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1 Portfolio Management Final Exam

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[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import pickle
from statsmodels.formula.api import ols
import statsmodels.api as sm
from scipy.stats import gaussian_kde
import scipy
import scipy.sparse
import patsy
from statistics import median
import bz2
from tqdm import tqdm
import os
from collections import defaultdict
import time
```

1.1 Problem 3.1

See implementation of (a)-(d) in the class `FactorReturnEstimator`

[2]:
```
class FactorReturnEstimator:
    '''
    Fractor Return Estimator solving Problem 3.1 (a) - (d)

    Parameters
    --------------------

    input_dir: str
        the directory of the raw datasets containing compressed
        pickle files

    cache_dir: str
        the directory of the cached files. In this program we
```

```python
        will save each date's factors as a csv file on the disk
        to avoid filling out the memory.

    disable_progress: bool, default=False
        disable the progress bar. The whole process would take
        several minutes for a single alpha factor.

    '''

    def __init__(self, input_dir, cache_dir, disable_progress=False):
        '''
        ctor
        '''
        self.disable = disable_progress
        cache_dir = cache_dir.rstrip('/')
        input_dir = input_dir.rstrip('/')

        if os.path.isdir(cache_dir) == False:
            # cache the data so that each date have a csv file
            # stored in the disk, so that we don't need to feed
            # all the data into the memory
            os.makedirs(cache_dir)
            self.dates = []
            for year in range(2003, 2011):
                fil = input_dir + "/pandas-frames." + str(year) + ".pickle.bz2"
                # unzip the compressed pickle files
                frames = pickle.load(bz2.open(fil, "rb"))
                # a tqdm progress bar iterator
                loop = tqdm(frames.items(), leave=False, disable=self.disable)
                for date, df_date in loop:
                    self.dates.append(date)
                    filepath = cache_dir+'/'+date+'.csv'
                    if os.path.isfile(filepath) == False:
                        df_date.to_csv(filepath)
                    loop.set_description('extracting year={:d}'.format(year))
        else:
            # directly read the dates from the disk file names
            self.dates = [filename.replace('.csv', '') for filename
                          in os.listdir(cache_dir) if filename[-4:] == '.csv']

        self.cache_dir = cache_dir
        self.dates = sorted(self.dates)

    def wins(self, x, a, b):
        '''
        Winsorize the array to [a, b] to dampen the effect of outliers
        '''
```

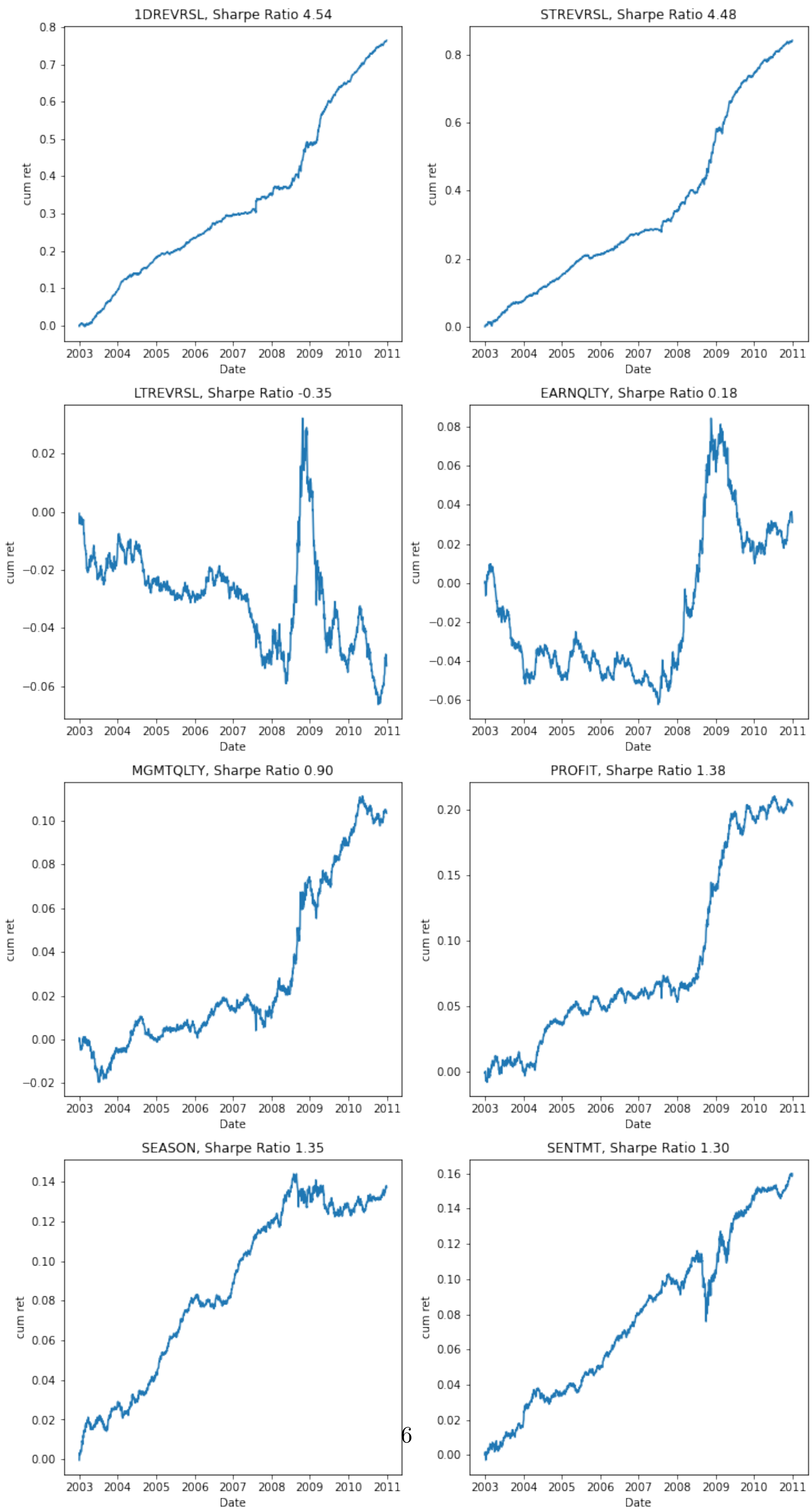
```python
        return (np.where(x <= a, a, np.where(x >= b, b, x)))

