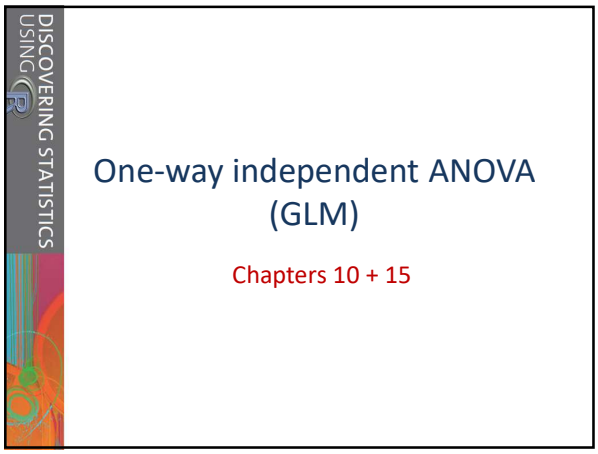
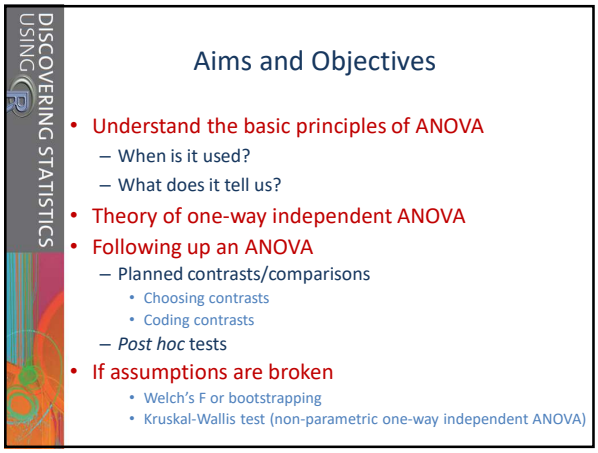
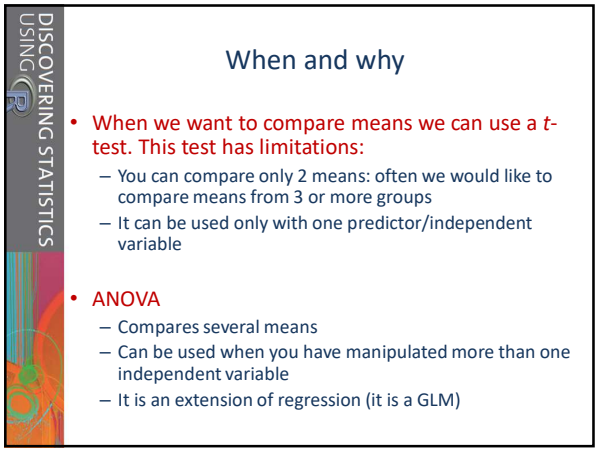

DISCOVERING STATISTICS USING R
One-way independent ANOVA (GLM)
Chapters 10 + 15

1


DISCOVERING STATISTICS USING R
Aims and Objectives
Understand the basic principles of ANOVA
When is it used?
What does it tell us?
Theory of one-way independent ANOVA
Following up an ANOVA
Planned contrasts/comparisons
Choosing contrasts
Coding contrasts
Post hoc tests
If assumptions are broken
Welch's F or bootstrapping
Kruskal-Wallis test (non-parametric one-way independent ANOVA)

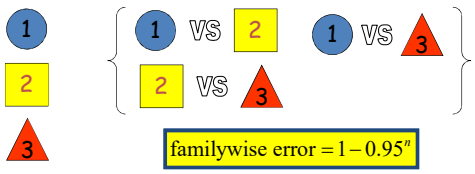
2


DISCOVERING STATISTICS USING R
When and why
When we want to compare means we can use a t-test. This test has limitations:
You can compare only 2 means: often we would like to compare means from 3 or more groups
It can be used only with one predictor/independent variable
ANOVA
Compares several means
Can be used when you have manipulated more than one independent variable
It is an extension of regression (it is a GLM)

3

Why not use lots of *t*-tests?

