Worksheet 2: Applications of Integration

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Arc Length.

The concepts of the method of Riemann Sum and integration are both directed at measuring total accumulation. The length of a curve can be described as the total length of a combination of short curves. Any *smooth* curve that is short enough would look like a line segment, and the length would be approximately $\sqrt{(dx)^2 + (dy)^2}$. Thus we can get an arc length formula if we integrate the expression.

The Arc Length Formula for Graphs: Consider the graph y = f(x). If f' is continuous on the closed interval [a, b], then the length of y = f(x), $a \le x \le b$, is

$$L = \int_{a}^{b} \sqrt{1 + [f'(x)]^{2}} \, dx = \int_{a}^{b} \sqrt{1 + \left[\frac{dy}{dx}\right]^{2}} \, dx.$$

A further application of the arc length formula lies in the arc length function defined by

$$s(x) = \int_{a}^{x} \sqrt{1 + [f'(t)]^2} dt.$$

The length function is useful in finding out the exact point you would arrive at after travelling for a fixed distance.

Exercise 1. Find the exact length of the curve

$$y = \frac{x^3}{3} + \frac{1}{4x}, \ 1 \le x \le 2.$$

$$y = e^x, \ 0 \le x \le 1.$$

$$\int_{0}^{\infty} \int_{0}^{\infty} \left(\frac{dx}{dx} \right) dx$$

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- (a) Find the exact length.
- (b) Find the length function s(x). Does it have an inverse?
- (c) Find an interval for x where the exact length of the corresponding curve is equal to 1.

(a)
$$y = \frac{2}{3} x^{\frac{3}{2}} \Rightarrow y' = x'^{\frac{1}{2}}$$

Let $x = \int_{1}^{4} \sqrt{1+x} \, dx$

$$y = \frac{2}{3} (|x|)^{\frac{3}{2}}$$

$$y = \frac{2}{3} (|x|)^{\frac{3}{2}} - (|x|)^{\frac{3}{2}}$$

$$y = \frac{2}{3} (|x|)^{\frac{3}{2}} - (|x|)^{\frac{3}{2}} - (|x|)^{\frac{3}{2}} - (|x|)^{\frac{3}{2}} + (|x|)^{\frac{3}{$$

Average Value and Center of Mass.

Finding the average value of a function is a natural topic in many real world scenarios. It is important to remember that a clear domain must be given before an average can be computed.

The average value of the function f over the interval [a,b] is

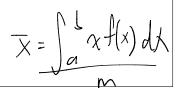
$$\frac{1}{b-a} \int_a^b f(x) \ dx.$$

The center of mass of an object with a given density function is the average position of all point masses. We treat the whole object as a system and we use the *moment* of the system about the origin to find the center of mass. For a pipe-like object that covers the interval [a, b] of a given density $\rho(x)$, the mass equals

$$m = \int_{a}^{b} \rho(x) \, dx$$

and the position of its center of mass equals

$$\overline{x} = \frac{1}{m} \int_{a}^{b} x \rho(x) \ dx.$$



Exercise 4.

- (a) Find the average value of $f(x) = 25 x^2$ on the interval [0, 2].
- (b) Find all values of c in the interval [0,2] such that f(c) is equal to the average value.

(a)
$$\beta h = \frac{1}{2} \int_{0}^{2} (-\chi^{2} + 25) d\chi$$

= $\frac{1}{2} \left(-\frac{\chi^{3}}{3} + 2JX \right) \Big|_{0}^{2} = \frac{1}{2} \left(-\frac{8}{3} + 50 \right) = -\frac{4}{3} + 2J = \frac{1}{3} + 2J$

Exercise 5. Find the mass and the center of mass of a steel pipe with density function $\rho(x) = \frac{1}{1+x^2}$ over the interval $[0,\sqrt{3}]$.

$$X = \frac{1}{m} \int_{0}^{13} x f(x) dx$$

$$N = \int_{0}^{13} f(x) dx$$

$$\int_{0}^{13} \frac{1}{x^{2}} dx = \frac{1}{m} (x) \int_{0}^{13} = \frac{\pi}{3}$$

$$\int_{0}^{13} \frac{1}{x^{2}} dx = \frac{\pi}{1} \int_{0}^{13} \frac{x}{1 + x^{2}} dx$$

$$= \frac{3}{2\pi} \int_{0}^{14} \frac{1}{1 + x^{2}} d(hx^{2}) = \frac{3}{2\pi} \int_{0}^{14} (hx^{2}) \Big|_{0}^{14} = \frac{3 \ln x}{\pi}$$

Exercise 6. (Optional) Prove the Mean Value Theorem for Integrals: if f is continuous on [a, b], then there exists c in [a, b] such that

$$\frac{1}{b-a} \int_a^b f(x) \, dx = f(c).$$

Interpret this result geometrically. $\,$

let
$$F(x) = \int_{\alpha}^{x} f(t) dt$$
 for some $X \in [a,b]$, F is confinuous and differentiable on (a,b)
By MJT, Here exist c in (a,b) s.t.
$$F'(c) = \frac{F(b) - F(a)}{b - a} = \int_{\alpha}^{b} f(t) dt - \int_{\alpha}^{a} f(t) dt$$
Where $F(c) = f(c)$

$$\int_{-1}^{1} f(c) = f(c) = \frac{f(b) - f(a)}{b - a} = \int_{-1}^{1} f(x) dx \Rightarrow f(c) = \int_{0}^{1} f(x) dx$$