

Unsupervised Feature Selection with Adaptive Structure Learning

Liang Du and Yi-Dong Shen Institute of Software, CAS {duliang,ydshen}@ios.ac.cn

Adaptive local structure learning with probabilistic neighborhood

Local Structure Learning

$$\min_{\mathbf{P}} \sum_{i,j} (||\mathbf{x}_i - \mathbf{x}_j||_2^2 \mathbf{P}_{ij} + \mu \mathbf{P}_{ij}^2), \text{ s.t. } \mathbf{P} \mathbf{1}_n = \mathbf{1}_n, \mathbf{P} \ge 0$$

Adaptive Local

Structure Learning

$$egin{aligned} \min_{\mathbf{P},\mathbf{W}} & \sum_{i,j}^n (||\mathbf{W}^T\mathbf{x}_i - \mathbf{W}^T\mathbf{x}_j||_2^2 \mathbf{P}_{ij} + \mu \mathbf{P}_{ij}^2) + \gamma ||\mathbf{W}||_{21} \ & ext{s.t.} & \mathbf{P}\mathbf{1}_n = \mathbf{1}_n, \mathbf{P} \geq \mathbf{0}, \mathbf{W}^T \mathbf{X} \mathbf{X}^T \mathbf{W} = \mathbf{I} \end{aligned}$$

Unsupervised Feature Selection with Adaptive Structure Learning (FSASL)

$$\begin{aligned} & \underset{\mathbf{W},\mathbf{S},\mathbf{P}}{\text{min}} & \left(||\mathbf{W}^T\mathbf{X} - \mathbf{W}^T\mathbf{X}\mathbf{S}||^2 + \alpha ||\mathbf{S}||_1 \right) \\ & + & \beta \sum_{i,j}^n \left(||\mathbf{W}^T\mathbf{x}_i - \mathbf{W}^T\mathbf{x}_j||^2 \mathbf{P}_{ij} + \mu \mathbf{P}_{ij}^2 \right) + \gamma ||\mathbf{W}||_{21} \\ & \text{s.t.} & \mathbf{S}_{ii} = 0, \mathbf{P}\mathbf{1}_n = \mathbf{1}_n, \mathbf{P} \geq \mathbf{0}, \mathbf{W}^T\mathbf{X}\mathbf{X}^T\mathbf{W} = \mathbf{I} \end{aligned}$$

Background

- Feature selection techniques keep few informative features to alleviate the great challenges presented by data with high dimensionality.
- ➤ Without class label, unsupervised feature selection methods chooses those features that can reveal or maintain the underlying structure of data.

Related work

- ➤ Most existing methods use all the input features (including redundant and noisy ones) to capture the various structures of data.
- > Such structures are further used to guide the search of relevant features.
- ➤ A lot of methods are developed: LapScore, SPEC, EVSC, TraceRatio, UDFS, MCFS, MRSF, SPFS, FSSL, GLSPFS, JELSR, NDFS, RUFS, CGSSL, RSFS, LLCFS etc.

Motivation

- > On the one hand, one need the true structures of data to identify the informative features;
- > On the other hand, one need the informative features to accurately estimate the structures;
- We have to face the *chicken-and-egg dilemma* between **structure characterization** and **feature learning**.

Basic idea

- ➤ We use the selected informative features to accurately re-estimate the underlying structures.
- We develop a unified framework to integrate these two essential sub-steps: structure characterization and feature learning.

Our method: FSASL

Adaptive global structure learning via sparse representation

Global Structure Learning

$$\min_{\mathbf{S}} \sum_{i=1}^{n} (||\mathbf{x}_i - \mathbf{X}\mathbf{s}_i||^2 + \alpha ||\mathbf{s}_i||_1) \quad \text{s.t.} \quad \mathbf{S}_{ii} = 0$$

Adaptive Global | Structure Learning

$$\min_{\mathbf{S}, \mathbf{W}} \sum_{i=1}^{n} ||\mathbf{W}^T \mathbf{x}_i - \mathbf{W}^T \mathbf{X} \mathbf{s}_i||^2 + \alpha ||\mathbf{S}||_1 + \gamma ||\mathbf{W}||_{21}$$

s.t. $\mathbf{S}_{ii} = 0, \mathbf{W}^T \mathbf{X} \mathbf{X}^T \mathbf{W} = \mathbf{I}$

Experiments

> K-means clustering with selected features

Table 1: Aggregated clustering results measured by Accuracy (%) of the compared methods.

