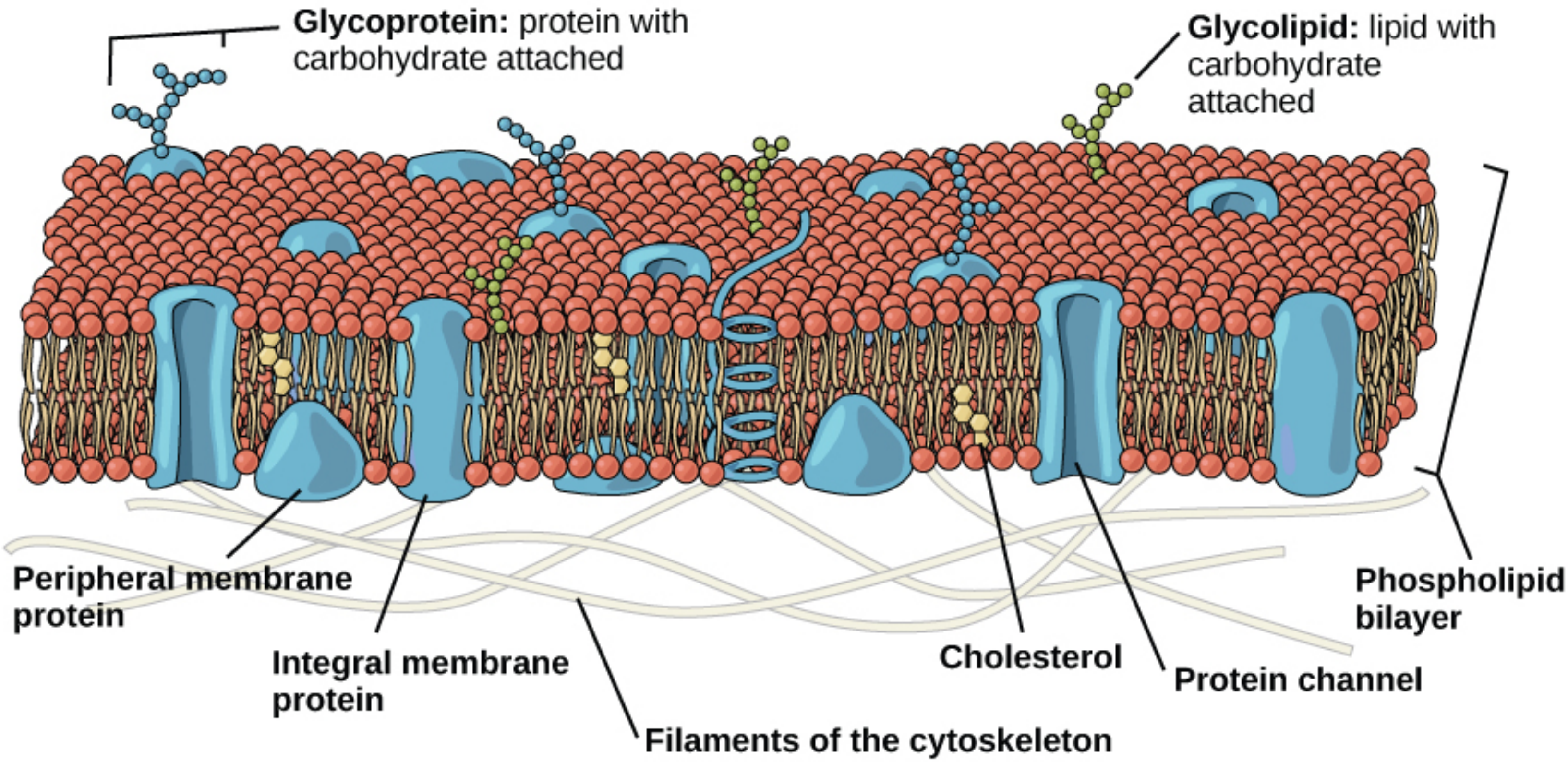
Introduction to neuroinformatics

Passive membrane

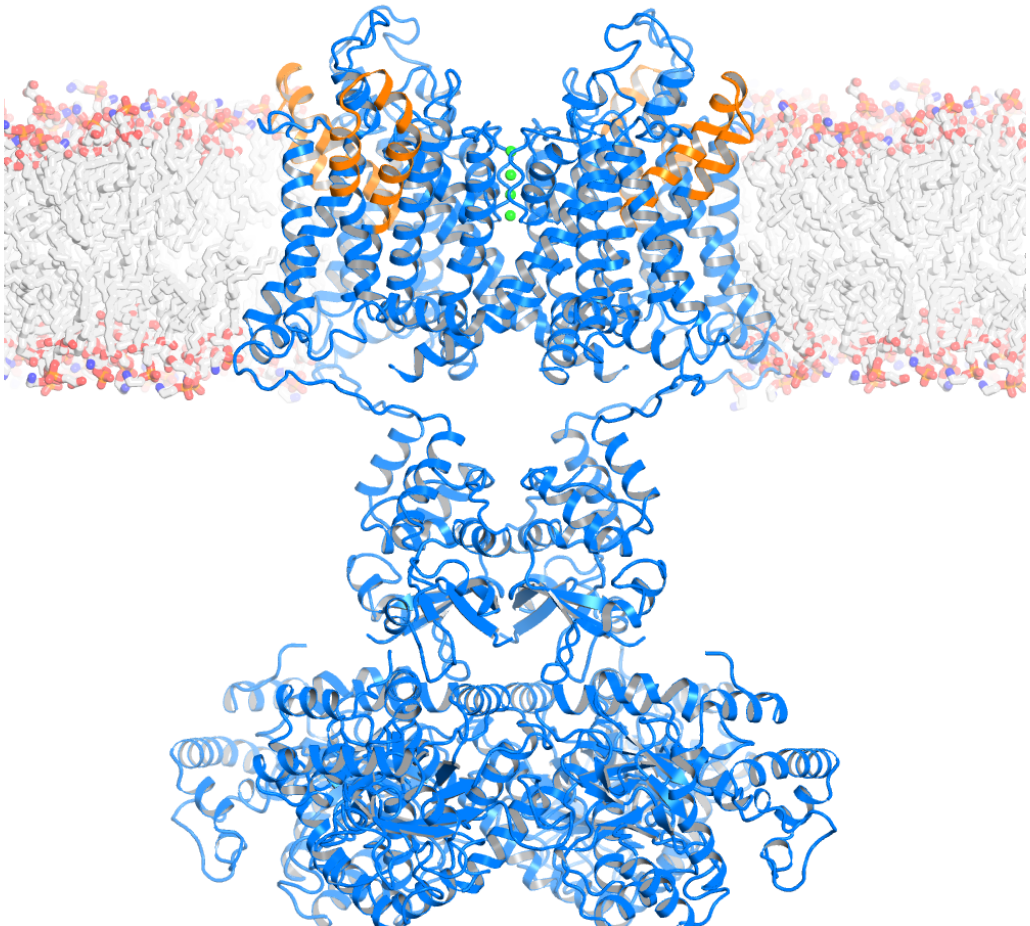
Valerio Mante
valerio@ini.uzh.ch

Cell membrane

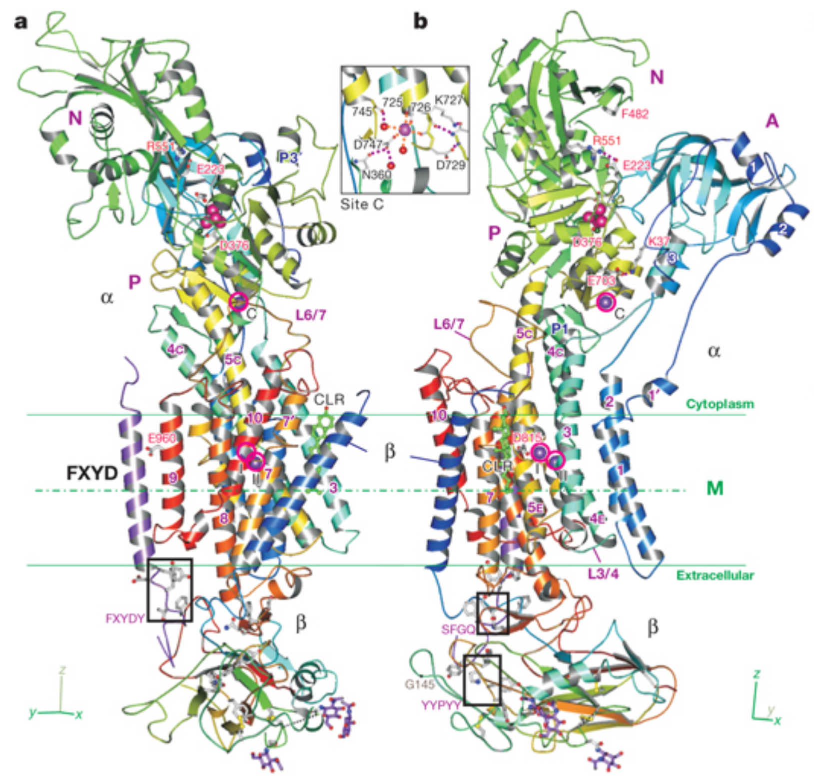


Channels and pumps

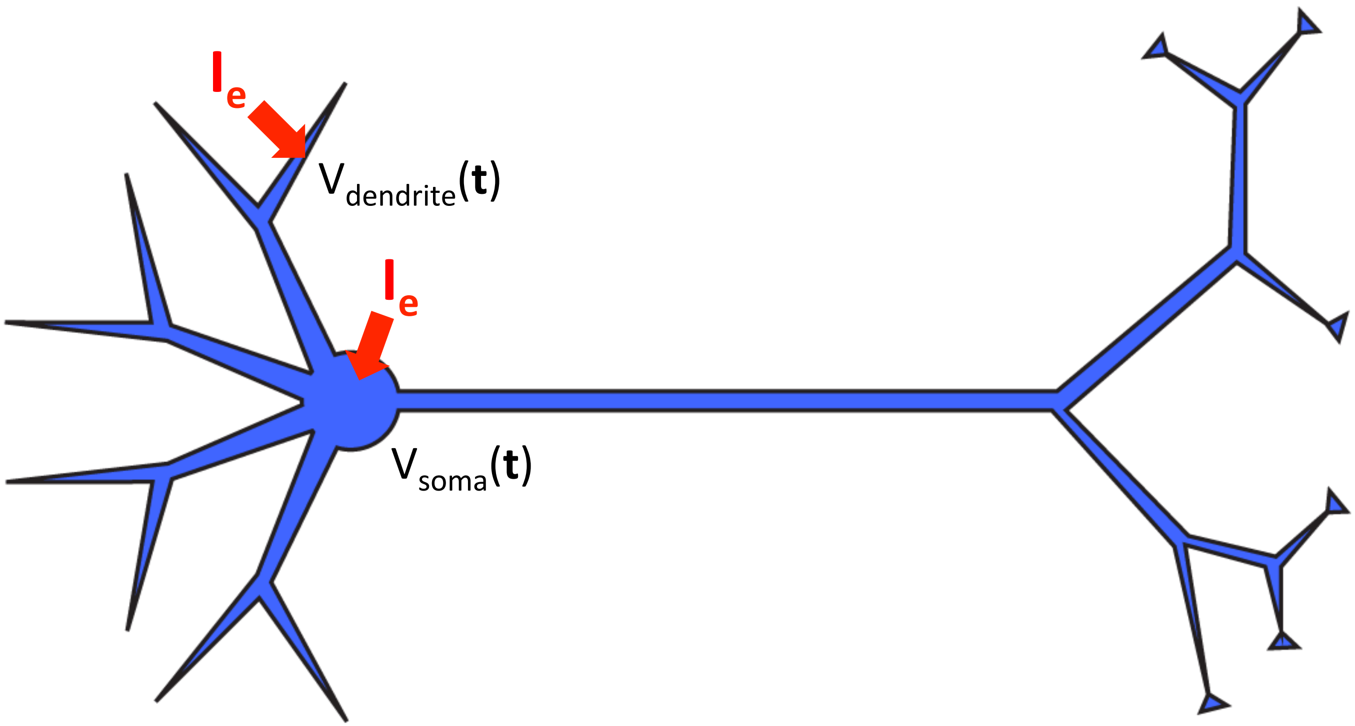
Potassium channel