    def sharpe(self, x):
        '''
        compute the sharpe ratio of the array of daily returns

                        annualized return
        Sharpe Ratio =  ----------------------
                        annualized volatility
        '''
        # assume a year consists of 252 trading days
        annualized_ret = np.prod(x+1)**(252/len(x))-1
        annualized_vol = x.std() * np.sqrt(252)
        return annualized_ret / annualized_vol

    def estimate_factor_returns(self, alpha_name, risk_names, plot=True,␣
↪ax=None, **plot_kwargs):
        '''
        estimatr the factor returns

        Parameters
        -----------------
        alpha_name: str
            the name of your potential alpha factor

        risk_names: list of str
            the names of the canonical risk factors

        plot: bool, default=True
            if True, plot the cumulative sum of the coefficients of the factor

        ax: matplotlib.axes._subplots.AxesSubplot
            the axes of matplotlib to plot on if provided

        plot_kwargs: dict, default={}
            the keyword arguments of the ax.plot()

        Return
        -----------------
        df: pd.Dataframe
            the daily coefficients of the factor (daily factor returns)

        sharpe_ratio: flaot
            the sharpe ratio of the factor return
        '''
        ret = []
        # a tqdm progress bar iterator
```

```python
        loop = tqdm(self.dates, disable=self.disable, leave=False)
        for date in loop:
            # first read the daily factors from the disk
            filename = self.cache_dir+'/'+date+'.csv'
            df = pd.read_csv(filename)
            # only use rows with IssuerMarketCap > 1 billion dollars
            estu = df.loc[df.IssuerMarketCap > 1e9].copy(deep=True)
            estu = estu[['Ret', alpha_name]+risk_names]
            # clean out the NaN values
            estu.dropna(inplace=True)

            # Problem 3.1(a)
            # build a matrix X containing the single alpha factor
            # and the canonical risk factors
            X = estu[[alpha_name]+risk_names]

            # Problem 3.1(b)
            # winsorize the returns to [-0.25, 0.25]
            # to dampen the effect of outliers
            y = self.wins(estu['Ret'], -0.25, 0.25)

            # Problem 3.1(c)
            # estimate the model y = Xf + e using multivariate OLS
            model = sm.OLS(y, sm.add_constant(X))
            results = model.fit()

            # Problem 3.1(d)
            # get the coefficient of the alpha factor
            ret.append(results.params[alpha_name])
            loop.set_description('estimating {:s}'.format(alpha_name))

        # and stuff the alpha coef in a time series associated to the given date
        df = pd.DataFrame({'Date': self.dates, alpha_name: ret})
        df['Date'] = pd.to_datetime(df['Date'])
        df.set_index('Date', inplace=True)

        # compute the sharpe ratio
        sharpe_ratio = self.sharpe(df[alpha_name])

        if plot == True:
            # if the axes of plot is not provided
            # we create one inside this method
            if ax is None:
                fig, ax = plt.subplots(figsize=(12, 4))
            # plot the cumulative sum of the coefficients
            ax.plot(df[alpha_name].cumsum(), **plot_kwargs)
            ax.set_title('{:s}, Sharpe Ratio {:.2f}'.format(
```

