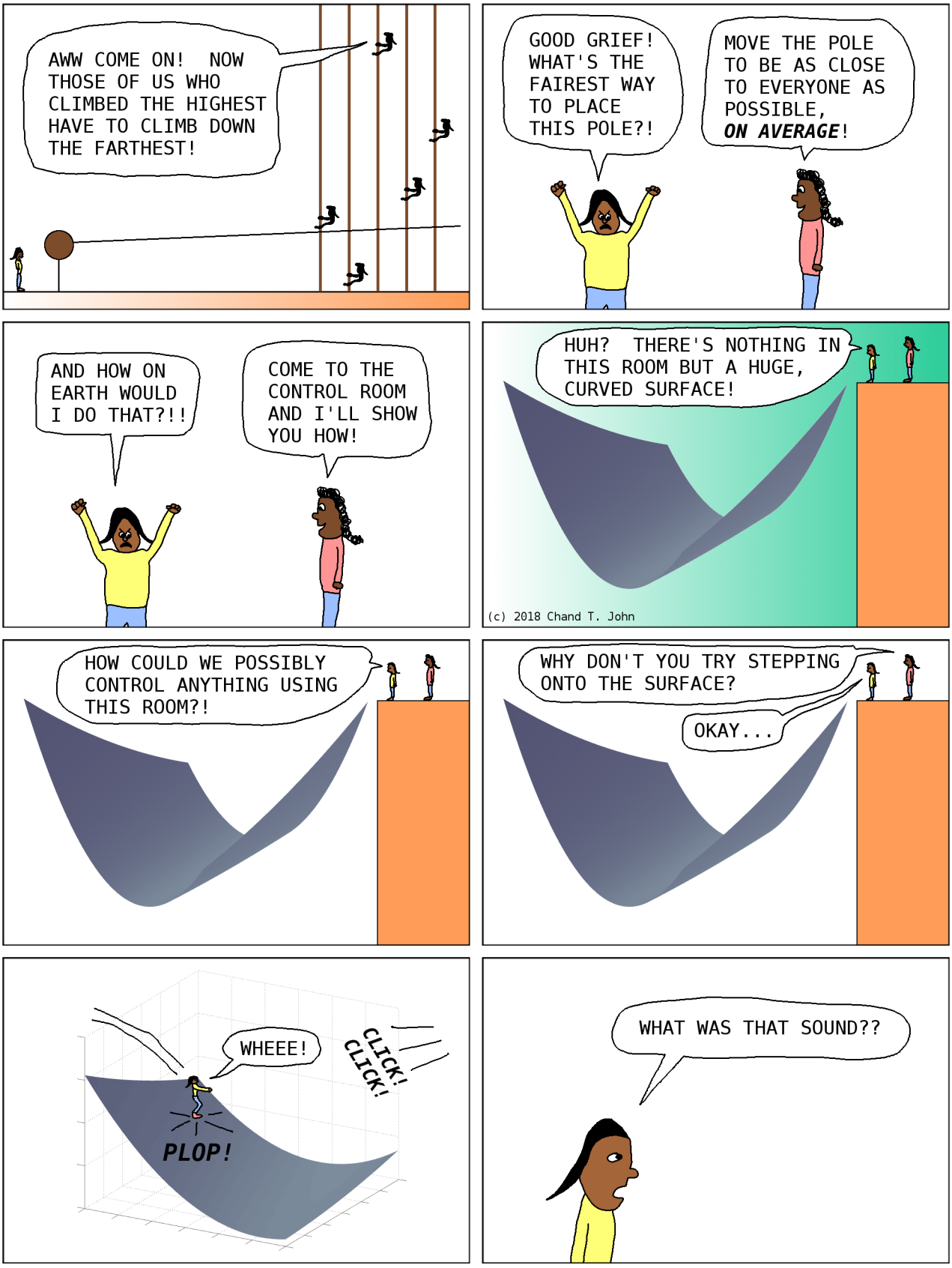
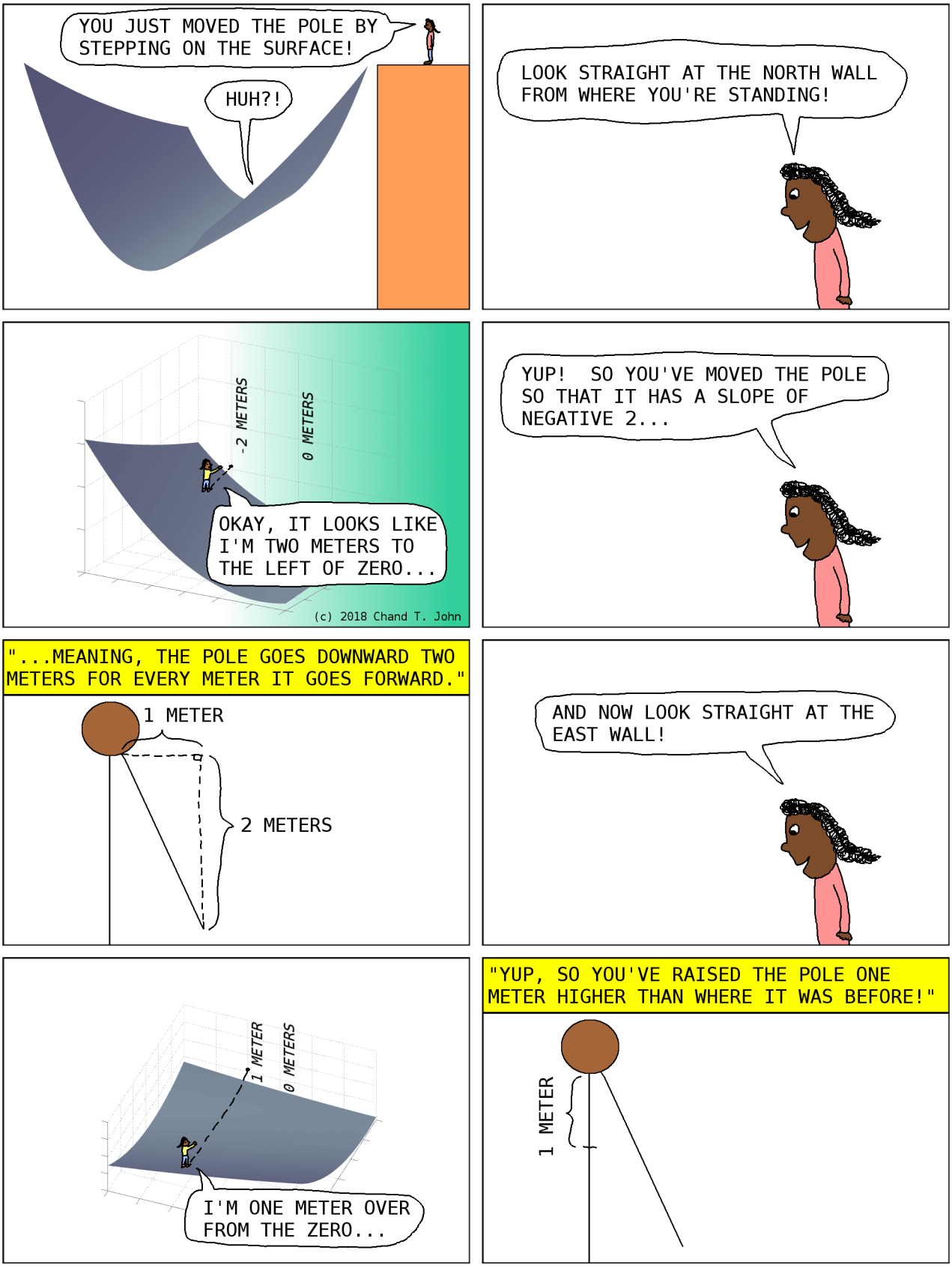