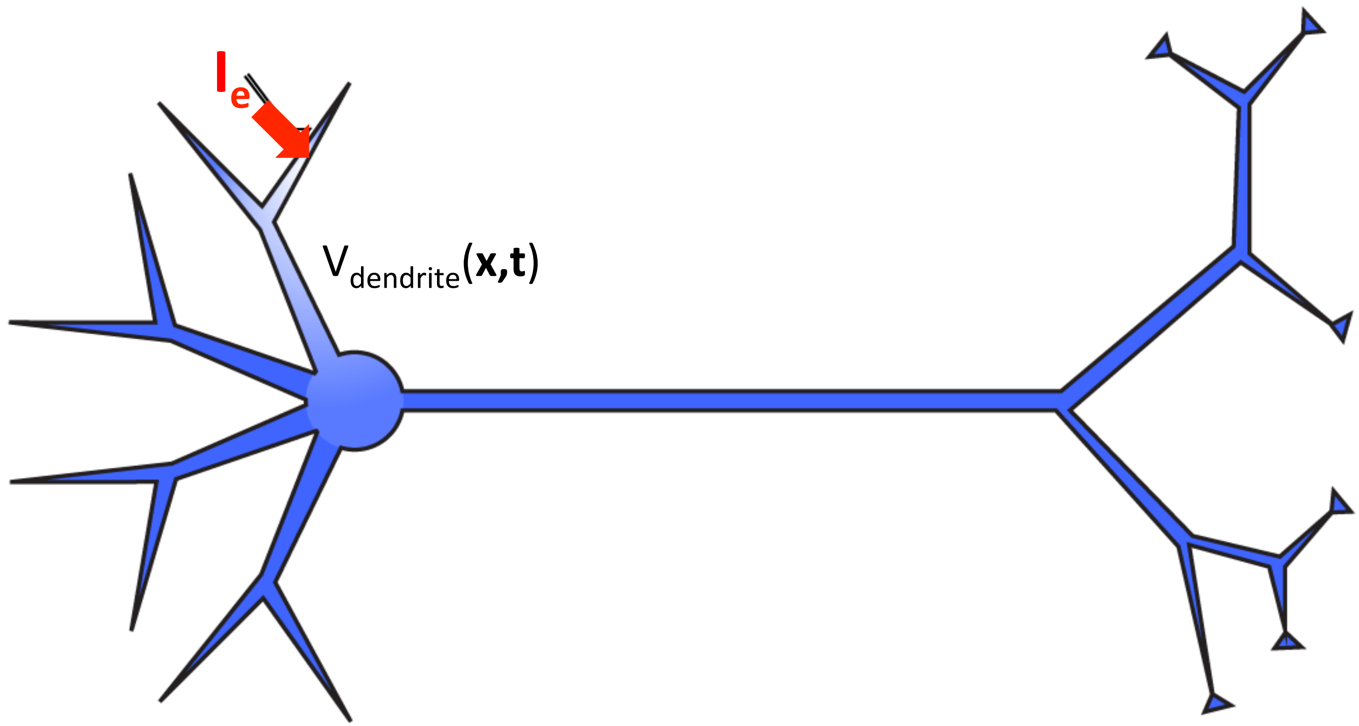
Sodium-potassium pump



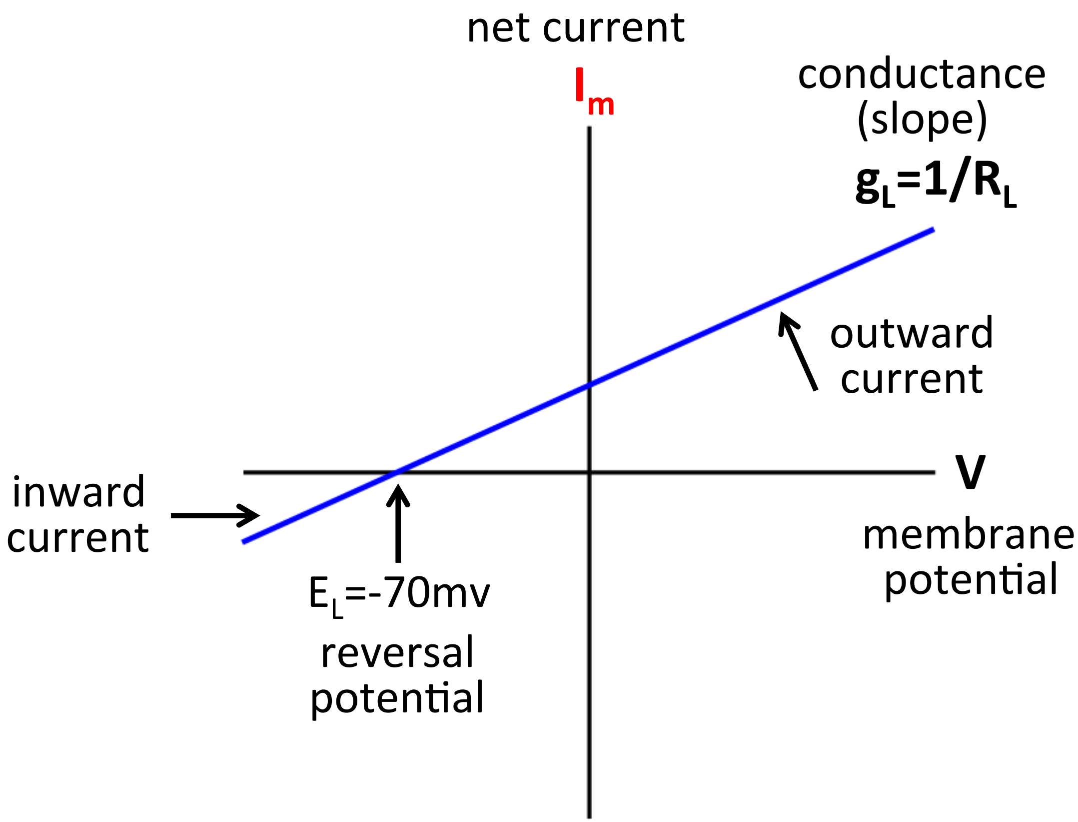
Potential as a function of time

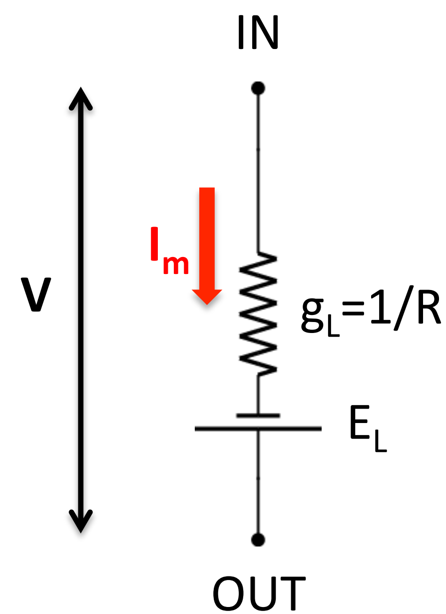


Potential as a function of space



Ionic currents and Ohm's law





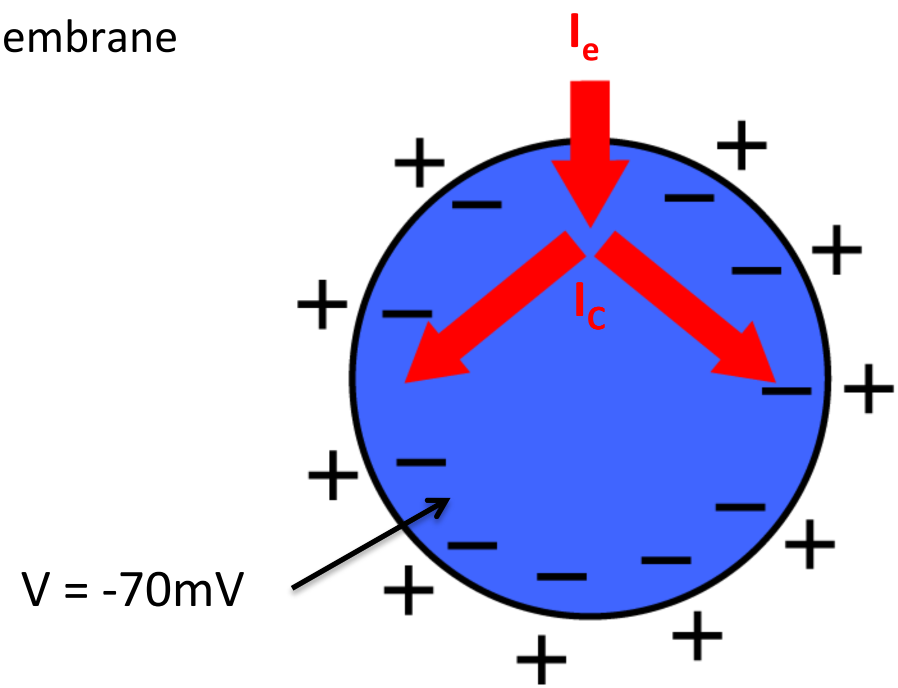
$$I_m = g_L\left(V - E_L\right)$$

What about time?