```
            alpha_name, sharpe_ratio))
        ax.set_xlabel('Date')
        ax.set_ylabel('cum ret')

    return df, sharpe_ratio
```

Running one OLS per day over several years, Plot the cumulative sum of the coefficients and calculate the sharpe ratio (in the title of the plot).

[3]:
```
alphas = ['1DREVRSL', 'STREVRSL', 'LTREVRSL', 'EARNQLTY',
          'MGMTQLTY', 'PROFIT', 'SEASON', 'SENTMT']

industry_factors = [
    'AERODEF', 'AIRLINES', 'ALUMSTEL', 'APPAREL', 'AUTO',
    'BANKS', 'BEVTOB', 'BIOLIFE', 'BLDGPROD', 'CHEM',
    'CNSTENG', 'CNSTMACH', 'CNSTMATL', 'COMMEQP', 'COMPELEC',
    'COMSVCS', 'CONGLOM', 'CONTAINR', 'DISTRIB', 'DIVFIN',
    'DIVYILD', 'DWNRISK', 'ELECEQP', 'ELECUTIL', 'FOODPROD',
    'FOODRET', 'GASUTIL', 'HLTHEQP', 'HLTHSVCS', 'HOMEBLDG',
    'HOUSEDUR', 'INDMACH', 'INSURNCE', 'INTERNET', 'LEISPROD',
    'LEISSVCS', 'LIFEINS', 'MEDIA', 'MGDHLTH', 'MULTUTIL',
    'OILGSCON', 'OILGSDRL', 'OILGSEQP', 'OILGSEXP', 'PAPER',
    'PHARMA', 'PRECMTLS', 'PSNLPROD', 'REALEST', 'RESTAUR',
    'ROADRAIL', 'SEMICOND', 'SEMIEQP', 'SOFTWARE', 'SPLTYRET',
    'SPTYCHEM', 'SPTYSTOR', 'TELECOM', 'TRADECO', 'TRANSPRT',
    'WIRELESS'
]

style_factors = ['BETA', 'SIZE', 'MOMENTUM', 'VALUE']

df_ret = {}


estimator = FactorReturnEstimator(input_dir='APANPS5440', cache_dir='frames')

fig, axes = plt.subplots(nrows=4, ncols=2, figsize=(12, 4*6))

for alpha_name, ax in zip(alphas, axes.flat):

    df, SR = estimator.estimate_factor_returns(
        alpha_name=alpha_name,
        risk_names=industry_factors + style_factors,
        plot=True, ax=ax)
    df_ret[alpha_name] = df
```


1DREVRSL, Sharpe Ratio 4.54
STREVRSL, Sharpe Ratio 4.48
LTREVRSL, Sharpe Ratio -0.35
EARNQLTY, Sharpe Ratio 0.18
MGMTQLTY, Sharpe Ratio 0.90
PROFIT, Sharpe Ratio 1.38
SEASON, Sharpe Ratio 1.35
SENTMT, Sharpe Ratio 1.30
cum ret
Date
2003 2004 2005 2006 2007 2008 2009 2010 2011

[4]:
```
df_ret = pickle.load(open('pandas-frames.alphas.pickle', 'rb'))
```

Comment on whether this time series is achievable as the returns on an investment strategy, both with and without trading-costs.

If we trade without trading-costs, then `1DREVRSL`, `STREVRSL`, and `PROFIT` are achievable since they have a good Shapre ratio and a low maximized drawback. The other factors are either low Sharpe ratio or have a large drawback during the backtest period.

If we further consider the trading-cost, `1DREVRSL` and `STREVRSL` coule be problematic since the factors move so fast, which will bring us huge T-cost.

[5]:
```
pickle.dump(df_ret, open('pandas-frames.alphas.pickle', 'wb'))
from functools import reduce
dfs = [df for key, df in df_ret.items()]
df_alpha = reduce(lambda left, right: left.join(right), dfs)
df_alpha[int(2015*2/3):].index
```

[5]:
```
DatetimeIndex(['2008-05-05', '2008-05-06', '2008-05-07', '2008-05-08',
               '2008-05-09', '2008-05-12', '2008-05-13', '2008-05-14',
               '2008-05-15', '2008-05-16',
               …
               '2010-12-17', '2010-12-20', '2010-12-21', '2010-12-22',
               '2010-12-23', '2010-12-27', '2010-12-28', '2010-12-29',
               '2010-12-30', '2010-12-31'],
              dtype='datetime64[ns]', name='Date', length=672, freq=None)
```

1.2 Problem 3.2

See the implementation of (a)-(c) in `CostlessPortfolioOptimizer`

[24]:
```
class CostlessPortfolioOptimizer:

