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Problem Set 4
EECS 649
2-12-22

4.1
a. Hill-climbing search
c. First-choice hill climbing
e. A random walk that moves to a successor state without concern for the value

4.2
Genotype refers to what genes an organism might have (e.g. xX genes) while phenotype refers to how those genes are expressed physically. For example, an organism might have a genotype of bB for iris color which results in a phenotype of blue irises.

For genetic algorithms, the specific values of weights and biases could be referred to as the genotype because these values may vary from organism to organism and may change via mutation. Furthermore, they are an internal value (like genes). The performance resulting from those weights and biases (genotype) can be referred to as a phenotype because it's the outward expression of the genotype.

4.3
a. A fitness function for an elevator would minimize wait time for both people who are on the elevator on their way to their destination floor and who are waiting for the elevator to come to them. People who have been waiting for longer amounts of time should have some priority over those who haven't been waiting as long.

b. A fitness function for use in evolving agents that control stop lights on a city main street would minimize wait time for cars at red lights, proportional to the number of cars waiting. For example, at the Massachusetts Street and 11^{th} Street intersection, the wait time for drivers at a red light on Mass is lower than the wait time for drivers at a red light on 11^{th} because Mass is a busier street. It is also important that 11^{th} Street doesn't have to wait too long at a red light, so a maximum waiting time should be implemented as well.

4.4
X is the set of classes: $\{Cn \mid 1 \leq n \leq N\}$ where N is the # of classes.

D is a set of professor-room-time tuples: $\{ (Pa, Rb, Tc) \mid 1 \leq a \leq A, 1 \leq b \leq B, 1 \leq c \leq C \}$ where A is the # of professors, B is the # of rooms, and C is the # of possible time slots for classes.

C: For any two classes, Cx and Cy, who have values (Px, Rx, Tx) and (Py, Ry, Ty).
If (Px = Py OR Rx = Ry) then $Tx \neq Ty$.

4.5

The first action I made was to find the MRV which was F because it only had 2 possible values, 1 or 0. I used F = 1 because it was the least constraining value (it ruled out the fewest values in the remaining variables).

With F = 1, C3 = 1 and the next MRV, T, could be 5, 6, 7, 8, or 9 for C2 + 2T = O + 10 to hold. The possible O values for all T are shown below. I used these values to determine a T value. The least constraining values were T = 7 and T = 8.

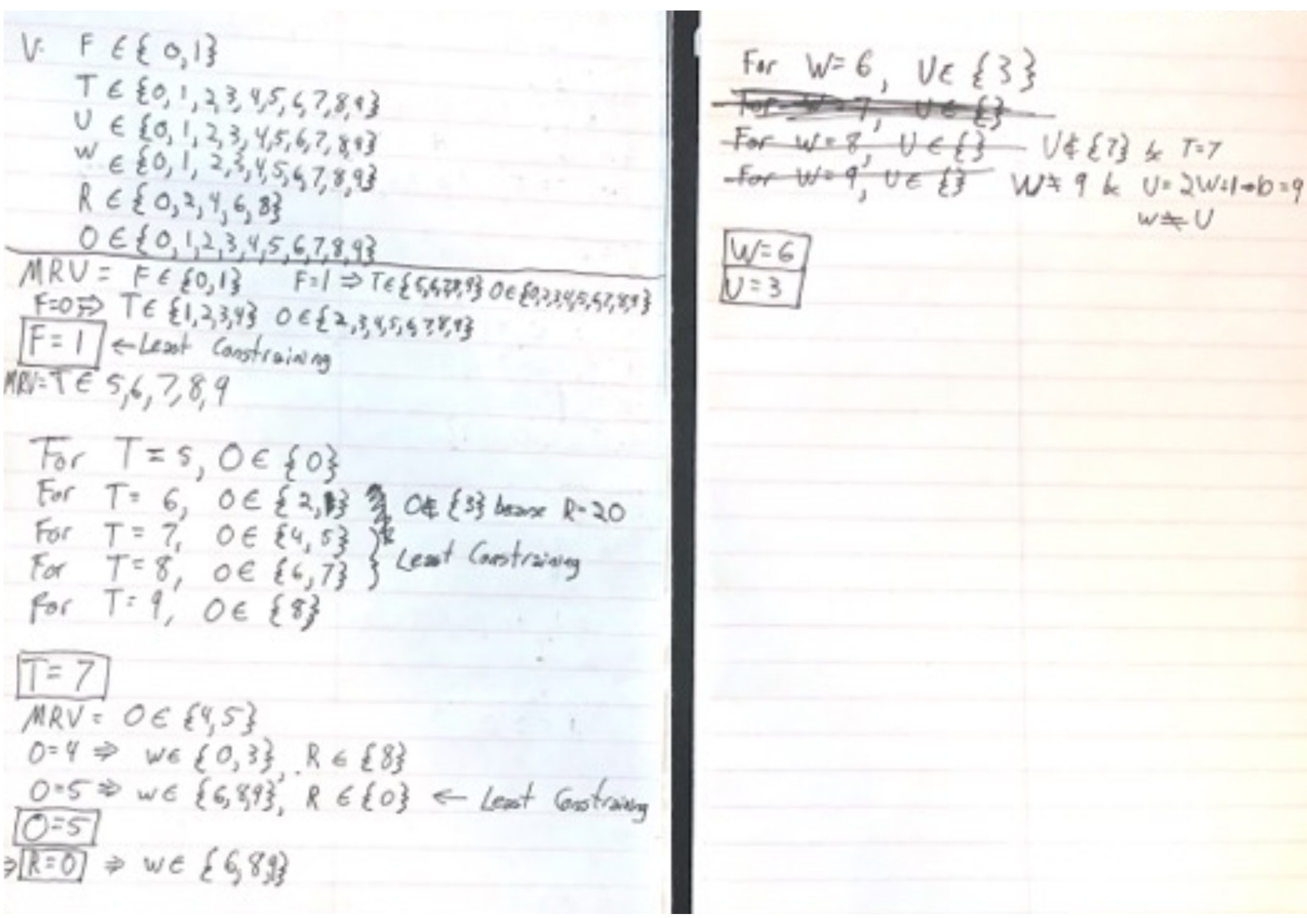
For T = 7, I found the MRV to be O. O must be 4 or 5 for C2 + 14 = O + 10 to hold. The least constraining value for O was 5 (based on W values for each O value).

