Algorithmic Methods for Mathematical Models

– COURSE PROJECT –

A logistics company needs to place a set P of packages inside a set T of trucks that will later transport these items. All trucks have identical length ($xTruck$), width ($yTruck$), and maximum capacity ($wTruck$ kg.), all of which are integer numbers.

Similarly, for each package p we know its length ($xDim_p$), width ($yDim_p$), and weight w_p in kg., all of which are also integer numbers. Due to the design of the packages, they cannot be rotated when placed in a truck. As a result, a 4x2 package does not fit in a 3x10 truck. Another important constraint is that packages cannot be put one on top of another. If we assume that the height of the truck is large enough for all packages, we can ignore the height of both the truck and the packages and focus on a 2-dimensional placement problem (see Figure 1-a).

For a number of reasons, there are pairs of packages that cannot be in the same truck. For that purpose, we are given a symmetric Boolean matrix $incomp$ such that $incomp[p_1][p_2] = 1$ if and only if packages p_1 and p_2 cannot be in the same truck.

The goal is to find out how to distribute the packages into the trucks so that transportation cost is minimized. More precisely, we want to minimize the load of the truck with the highest load, in addition to the number of trucks to be used. The multiobjective function would be as follows:

$$wTruck * \text{number of used trucks} + \text{load of the truck with the highest load}$$

1. **Work to be done:**
   - Formally state the problem
   - Devise an integer linear programming model for the optimization problem and implement it in OPL. The following Boolean variables will be useful:
     - $pt_{p,t}$ = package p is placed in truck t
     - $pxy_{p,x,y}$ = package p is in the cell with upper-right coordinates (x, y) (see Figure 1-b)
     - $pbl_{p,x,y}$ = the bottom left cell of p has upper-right coordinates (x, y) (see Figure 1-b)
     - $used_t$ = truck t is used

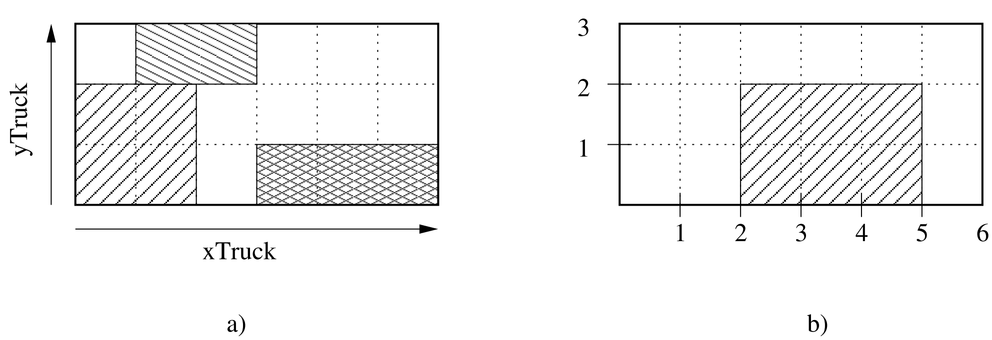

Figure 1: In a) we can see the placement of 3 packages of dimensions: 2x2, 2x1 and 3x1 inside a 6x3 truck. In b), the placement of package p makes the variables $pbl_{p,3,1}$, $pxy_{p,3,1}$, $pxy_{p,4,1}$, $pxy_{p,5,1}$, $pxy_{p,3,2}$, $pxy_{p,4,2}$ and $pxy_{p,5,2}$ true.

- Because of the complexity of the optimization problem, heuristic algorithms are needed. We are considering both GRASP and BRKGA meta-heuristics. Implement them in the programming language you prefer.
- Compare the performance of solving the model and the heuristics in terms of computation time and quality of the solutions. To that end, generate increasingly larger problem instances until solving takes around 2 hours.
- Compare the performance of the two meta-heuristics in terms of solving time and quality of the solution for even larger problem instances.

2. **Report**

   Prepare a report (8-10 pages) including:

   - Problem statement.
   - Integer lineal model, including the definition of the sets and parameters, the model itself and a short description of the objective function and every constraint.
   - For the meta-heuristics, the pseudo-code of the GRASP *constructive* and *local search phases* algorithms, the greedy function and the equation describing the RCL. For BRKGA, the chromosome structure and the pseudo-code of the *decoder* algorithm.
   - Comparative results.
   - Together with the report, you should also provide all sources and instructions on how to use them, so that results can be easily reproduced. If you implemented an instance generator, please provide it as well.