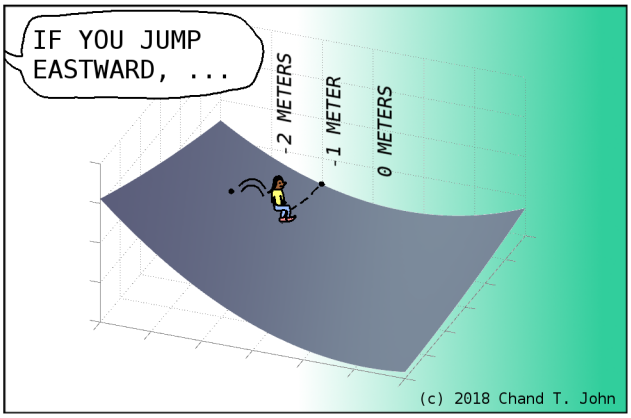
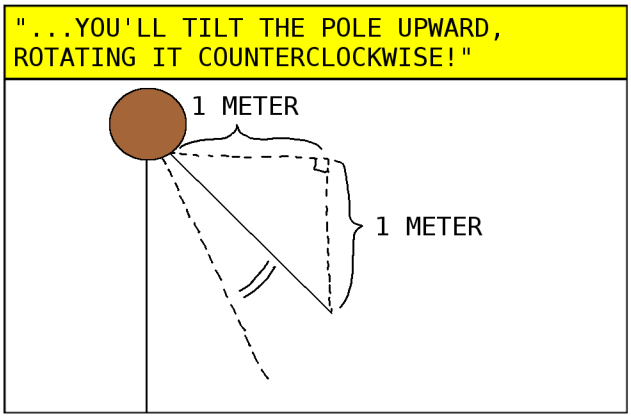
Linear Regression & Gradient Descent

