Finding Shortest Path on Land Surface

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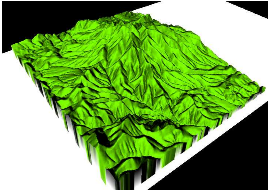
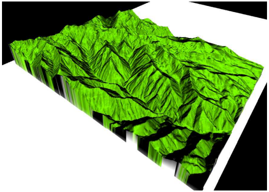
Introduction

Land Surface

- Land surfaces are modeled as *terrains*
- A *terrain* is the graph of a continuous function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ that assigns every point on a horizontal plane to an elevation[3]
- Property of a terrain: Every vertical line intersects it at no more than one point



(a) Eagle Peak, U.S.A



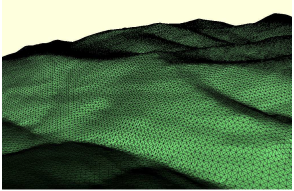
(b) Bear Head, U.S.A

Figure: Land surface

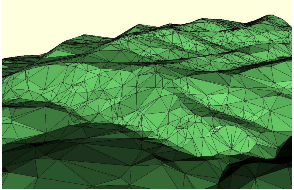
Introduction

TIN model

- Terrains can be represented by the *TIN* (Triangular Irregular Network) model
- A TIN is a graph $G = (V, E, F)$ formed by a set F of adjacent non-overlapping triangular faces, where V is the vertex set and E is the edge set
- The complexity of a TIN is represented by $n = |F|$ in this paper



(a) A TIN with more faces



(b) A TIN with less faces

Figure: TIN model

Introduction

Different Types of Shortest Paths

- Euclidean shortest path (p_e)
- Surface shortest path (p_s)
- As indicated in previous papers[7], there is $|p_s| \geq |p_e|$

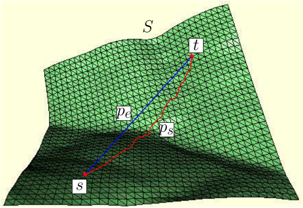
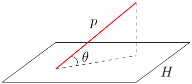


Figure: Euclidean shortest path vs. Surface shortest path

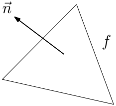
Introduction

Shortest Gentle Path

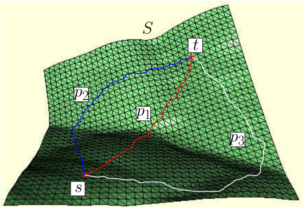
- In TIN model, a Shortest Gentle Path (SGP) p is formed by a sequence of adjacent straight line segments
- The slope of all segments in p is $\leq \theta_m$, where $\theta_m \in (0, \pi/2]$ is a parameter specified by the user
- There does not exist another SGP p' such that $|p'| < |p|$



(a) A segment



(b) A face



(c) Shortest gentle path

Figure: Shortest gentle path

Introduction

Problem Definition

Given a source point s, a destination point t on a terrain surface S, maximum slope value θ_m and approximation factor ϵ. Our goal is to find a path p between s and t such that p satisfies the following two requirements:

- Slope requirement: p is a gentle path with maximum slope θ_m
- Distance requirement: $|p| \leq (1+\epsilon)|p_o|$, where p_o is the SGP between s and t

When $\theta_m = \pi/2$, there is no slope constraint, so our problem degenerates to the conventional approximate surface shortest path problem

Introduction

Motivation

- High complexity
  - $O(n^2)$ complexity[2] if the slope constraint is not considered
  - At least as hard as the surface shortest path problem

Applications

- Application to the industry
  - Navigation
  - Path planning
- Application to the academia
  - Surface k-nearest neighbor search
  - Natural sciences

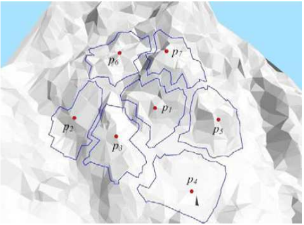


Figure: Shahabi et al., "Indexing land surface for efficient kNN query", PVLDB '08[7]

Related Work

Finding Shortest Paths

- Exact algorithms

Ref	Author	Time	Technique
[5]	Mitchell et al.	$O(n^2 \log n)$	Continuous Dijkstra
[2]	Chen, Han	$O(n^2)$	One angle one split