- If we want to compare several means why don't we compare pairs of means with *t*-tests?
  - Inflates the Type I error rate
  - Can't look at several independent variables

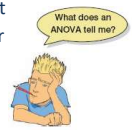


$$\text{familywise error} = 1 - 0.95^{n}$$

4

What does ANOVA tell us?

- Null hypothesis:
  - Like a *t*-test, ANOVA tests the null hypothesis that the means are the same
- ANOVA is an omnibus test
  - It tests for an overall difference between groups
  - It can tell us that group means are different
  - It doesn't tell us exactly which means differ



5

Example

- Testing the effect of Viagra on libido using 3 groups
  - Placebo (sugar pill)
  - Low dose of Viagra
  - High dose of Viagra

- Participants
  - 15 (frustrated) men, 5 in each treatment
- Outcome
  - An objective measure of libido

6

ANOVA as a linear model

$$\text{outcome}_i = (\text{model}) + \text{error}_i$$

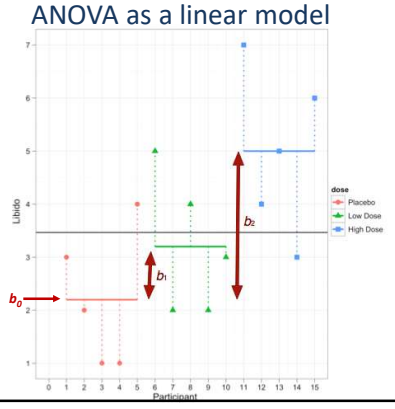
$$\text{libido}_i = b_0 + b_1\text{low}_i + b_2\text{high}_i + \varepsilon_i$$

Table 10.2: Dummy coding for the three-group experimental design

Group	Dummy Variable 1 (Low)	Dummy Variable 2 (High)
Placebo	0	0
Low Dose Viagra	1	0
High Dose Viagra	0	1

7

ANOVA as a linear model



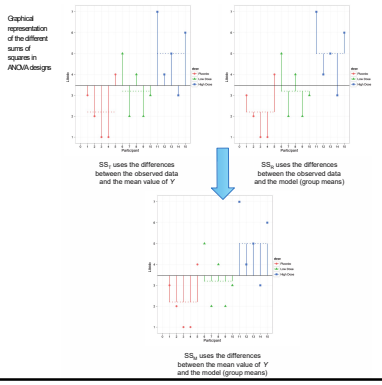
8

Theory of ANOVA

- Sums of Squares
  - Calculate how much variability there is between scores
    - Total Sum of Squares (SS_T)
  - Calculate how much of this variability can be explained by the model we fit to the data
    - Model Sum of Squares (SS_M): how much variability is due to the categorical predictor variable
  - … and how much cannot be explained
    - residual Sum of Squares (SS_R): how much variability is due to unsystematic differences

9

Theory of ANOVA



10

Theory of ANOVA

- How well does the model fit the data?
  - R^2
    - The proportion of the total variability in the data accounted for by the model

$$R^2 = \frac{SS_M}{SS_T}$$

  - F
    - *SS* are total values (*i.e.* dependent on sample size, Chapter 2 and 7), therefore we need *MS* to assess whether the model is a significant improvement over the null-model (*i.e.* the mean)

$$F = \frac{MS_M}{MS_R}$$

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Theory of ANOVA

- The *F*-statistic
  - Makes the same assumptions as all parametric test statistics based on the normal distribution:
    - Dependent variable is measured on at least the interval scale
    - The variances within each group are approximately equal (homogeneity of variance)
    - Observations are independent
  - Is fairly robust against violations of normality and homogeneity of variance, ***as long as sample sizes of the groups are equal***

12

Follow-up tests

- If the *F*-ratio tells us that group means are different
  - We next want to find out where these differences lie!
- How?
  - Planned contrasts *or* polynomial trend analysis
    - Hypothesis driven (planned *a priori*)
    - Orthogonal/non-orthogonal
  - *Post hoc* tests
    - Not planned (no hypothesis)
    - Compare all pairs of means