Single-compartment model

Model neuron:

1. Sphere of membrane
2. Isopotential



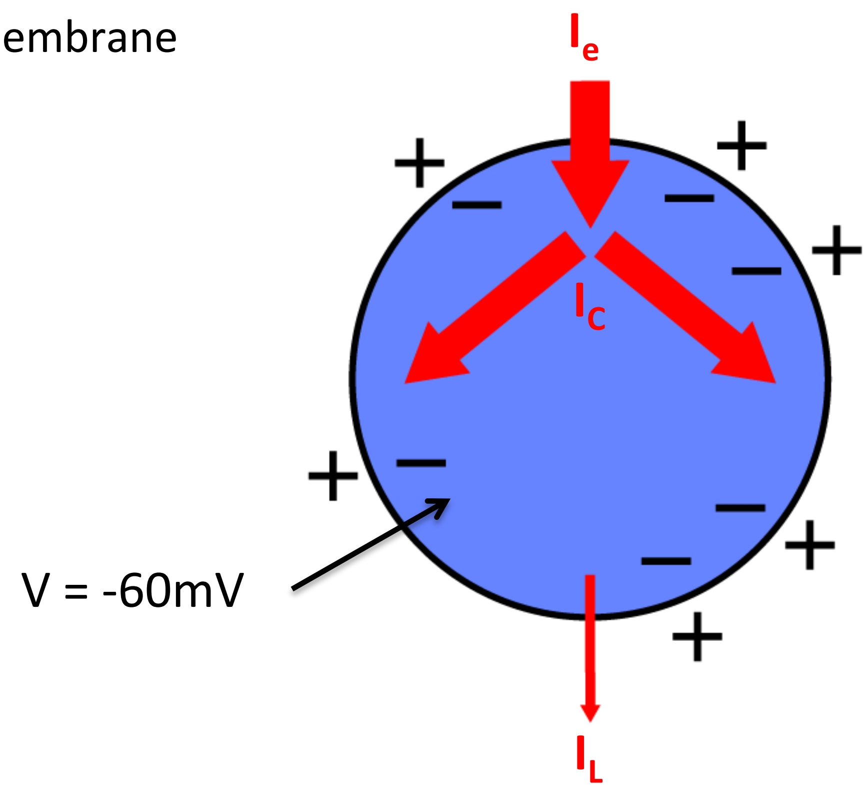
I_e = injected current

I_C = capacitive current
charges/discharges the membrane

Single-compartment model

Model neuron:

1. Sphere of membrane
2. Isopotential



I_e = injected current

I_C = capacitive current

charges/discharges the membrane

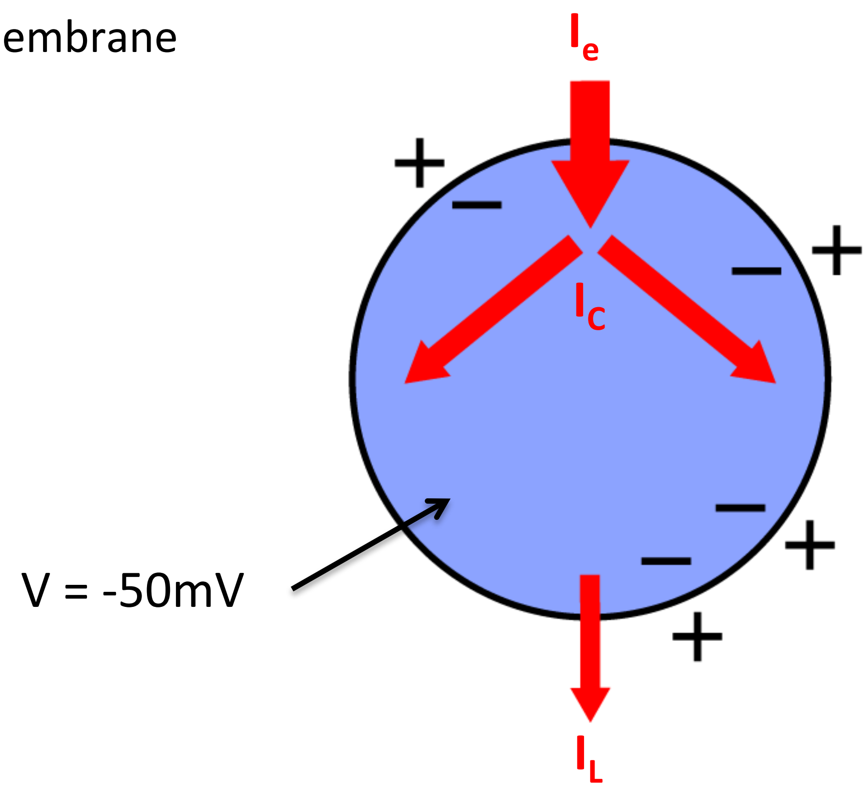
I_L = leak current

$$I_L = g_L\left(V - E_L\right)$$

Single-compartment model

Model neuron:

1. Sphere of membrane
2. Isopotential



I_e = injected current

I_C = capacitive current
charges/discharges the membrane

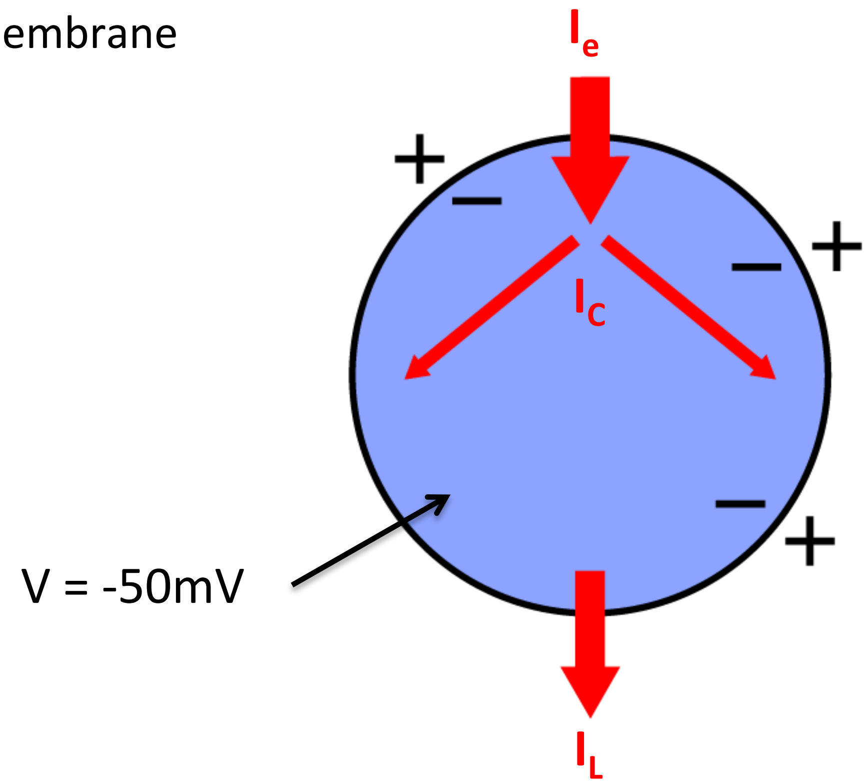
I_L = leak current

$$I_L = g_L\left(V - E_L\right)$$

Single-compartment model

Model neuron:

1. Sphere of membrane
2. Isopotential



I_e = injected current

I_C = capacitive current
charges/discharges the membrane

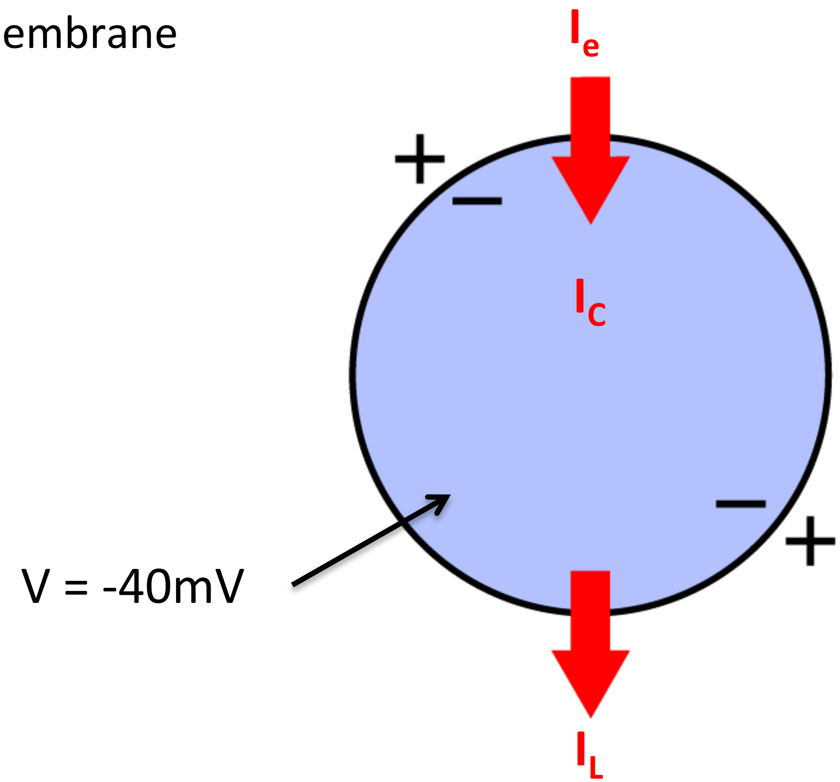
I_L = leak current

$$I_L = g_L\left(V - E_L\right)$$

Single-compartment model

Model neuron:

1. Sphere of membrane
2. Isopotential



I_e = injected current

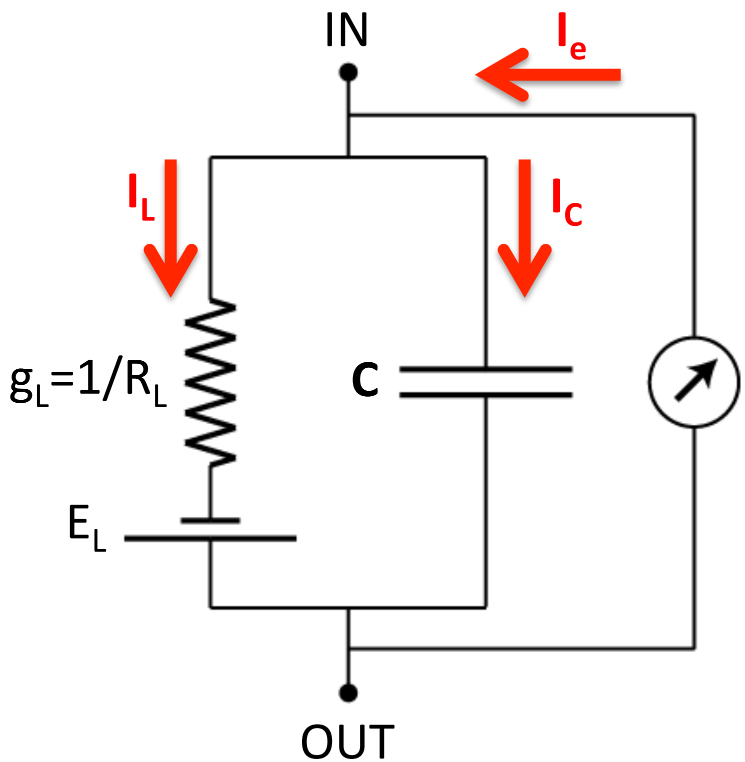
I_C = capacitive current

charges/discharges the membrane

I_L = leak current

$$I_L = g_L\left(V - E_L\right)$$

The membrane as an electrical circuit



C = capacitance

Ability to store charge

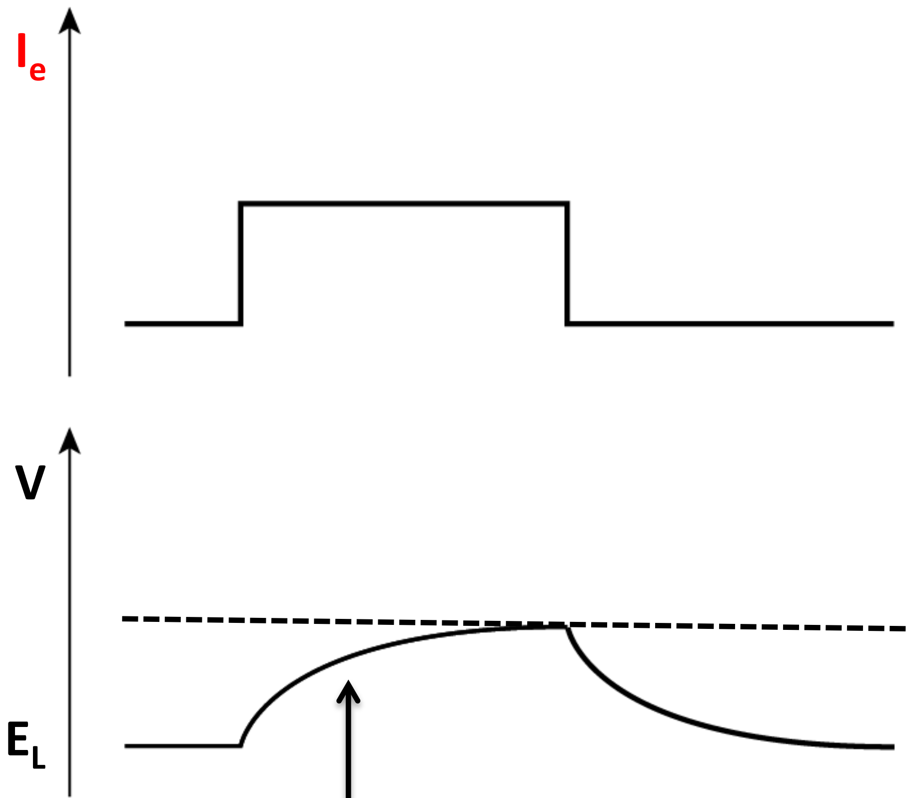
$$I_e = I_L + I_C$$

$$I_L = g_L\left(V - E_L\right)$$

$$C\frac{dV}{dt} = I_C$$

$$\boxed{V\left(t\right) = V_\infty + \left(V\left(0\right) - V_\infty\right)e^{-\frac{t}{\tau_m}}}$$

