

# Fairness in Decision Making

## Is Linear Regression Fair?

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(New York University)

Joint work with



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(Google Research)



Claire Mathieu  
(CNRS, Université de Paris)



Namrata  
(University of Warwick)

# PART I

## An Oversimplified Overview

# A Hypothetical Situation

Alice

Bob

# A Hypothetical Situation

Alice



Bob



# A Hypothetical Situation

Alice



Bob



# A Hypothetical Situation

Alice



Bob

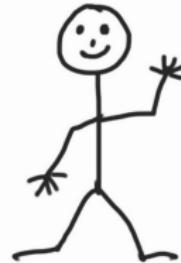


# A Hypothetical Situation

Alice



Bob



Math  
Science



# A Hypothetical Situation

Alice



9

Math

9

Science

Bob



10

7



# A Hypothetical Situation

Alice



9

Math

9

Science

Sum: 18

Bob



10

7



Sum: 17

# A Hypothetical Situation

Alice



9

Math

9

Science

Sum: 18

Max: 9

Bob



10

7



Sum: 17

Max: 10

# A Hypothetical Situation

Alice



9

Math

9

Science

Sum: 18 ✓

Max: 9

Bob



10

7



Sum: 17

Max: 10 ✓

# A Hypothetical Situation

Alice



Bob



Caucasian

9

Math

10

9

Science

7

Sum: 18 ✓

Max: 9

Sum: 17

Max: 10 ✓

UNIVERSITY



# A Hypothetical Situation

Alice



Bob



*African American*

9

Math

10

9

Science

7

Sum: 18 ✓

Max: 9

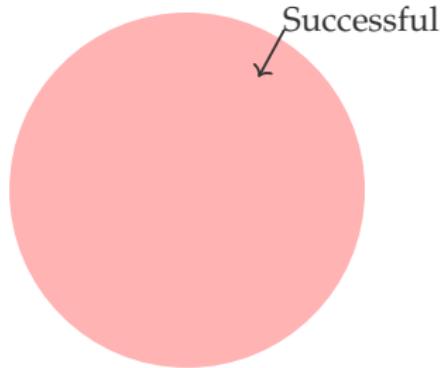
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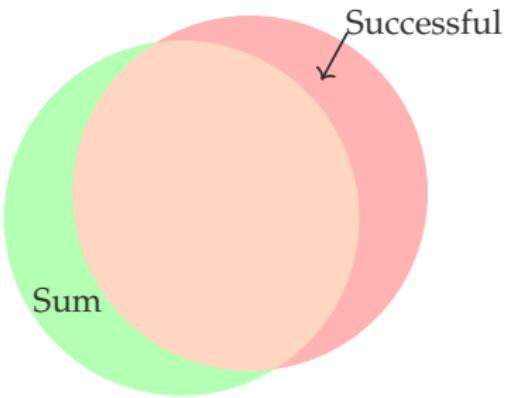
# A Hypothetical Situation: Extrapolation

Caucasian

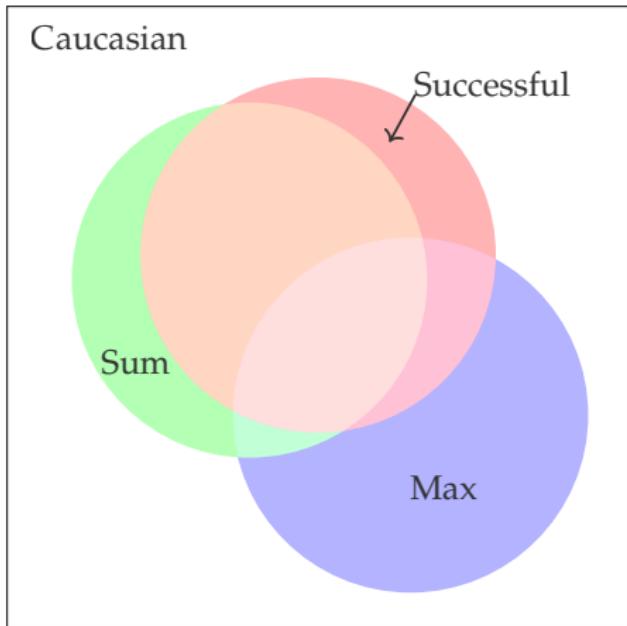


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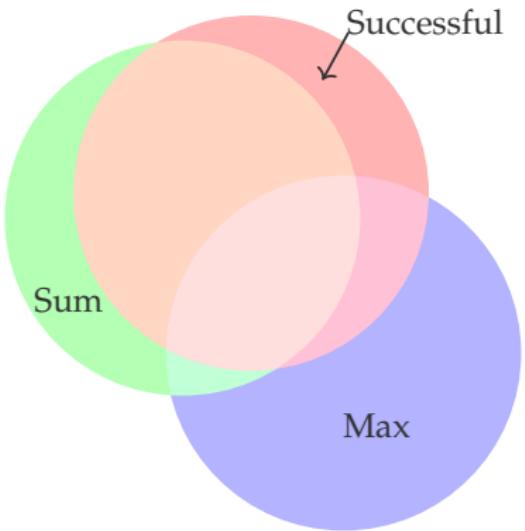


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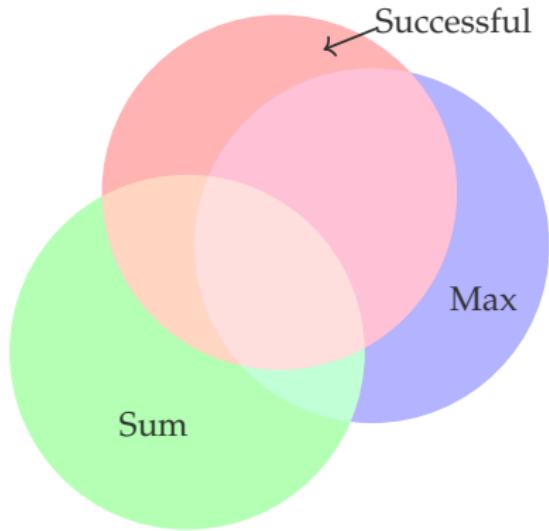


