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Lecture 9
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Exercise 9.1: Spike timing dependent plasticity

It is experimentally observed that the relative timing of pre-and post-synaptic action potentials can change the synaptic efficacy. Within a time window of 50ms, presynaptic spikes that precede postsynaptic action potentials produce LTP (strengthening of the synapse), whereas presynaptic spikes that follow postsynaptic APs produce LTD (weakening of the synapse).

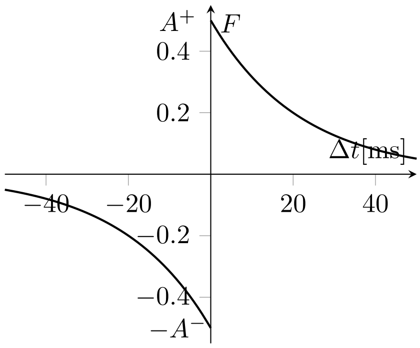

Figure 1: Spike-timing-dependent plasticity (STDP) modification function for a particular synapse, after Song *et al.* (2000).

$F(\Delta t)$ is the amount of synaptic weight modification observed after a pair of pre- and postsynaptic spikes, as function of $\Delta t = t_{post} - t_{pre}$.

$$F(\Delta t) = \begin{cases} A^+ \exp(-\Delta t/\tau^+) & \text{if } \Delta t \geqslant 0 \\ -A^- \exp(\Delta t/\tau^-) & \text{if } \Delta t < 0 \end{cases}$$

For the STDP window depicted in Fig 1, we have $A^+ = A^- = 0.5$ and $\tau^+ = \tau^- = 21.8$ms.

1. Why is it important – for any learning rule – to have mechanisms for both the strengthening and the weakening of the synaptic weight?
2. Re-express the learning rule in a differential equation form, i.e. $\dot{w} = \ldots$ as a function of the presynaptic spike train $X(t) = \sum_{t_{pre}} \delta(t - t_{pre})$ and postsynaptic spike train $Y(t) = \sum_{t_{post}} \delta(t - t_{post})$
3. Compute the average weight change $\langle \dot{w} \rangle$ as a function of the correlation function $C(s) = \langle X(t)Y(t+s) \rangle$.
4. When input and output spike trains are independent, show that the average weight change $\langle \dot{w} \rangle$ is proportional to the integral of the learning window ($\int_{-\infty}^{\infty} F(s)ds$). Use ρ_x as the presynaptic firing rate and ρ_y as the postsynaptic firing rate. What is the relation between this average learning rule and Hebb rule?
5. Assume that the correlation function is given by $C(s) = C_0 + H(s)C_1 \exp(-s/\tau_C)$. Compute the average weight change as a function of the correlation coefficient C_1. What happens at low C_1? What happens at high C_1? H is the Heaviside step function, defined as follows:

$$H(s) = \begin{cases} 1 & \text{if } s \geqslant 0 \\ 0 & \text{if } s < 0 \end{cases}$$

Exercise 9.2: Triplet STDP (optional)

1. Read paper [1] cited in the reference.
2. What is the computational advantage of this triplet rule? What is its biological relevance?

References

1. Pfister, J.-P., and Gerstner, W. (2006). Triplets of spikes in a model of spike timing-dependent plasticity. Journal of Neuroscience, 26(38), 9673-9682.