Potential time-course

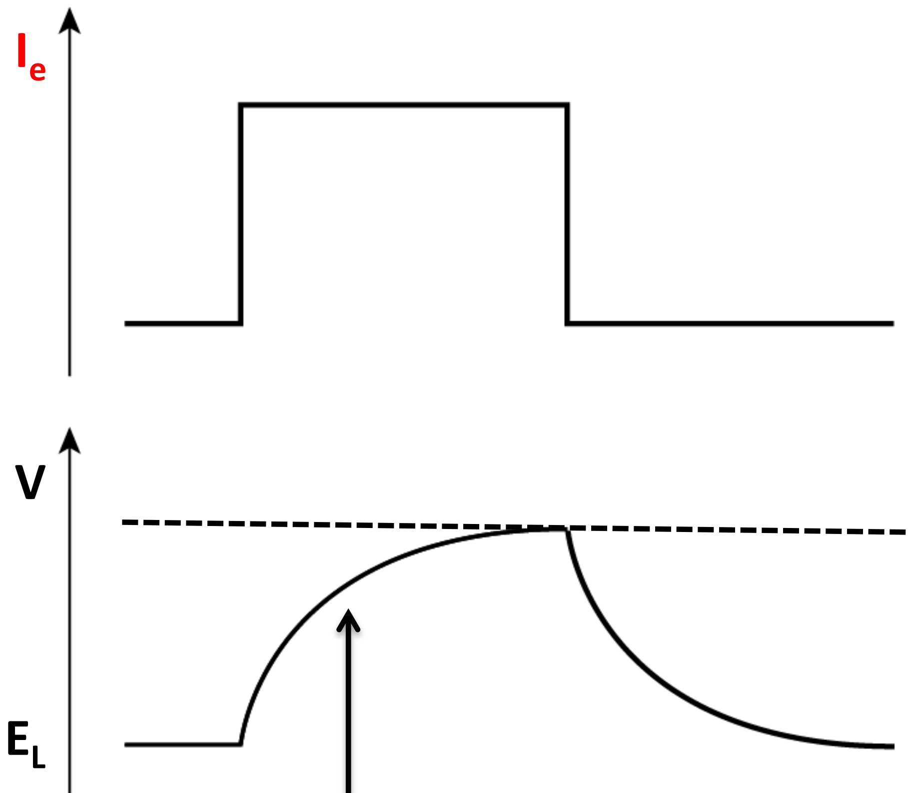


$$V_\infty = E_L + R_L I_e$$

$$\tau_m = R_L C$$

Membrane
time-constant

Spatial summation



larger current

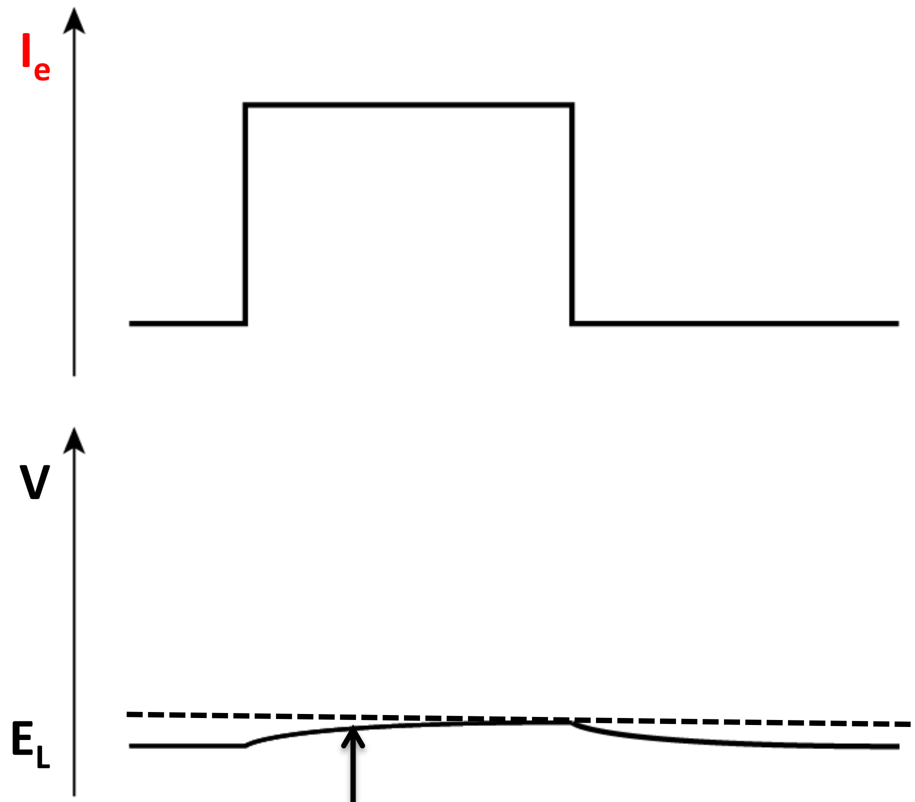
e.g. through spatial summation

$$V_\infty = E_L + R_L I_e$$

$$\tau_m = R_L C$$

Membrane time-constant

Input resistance



smaller resistance

e.g. through larger membrane area

$$V_\infty = E_L + R_L I_e$$

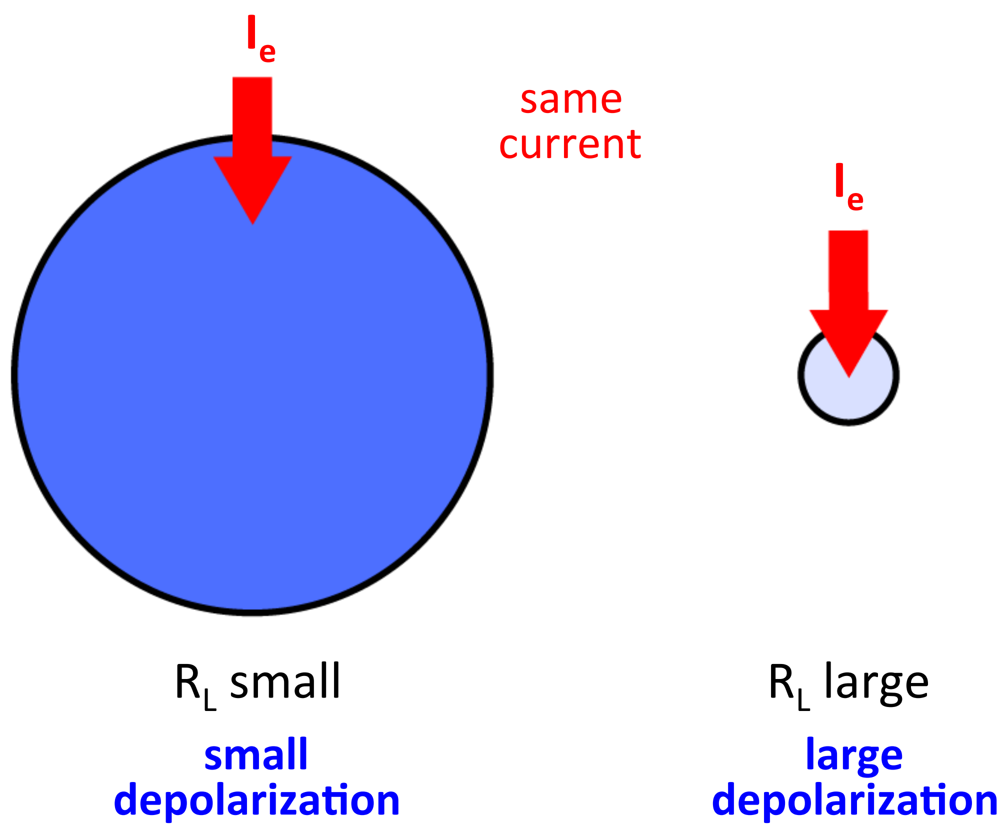
$$\tau_m = R_L C$$

Membrane time-constant

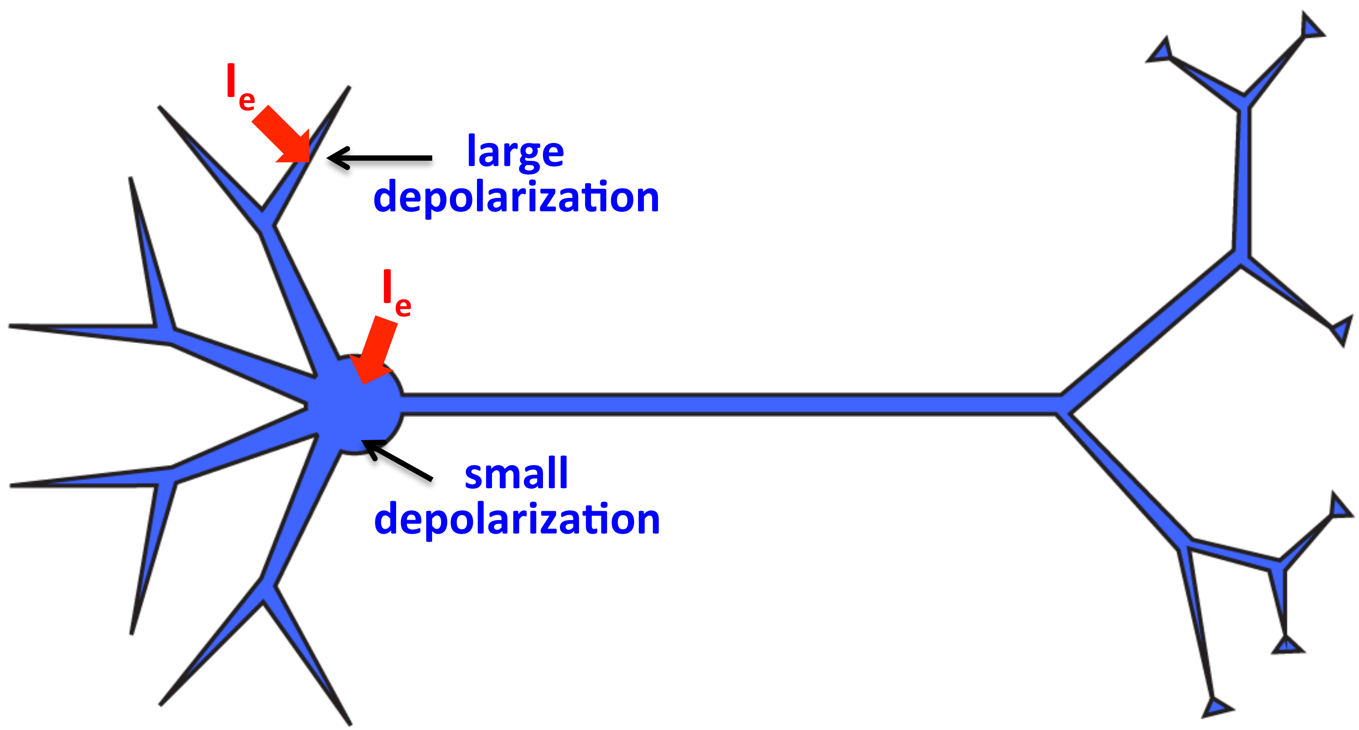
$$R_L = \frac{r_L}{Area}$$

Input resistance

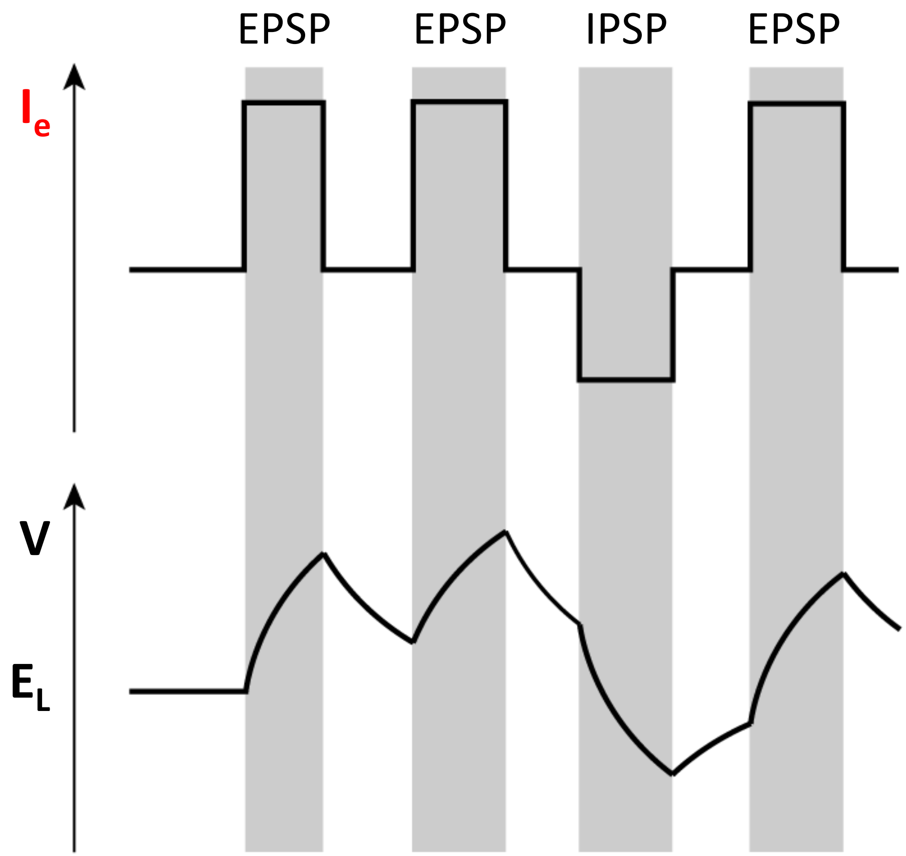
$$R_L = \frac{r_L}{Area}$$



Local differences in input resistance

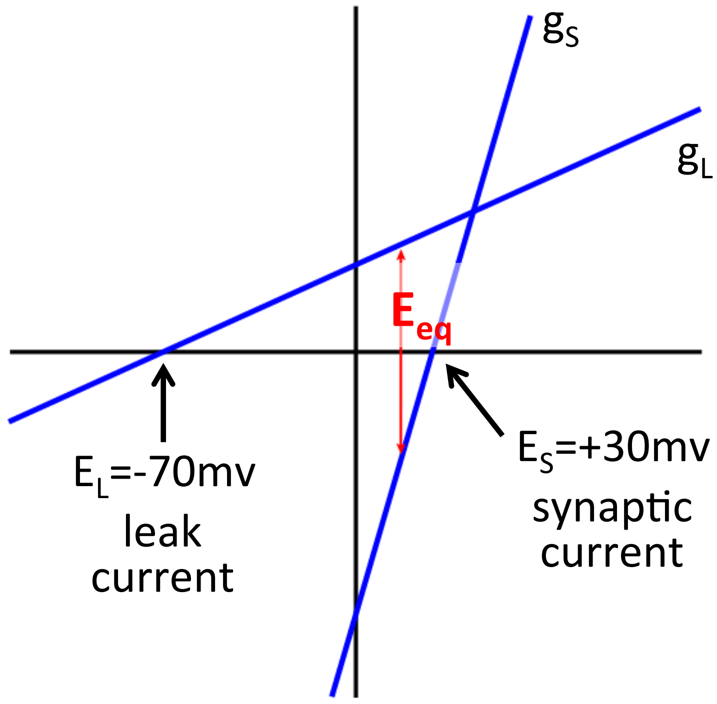


Temporal summation



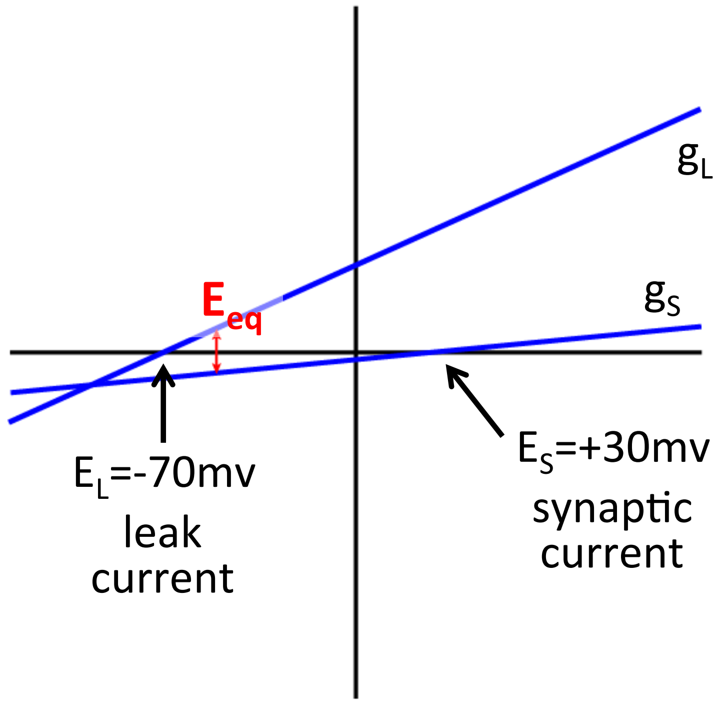
Holds for small currents
(until threshold for AP)