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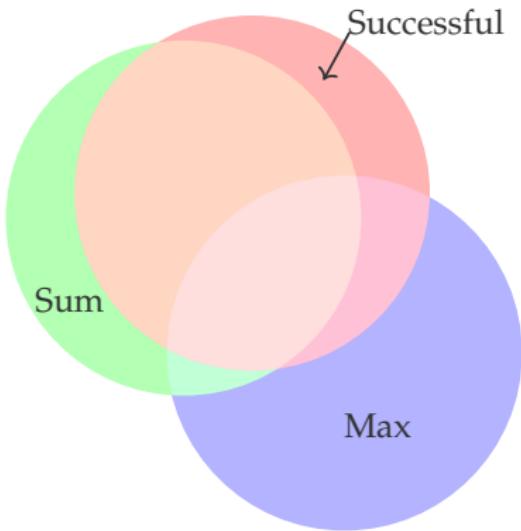


African American

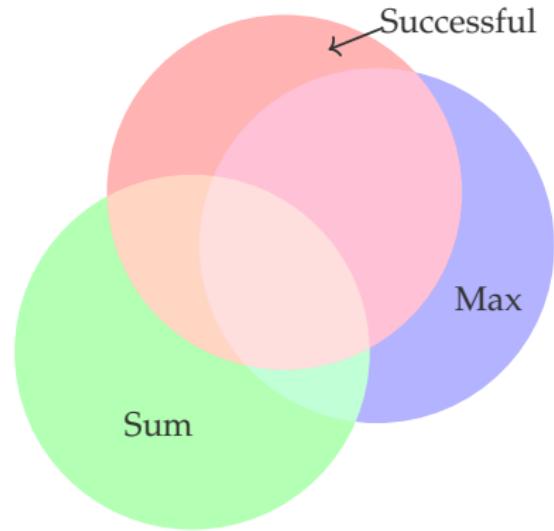


# A Hypothetical Situation: Extrapolation

Caucasian



African American



Decision Making System using **Sum** formula:  
Reinforces **Bias** against African Americans

# PART II

## The Real Deal

# Setting: Decision Making



Applications

# Setting: Decision Making



Applications



Admission Committee

# Setting: Decision Making



Applications



Admission Committee



Historical Data

# But what is Linear Regression !?

# But what is Linear Regression !?

Alice



9

9

Math

Science

Bob



10

7

# But what is Linear Regression !?

Alice



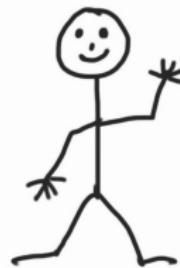
9

Math

9

Science

Bob



10

7

$$3 \times \text{Math} + \text{Science}$$

**Linear Regression**

Learn Best **Weighted Sum** Formula

# Data

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African American: 442 (30.88% successful)  
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Affirmative Action is practiced in College Admission  
in the US since 1970s

