

IN AND AROUND THE SENSITIVITY CONJECTURE

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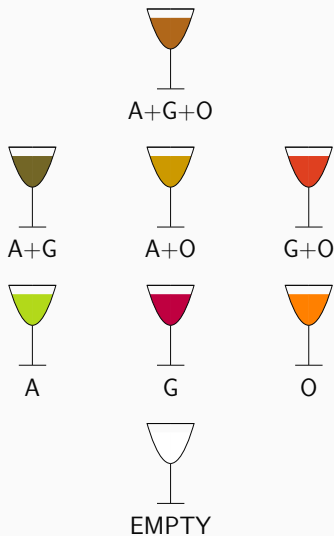
September 4, 2015

Joint work with Sébastien Tavenas.

AVAILABLE INGREDIENTS



THE OPTIONS



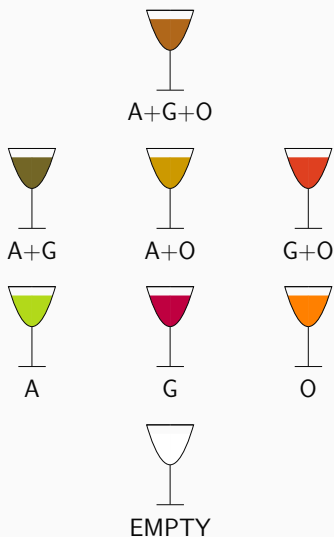
ALICE AND BOB



ALICE'S OPINION



ALICE'S OPINION



ALICE'S OPINION



EMPTY

BOB'S OPINION



Anything with
Grape juice is
fine.

BOB'S OPINION



Anything with
Grape juice is
fine.



$A+G+O$



$A+G$



$A+O$



$G+O$



A



G



O



EMPTY

BOB'S OPINION



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A

G

O

EMPTY

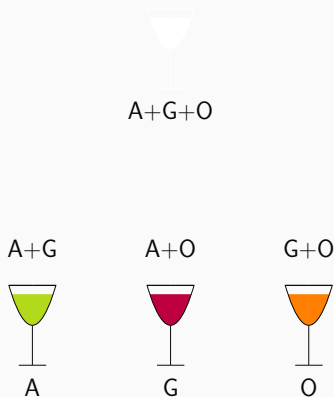
Sensitivity of a combination:

1. Fix a combination X . Set $s(X) = 0$.
2. For every ingredient i :
 - 2.1. If $i \in X$, let $Y = X - i$. Else, let $Y = X + i$.
 - 2.2. If Opinion of X is different from Y , increment $s(X)$ by 1.

Sensitivity:

$$s = \max_X s(X).$$

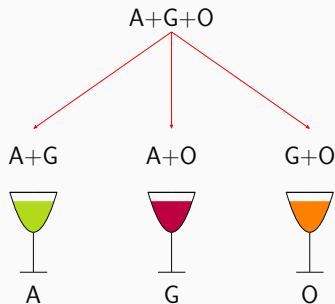
SENSITIVITY: ALICE'S OPINION



EMPTY

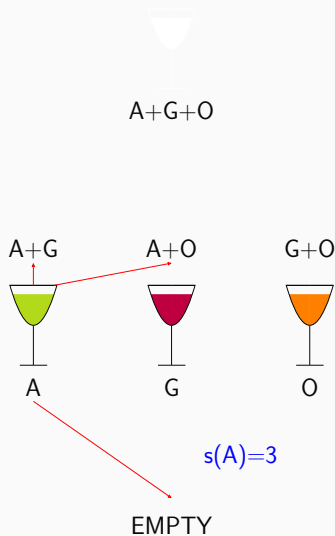
SENSITIVITY: ALICE'S OPINION

$$s(A+G+O)=0$$



EMPTY

SENSITIVITY: ALICE'S OPINION



BLOCK SENSITIVITY

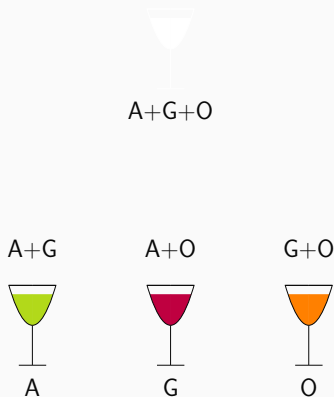
Block Sensitivity of a combination:

1. Fix a combination X .
2. For every partition P of Ingredients into blocks B_1, \dots, B_k .
 - 2.1 Set $bs^P(X) = 0$.
 - 2.2 For every block B_i :
 - 2.2.1. Let $Y = X$.
 - 2.2.2. For every ingredient j in B_i :
If $j \in X$, let $Y = Y - j$. Else, let $Y = Y + j$.
 - 2.2.3. If Opinion of X is different from Y , increment $bs^P(X)$ by 1.
3. Set $bs(X) = \max_X bs^P(X)$.