    '''
    Costless Portfolio Optimization solving Problem 3.2 (a) - (f)

    Parameters
    ----------------
    cov_dir: str
        the directory of the compressed pickle files storing covariance

    cache_frames_dir, str
        the directory of the cached frames csv files

    cache_dir, str
        the directory of the cached covariance csv files
```

```python
    df_alpha: dict or pd.Dataframe
        the alpha factor returns

    canonical_risk_factors, list of str
        the cannonical risk factors

    disable_progress, bool, default=False
        to disable the progressbar

    '''

    def __init__(self, cov_dir, cache_frames_dir, cache_dir, df_alpha,
                 canonical_risk_factors, disable_progress=False):
        self.cov_dir = cov_dir.rstrip('/')
        self.cache_frames_dir = cache_frames_dir.rstrip('/')
        self.cache_dir = cache_dir.rstrip('/')
        self.df_alpha = df_alpha
        self.alpha_names = list(df_alpha)
        # Problem 3.2(a)
        # calculate its mean factor return over the
        # first 2/3 of the date range (the "in-sample period")
        # and store these coefficients in a vector
        self.alpha = df_alpha[:int(len(df_alpha)*2/3)].mean().values
        self.canonical_risk_factors = canonical_risk_factors
        self.p = len(canonical_risk_factors)
        self.disable = disable_progress
        # create a lookup table of the factor names
        # to make the code run faster
        self.lookup_table = defaultdict(
            lambda: -1,
            zip(canonical_risk_factors, range(self.p))
        )

        if os.path.isdir(self.cache_dir) == False:
            # cache the data so that each date have a csv file
            # stored in the disk, so that we don't need to feed
            # all the data into the memory
            os.makedirs(self.cache_dir)
            self.dates = []
            for year in range(2003, 2011):
                fil = self.cov_dir + "/covariance." + str(year) + ".pickle.bz2"
                # unzip the compressed pickle files
                cov_dict = pickle.load(bz2.open(fil, "rb"))
                # a tqdm progress bar iterator
                loop = tqdm(cov_dict.items(), leave=False,
                            disable=self.disable)
```

```
                for date, df_date in loop:
                    self.dates.append(date)
                    filepath = self.cache_dir+'/'+date+'.csv'
                    if os.path.isfile(filepath) == False:
                        df_date.to_csv(filepath)
                    loop.set_description('extracting year={:d}'.format(year))
        else:
            # directly read the dates from the disk file names
            self.dates = [filename.replace('.csv', '') for filename
                          in os.listdir(self.cache_dir) if filename[-4:] == '.
↪csv']

        # only calculate over the last 1/3 of the dates (the "out of sample␣
↪period").
        self.dates = sorted(self.dates)[int(len(df_alpha)*2/3):]

    def get_F(self, df_cov):
        '''
        Subsets the covariance matrix dataframe to the canonical risk factors.
        Hence you will be able to calculate F on any given date.

        Parameters
        ------------------
        df_cov, pd.Dataframe
            the dataframe of the factors on a given date
        '''

        F = np.zeros((self.p, self.p))

        for i in range(len(df_cov)):
            factor1 = df_cov['Factor1'].iloc[i]
            factor2 = df_cov['Factor2'].iloc[i]

            idx1 = self.lookup_table[factor1]
            idx2 = self.lookup_table[factor2]
            if idx1 == -1 or idx2 == -1:
                continue
            varcovar = df_cov['VarCovar'].iloc[i]
            F[idx1][idx2] = varcovar
            if idx1 != idx2:
                F[idx2][idx1] = varcovar

        # assume the covariance matrix is annualized percentage squared
        # so we need to divid by (100*sqrt(252)) ** 2
        return F/10000/252

    def f(self, h):
```

```
        '''
        f = 0.5k hT QT Q h + 0.5k hT D h - alphaT h
        '''
        tmp = 0.0
        tmp += 0.5 * self.kappa * np.sum(np.matmul(self.Q, h) ** 2)
        tmp += 0.5 * self.kappa * np.dot(h ** 2, self.specVar)
        tmp -= np.dot(h, self.alpha_vec)

        return(tmp)

    def grad(self, h):
        '''
        grad = k QTQ h + kDh - alpha,

        we don't compute QTQ since it's an nxn matrix

        we compute QT @ ( Q @ h)
        '''
        tmp = - self.alpha_vec
        Qh = np.matmul(self.Q, h)
        tmp += self.kappa * np.matmul(self.QT, Qh)
        tmp += self.kappa * (self.specVar * h)

        return tmp

    def optimize(self, kappa):
        '''
        For each date in the out-of-sample period,
        find h* that minimizes the objective
        '''