13

Planned contrasts

- Basic idea
  - The variability explained by the model (SS_M) is due to participants being assigned to different groups
  - This variability can be broken down further to test specific hypotheses about which groups might differ
  - We break down the variance according to hypotheses made *a priori* (before the experiment/analysis)

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Planned contrasts

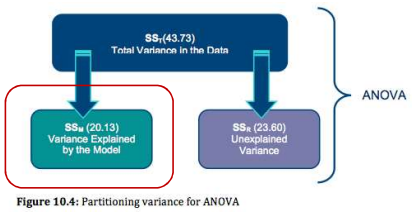
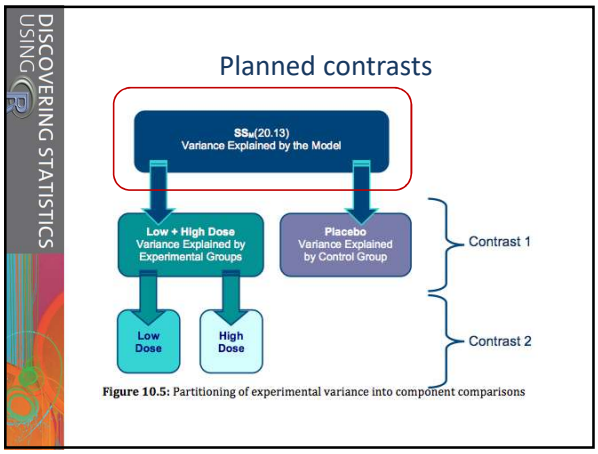


Figure 10.4: Partitioning variance for ANOVA

15


DISCOVERING STATISTICS USING R
Planned contrasts
SSM(20.13)
Variance Explained by the Model
Low + High Dose
Variance Explained by Experimental Groups
Placebo
Variance Explained by Control Group
Low Dose
High Dose
Contrast 1
Contrast 2
Figure 10.5: Partitioning of experimental variance into component comparisons

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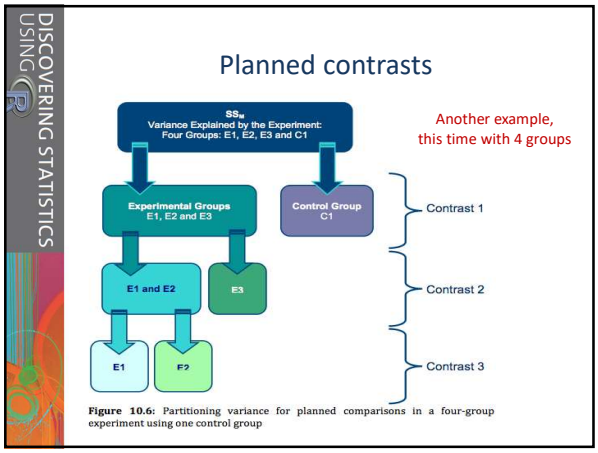
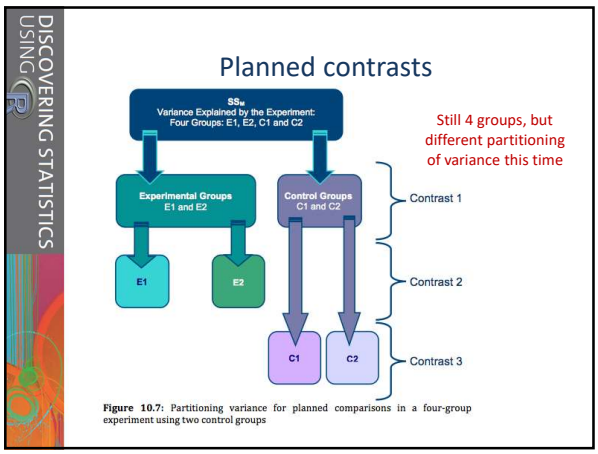

DISCOVERING STATISTICS USING R
Planned contrasts
SSM
Variance Explained by the Experiment:
Four Groups: E1, E2, E3 and C1
Another example, this time with 4 groups
Experimental Groups
E1, E2 and E3
Control Group
C1
E1 and E2
E3
E1
E2
Contrast 1
Contrast 2
Contrast 3
Figure 10.6: Partitioning variance for planned comparisons in a four-group experiment using one control group