# Fitting to a Line

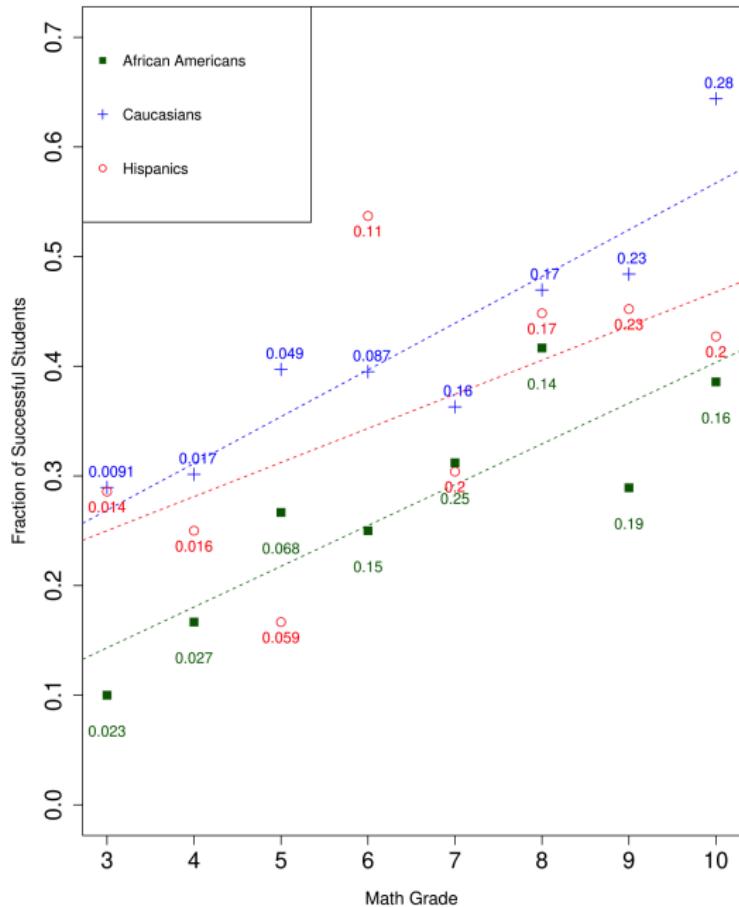
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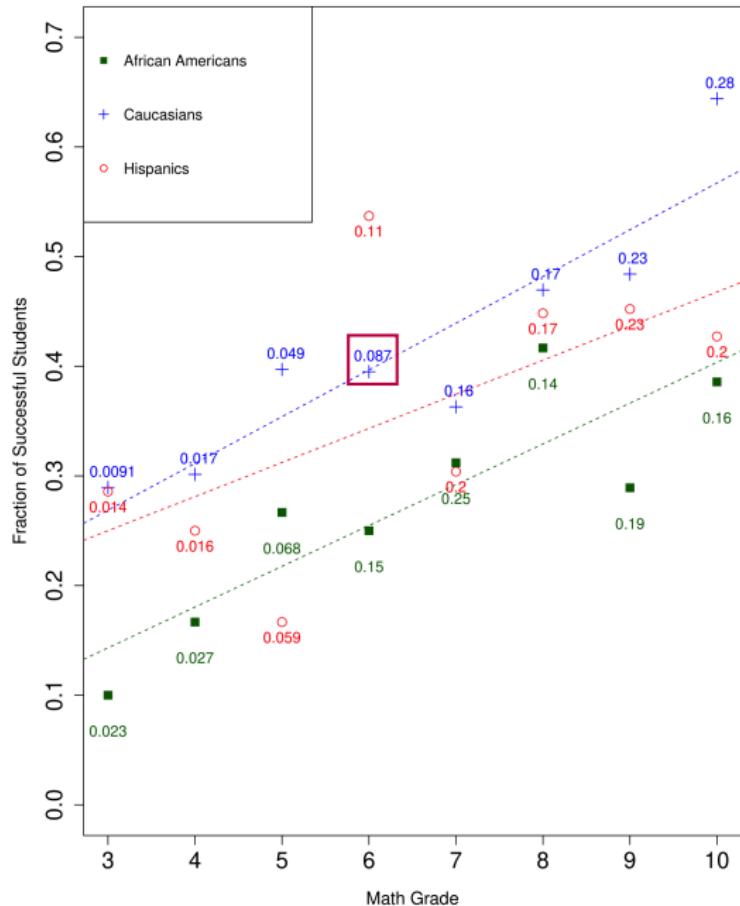
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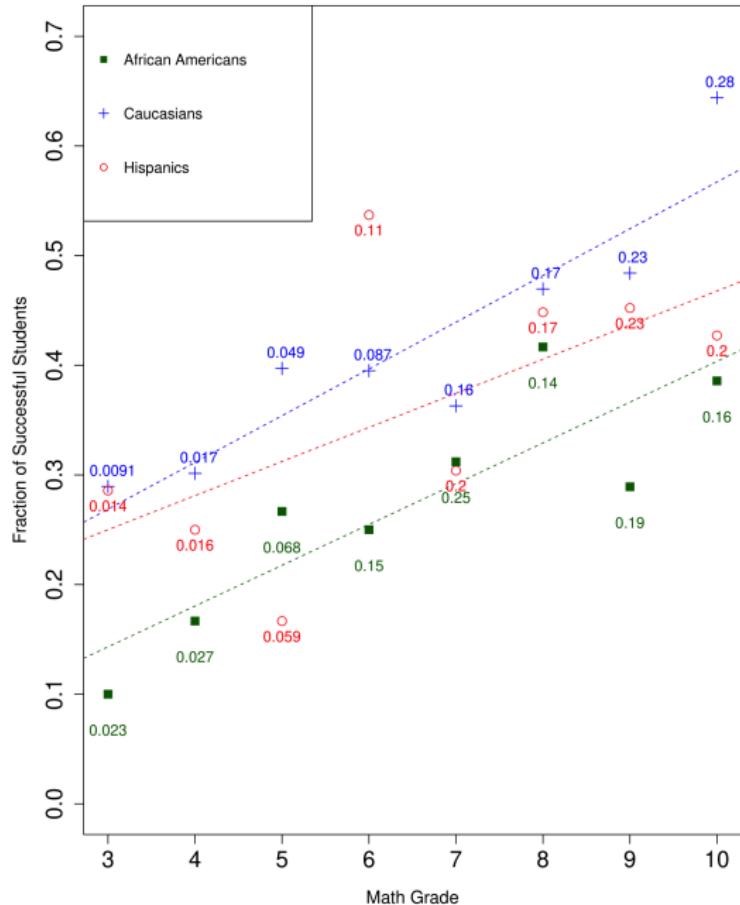
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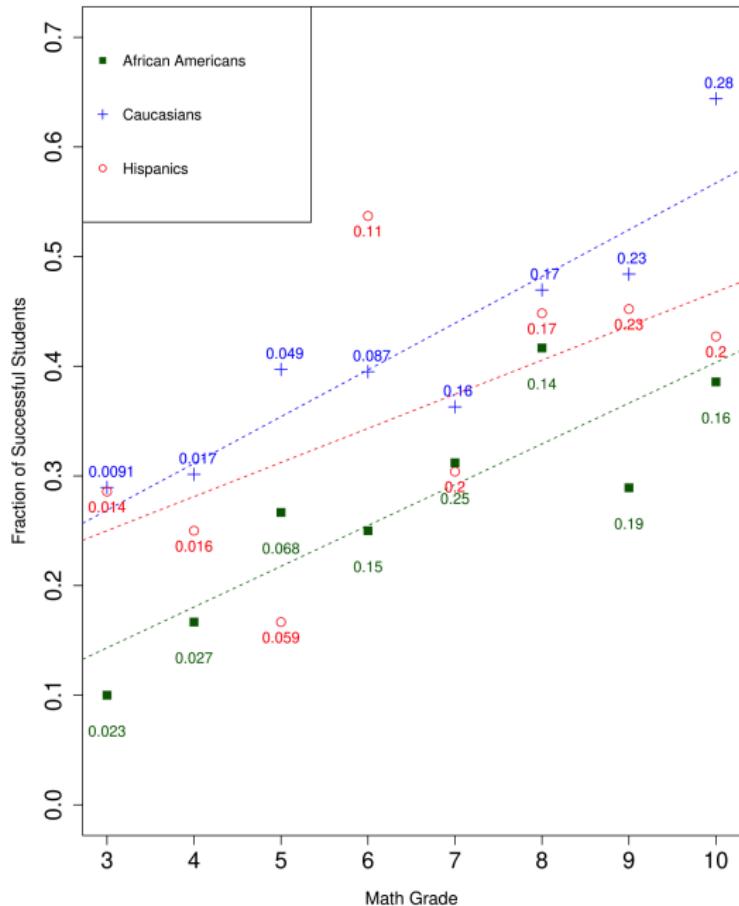
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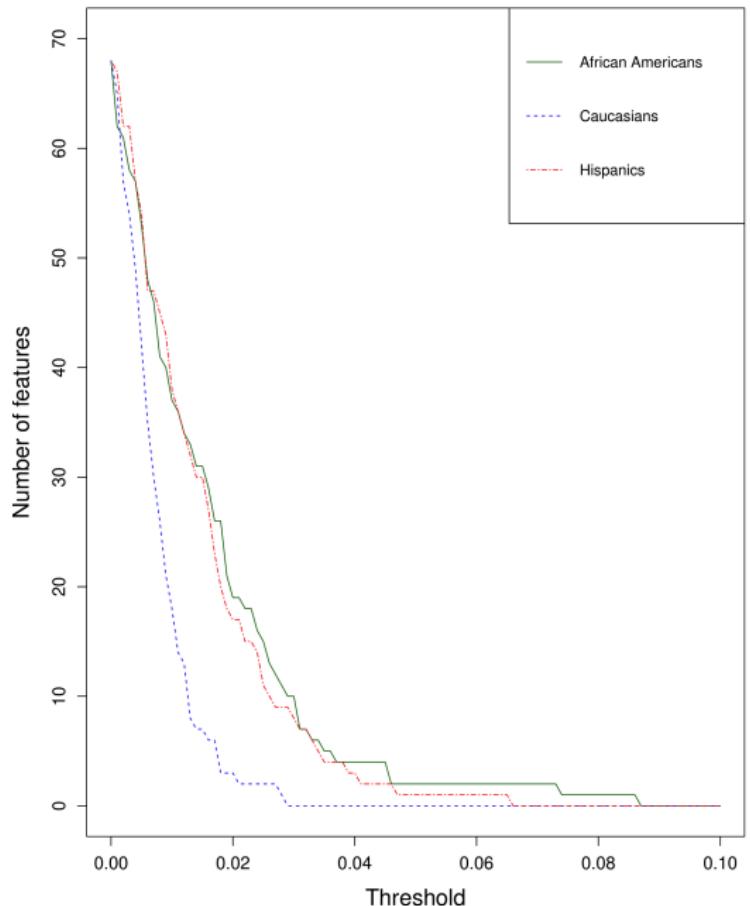