Two ionic currents



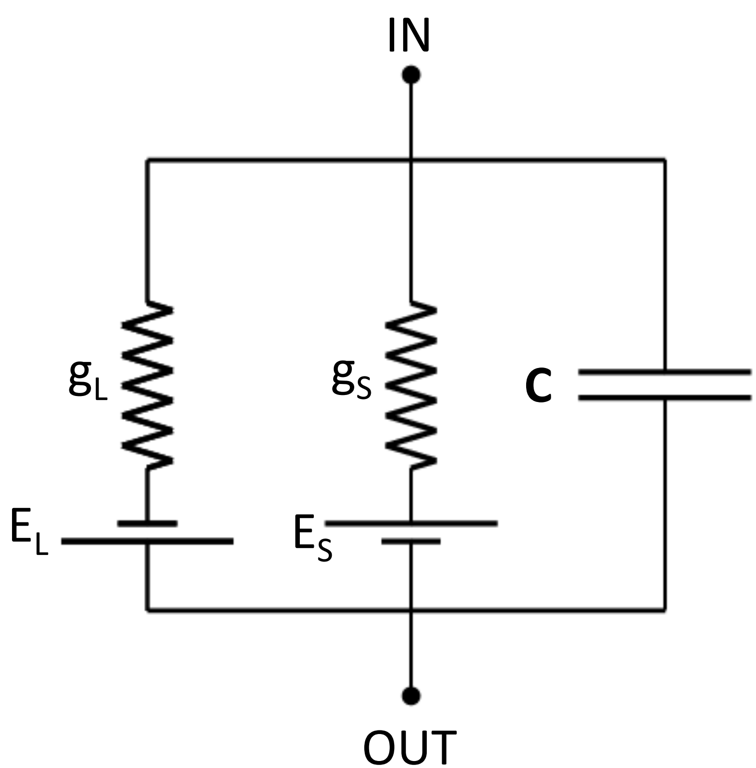
@ equilibrium:
$I_L = I_S$

Two ionic currents



@ equilibrium:
$I_L = I_S$

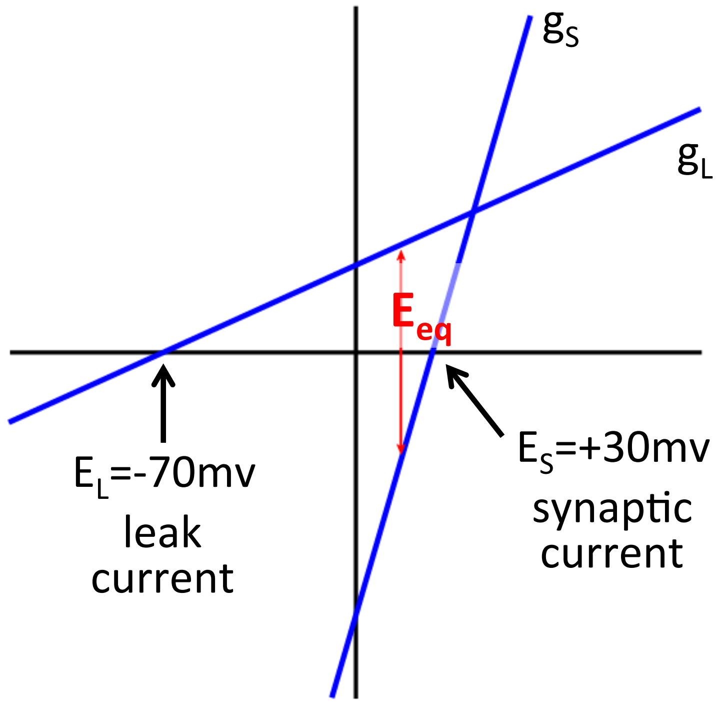
The membrane as an electrical circuit



$$V_\infty = \frac{g_L E_L + g_S E_S}{g_L + g_S}$$

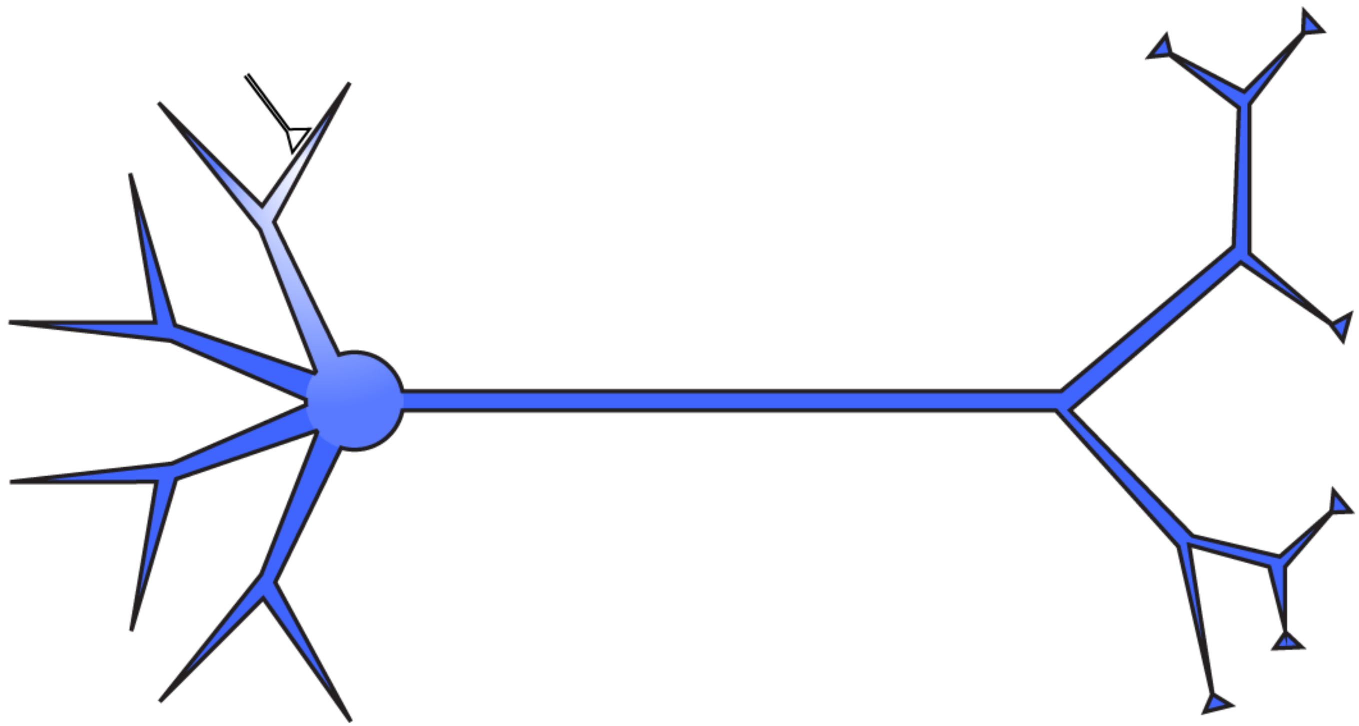
$$\tau_m = \frac{C}{g_L + g_S}$$

Two ionic currents

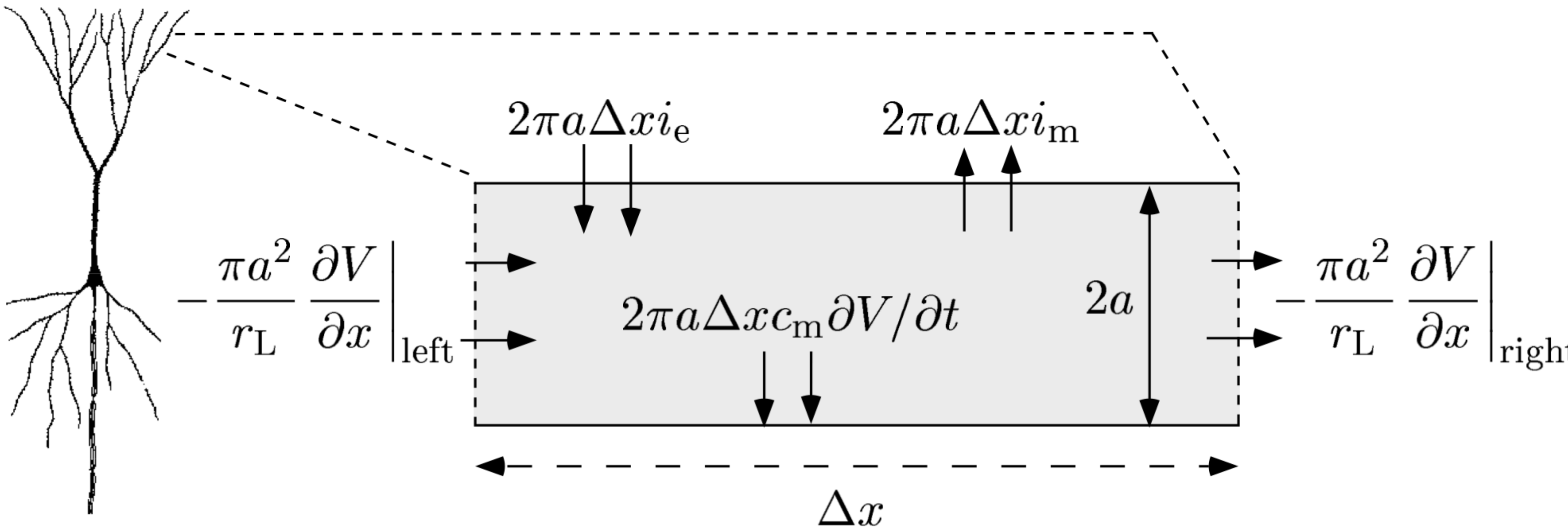


@ equilibrium:
$I_L = I_S$

Spatial potential gradients