        self.kappa = kappa
        profits = []
        profits_exact = []
        positions = []
        vols = []
        ido_frac = []
        industry_frac = []
        style_frac = []
        # for each date in the out-sample period
        loop = tqdm(self.dates, disable=self.disable, leave=False)
        for date in loop:
            # Problem 3.2(b)
            # calculate F on that  date
            df_cov = pd.read_csv(self.cache_dir+'/'+date+'.csv')
            F = self.get_F(df_cov)
            # get X and specVar
```

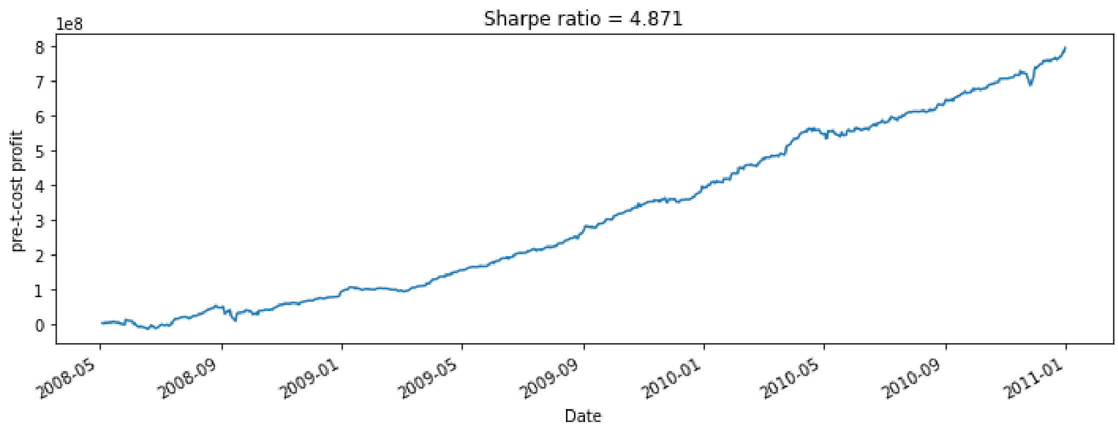
```
        df_frames = pd.read_csv(self.cache_frames_dir+'/'+date+'.csv')
        df_frames = df_frames.loc[df_frames.IssuerMarketCap > 1e9].copy(
            deep=True)
        X = df_frames[self.canonical_risk_factors].values
        # compute Q using a simple matrix factorization
        XT = X.transpose()
        self.Q = np.matmul(scipy.linalg.sqrtm(F), XT)
        self.QT = self.Q.transpose()

        # notice that the SpecRisk column is annulized percent volatility
        annualized_vol = df_frames['SpecRisk'].values / 100
        # convert annulaized vol to daily variance
        # assume 1 year = 252 trading days
        # again we need to divid by (100*sqrt(252)) ** 2
        self.specVar = annualized_vol ** 2 / 252

        # get combined alpha
        X_alpha = df_frames[self.alpha_names].values
        self.alpha_vec = np.matmul(X_alpha, self.alpha)

        # initailize h to be equal weighted portfolio
        h0 = np.zeros(len(df_frames))

        # Problem 3.2(c)
        # get the optimized result
        optimizer_result = scipy.optimize.fmin_l_bfgs_b(
            self.f, h0, self.grad)
        h_star = optimizer_result[0]
        positions.append(h_star)
        # solve the optimization in a closed form
        A = kappa * (np.matmul(self.Q.transpose(), self.Q) +
                     np.diag(self.specVar))
        B = np.dot(self.alpha, X_alpha.transpose())

        h_star_exact = np.linalg.solve(A, B)
        R = df_frames['Ret'].values

        # Problem 3.2(d)
        # For every day in the out of sample period, compute the pre-t-cost␣
↪profit
        profits.append(np.dot(h_star, R))
        profits_exact.append(np.dot(h_star_exact, R))

        # Problem 3.2(e)
        var_ido = np.dot(h_star ** 2, self.specVar)

        x = np.matmul(XT, h_star)
```

```
            Fx = np.matmul(F, x)
            var_factor = x * Fx

            # the last four factors are style
            var_style = np.sum(var_factor[-4:])
            # the rest of them are industry
            var_industry = np.sum(var_factor[:-4])

            var_total = (var_ido+var_industry+var_style)
            vol = np.sqrt(var_total)

            vols.append(vol)
            ido_frac.append(var_ido/var_total)
            industry_frac.append(var_industry/var_total)
            style_frac.append(var_style/var_total)

            loop.set_postfix(warnflag=optimizer_result[2]['warnflag'])