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- What happens if we admit top 10% students?
- Error is not Uniform Across Race groups

# Number of Influential Features

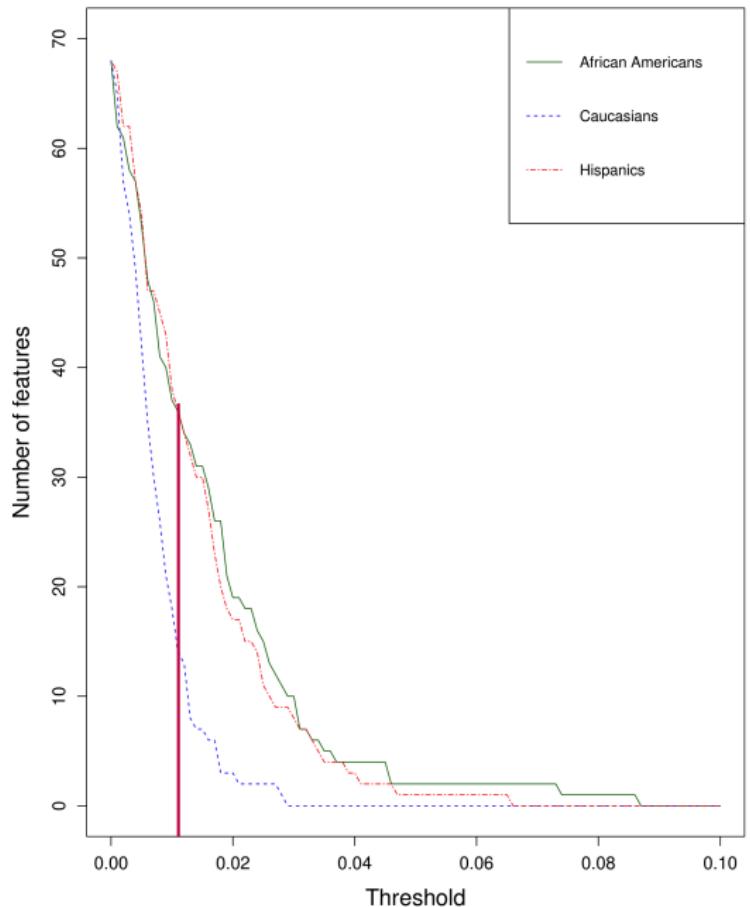
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Extra Curriculars, ...