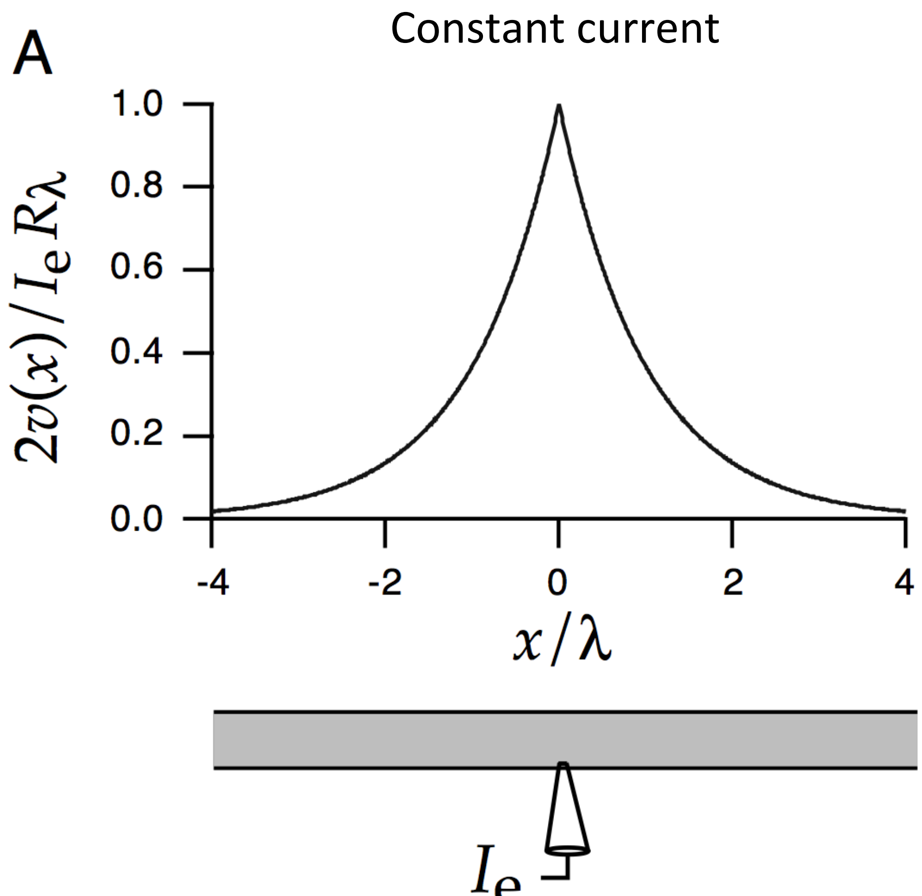


The cable equation



$$c_m \frac{\partial V}{\partial t} = \frac{1}{2ar_L} \frac{\partial}{\partial x}\left(a^2 \frac{\partial V}{\partial x}\right) - i_m + i_e \, .$$

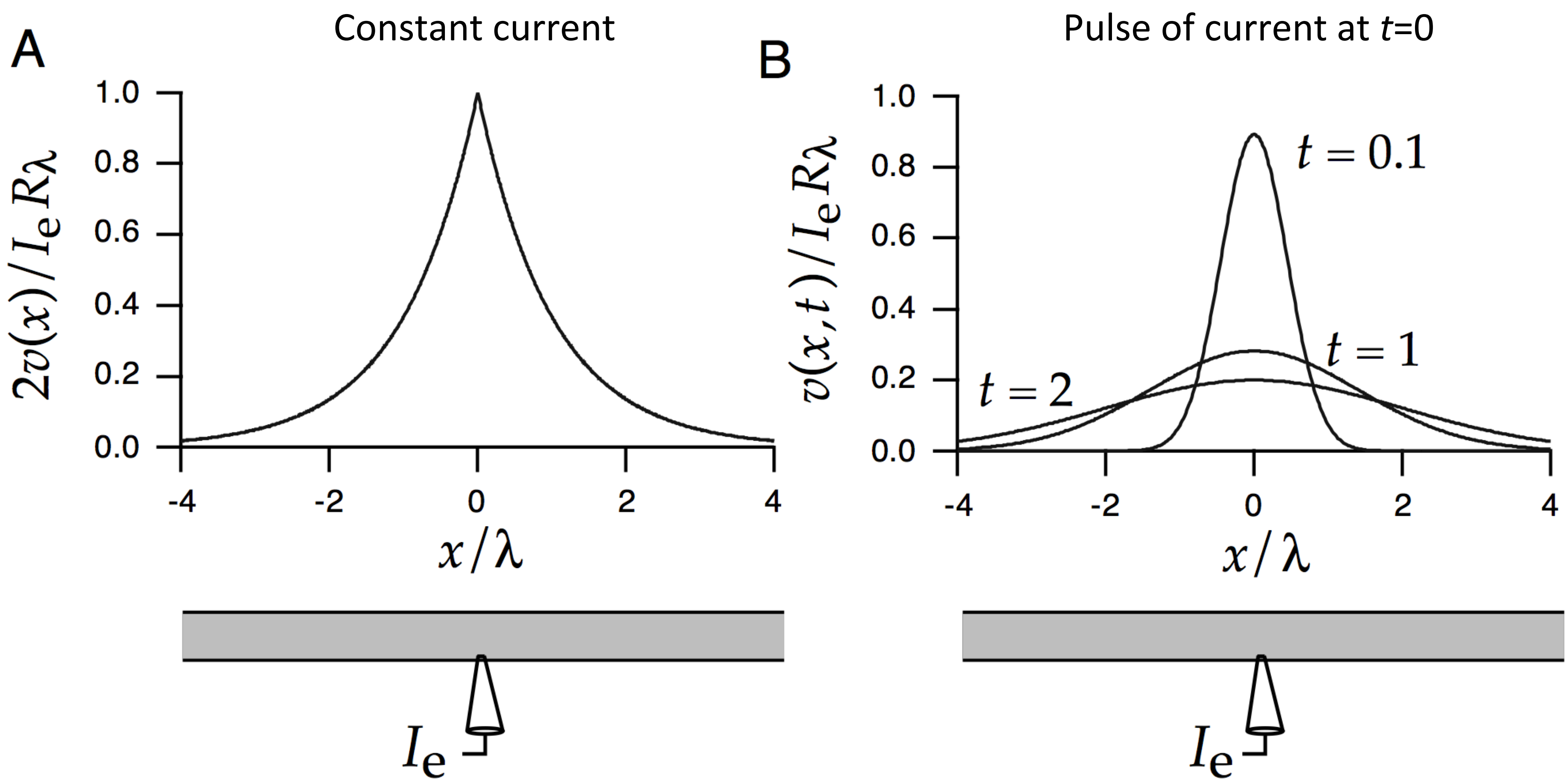
Infinite cable



$$v(x) = \frac{I_e R_\lambda}{2} \exp\left(-\frac{|x|}{\lambda}\right)$$

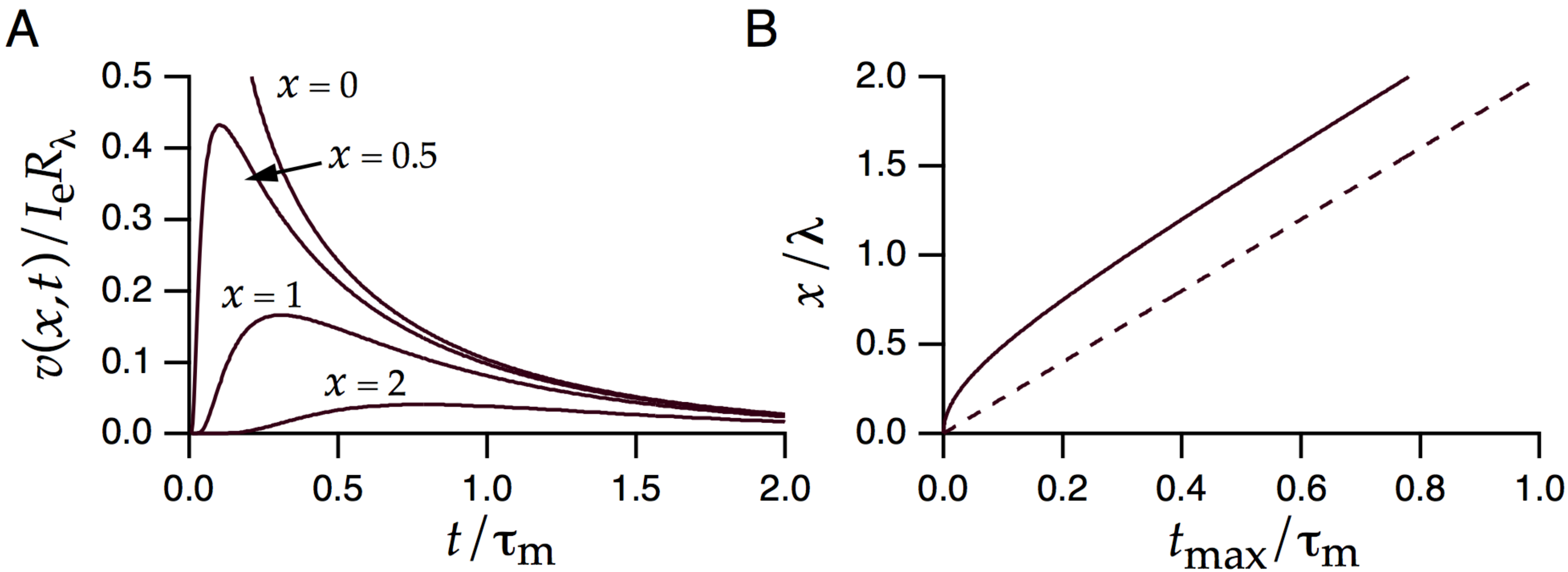
$$R_\lambda = \frac{r_m}{2\pi a \lambda} = \frac{r_L \lambda}{\pi a^2} \qquad \lambda = \sqrt{\frac{a r_m}{2 r_L}}$$