Block Sensitivity:

$$bs = \max_X bs(X).$$

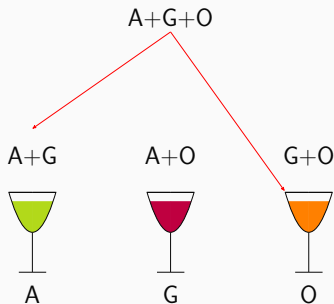
BLOCK SENSITIVITY: ALICE'S OPINION



EMPTY

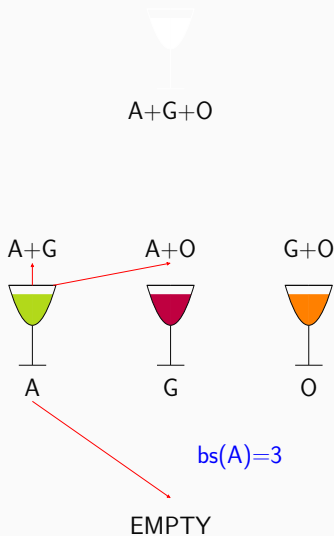
BLOCK SENSITIVITY: ALICE'S OPINION

$$bs(A+G+O)=1$$



EMPTY

BLOCK SENSITIVITY: ALICE'S OPINION

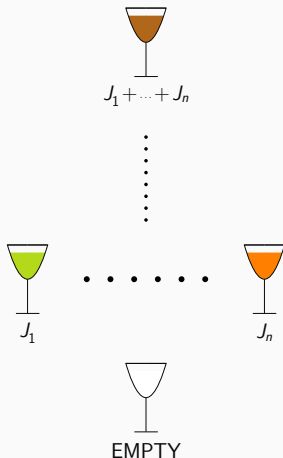


BOOLEAN FUNCTION

Ingredients: J_1, J_2, \dots, J_n .

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Ingredients: J_1, J_2, \dots, J_n .



$$f : \mathcal{P}(\{J_1, \dots, J_n\}) \rightarrow \{\text{Yes}, \text{No}\}$$

SENSITIVITY AND BLOCK SENSITIVITY

- Combination $X = J_1 + \dots + J_r$.
- Sensitivity: # of times opinion changes when either:
 - ★ $J \in \{J_{r+1}, \dots, J_n\}$ is added to X .
 - ★ $J \in \{J_1, \dots, J_r\}$ is removed from X .
- Block Sensitivity: # of times opinion changes for best partition P when for every block B_i :
 - ★ All $J \in B_i$ is either added to/removed from X .
- For every combination X we have:

$$0 \leq s(X) \leq bs(X) \leq n.$$

Conjecture (Nisan and Szegedy, 1992)

There exist constants c and δ such that for all boolean functions we have:

$$bs \leq c \cdot s^\delta.$$

SOME KNOWN RESULTS

Theorem (Nisan, 1991)

For all monotone boolean functions we have:

$$bs = s.$$

Theorem (Ambainis and Sun, 2011)

There exists a boolean function for which we have:

$$bs = \frac{2}{3}s^2 - \frac{1}{3}s.$$

DISJUNCTIVE NORMAL FORM

Alice's Opinion

A+G



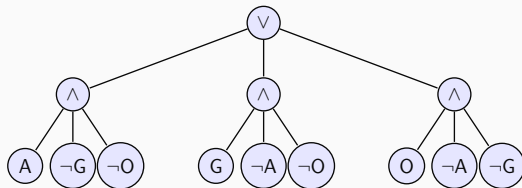
A+O



G+O

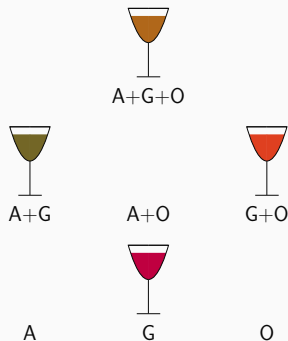


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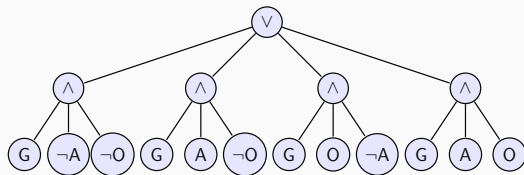


DISJUNCTIVE NORMAL FORM

Bob's Opinion

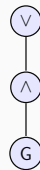
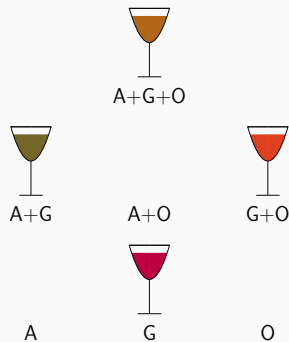


EMPTY



DISJUNCTIVE NORMAL FORM

Bob's Opinion



EMPTY

Definition

Every Juice ingredient appears **positively** in at most **one** clause.

Theorem

Any boolean function admitting the block property¹ has:

$$bs \leq 4s^2.$$

¹Conditions apply.

Lemma

Any boolean function admitting the block property has:

$$bs = \# \text{ of clauses.}$$

.

Lemma

Any boolean function admitting the block property has:

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Any boolean function admitting the block property has:

$$s \geq \frac{1}{2} \cdot \frac{\# \text{ of clauses}}{\text{Size of largest clause}}.$$

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Thank you!