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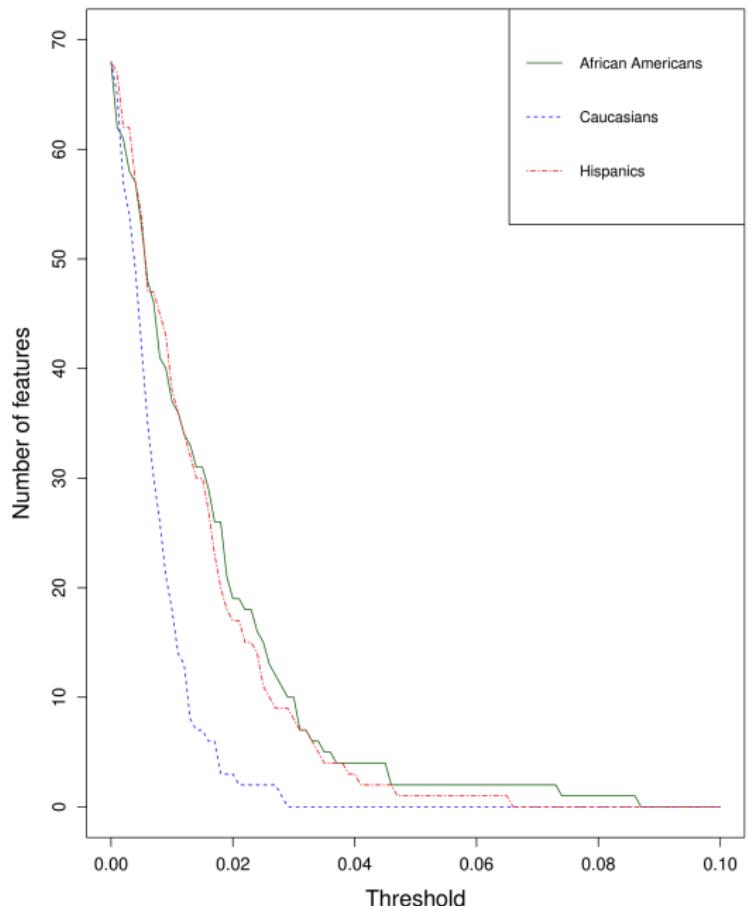
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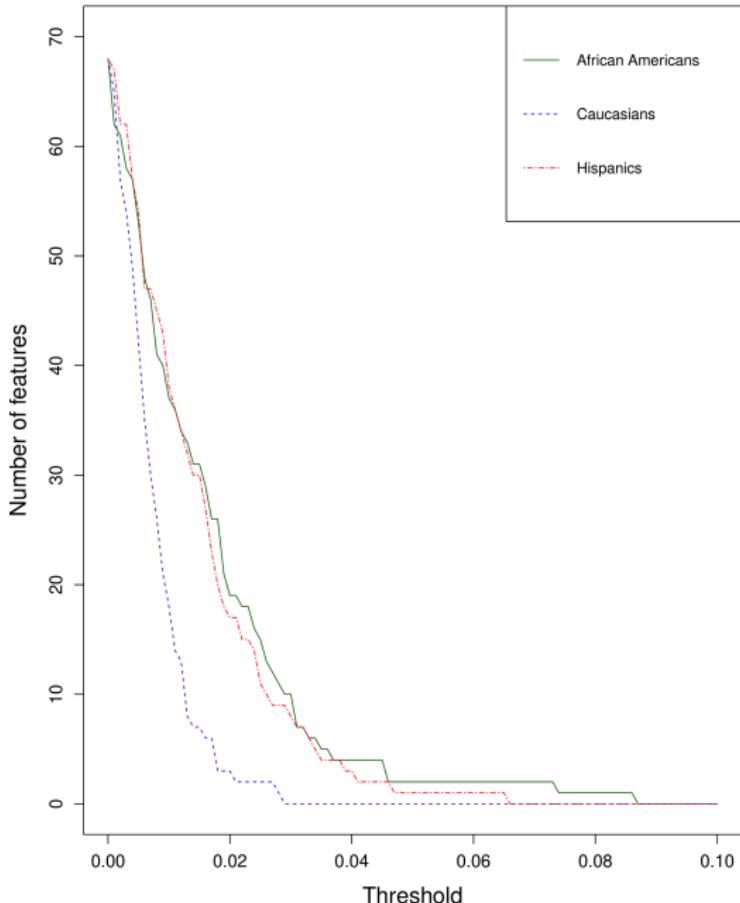
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# Number of Influential Features



- 9<sup>th</sup> and 11<sup>th</sup> grades,  
Standardized tests,  
Extra Curriculars, ...
- Caucasians have **few**  
**influential** features
- The regressor formula  
of **minority** groups  
is **complex**