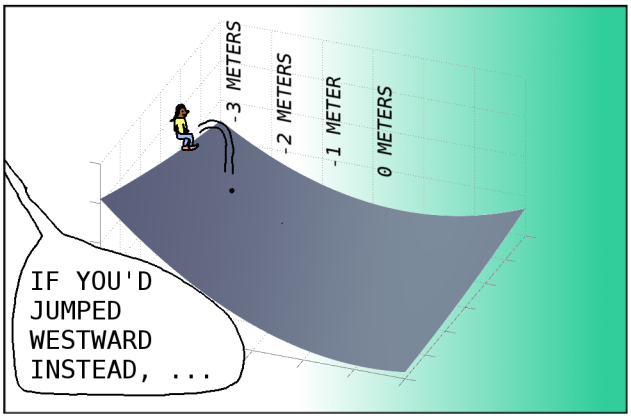


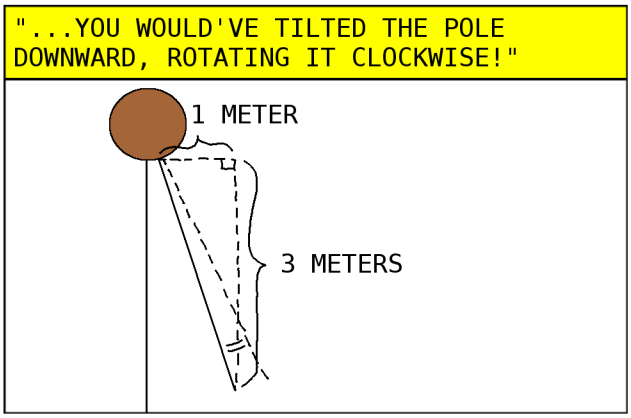

AWW COME ON! NOW THOSE OF US WHO CLIMBED THE HIGHEST HAVE TO CLIMB DOWN THE FARTHEST!
GOOD GRIEF! WHAT'S THE FAIREST WAY TO PLACE THIS POLE?!
MOVE THE POLE TO BE AS CLOSE TO EVERYONE AS POSSIBLE, ON AVERAGE!
AND HOW ON EARTH WOULD I DO THAT?!!
COME TO THE CONTROL ROOM AND I'LL SHOW YOU HOW!
HUH? THERE'S NOTHING IN THIS ROOM BUT A HUGE, CURVED SURFACE!
(c) 2018 Chand T. John
HOW COULD WE POSSIBLY CONTROL ANYTHING USING THIS ROOM?!
WHY DON'T YOU TRY STEPPING ONTO THE SURFACE?
OKAY...
WHEEE!
CLICK! CLICK!
PLOP!
WHAT WAS THAT SOUND??

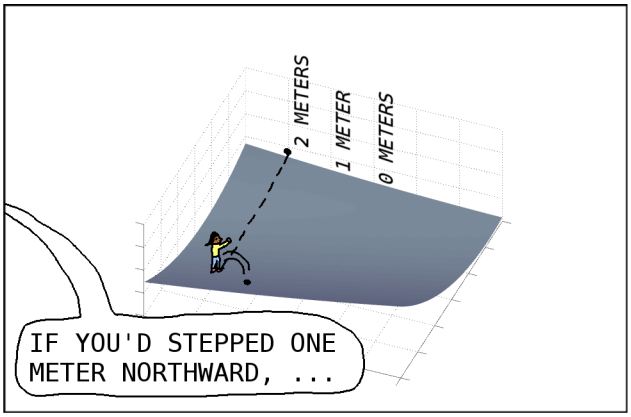

YOU JUST MOVED THE POLE BY STEPPING ON THE SURFACE!
HUH?!
LOOK STRAIGHT AT THE NORTH WALL FROM WHERE YOU'RE STANDING!
-2 METERS
0 METERS
OKAY, IT LOOKS LIKE I'M TWO METERS TO THE LEFT OF ZERO...
(c) 2018 Chand T. John
YUP! SO YOU'VE MOVED THE POLE SO THAT IT HAS A SLOPE OF NEGATIVE 2...
"...MEANING, THE POLE GOES DOWNWARD TWO METERS FOR EVERY METER IT GOES FORWARD."
1 METER
2 METERS
AND NOW LOOK STRAIGHT AT THE EAST WALL!
1 METER
0 METERS
I'M ONE METER OVER FROM THE ZERO...
"YUP, SO YOU'VE RAISED THE POLE ONE METER HIGHER THAN WHERE IT WAS BEFORE!"
1 METER