Data Sets	AllFea	LapScore	MCFS	LLCFS	UDFS	NDFS	SPFS	RUFS	JELSR	GLSPFS	FSASL
MFEA		51.78	51.04	60.38	48.94	67.13	68.20	64.58	67.01	61.00	69.94
	68.73	$\pm \ 5.51$	$\pm \ 8.13$	$\pm \ 8.58$	$\pm \ 3.32$	± 7.53	\pm 9.43	± 7.99	$\pm \ 8.37$	$\pm \ 8.70$	\pm 7.19
		0.00	0.00	0.00	0.00	0.01	0.22	0.00	0.01	0.00	1.00
USPS49		69.21	53.74	94.96	94.05	68.12	83.43	85.86	95.16	94.75	95.95
	77.70	± 8.95	$\pm \ 3.50$	$\pm \ 1.44$	$\pm \ 1.13$	$\pm \ 8.18$	± 6.66	$\pm \ 2.58$	$\pm \ 0.55$	$\pm~0.61$	\pm 0.48
		0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	1.00
UMIST		36.73	44.46	47.31	48.04	52.80	46.72	50.87	53.52	50.53	54.92
	42.40	± 1.18	$\pm \ 3.26$	$\pm \ 0.83$	$\pm \ 1.92$	$\pm \ 2.26$	$\pm \ 1.70$	± 1.95	$\pm \ 1.54$	$\pm \ 0.59$	\pm 1.89
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	1.00
JAFFE		67.62	73.56	64.79	75.48	74.98	73.93	75.75	77.77	75.46	79.29
	71.57	± 8.49	$\pm \ 4.83$	$\pm \ 4.08$	$\pm \ 1.63$	$\pm \ 2.15$	$\pm \ 2.85$	$\pm \ 2.53$	$\pm \ 1.87$	± 1.61	\pm 2.24
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
AR		25.29	29.05	34.22	30.87	32.34	31.06	34.84	34.19	34.12	36.11
	30.26	± 2.89	± 1.19	\pm 2.70	$\pm \ 0.35$	$\pm \ 1.52$	$\pm \ 2.14$	± 1.90	$\pm \ 2.52$	$\pm \ 1.60$	\pm 0.75
		0.00	0.00	0.05	0.00	0.00	0.00	0.04	0.02	0.00	1.00
COIL		45.60	51.50	50.84	31.40	44.22	56.94	59.20	59.53	57.96	60.93
	59.17	± 6.16	$\pm \ 5.38$	$\pm \ 3.76$	± 16.89	± 6.33	$\pm \ 3.43$	$\pm \ 3.28$	$\pm \ 4.01$	$\pm \ 2.27$	\pm 2.50
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	1.00
LUNG		58.97	70.42	71.58	65.46	75.52	73.49	77.35	77.86	77.83	81.93
	72.46	$\pm \ 5.24$	$\pm \ 3.41$	$\pm \ 5.85$	$\pm \ 3.88$	$\pm \ 1.57$	$\pm \ 3.43$	$\pm \ 2.62$	$\pm \ 3.12$	$\pm \ 2.70$	\pm 1.63
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
TOX		40.25	43.10	39.28	47.14	38.28	39.93	47.67	43.96	47.38	49.17
	43.65	± 0.65	$\pm \ 1.86$	$\pm \ 0.49$	$\pm \ 0.75$	± 1.64	± 1.13	$\pm \ 0.83$	$\pm \ 1.56$	± 1.93	$\pm~0.67$
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Average	58.24	49.43	52.11	57.92	55.17	56.67	59.21	62.02	63.63	62.38	66.03

> The effect of adaptive structure learning

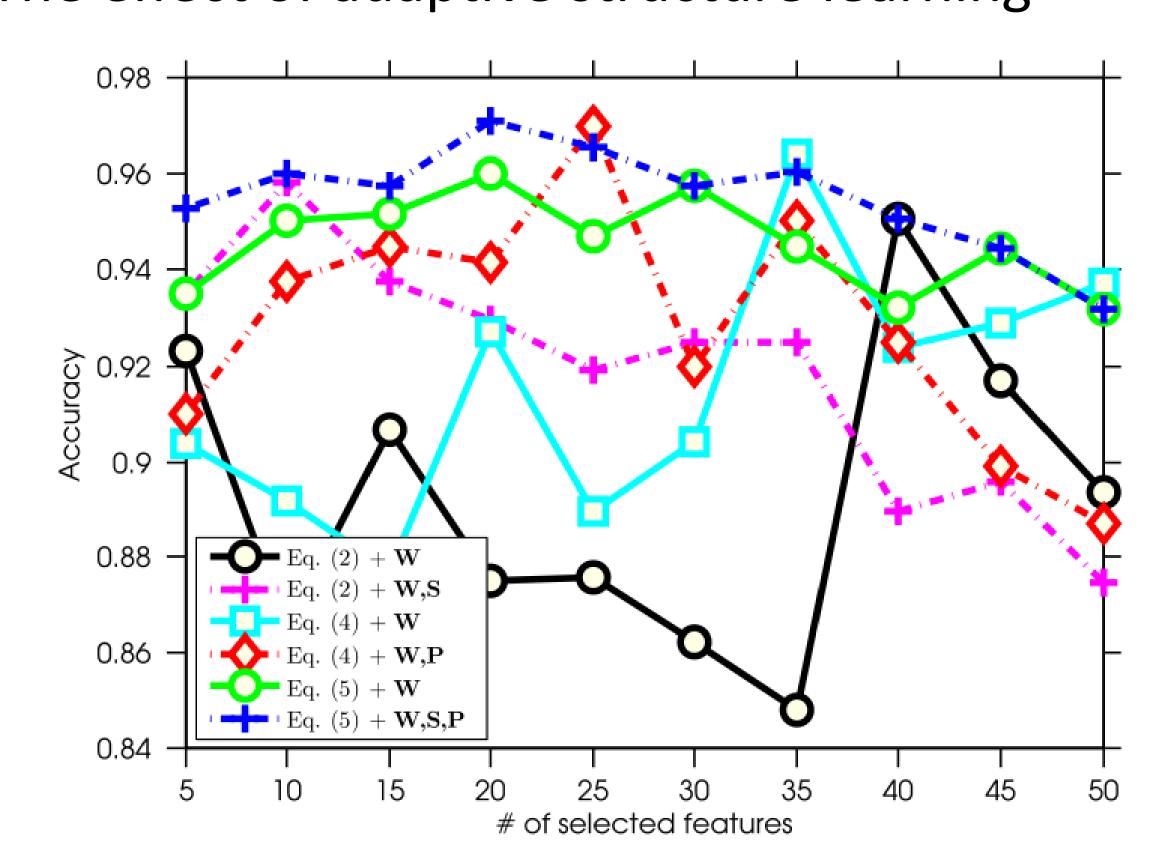


Figure 3: Clustering accuracy w.r.t. 6 different settings of FSASL on USPS200.

Code: https://github.com/csliangdu/FSASL