17


DISCOVERING STATISTICS USING R
Planned contrasts
SSM
Variance Explained by the Experiment:
Four Groups: E1, E2, C1 and C2
Still 4 groups, but different partitioning of variance this time
Experimental Groups
E1 and E2
Control Groups
C1 and C2
E1
E2
C1
C2
Contrast 1
Contrast 2
Contrast 3
Figure 10.7: Partitioning variance for planned comparisons in a four-group experiment using two control groups

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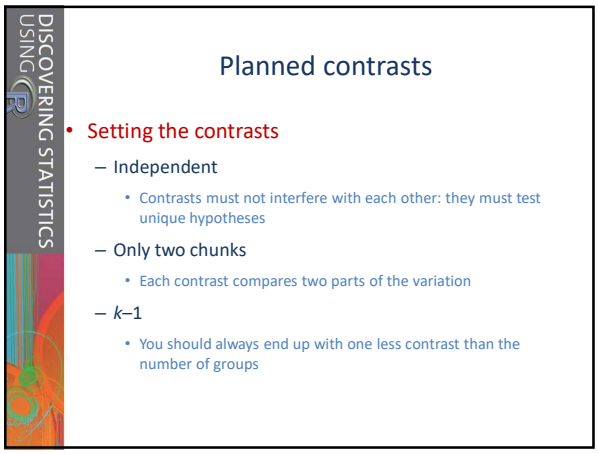
DISCOVERING STATISTICS USING R
Planned contrasts
Setting the contrasts
Independent
Contrasts must not interfere with each other: they must test unique hypotheses
Only two chunks
Each contrast compares two parts of the variation
k–1
You should always end up with one less contrast than the number of groups

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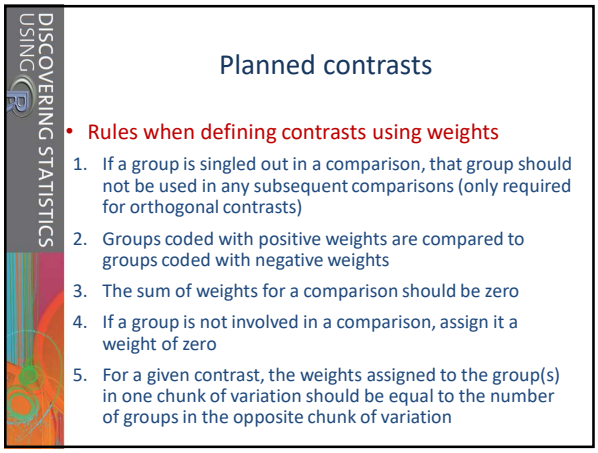
DISCOVERING STATISTICS USING R
Planned contrasts
Rules when defining contrasts using weights
1. If a group is singled out in a comparison, that group should not be used in any subsequent comparisons (only required for orthogonal contrasts)
2. Groups coded with positive weights are compared to groups coded with negative weights
3. The sum of weights for a comparison should be zero
4. If a group is not involved in a comparison, assign it a weight of zero
5. For a given contrast, the weights assigned to the group(s) in one chunk of variation should be equal to the number of groups in the opposite chunk of variation

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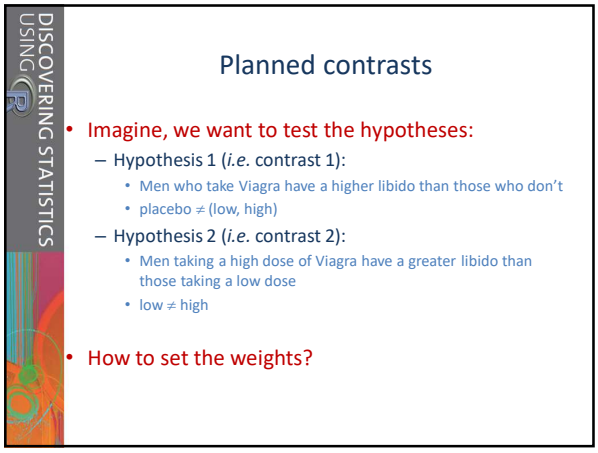
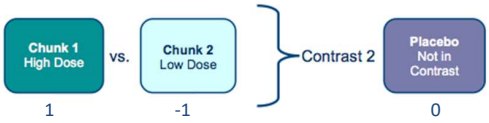