        # and stuff the profits in a time series associated to the given date
        df = pd.DataFrame({'Date': self.dates,
                           'profit': profits,
                           'exact': profits_exact,
                           'vols': vols,
                           'ido_frac': ido_frac,
                           'industry_frac': industry_frac,
                           'style_frac': style_frac})
        df['Date'] = pd.to_datetime(df['Date'])
        df.set_index('Date', inplace=True)

        return df, positions


canonical_risk_factors = industry_factors + style_factors


optimizer = CostlessPortfolioOptimizer(cov_dir='APANPS5440/',
                                       cache_dir='covariances/',
                                       cache_frames_dir='frames/',
                                       df_alpha=df_alpha,
                                       disable_progress=False,
                                       ␣
 ↪canonical_risk_factors=canonical_risk_factors)
df, hs = optimizer.optimize(kappa=1e-6)
```

For every day in the out of sample period, compute the pre-t-cost profit and print out the sharpe ratio.

[25]:
```python
# Problem 3.2(d)
# Plot the cumulative sum, and print out the sharpe ratio of the out-of-sample
fig, ax = plt.subplots(figsize=(12, 4))
df['profit'].cumsum().plot(ax=ax)
ax.set_ylabel('pre-t-cost profit')
mean = df['profit'].mean() * 252
std = df['profit'].std() * np.sqrt(252)