Infinite cable

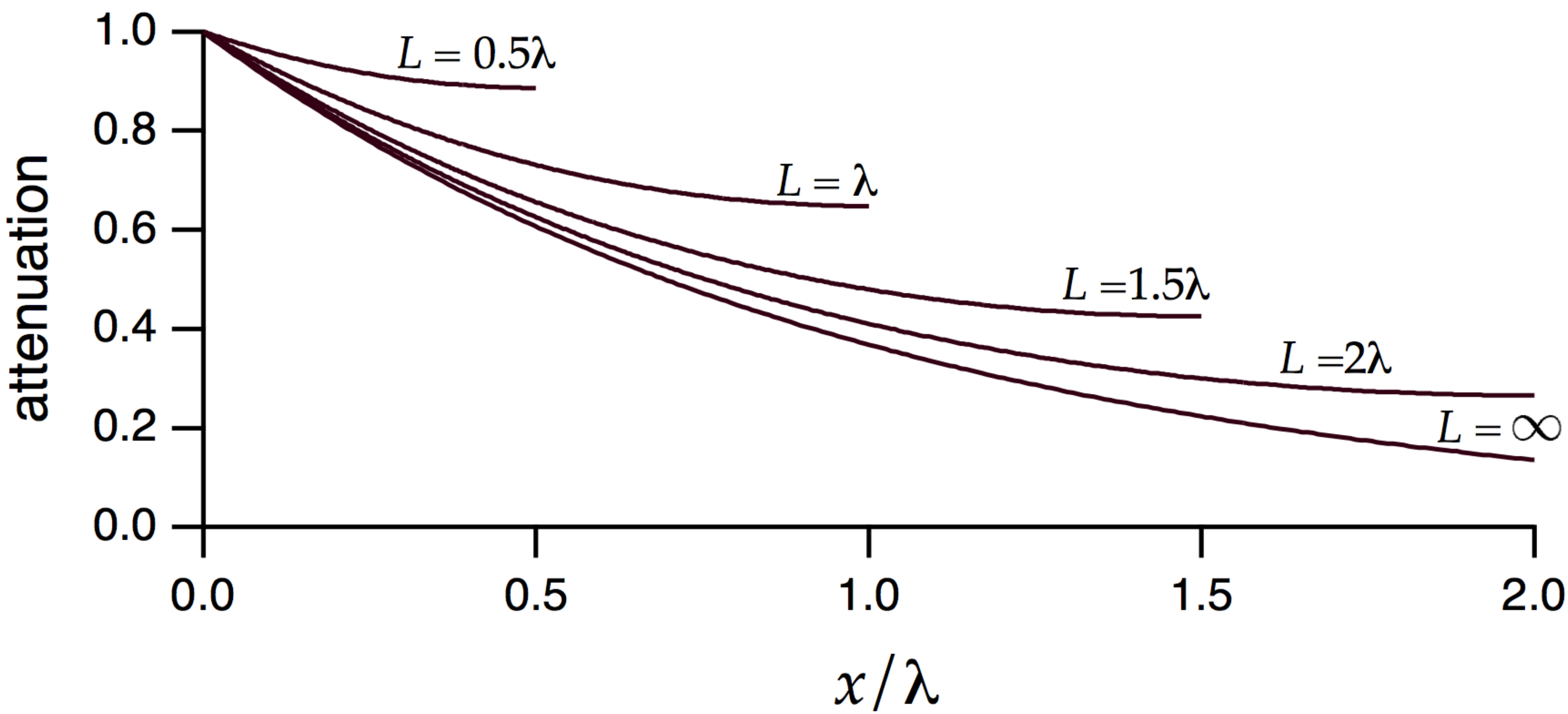


$$\tau_m = r_m c_m \qquad v(x,t) = \frac{I_e R_\lambda}{\sqrt{4\pi\lambda^2 t/\tau_m}} \exp\left(-\frac{\tau_m x^2}{4\lambda^2 t}\right) \exp\left(-\frac{t}{\tau_m}\right)$$

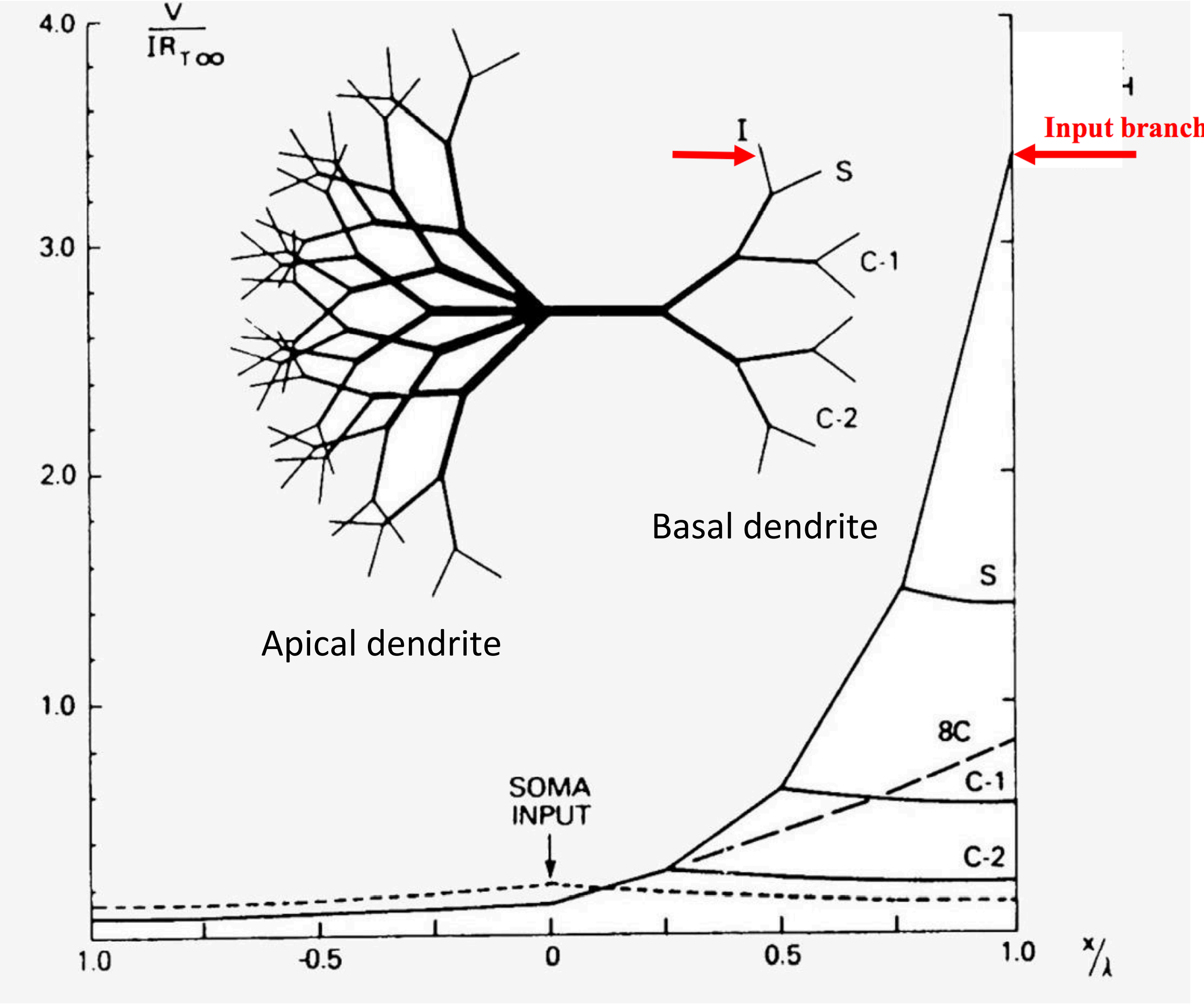
Spatio-temporal response to current pulse



Finite cable



Passive currents in a branching neuron



Rall and Rinzel, 1973

The appropriate level of description

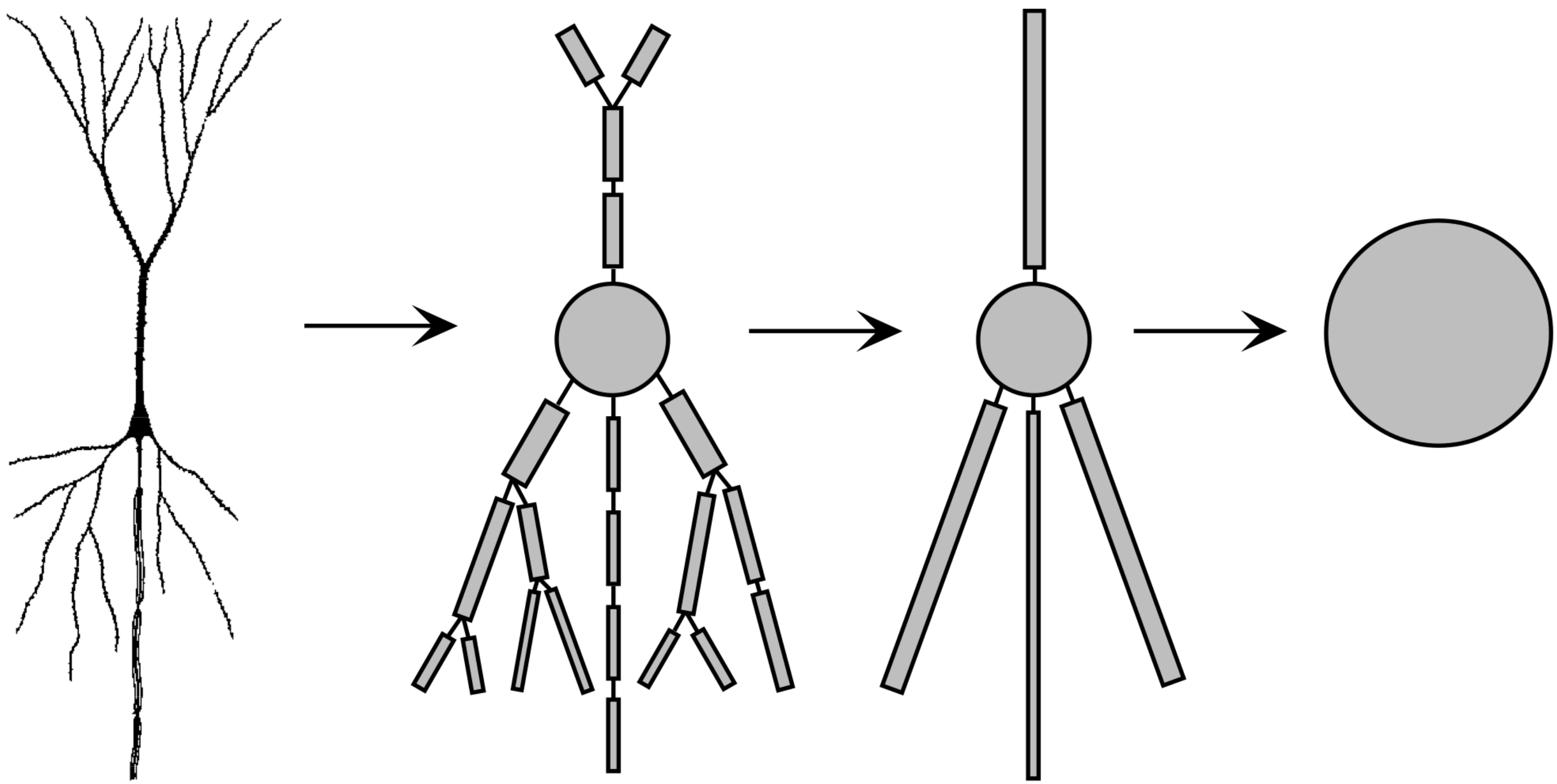


Figure 6.15: A sequence of approximations of the structure of a neuron. The neuron is represented by a variable number of discrete compartments each representing a region that is described by a single membrane potential. The connectors between compartments represent resistive couplings. The simplest description is the single-compartment model furthest to the right. (Neuron diagram from Haberly, 1990.)