# $\ell_p$ -norm Rankings

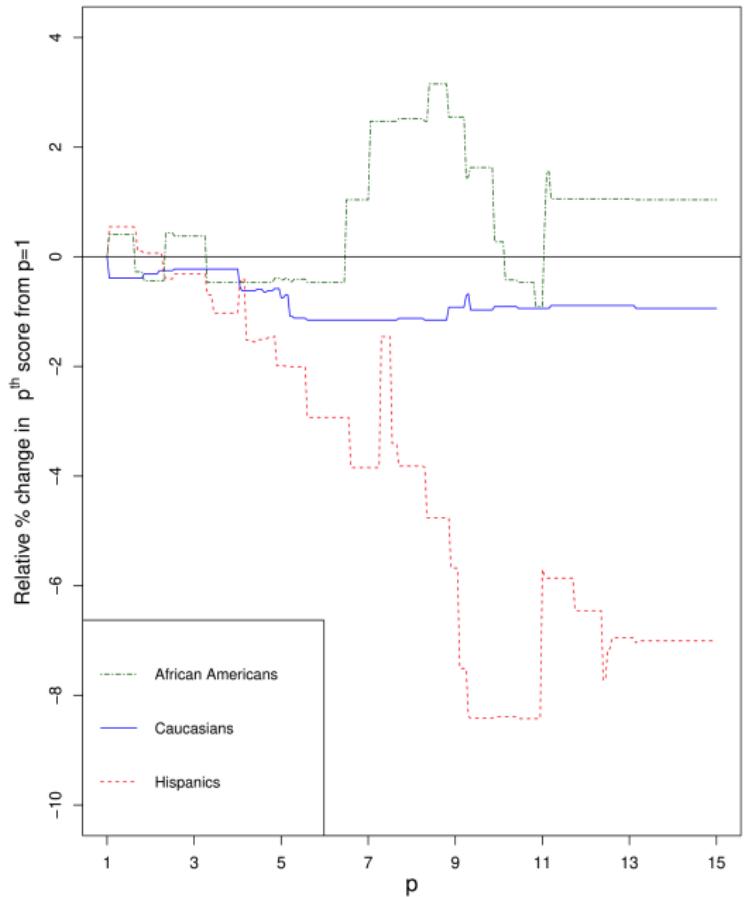
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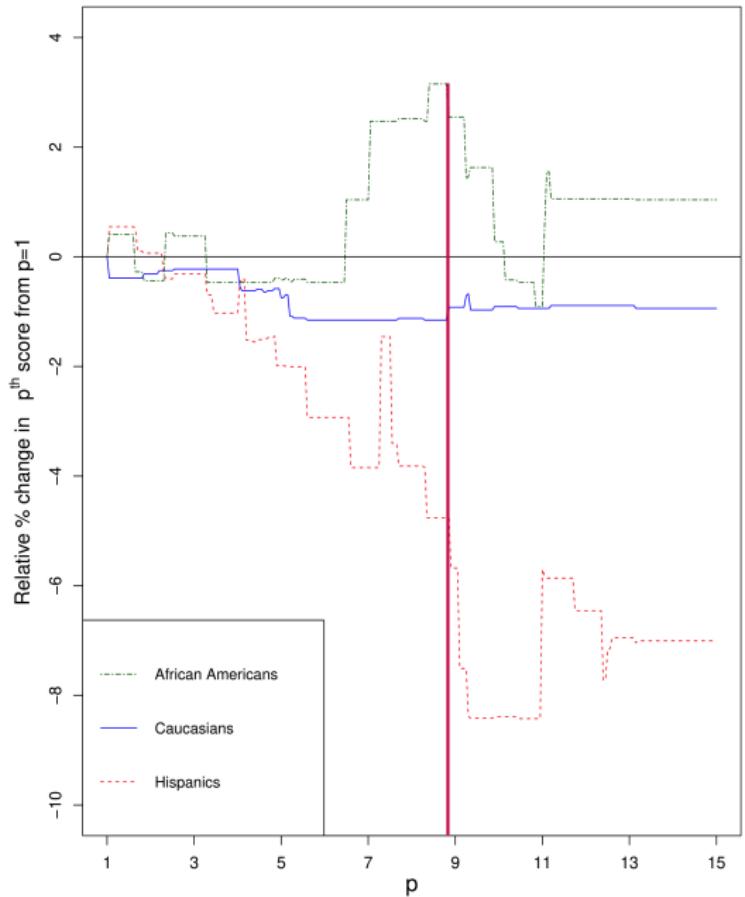
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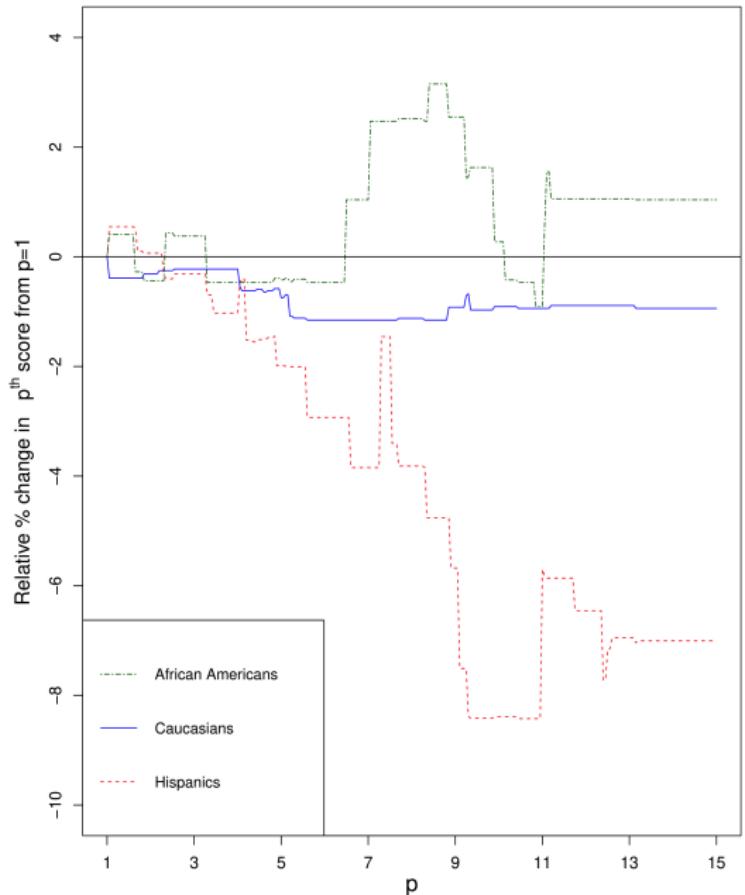
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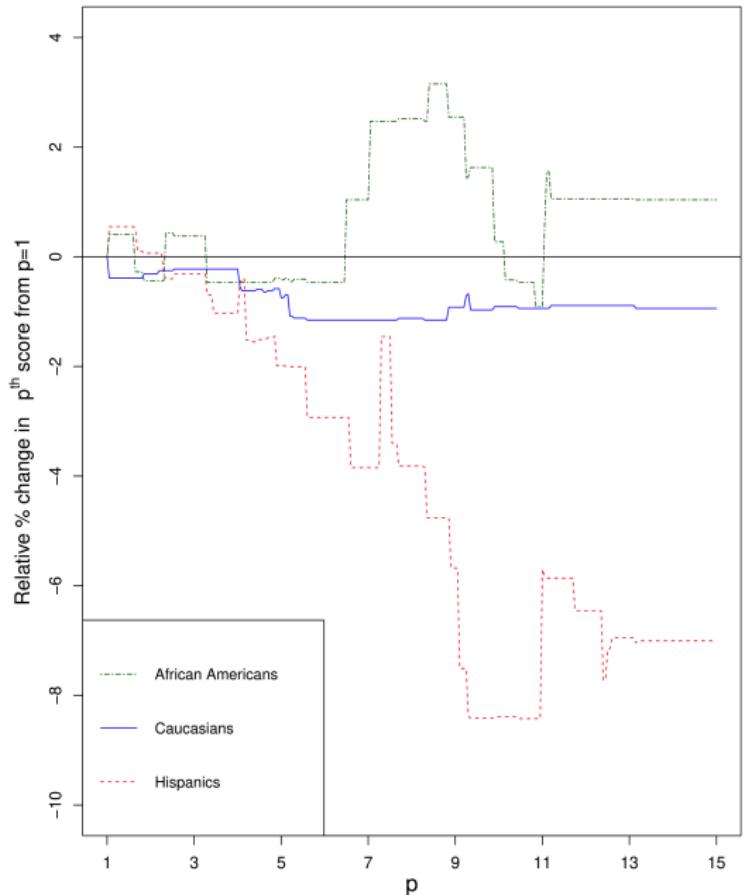
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→ Hispanics "prefer" 1-norm