It immediately followed that R = 0 and C1 = 1 because 10*C1 + R = O + O.

For R = 0 and O = 5, W must be 6 for 1 + 2W = U + 10 because the other possible W values (8,9) result in U values that violate the *Alldiff*(F, T, U, W, R, O) constraint:

> If W = 8, U = 7. U cannot be 7 because T = 7, so W cannot be 8.
> If W = 9, U = 9. U cannot be 9 because W = 9, so W cannot be 9.

Final Result: F = 1, T = 7, O = 5, R = 0, W = 6, U = 3



V: F ∈ {0,1}
T ∈ {0,1,2,3,4,5,6,7,8,9}
U ∈ {0,1,2,3,4,5,6,7,8,9}
W ∈ {0,1,2,3,4,5,6,7,8,9}
R ∈ {0,2,4,6,8}
O ∈ {0,1,2,3,4,5,6,7,8,9}

MRV = F ∈ {0,1}　F=1 ⇒ T ∈ {5,6,7,8,9} O ∈ {0,2,3,4,5,6,7,8,9}
F=0 ⇒ T ∈ {1,2,3,4} O ∈ {2,3,4,5,6,7,8,9}
F = 1 ← Least Constraining
MRV=T ∈ 5,6,7,8,9

For T = 5, O ∈ {0}
For T = 6, O ∈ {2,3}　O ∉ {3} because R=20
For T = 7, O ∈ {4,5}
For T = 8, O ∈ {6,7}　Least Constraining
For T = 9, O ∈ {8}

T = 7
MRV = O ∈ {4,5}
O=4 ⇒ W ∈ {0,3}, R ∈ {8}
O=5 ⇒ W ∈ {6,8,9}, R ∈ {0} ← Least Constraining
O=5
⇒ R=0 ⇒ W ∈ {6,8,9}

For W=6, U ∈ {3}
~~For W=7, U ∈ {}~~
~~For W=8, U ∈ {}~~　U ∉ {7} b/c T=7
~~For W=9, U ∈ {}~~　W ≠ 9 b/c U=2W+1−10=9　W ≠ U

W=6
U=3

4.6

```
from math import sin, pow
import random

def F(x):
    return 4 + 2*x + 2*sin(20*x) - 4*pow(x,2)

def roulette_selection(population):
    while True: # continue until return
        max     = sum([F(x) for x in population])
        pick    = random.uniform(0, max)
        current = 0
        # Return proportional to fitness
        for x in population:
            current += F(x)
            if current > pick:
                return x

population = [i*0.01 for i in range(1,100)] # Initial population
N = 10
best = 0

for i in range(50): # For 50 generations
    # Roulette Selection
    select = []
    while len(select) < N:  # Ensure at least n individuals in population
        select.append(roulette_selection(population))
    population = select.copy()

    # Crossover
    cross = []
    for i in range(N):
        # pick two parents
        x = roulette_selection(population)
        y = roulette_selection(population)
        while x == y: # select unique parents
            y = roulette_selection(population)
        a = random.random()
        cross.append(a*x+(1-a)*y)
    population = cross.copy()

    # Mutation
    mut = []
    epsilon = 0.01
    for x in population:
        r = random.random()
        if r <= 0.3:            # x-epsilon w/ probability 0.3
```

```
            if x-epsilon >= 0: mut.append(x-epsilon)
            else: mut.append(0) # Clip to remain in [0,1]
        elif 0.3 < r <= 0.7:    # x w/ probability 0.4
            mut.append(x)       # copy w/ probability 0.4
        else:                   # x+epsilon w/ probability 0.3
            if x+epsilon <= 1: mut.append(x+epsilon)
            else: mut.append(1) # Clip to remain in [0,1]
    population = mut.copy()

    for x in population:
        if F(x) > F(best):
            best = x
print('Best x = {}; F({}) = {}'.format(best,best,F(best)))
```

