# Algorithm Ver1.0

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April 2019

## 1 The aggregation error of perceived data privacy protection(How to strengthen $\alpha$)

$$Pr[|\hat{s} - s| \geq \alpha] \leq 1 - \delta \tag{1}$$

The aggregation error of the randomized sensing data can be expressed as:

$$\hat{s} - s = \frac{1}{n} \sum_{i=1}^{N} \eta_i \tag{2}$$

Then we can derive that:

$$D(\frac{1}{n} \sum_{i=1}^{N} \eta_i) = \frac{2}{n^2} \sum_{i=1}^{n} b_i^2 \tag{3}$$

If we introduce the regular chebyshev inequality:

$$P(|X - E(X)| \geq \alpha) \leq \frac{Var(X)}{\alpha^2} \tag{4}$$

Thus, we can calculate the aggregation error of perceived data privacy protection:

$$\alpha = \frac{\sqrt{2}\gamma}{M\sqrt{1-\delta}} \sqrt{\sum_{i=1}^{M} \frac{1}{\epsilon_i^2}} \tag{5}$$

Next, we introduce the generalized form of chebyshev inequality combined with Markov inequality:

$$P(|X - E(X)| \geq \alpha) = P(|X| \geq \alpha) \leq \lambda \frac{Var(X)}{\alpha^2} + (1-\lambda)\frac{E(|x|)}{\alpha}, \lambda \in (0,1) \tag{6}$$

thus we have:

$$(1-\delta)\alpha^2 - [(1-\lambda)\frac{\gamma}{M} \sum_{i=1}^{M} \frac{1}{\epsilon_i}]\alpha - \frac{2\lambda\gamma^2}{M^2} \sum_{i=1}^{M} \frac{1}{\epsilon_i^2} = 0 \tag{7}$$

From this we establish the expression for the aggregate error (we can see that the original expression is a special case when it is equal to 1):

$$\alpha = \gamma \frac{(1-\lambda)\sum_{i=1}^{M} \frac{1}{\epsilon_i} + \sqrt{(1-\lambda)^2 (\sum_{i=1}^{M} \frac{1}{\epsilon_i})^2 + 8(1-\delta)\lambda \sum_{i=1}^{M} \frac{1}{\epsilon_i^2}}}{2M(1-\delta)}, \lambda \in (0,1) \tag{8}$$

when $\lambda = 0$, we got that:

$$\alpha = \gamma \frac{\sum_{i=1}^{M} \frac{1}{\epsilon_i}}{M(1-\delta)} \tag{9}$$

# 2 Nash equilibrium

$$u_s(\epsilon, p) = \frac{1}{\alpha} - \sum_{i=1}^{M} p_{\epsilon_i} \tag{10}$$

$$u_i(\epsilon_i, p) = p_{\epsilon_i} - \epsilon_i c_i - c \tag{11}$$

## 2.1 The static state of $\epsilon_i$

The income of mobile station and base station is set up as the equations to find the Nash equilibrium point.When the Nash equilibrium tends to converge, we can easily get the total utility function $u_s(\epsilon, p) + \sum_{i=1}^{M} u_i(\epsilon_i, p)$ tends to a fixed value, which can facilitate the privacy protection level under the final stable state:

$$\frac{\partial [u_s(\epsilon, p) + \sum_{i=1}^{M} u_i(\epsilon_i, p)]}{\partial \epsilon_i} = \frac{\partial (\frac{1}{\alpha} - \sum_{i=1}^{M} \epsilon_i c_i - Mc)}{\partial \epsilon_i} = 0, i = 1, 2, 3...M \tag{12}$$

if we use $\alpha$ in equation (5):

$$\epsilon_i^* = (\frac{M\sqrt{1-\delta}}{c_i\sqrt{2}\gamma})^{\frac{1}{3}} (\sum_{i=1}^{M} \frac{1}{\epsilon_i^2})^{-\frac{1}{2}}, i = 1, 2, 3...M \tag{13}$$

if we use $\alpha$ in equation (9):

$$\epsilon_i^* = \sqrt{\frac{M(1-\delta)}{c_i\gamma}} (\sum_{i=1}^{M} \frac{1}{\epsilon_i})^{-1} \tag{14}$$

## 2.2 How to determine $p$

when the NE tends to converge, the $u_i$ tends to be a fixed value:

$$\frac{\partial p_{\epsilon_i}}{\partial \epsilon_i}|_{\epsilon_i = \epsilon_i^*} - c_i = 0, i = 1, 2, 3...M \tag{15}$$

$$\frac{\partial^2 p_{\epsilon_i}}{\partial^2 \epsilon_i}|_{\epsilon_i = \epsilon_i^*} \leq 0, i = 1, 2, 3...M \tag{16}$$

According to the slope transformation in different intervals, we can fit this piecewise payment functionand then conduct sampling and quantization operations.