# Polynomial Regression

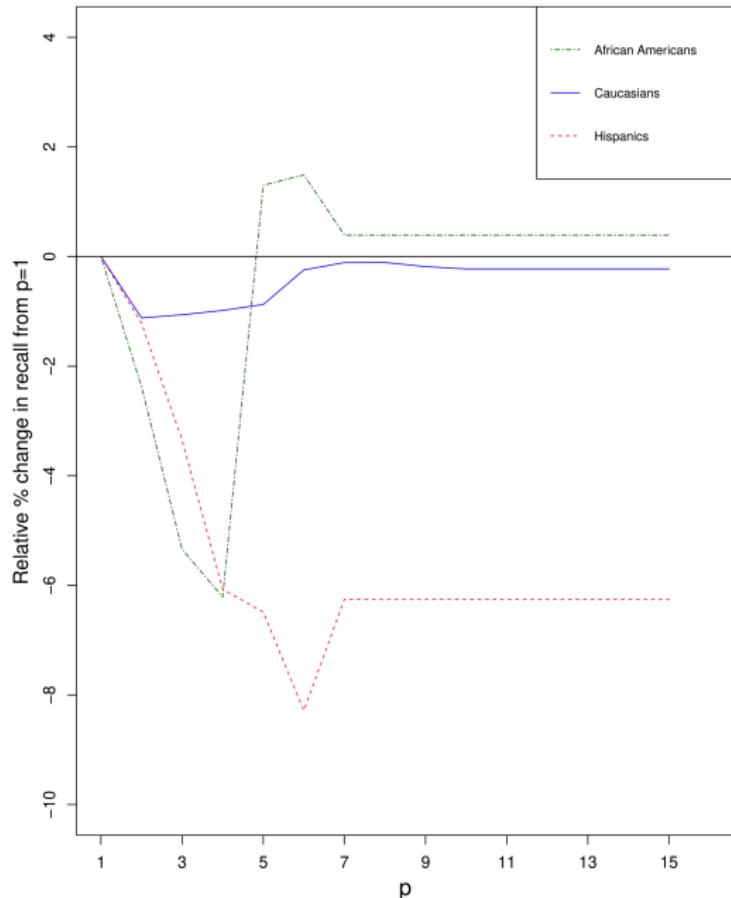
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# Polynomial Regression

→ From **Geometric** Insights  
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→ Training/Test : 50/50