ax.set_title('Sharpe ratio = {:.3f}'.format(mean/std))
```

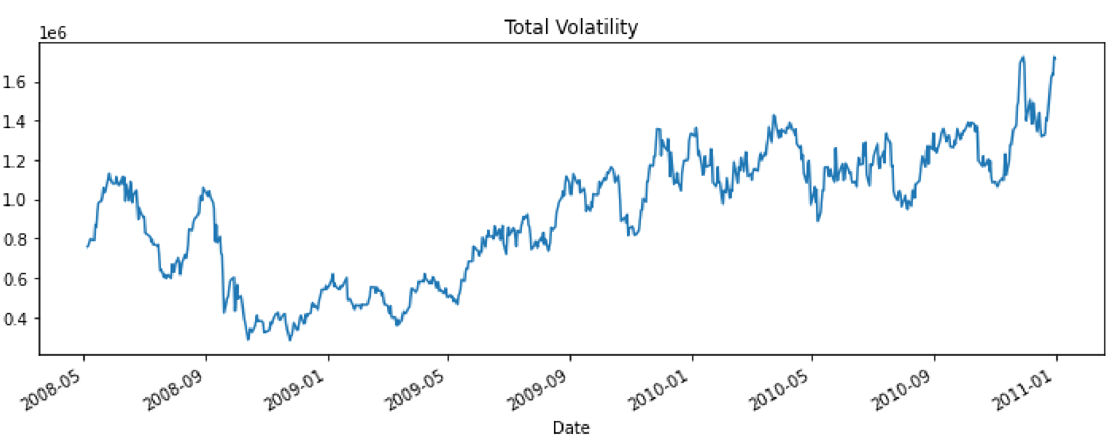
[25]:
```
Text(0.5, 1.0, 'Sharpe ratio = 4.871')
```



For every day in the out of sample period, do a variance decomposition on h^*

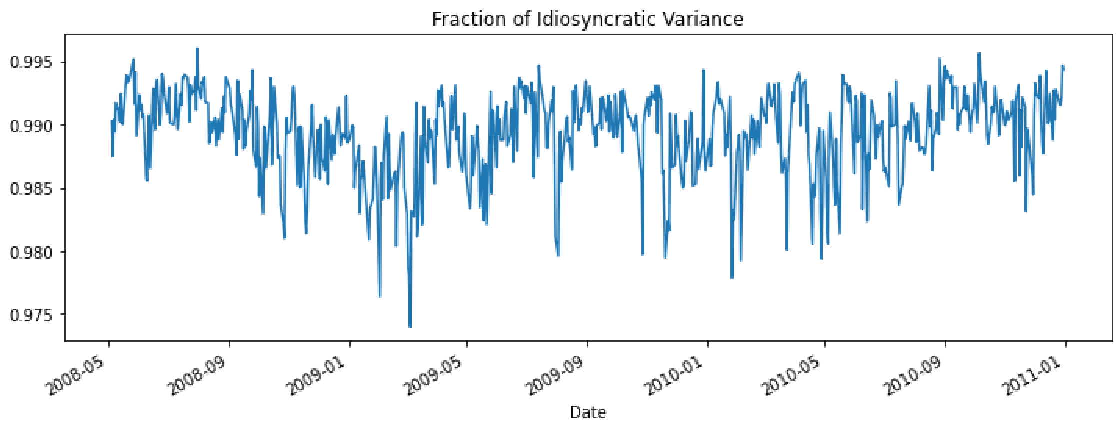
[26]:
```python
fig, ax = plt.subplots(figsize=(12, 4))
df['vols'].plot(ax=ax)
ax.set_title('Total Volatility')
```

[26]:
```
Text(0.5, 1.0, 'Total Volatility')
```



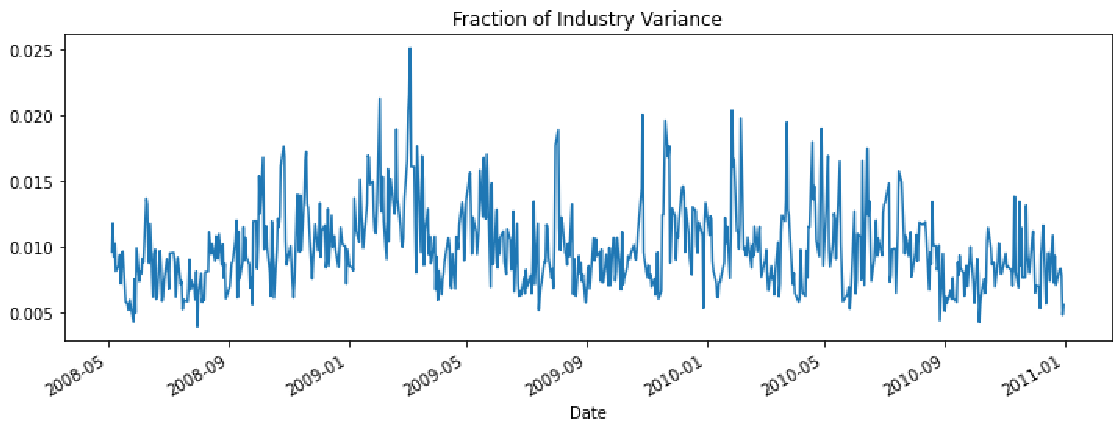
[27]:
```
fig, ax = plt.subplots(figsize=(12, 4))
df['ido_frac'].plot(ax=ax)
ax.set_title('Fraction of Idiosyncratic Variance')
```

[27]: Text(0.5, 1.0, 'Fraction of Idiosyncratic Variance')



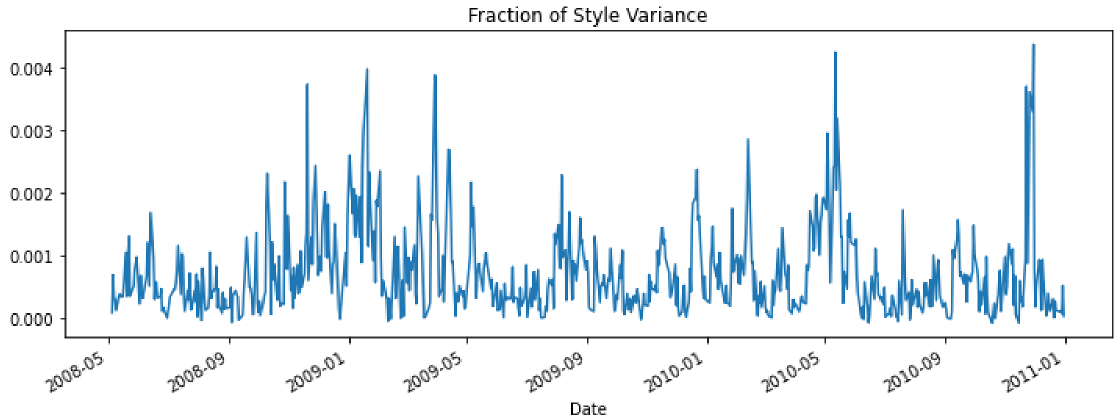
[28]:
```
fig, ax = plt.subplots(figsize=(12, 4))
df['industry_frac'].plot(ax=ax)
ax.set_title('Fraction of Industry Variance')
```

[28]: Text(0.5, 1.0, 'Fraction of Industry Variance')



[29]:
```
fig, ax = plt.subplots(figsize=(12, 4))
df['style_frac'].plot(ax=ax)
ax.set_title('Fraction of Style Variance')
```

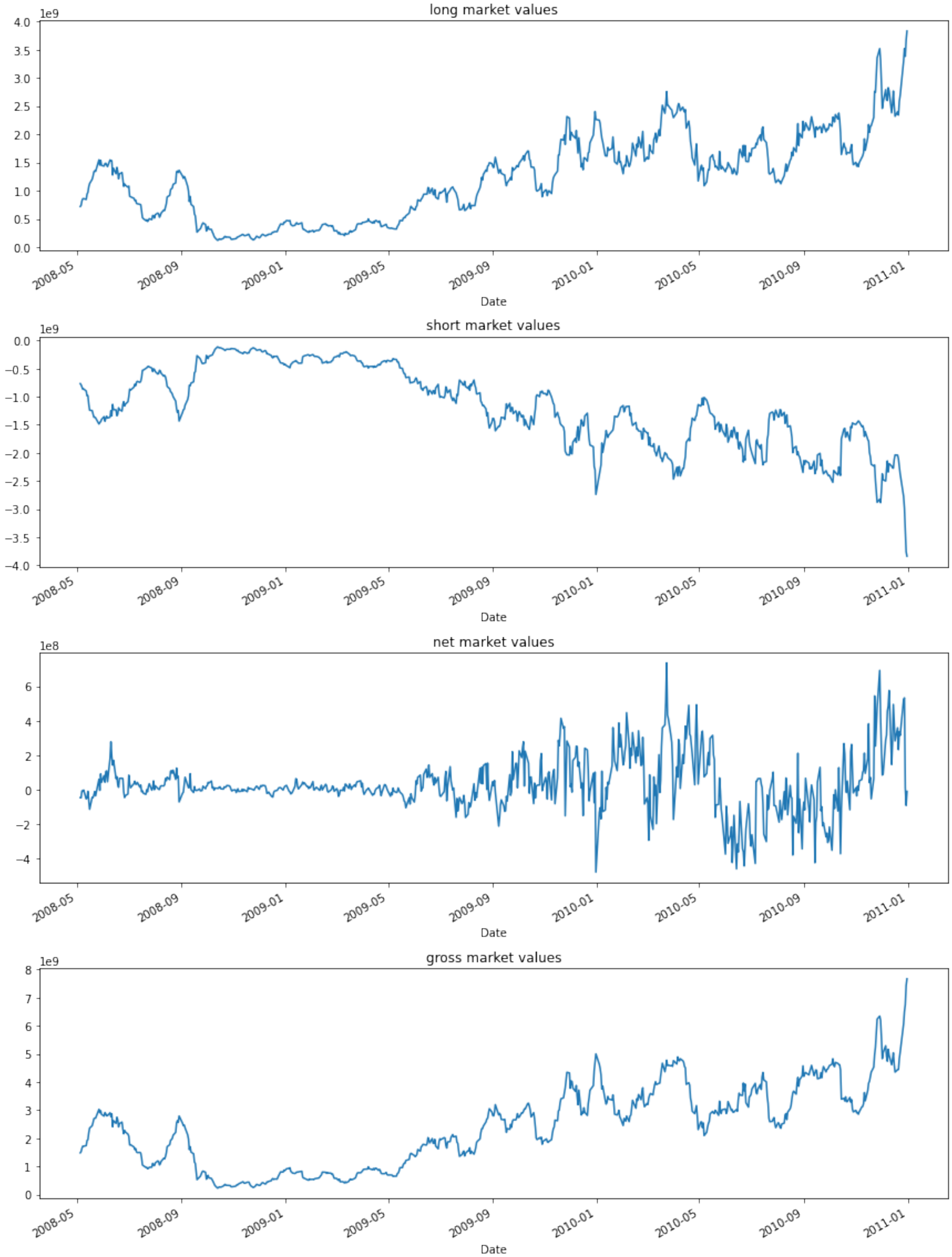
[29]: Text(0.5, 1.0, 'Fraction of Style Variance')



Plot the long, short, net and gross market values, in dollars, for every optimal portfolio in the out of sample period.

[30]:
```
# Problem (f)
long = []
short = []
net = []
gross = []
for h in hs:
    long.append(np.sum(h[h > 0]))
    short.append(np.sum(h[h < 0]))
    net.append(np.sum(h))
    gross.append(np.sum(np.abs(h)))