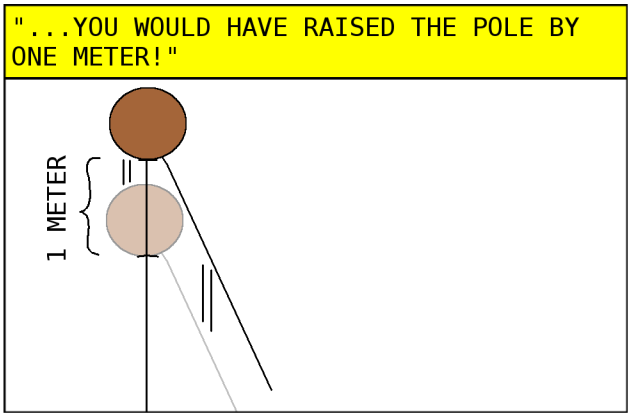

IF YOU JUMP EASTWARD, ...
-2 METERS
-1 METER
0 METERS
(c) 2018 Chand T. John

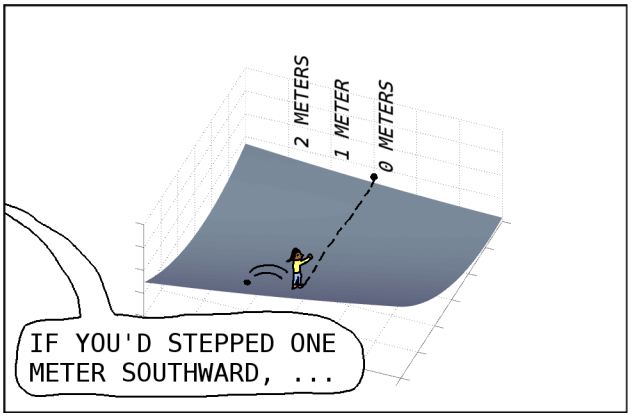

"...YOU'LL TILT THE POLE UPWARD, ROTATING IT COUNTERCLOCKWISE!"
1 METER
1 METER

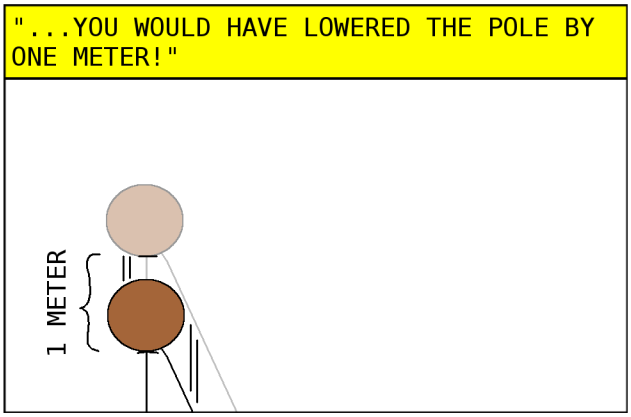

-3 METERS
-2 METERS
-1 METER
0 METERS
IF YOU'D JUMPED WESTWARD INSTEAD, ...