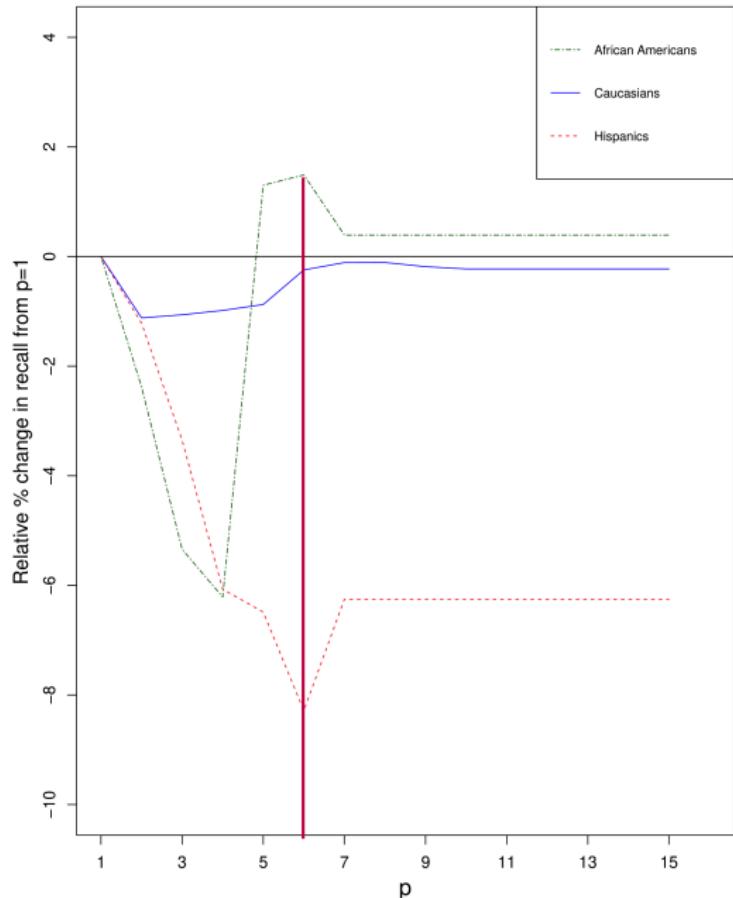
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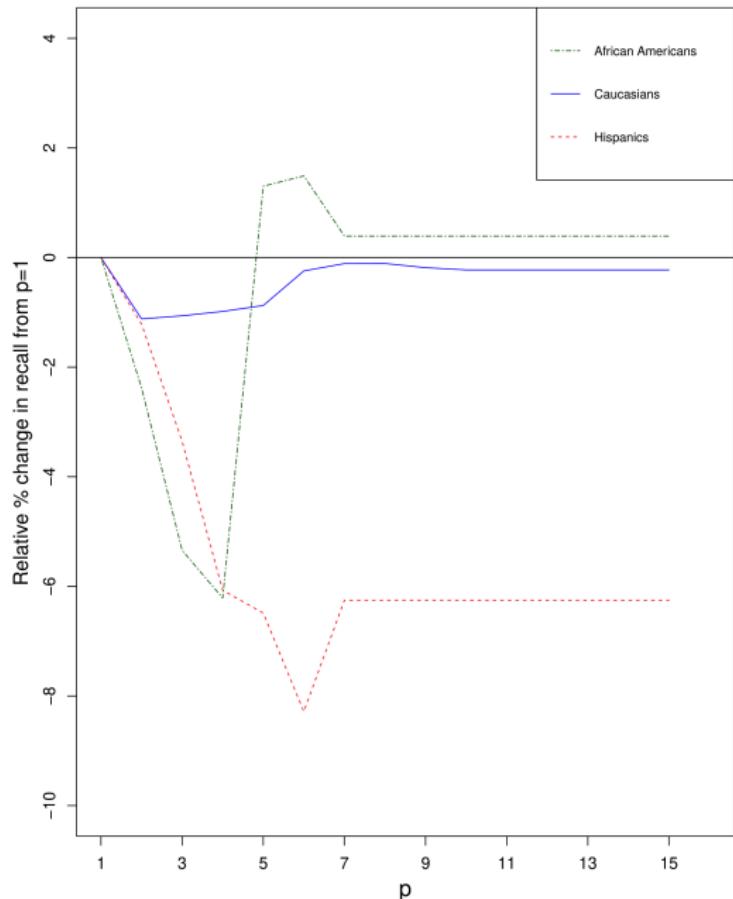
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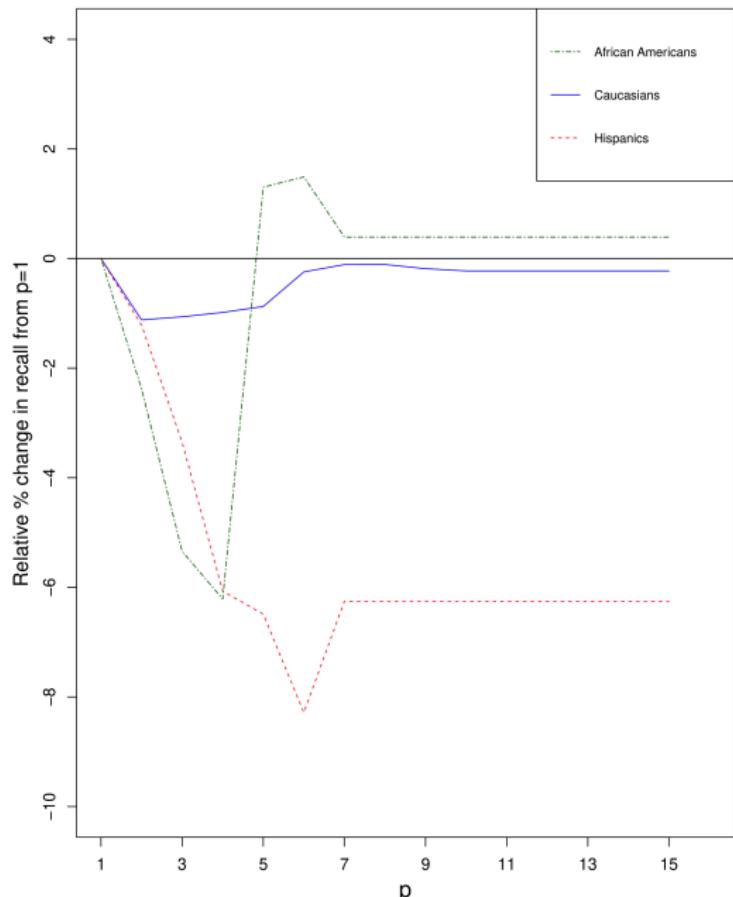
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# Polynomial Regression



- From Geometric Insights to Algebraic Tools
- Training/Test : 50/50
- Poly Regression captures more complex relationships
- A proof of concept  
Not a solution

# Putting things Together

*Each **minority** group must have its **own** predictor*  
(formalized in Kleinberg-Ludwig-Mullainathan-Rambachan '18)

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**Linear Regression** cannot capture  
the **complexity** of minority group data

THANK  
YOU!