

# **Efficient Solver**

by

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*Please insert your dedication here.*

## ACKNOWLEDGMENTS

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*It is customary for authors of academic books to include in their prefaces statements such as this: "I am indebted to ... for their invaluable help; however, any errors which remain are my sole responsibility." Occasionally an author will go further. Rather than say that if there are any mistakes then he is responsible for them, he will say that there will inevitably be some mistakes and he is responsible for them....*

*Although the shouldering of all responsibility is usually a social ritual, the admission that errors exist is not — it is often a sincere avowal of belief. But this appears to present a living and everyday example of a situation which philosophers have commonly dismissed as absurd; that it is sometimes rational to hold logically incompatible beliefs.*

— DAVID C. MAKINSON (1965)

Above is the famous "preface paradox," which illustrates how to use the `wbepi` environment for epigraphs at the beginning of chapters. You probably also want to thank the Academy.

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## ABSTRACT

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Numerical simulation of physical phenomena has long been of the interests in the area of mechanical engineering, physics, and in recent years computer graphics. The research in the field usually falls into two categories: modeling and solver. Research in modeling intends in finding mathematical equations, sometimes with better accuracy, sometimes are easier to solve, that can be used to describe the physical phenomena. Research in solvers, on the other hand, is to develop algorithms that are able to solve larger equations faster, for iterative approximate solvers, solve to higher accuracy. This dissertation focuses on designing efficient solvers, centering on a novel data structure namely SPGrid, which provides the flexibility of a sparse and adaptive data structure while retains the performance of a uniform grid. Furthermore, it will present the use of modern Single Instruction Multiple Data (SIMD) hardware and heterogeneous platforms that exploits their high computation throughputs to saturate memory utility. Finally, the efficiency of those design principles and practices are demonstrated in the Poisson and Linear elasticity solvers.



## 1 INTRODUCTION

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This documents presents a set of techniques and algorithms in solving large scale linear equations that emerged from the simulations of physical phenomenal. Those techniques has enabled large scale simulations that provides not only significantly more visual details but also higher fidelity solutions, and exposes challenges that were previously unrecognized in small scale simulations.

Those techniques are demonstrated in the context of heterogenous poisson equation, 2nd order accurate fluid simulation, and large scale linear elasticity solver for topology optimization.

**DISCARD THIS PAGE**

## COLOPHON

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This template uses Gyre Pagella by default. (I used Arno Pro in my dissertation.)

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