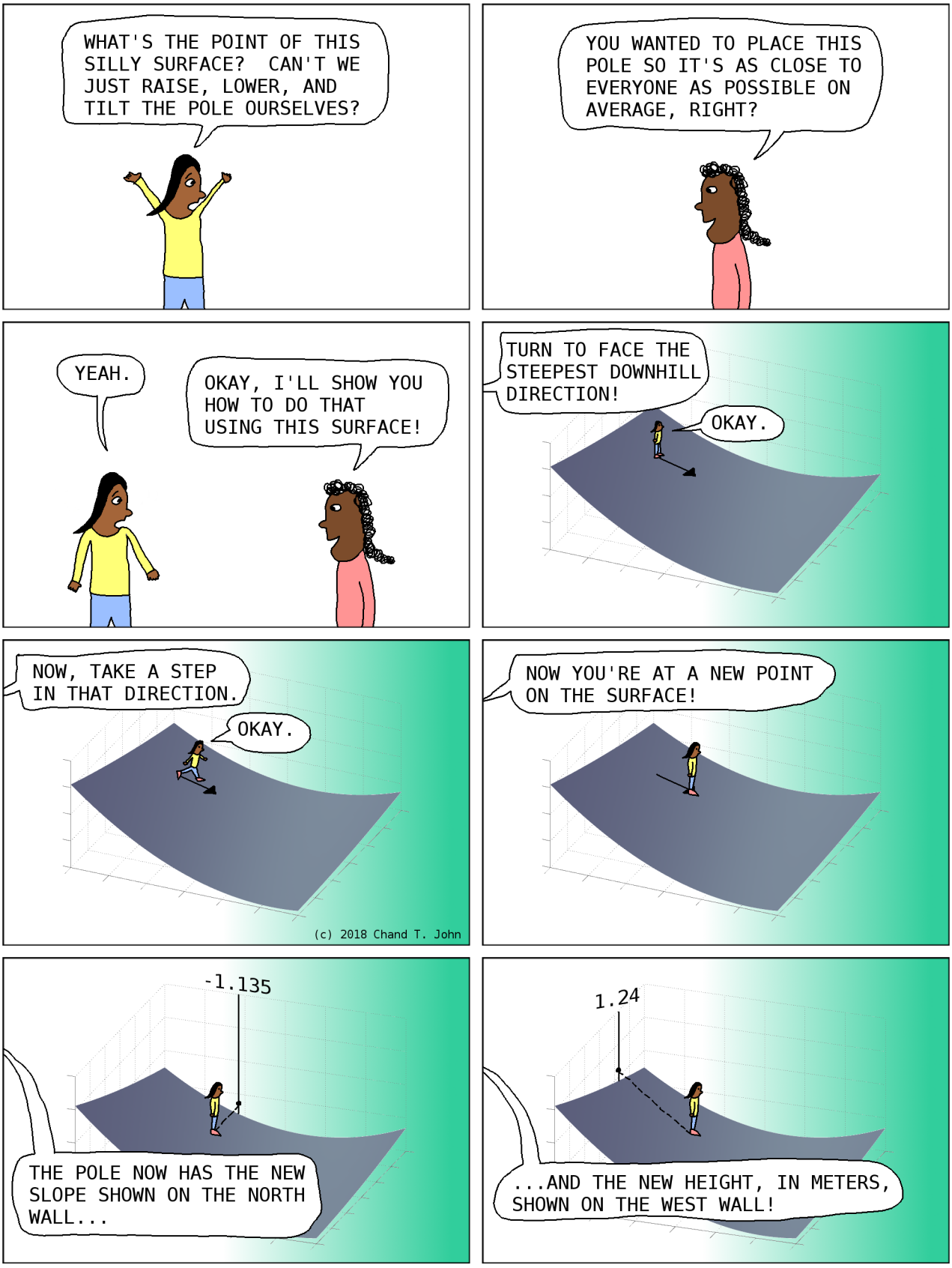

"...YOU WOULD'VE TILTED THE POLE DOWNWARD, ROTATING IT CLOCKWISE!"
1 METER
3 METERS

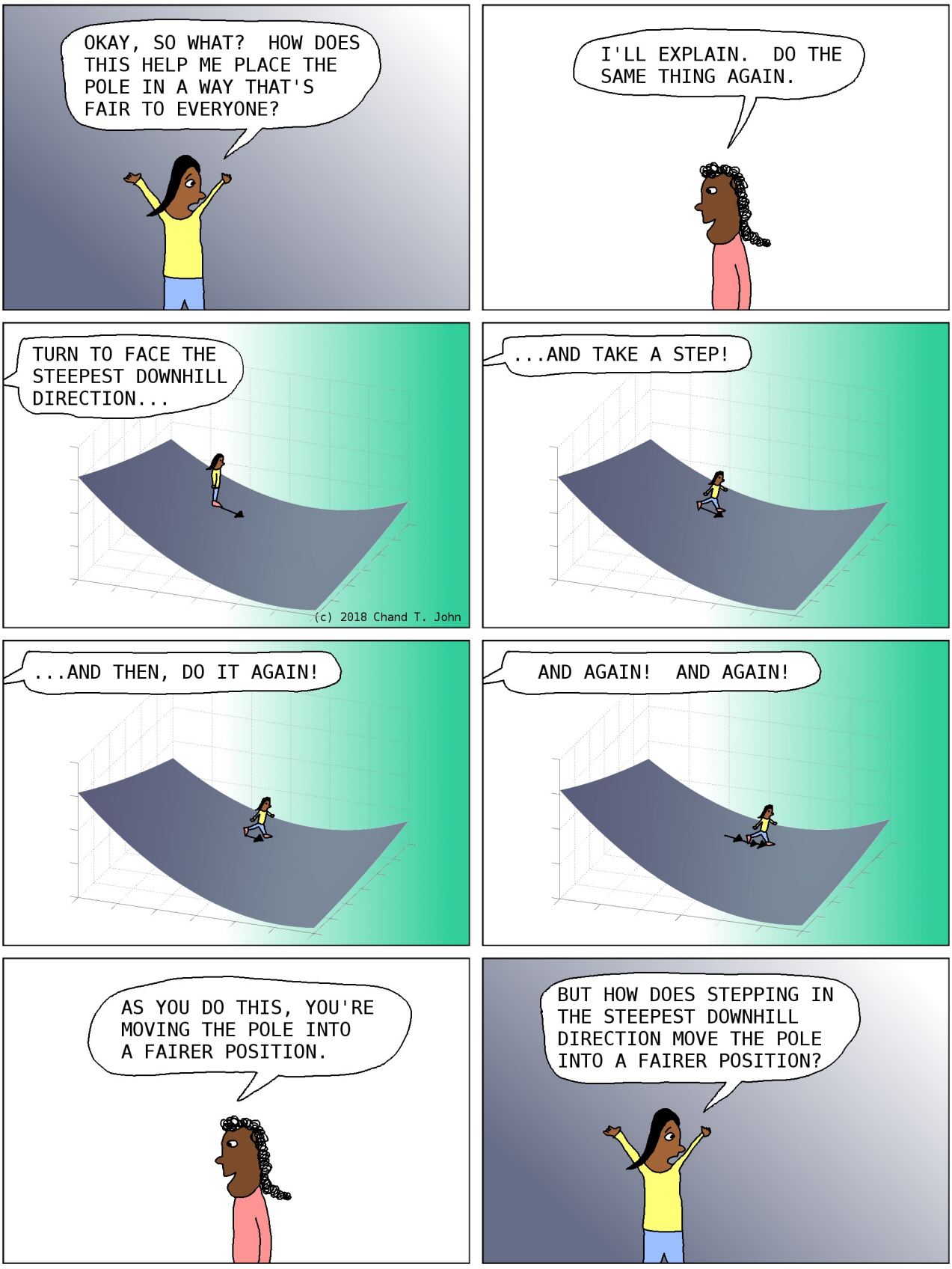

2 METERS
1 METER
0 METERS
IF YOU'D STEPPED ONE METER NORTHWARD, ...


"...YOU WOULD HAVE RAISED THE POLE BY ONE METER!"
1 METER

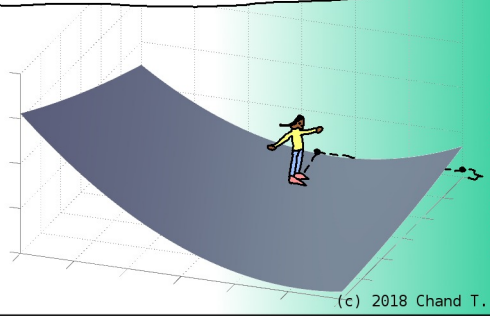

2 METERS
1 METER
0 METERS
IF YOU'D STEPPED ONE METER SOUTHWARD, ...


"...YOU WOULD HAVE LOWERED THE POLE BY ONE METER!"
1 METER


WHAT'S THE POINT OF THIS SILLY SURFACE? CAN'T WE JUST RAISE, LOWER, AND TILT THE POLE OURSELVES?
YOU WANTED TO PLACE THIS POLE SO IT'S AS CLOSE TO EVERYONE AS POSSIBLE ON AVERAGE, RIGHT?
YEAH.
OKAY, I'LL SHOW YOU HOW TO DO THAT USING THIS SURFACE!
