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Lecture 3
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Exercise 3.1: Diffusion

The cell membrane is a biological membrane that separates the interior environment of cells from the outside environment. The cell membrane is selectively permeable to some ions and organic molecules and controls the movement of substances in and out of cells. Choose the right one among the options [1] / [2].

We start an experiment with a membrane separating two solutions (ions, molecules and water), electrically neutral. Suppose that the membrane is only permeable to one type of ion. If we add a certain quantity of this ion (of course with another non-diffusible ion of the opposite charge) on one side of the membrane, so that the concentration becomes higher in the compartment A than in B, there will be a net diffusion of this ion [from A to B] / [from B to A]. If our ion carries a positive charge, the difference of concentration will result in an electrical potential difference across that membrane, with side A being more [positive] / [negative]. Two forces of [opposite] / [the same] direction act on the ion: the chemical and the electrical one.

Exercise 3.2: The Nernst Equation

The Nernst equation gives the voltage E_{ion} (reversal potential) that balances a given difference in chemical concentration across a cell membrane ($[\text{Ion}]_{\text{intracellular}}$ and $[\text{Ion}]_{\text{extracellular}}$):

$$E_{ion} = \frac{RT}{zF} \cdot \ln\left(\frac{[\text{Ion}]_{\text{extracellular}}}{[\text{Ion}]_{\text{intracellular}}}\right)$$

where E_{ion} is measured in [V], R is the gas constant [8.31 J / (K · mol)], T is the absolute temperature in [K], F is the Faraday's constant [96′500 C / mol], z (called valence) is the charge of the ion (positive number for positive ion charge) and ln denotes the natural logarithm. If natural logs are replaced by base 10 logarithms, these must be multiplied by a conversion factor of ln(10) = 2.303. Since the numerical value of $2.303RT/F$ is about 58mV at 20°C we can use:

$$E_{ion} = \frac{58\text{mV}}{z} \cdot \log_{10}\left(\frac{[\text{Ion}]_{\text{extracellular}}}{[\text{Ion}]_{\text{intracellular}}}\right)$$

1. If $[\text{K}^+]_{\text{extracellular}} = 5\text{mM}$, $[\text{K}^+]_{\text{intracellular}} = 148\text{mM}$, $[\text{Na}^+]_{\text{extracellular}} = 142\text{mM}$, $[\text{Na}^+]_{\text{intracellular}} = 10\text{mM}$, calculate E_K and E_{Na}.
2. What condition will have the more dramatic effect: an increase of 5mM of the extracellular concentration of Na^+ or of K^+? By how much? (Supplementary question: knowing that cardiac cells have electrical properties very similar to neurons, which one of these two ions is more dangerous when injected rapidly?).

Exercise 3.3: The Goldman-Hodgkin-Katz Equation

The membrane potential of our cells is the result of the concentration gradients established for many different ions across the cell membrane (such as Na^+, K^+, Ca^{++} and Cl^-). The distribution of the various ions is dependent not only on the electrochemical gradient for each ion, but also on the ability of a given ion to cross the membrane, or permeability. Therefore the permeability of the membrane for a given ion also determines the value of the membrane potential. The dependence of membrane potential on permeability and concentration is given by the Goldman-Hodgkin-Katz equation, here for the monovalent ions K^+, Na^+ and Cl^-:

$$V_{membrane} = \frac{RT}{F} \cdot \ln\left(\frac{P_{\text{K}} \cdot [\text{K}^+]_{out} + P_{\text{Na}} \cdot [\text{Na}^+]_{out} + P_{\text{Cl}} \cdot [\text{Cl}^-]_{in}}{P_{\text{K}} \cdot [\text{K}^+]_{in} + P_{\text{Na}} \cdot [\text{Na}^+]_{in} + P_{\text{Cl}} \cdot [\text{Cl}^-]_{out}}\right)$$

where P_{K}, P_{Na}, P_{Cl} are the permeabilities, $[\cdot]_{in}$ and $[\cdot]_{out}$ the intracellular and extracellular concentrations of the respective monovalent ion. R, T and F are as for the Nernst equation above.

1. If we take only Na^+ and K^+ into account ($P_{\text{Cl}} = 0$), and if V_m is experimentally observed to be at -77 mV, for which one of these two ions is the membrane more permeable? Use the ion concentrations from exercise 6: 5, 148, 142, 10 mM. By using the algebraic transformations form exercise 6, find the ratio P_K/P_{Na}.