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Lecture 5
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Solution 5.1: Deriving the Hodgkin-Huxley Model

(a) n_{ion} lies between 0 (no conductance) and 1 (maximal conductance).

(b) The probability of two independent events a and b that have probabilities $p(a)$ and $p(b)$ to occur simultaneously is $p(a) \cdot p(b)$. With this we get n^x as the total probability.

(c) The rate at which n changes is equal to the rate at which gates open, minus the rate at which they close.

$$\frac{dn}{dt} = \alpha_n \cdot (1 - n) - \beta_n \cdot n$$

(d) In steady state, we have

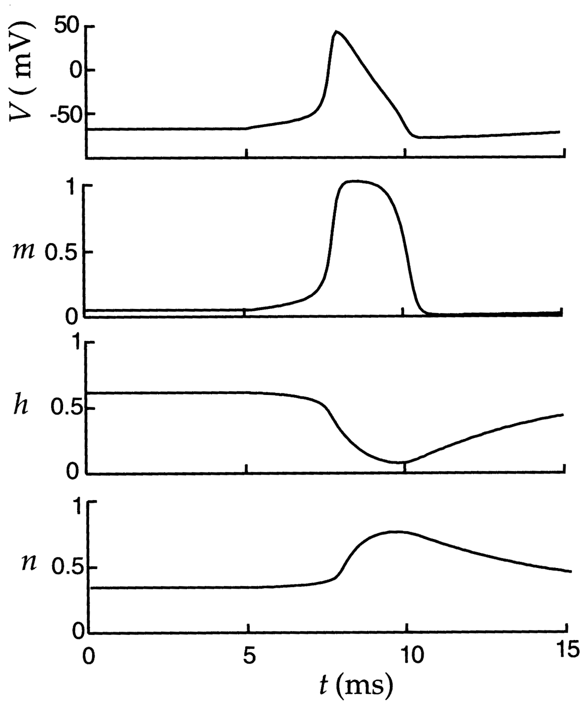
$$\frac{dn_\infty}{dt} = \alpha_n \cdot (1 - n_\infty) - \beta_n \cdot n_\infty = 0$$
$$\iff \frac{1}{n_\infty} = \frac{\beta_n}{\alpha_n} + 1$$
$$\iff n_\infty = \frac{\alpha_n}{\alpha_n + \beta_n}$$

(e) Keep in mind that we are looking at voltage sensitive ion channels. So yes, α_n and β_n are voltage dependent (they are not time dependent). Via α_n and β_n, the gating variable n is voltage *and* time dependent. Note that n_∞ is voltage, but not time dependent.

(f) n is the probability that a gate in the K^+ channel is open. Because it is modeled by 4 gates in series, n^4 is the probability that the whole channel is open; this is also the fraction of open channels. Multiplying this fraction with the maximal conductance $\overline{g_K}$ gives the actual conductance for K^+. This term is then multiplied with the driving potential for K^+, giving I_K.

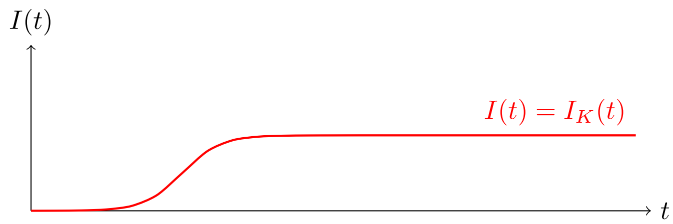
(g) In this equation, $\overline{g_{Na}}$ is the maximum sodium conductance of the axon membrane. Sodium traverses the membrane through protein channels in the membrane, and these can open and close. $\overline{g_{Na}}$ is the conductance for sodium that is seen if all sodium channels are open. The gating variables m and h each change between 0 and 1 as functions of time and voltage. The product m^3h represents the fraction of the total sodium conductance at any given time. The sodium channel behaves as if it has two sets of gates. One set, the 'activation' gates (described by m), open rapidly when the cell is depolarised above a threshold voltage. The other gate, the 'inactivation' gate (described by h), closes slowly when the cell is depolarised. So m changes quickly and h changes more slowly.

(h) Taken from Peter Dayan and L. F. Abbott, Theoretical Neuroscience, The MIT Press.