TURN TO FACE THE STEEPEST DOWNHILL DIRECTION!
OKAY.
NOW, TAKE A STEP IN THAT DIRECTION.
OKAY.
(c) 2018 Chand T. John
NOW YOU'RE AT A NEW POINT ON THE SURFACE!
-1.135
THE POLE NOW HAS THE NEW SLOPE SHOWN ON THE NORTH WALL...
1.24
...AND THE NEW HEIGHT, IN METERS, SHOWN ON THE WEST WALL!

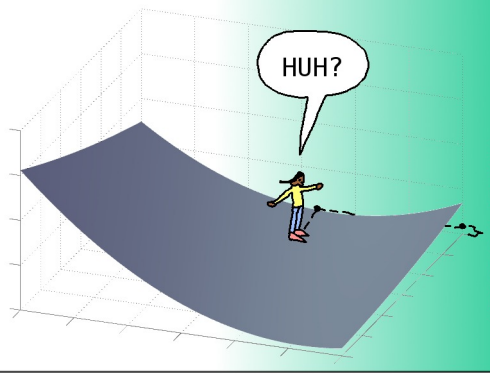

OKAY, SO WHAT? HOW DOES THIS HELP ME PLACE THE POLE IN A WAY THAT'S FAIR TO EVERYONE?
I'LL EXPLAIN. DO THE SAME THING AGAIN.
TURN TO FACE THE STEEPEST DOWNHILL DIRECTION...
(c) 2018 Chand T. John
...AND TAKE A STEP!
...AND THEN, DO IT AGAIN!
AND AGAIN! AND AGAIN!
AS YOU DO THIS, YOU'RE MOVING THE POLE INTO A FAIRER POSITION.
BUT HOW DOES STEPPING IN THE STEEPEST DOWNHILL DIRECTION MOVE THE POLE INTO A FAIRER POSITION?