DISCOVERING STATISTICS USING R
Planned contrasts
Imagine, we want to test the hypotheses:
Hypothesis 1 (i.e. contrast 1):
Men who take Viagra have a higher libido than those who don't
placebo ≠ (low, high)
Hypothesis 2 (i.e. contrast 2):
Men taking a high dose of Viagra have a greater libido than those taking a low dose
low ≠ high
How to set the weights?

21

Planned contrasts

- Setting the weights for the 1st contrast:

- And the 2nd contrast:



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Planned contrasts

Table 10.4: Orthogonal contrasts for the Viagra data

Group	Dummy Variable 1 (Contrast$_1$)	Dummy Variable 2 (Contrast$_2$)
Placebo	−2	0
Low Dose	1	−1
High Dose	1	1

Table 10.5: Non-orthogonal contrasts for the Viagra data

Group	Dummy Variable 1 (Contrast$_1$)	Dummy Variable 2 (Contrast$_2$)
Placebo	−2	−1
Low Dose	1	0
High Dose	1	1

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Planned contrasts

- Standard contrasts
  - Often you will code the weights for your own contrasts
  - However, R has a couple of commonly used contrasts built-in (orthogonal and non-orthogonal)
    - dummy (default for unordered factors), treatment, Helmert, …
- Polynomial contrasts
  - When interested in trends across the means of groups:
    - linear, quadratic, cubic, … (default for ordered factors)
    - maximum order of polynomial= *k*-1

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Post hoc tests

- If we have no predictions *a priori* about our data
  - Compare all means using pairwise comparisons
  - Need to control the family-wise error rate
    - Use stricter criterion to accept a difference as significant
    - However, this goes at the expense of statistical power
    - Simplest example is the Bonferroni method:

$$p_{crit} = \frac{\alpha}{n\ comparisons}$$

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One-way independent ANOVA

- General procedure:
  - Enter/import data
    - Make sure R recognizes variables (numeric, factor, date…)
  - Explore data
    - Compute descriptive statistics
    - Plot the data
    - Check parametric assumptions
  - Perform analysis
    - Define contrasts *a priori* and compute the ANOVA, or…
    - … compute the ANOVA and perform *post hoc* comparisons
  - Validate and interpret model
    - Check additional parametric assumptions

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One-way independent ANOVA

- Two equivalent functions in R
  - *'lm()'*:

```
> viagraModel<- lm(libido~ dose, data= viagraData)
> anova(viagraModel)
> summary(viagraModel)
> par(mfrow= c(2,2)); plot(viagraModel)
```

  - *'aov()'*:

```
> viagraModel<- aov(libido~ dose, data=viagraData)
> summary(viagraModel)
> summary.lm(viagraModel)
> par(mfrow= c(2,2)); plot(viagraModel)
```