df_mkt_values = pd.DataFrame({'Date': df.index,
                              'long': long,
                              'short': short,
                              'net': net,
                              'gross': gross
                              }).set_index('Date')

fig, ax = plt.subplots(4,1,figsize=(12, 16))
df_mkt_values['long'].plot(ax=ax[0])
df_mkt_values['short'].plot(ax=ax[1])
df_mkt_values['net'].plot(ax=ax[2])
df_mkt_values['gross'].plot(ax=ax[3])
ax[0].set_title('long market values')
ax[1].set_title('short market values')
ax[2].set_title('net market values')
ax[3].set_title('gross market values')
```

```
plt.tight_layout()
```



1.3 Problem 3.3

The formula would be

$$h^* = \left(\kappa Q^T Q + \kappa D\right)^{-1} \alpha$$

In `CostlessPortfolioOptimizer` class we implemented this formula by using `np.linage.solve` to avoid inverting a huge matrix.

As we can see it's very close to the numerical optimized result.

[32]:
```
# Problem 3.3
# Plot the cumulative sum,
fig, ax = plt.subplots(figsize=(12, 4))
df[['exact', 'profit']].cumsum().plot(ax=ax)
ax.set_ylabel('pre-t-cost profit')
```

[32]:
```
Text(0, 0.5, 'pre-t-cost profit')
```