*** For my implementation without crossover, I just didn't include the chunk of code starting with "# Crossover" and ending with "population = cross.copy()"

I used N = 10 and found a maximum F(x) at x = ~0.39 and F(0.39) = ~6.169.

The implementation without crossover was more inconsistent and wouldn't always reach the same maximum as my crossover implementation; sometimes stopping around F(0.8) = 6.13.

The implementation with crossover would consistently find a maximum at x = ~0.39 with F(0.39) = ~6.169.

For each implementation I ran the algorithm for 50 generations, which is where convergence was typically found.

4.7

a. Random restart hill climbing:

```
import numpy as np
import random
# Return the number of queens conflicting in grid.
def fitness(queens, N):
    c = 0 # Number of conflicts
    # For each queen
    for q in range(len(queens)):
        for q_y in range(len(queens)):
            q_x = queens[q_y]
            if q_y != q: # Check all other queens
                if q_x == queens[q]: # If queens are in same column
                    c += 1
                if abs(q - q_y) == abs(queens[q] - q_x): # If queens share a diagonal
                    c += 1
    return (N*(N-1))/2 - c

def print_grid(queens, N):
    grid = np.full(shape=(N, N), fill_value='-')
    q_x = 0
    for q_y in queens:
        grid[q_x][q_y] = '*'
        q_x += 1
    for row in np.flip(grid, 0):
        for cell in row:
            print(cell, end=' ')
        print()

N = 8       # Grid size, number of queens
queens = [] # list of queen x-coordinates; queen[y] = x
i = 0       # Epoch count

success = False # Conflict exists

while not success:
    i+=1
    # Random start state for queen n
    for n in range(N):
        queens.append(random.randint(0,N-1))
    # If a solution is found, break loop
    if fitness(queens, N) == N*(N-1)/2:
        print_grid(queens, N) # Print grid
        success = True

print('SUCCESS in {} iterations'.format(i))
```

Simulated Annealing:

```
from math import exp, inf
import numpy as np
import random
def fitness(queens, N): # Same as random restart hill climbing
def print_grid(queens, N): # Same as random restart hill climbing

def schedule(t): # Test schedule function
    # calculate temperature for current epoch
    return 0.75*(pow(t,-0.5)-0.025)

def rand_successor(queens, N):  # Return a set of queens with a random queen moved
    q_copy = queens.copy()      # copy of queen positions
    q = random.randint(0,N-1)   # random queen, q
    q_y = q_copy[q]             # q position
    while q_y == q_copy[q]:     # choose new position for q
        q_y = random.randint(0,N-1)
    q_copy[q] = q_y
    return q_copy

N = 8       # Grid size, number of queens
queens = [] # list of queen x-coordinates; queen[y] = x
for n in range(N): # Random start states for all N queens
    queens.append(random.randint(0,N-1))

T = inf # Temperature
t = 1 # Epoch
while True:
    T = schedule(t)
    if T == 0: # Stop annealing when Temperature reaches 0
        print('T == 0. No more annealing to be done')
        print('There are {} conflicts.'.format((N*(N-1)/2) - fitness(queens,N)))
        print_grid(queens, N)
        break
    next = rand_successor(queens, N) # Pick a random successor
    delta = fitness(next,N) - fitness(queens,N) # Calculate change in fitness
    r = random.random() # Random number 0-1
    p = exp(delta/T)    # Probability of accepting next set of queens
    if delta > 0:   # If next has higher fitness than current, update queens
        queens = next.copy()
    elif r <= p:    # Set next to queens anyway w/ probability e^(delta/T)
        queens = next.copy()
    if fitness(queens,N) == N*(N-1)/2: # Break loop if solution has been found
        print_grid(queens, N)
        print('SUCCESS in {} iterations.'.format(t))
        break
    t += 1 # Iterate epoch
```

Method	Average Number of Evaluations
Random Restart Hill Climbing	158,126
Simulated Annealing schedule = $0.998*T_{t-1}$ $T_0 = 30$	2173
Simulated Annealing schedule = $0.75(t^{-0.5} - 0.025)$ t := epoch	512
Simulated Annealing schedule = $0.75(T_{t-1}^{-0.5} - 0.025)$ $T_0 = 100$	673