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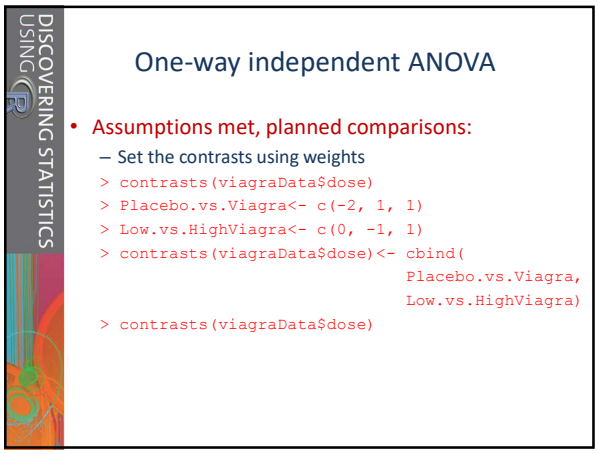
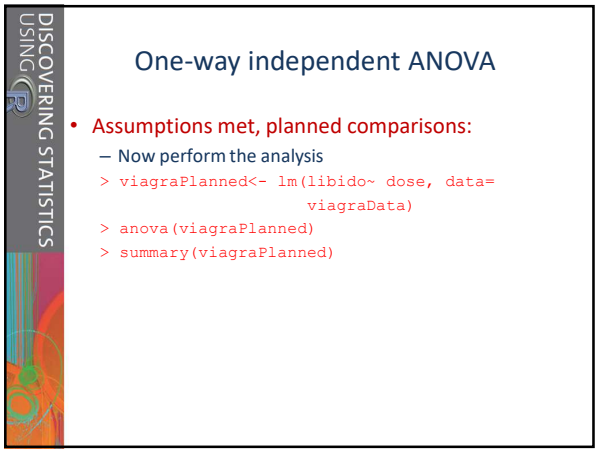
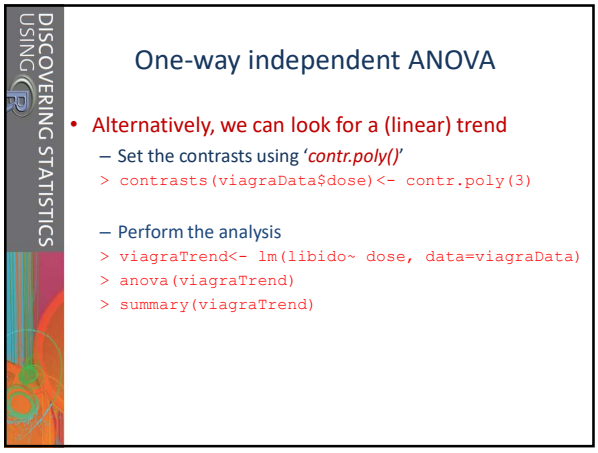

DISCOVERING STATISTICS USING R
One-way independent ANOVA
• Assumptions met, planned comparisons:
– Set the contrasts using weights
> contrasts(viagraData$dose)
> Placebo.vs.Viagra<- c(-2, 1, 1)
> Low.vs.HighViagra<- c(0, -1, 1)
> contrasts(viagraData$dose)<- cbind(
Placebo.vs.Viagra,
Low.vs.HighViagra)
> contrasts(viagraData$dose)

28


DISCOVERING STATISTICS USING R
One-way independent ANOVA
• Assumptions met, planned comparisons:
– Now perform the analysis
> viagraPlanned<- lm(libido~ dose, data=
viagraData)
> anova(viagraPlanned)
> summary(viagraPlanned)

29


DISCOVERING STATISTICS USING R
One-way independent ANOVA
• Alternatively, we can look for a (linear) trend
– Set the contrasts using '*contr.poly()*'
> contrasts(viagraData$dose)<- contr.poly(3)
– Perform the analysis
> viagraTrend<- lm(libido~ dose, data=viagraData)
> anova(viagraTrend)
> summary(viagraTrend)

30

One-way independent ANOVA

- Or, if we did not have any explicit hypotheses, we can perform *post hoc* comparisons
  - We already have performed the main analysis

```
> anova(viagraModel)
```

  - Now, we can use '*pairwise.t.test()*' for Bonferroni and related tests…

```
> pairwise.t.test(viagraData$libido,
                  viagraData$dose,
                  p.adjust.method= "bonferroni")
```

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One-way independent ANOVA

- Or, if we did not have any explicit hypotheses, we can perform *post hoc* comparisons
  - We already have performed the main analysis

```
> anova(viagraModel)
```

  - … or '*glht()*' for Tukey and Dunnett

```
> postHocs<- glht(viagraModel, linfct= mcp(dose=
                  "Tukey"))
> summary(postHocs)
> confint(postHocs)
```