SEE HOW HIGH YOU ARE FROM THE GROUND RIGHT NOW?

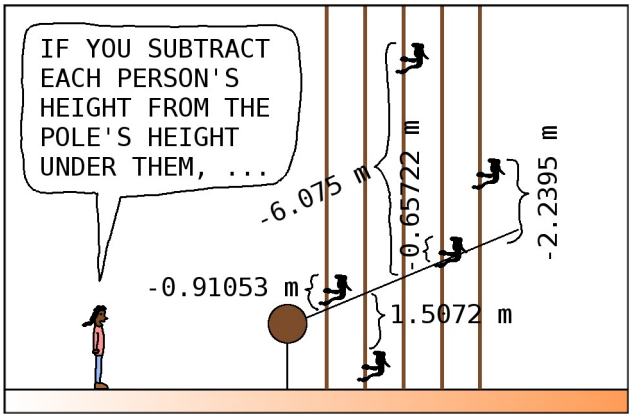


YOUR HEIGHT FROM THE GROUND IS ACTUALLY A MEASURE OF HOW UNFAIR THE POLE'S POSITION IS!

HUH?



IF YOU SUBTRACT EACH PERSON'S HEIGHT FROM THE POLE'S HEIGHT UNDER THEM, ...



...SQUARE THOSE DISTANCES, ...

$(-0.91053)^2 = 0.82906$

$(1.5072)^2 = 2.2717$

$(-6.0750)^2 = 36.906$

$(-0.65722)^2 = 0.43194$

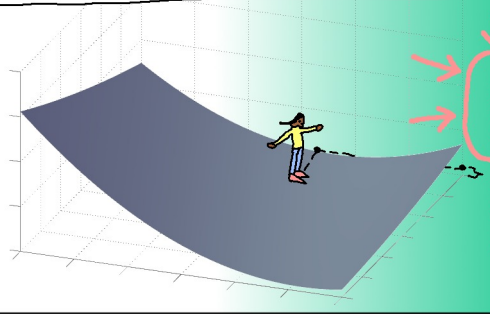
$(-2.2395)^2 = 5.0154$

...AVERAGE THEM, AND DIVIDE BY TWO, ...

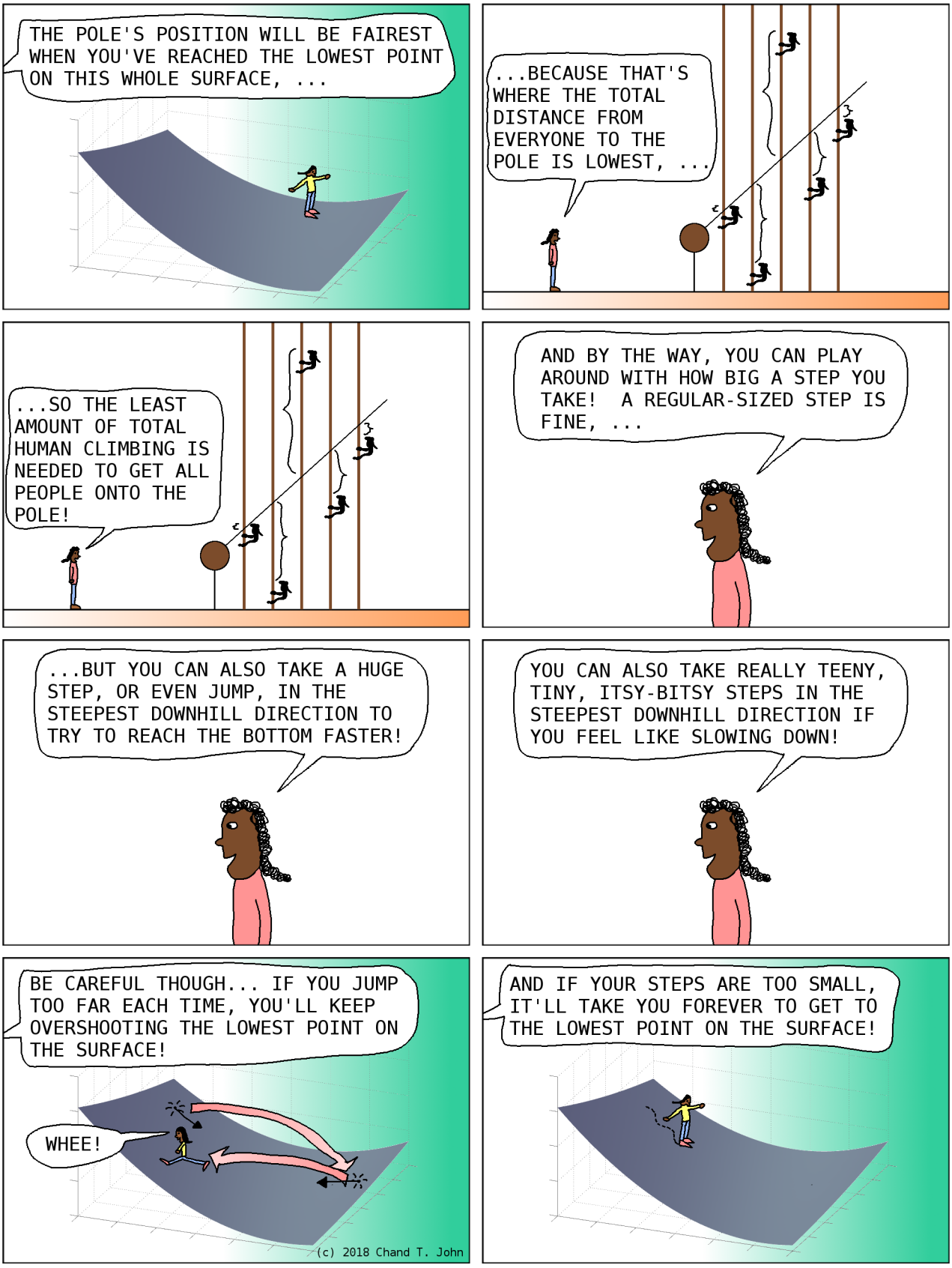
$$0.82906 + 2.2717 + 36.906 + 0.43194 + 5.0154 \div 5 = 9.0908$$

