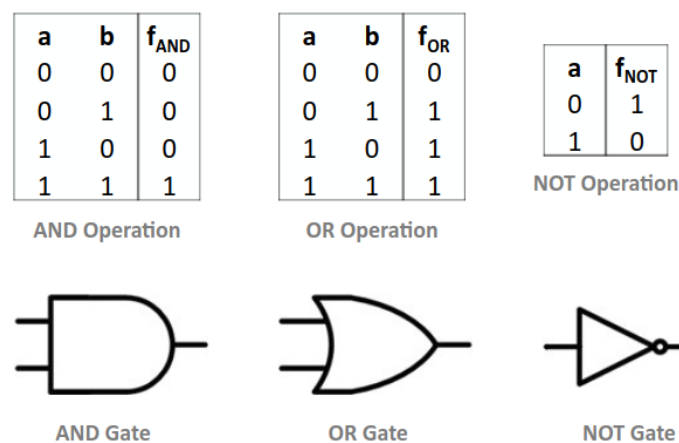


# 1 Digital Logic

We don't learn how the brain works by studying neurons, the same way that just by studying transistors we do not know how computer works. We know the brain does processing but we don't know how it works. The bottleneck to understand the brain is probably that we do not have the right abstractions to understand it.

## 1.1 (Basic) Digital Logic

Gates are processing units.



Circuits are a combination of gates.

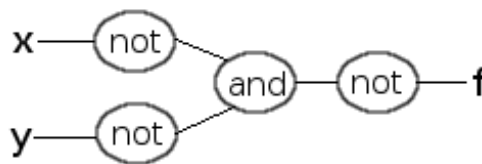


Figure 1: A basic circuit implementing an OR gate.

The circuit in Fig 1 produces the same as an OR gate. With NOT and AND gates we can build an OR gate, but with AND and OR we can't build a NOT gate.

**XOR gates = exclusive OR** They are exclusive in the case of two inputs. For more than two, XOR counts the number of "active" (1's) inputs and returns 0 for even and 1 for odd.

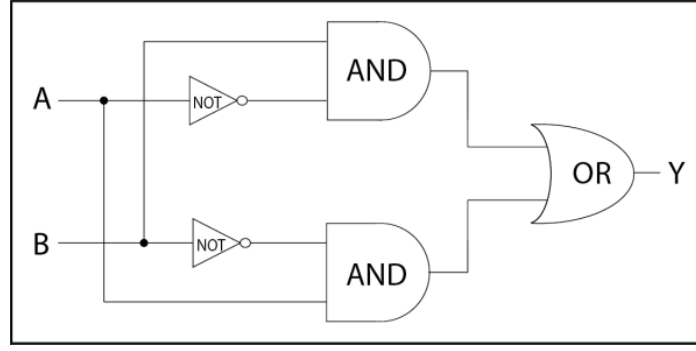


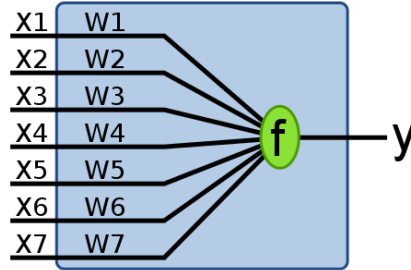
Figure 2: XOR gate built out other gates. Image from <https://blog.digilentinc.com/building-logic-gates-with-transistors/>

A table with  $N$  inputs has  $2^N$  rows.

Other gates

Input		Output						
A	B	AND	OR	NOT	XOR	NAND	NOR	XNOR
0	0	0	0	1	0	1	1	1
0	1	0	1	1	1	1	0	0
1	0	0	1	0	1	1	0	0
1	1	1	1	0	0	0	0	1

## 1.2 Linear Threshold (LT) Unit/Gates (Perceptron)



This model represents a neuron with inputs  $x$  and one output  $y$ . Weights  $w$  determine the influence of the inputs.  $f$  is a function determining the output: if the influence of all the inputs combined crosses a threshold, then the neuron becomes active. Active state:  $\sum_i (w_i \cdot x_i) \geq \theta$ . Otherwise, the neuron is inactive.

We add a bias input as  $-\theta$  so that  $w_0 + \sum_i (w_i \cdot x_i) \geq 0$  activates the neuron.

This model can create AND/OR/NOT-Gates. Not the XOR/XNOR-Gate however.

### 1.2.1 XOR impossibility with LT/Perceptrons

To compute XOR with LT, it is required that:

- $w_0 < 0$
- $x_2 \times w_2 + w_0 \geq 0$
- $x_1 \times w_1 + w_0 \geq 0$
- $x_1 \times w_1 + x_2 \times w_2 + w_0 < 0$ .

This causes a contradiction because  $x_1 \times w_1 + x_2 \times w_2 + 2 \times w_0 < 0$  and  $x_1 \times w_1 + x_2 \times w_2 + 2 \times w_0 \geq 0$  when adding up the constraints.

XOR is not a linear combination of the inputs.

## 1.3 McCulloch-Pitts Neuron / Perceptron

### Similarities to real neurons:

- Both can be active or inactive.
- The input/output is directed.
- The activation of a neuron is dependent on a weighted function of other neurons.

### Differences to real neurons:

- Real neurons exist in continuous time, whereas McCulloch-Pitts neurons operate in discrete time.
- Real neurons have degrees of activation, not just on/off.
- The activation as a function of the inputs of real neuron is typically not linear or threshold linear.