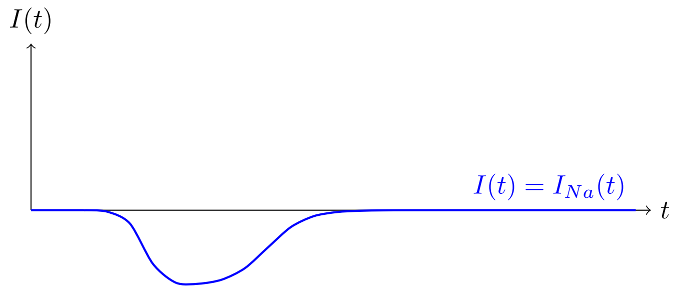


Solution 5.2: Voltage-Clamp

1. Blocking Na^+ channels means that $g_{Na} \approx 0$. Using $I_{Na} = g_{Na} \cdot (V - E_{Na})$ we obtain that the Sodium current is approximately zero and that the only relevant current is the potassium current, which looks as follows:



2. Similarly here $g_K \approx 0$ and $I_K = g_K \cdot (V - E_K) \approx 0$. Only the sodium current is relevant and it looks like this:



3. By virtue of the Nernst-Equation $E_{Na} = 0$ for equal concentrations inside and outside (there is no net diffusion current). When we now clamp V to zero, we again get $I_{Na} = g_{Na} \cdot (V - E_{Na}) = 0$, because of the second term this time and the plot looks the same as in the Tetrodotoxin case.

4. With the same reasoning as in 3 we obtain $I_K = g_K \cdot (V - E_K) = 0$ and the plot looks like in 2.

5. When the pump fails and doesn't maintain the ion concentration differences (here we assume that five minutes is enough to break down any transmembrane gradients), there cannot be any steady-state with currents, since the concentration gradients are not maintained (no battery anymore). Both currents are zero.

Solution 5.3: I-V Curve

See Fig. 1 for plots.

1. The conductance is the slope of the I-V relationship. $I_K(V)$ is a straight line and the slope g_K is the same for all V. Note that it is only an example for this exercise. In nature, g_K can very well be voltage-dependent (as in the Hodgkin Huxley model for the generation of action potenitals).
2. The slope I_K is twice a steep, I_{Na} is the same, I is shifted upwards
3. I_{Na} is also a straight line. It should intersect the axis at the sodium reversal potential.

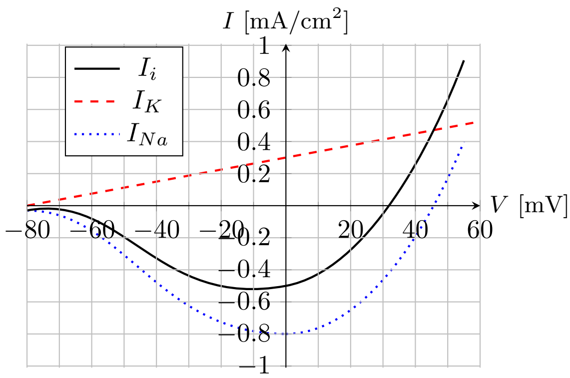
Solution 5.4: A Historical Figure

From resting potential to zero we have a step of 60 mV, so in the H&H frame of reference we are at –60 mV. On the graph we see that $n_\infty(V = -60\text{mV}) = 0.9$, which we insert:

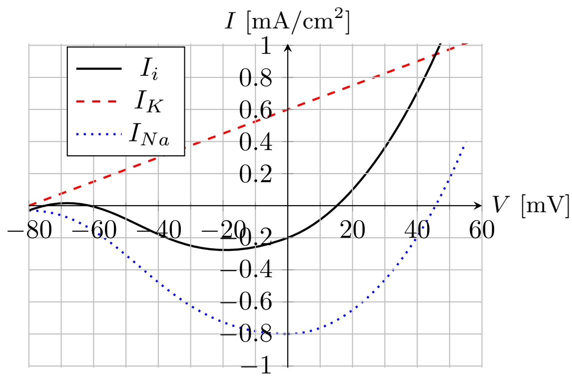
$$
\begin{aligned}
I_{K\infty} &= \overline{g_K} \cdot {n_\infty}^4 \cdot (V_m - E_K) \\
&= 36\frac{\text{mS}}{\text{cm}^2} \cdot 0.6561 \cdot 72\text{mV} \\
&= 1.7\frac{\text{mA}}{\text{cm}^2}
\end{aligned}
$$

Useful Links

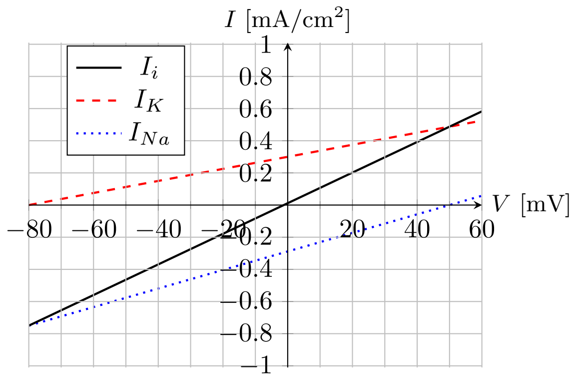
For nice and useful visualizations on this topic (but also other topics covered in the lecture) check-out the website.



(a) Original Relation



(b) g_K doubled



(c) g_{Na} voltage independent

Figure 1: Exercise 5.3: I-V Curve