$$9.0908 \div 2 = 4.545$$

...THAT NUMBER IS HOW HIGH UP YOU ARE FROM THE GROUND, IN METERS!



IN FACT, THAT'S THE WHOLE POINT OF THIS SURFACE! AS YOU MOVE AROUND, YOU MOVE THE POLE, AND HOW HIGH UP YOU ARE IS ALWAYS THE HALVED AVERAGE OF SQUARED DISTANCES FROM EACH PERSON TO THE POLE!


THE POLE'S POSITION WILL BE FAIREST WHEN YOU'VE REACHED THE LOWEST POINT ON THIS WHOLE SURFACE, ...
...BECAUSE THAT'S WHERE THE TOTAL DISTANCE FROM EVERYONE TO THE POLE IS LOWEST, ...
...SO THE LEAST AMOUNT OF TOTAL HUMAN CLIMBING IS NEEDED TO GET ALL PEOPLE ONTO THE POLE!
AND BY THE WAY, YOU CAN PLAY AROUND WITH HOW BIG A STEP YOU TAKE! A REGULAR-SIZED STEP IS FINE, ...
...BUT YOU CAN ALSO TAKE A HUGE STEP, OR EVEN JUMP, IN THE STEEPEST DOWNHILL DIRECTION TO TRY TO REACH THE BOTTOM FASTER!
YOU CAN ALSO TAKE REALLY TEENY, TINY, ITSY-BITSY STEPS IN THE STEEPEST DOWNHILL DIRECTION IF YOU FEEL LIKE SLOWING DOWN!
BE CAREFUL THOUGH... IF YOU JUMP TOO FAR EACH TIME, YOU'LL KEEP OVERSHOOTING THE LOWEST POINT ON THE SURFACE!
WHEE!
AND IF YOUR STEPS ARE TOO SMALL, IT'LL TAKE YOU FOREVER TO GET TO THE LOWEST POINT ON THE SURFACE!
(c) 2018 Chand T. John

OKAY! SO I'LL JUST TAKE SOME GOOD-SIZED STEPS DOWNHILL IN THE STEEPEST DIRECTION, ...

...AND LOOK! I'M AT THE BOTTOM OF THE SURFACE NOW!

AND NOW THE POLE IS AT THE FAIREST SLOPE AND HEIGHT! YAY!

EVERYONE CLIMB ONTO THE POLE AND THEN I'LL LOWER IT TO THE GROUND, AND WE'RE DONE!

THIS APPROACH, WHERE WE ***DESCENDED*** ALONG THE SURFACE...

...BY STEPPING REPEATEDLY IN THE STEEPEST DOWNHILL DIRECTION, ALSO CALLED THE ***GRADIENT*** DIRECTION, ...

...IS CALLED THE ***GRADIENT DESCENT ALGORITHM***!

THE GRADIENT DESCENT ALGORITHM IS ONE WAY TO DO ***LINEAR REGRESSION***: TO FIND A LINE THAT RUNS AS CLOSE TO A SET OF DATA POINTS AS POSSIBLE!