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Reporting results

- Depends a little on what exactly we did, but e.g.
  - There was a significant effect of Viagra on levels of libido ($R^2_{Adj.}$= 0.37, $F_{(2, 12)}$= 5.12, p< .05). Planned contrasts revealed that taking any dose of Viagra significantly increased libido compared to the placebo (t= 2.47, p< .05), but that taking a high dose did not significantly increase libido compared to taking a low dose (t= 2.03, p= .07).
  - There was a significant effect of Viagra on levels of libido ($R^2_{Adj.}$= 0.37, $F_{(2, 12)}$= 5.12, p< .05). A polynomial trend analysis revealed a significant linear increase (t= 3.16, p< .01) indicating that, as the dose of Viagra increased, libido increased proportionally.

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One-way independent ANOVA

- What if assumptions are broken…
  - If Levene's test is significant (variances are different between groups), calculate *Welch's F*

```
> oneway.test(libido~ dose, data= viagraData)
```

  - If other assumptions are also not met
    - Robust ANOVA
    - Non-parametric equivalent: the Kruskal-Wallis test.
      - Like the Wilcoxon tests of Chapter 9, this test ranks the data

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Kruskal-Wallis test

- Non-parametric one-way independent ANOVA
  - Perform the omnibus test

```
> kruskal.test(libido~ dose, data= viagraData)
> by(rank(viagraData$libido), viagraData$dose,
    mean)
```

  - *Post hoc* multiple comparisons (package "pgirmess")

```
> kruskalmc(libido~ dose, data= viagraData)
```

  - Or look for a linear trend in the group rank means using the Jonckheere-Terpstra test (package "clinfun")

```
> jonckheere.test(viagraData$libido,
                  as.numeric(viagraData$dose))
```

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Reporting results

- Kruskal-Wallis test
  - The libido of participants was significantly affected by Viagra, $H_{(2)}$= 6.20, p< .05. Focused comparisons of mean ranks between groups showed that libido was not significantly different between a low dose of Viagra and the placebo ($\Delta_{\text{mean rank}}$= 2.7). When a high dose of Viagra was administered, however, libido was significantly higher ($\Delta_{\text{mean rank}}$ = 6.9) than in the placebo group. No significant difference ($\Delta_{\text{mean rank}}$= 4.2) was apparent between the two Viagra treatments. For all comparisons, the critical difference (corrected for the number of comparisons) was 6.8.
  - The libido of participants was significantly affected by Viagra, $H_{(2)}$= 6.20, p< .05. The Jonckheere-Terpstra test, moreover, revealed a significant positive trend in the data: as the dose of Viagra increased, so did the median libido of participants (J= 61.5, p< .05).

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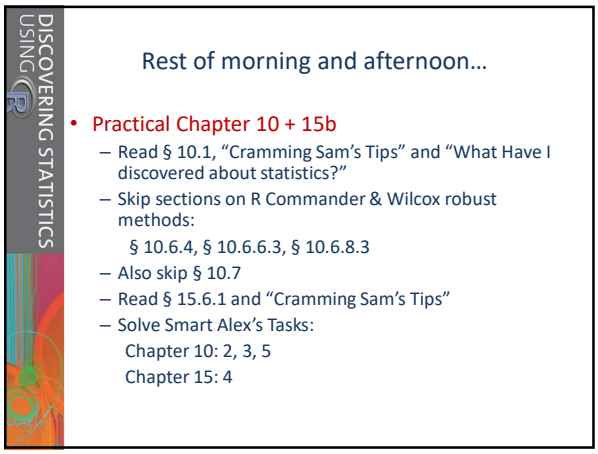
DISCOVERING STATISTICS USING R
Rest of morning and afternoon…
• Practical Chapter 10 + 15b
– Read § 10.1, "Cramming Sam's Tips" and "What Have I discovered about statistics?"
– Skip sections on R Commander & Wilcox robust methods:
§ 10.6.4, § 10.6.6.3, § 10.6.8.3
– Also skip § 10.7
– Read § 15.6.1 and "Cramming Sam's Tips"
– Solve Smart Alex's Tasks:
Chapter 10: 2, 3, 5
Chapter 15: 4

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