- Approximation algorithms

Ref	Author	Time	Technique
[1]	Aleksandrov et al.	$O(\frac{n}{\sqrt{\epsilon}} \log \frac{n}{\epsilon} \log \frac{1}{\epsilon})$	Steiner point insertion
[8]	Xing et al.	—	Shortest path oracle

Related Work

Surface Simplification

In computer graphics, *surface simplification* means transforming a complex, highly detailed model into a less detailed one while the shape of the model is maintained

- Vertex decimation[6]
- Edge contraction[4]

Figure: Surface simplification (Image cited from [4])

Algorithm

Framework

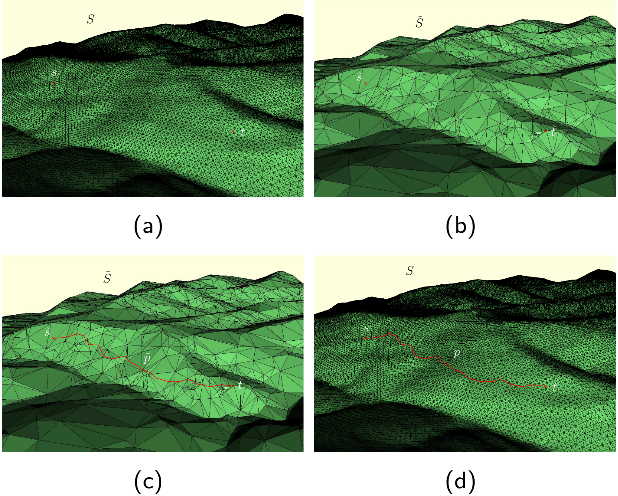
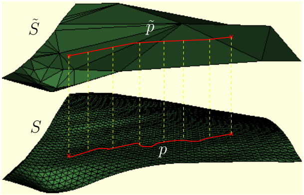


Figure: An overview of our algorithm

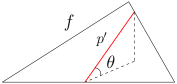
Algorithm

Path Mapping

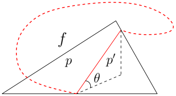
- Step 1: Shadowing
  - Compute the projective path of $\tilde{p}$ on S
  - From the property of a terrain, any path $\tilde{p}$ on $\tilde{S}$ has exactly one corresponding projective path p on S
- Step 2: Adjusting
  - For each path segment $p_i = \overline{ab}$, if its slope is larger than θ_m, then we apply the path finding algorithm $\mathcal{A}$ to find the SGP $\tilde{p}_i$ between a and b, and replace p_i with $\tilde{p}_i$



(a)



(b)



(c)

Figure: Path mapping

Algorithm

Distance Bound

Though the slope requirement is satisfied, the distance bound, $|p| \leq (1+\epsilon)|p_o|$, is required

The distance error is determined by the *difference* between $\tilde{S}$ and S, denoted by $\Delta(S, \tilde{S})$

- If $\tilde{S}$ is exactly the same as S, that is $\Delta(S, \tilde{S}) = 0$, no error will be caused, but the computation is slow
- If $\tilde{S}$ is over-simplified, $\tilde{S}$ contains much less faces than S, the computation is fast, but the error may be too large

Given a TIN terrain S, find a simplified surface $\tilde{S}$ such that

- For any pair of p_o and the mapped path p found by our algorithm, $|p| \leq (1+\epsilon)|p_o|$
- The number of faces in $\tilde{S}$ is minimized

Algorithm

Distance Bound

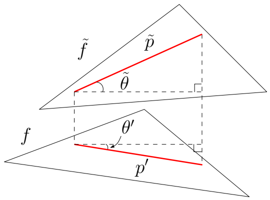
Due to the difference between the slope of $\tilde{f}$ and f, when a path segment $\tilde{p}$ is mapped from $\tilde{f}$ to f, error might be caused for two reasons:

- Error caused by shadowing

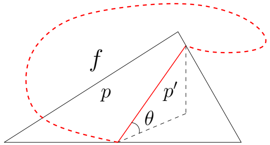
$$|p'| = \frac{\cos\tilde{\theta}}{\cos\theta'} \cdot |\tilde{p}| \qquad (1)$$

- Error caused by adjusting

$$|p| = \max\{1, \frac{\sin\theta'}{\sin\theta_m}\} \cdot |p'| \quad (2)$$



(a) Shadowing



(b) Adjusting

Figure: Causes of error

Algorithm

Distance Bound

We define $\Delta(S, \tilde{S})$, the *difference* between S and $\tilde{S}$ as follows:

$$\Delta(S, \tilde{S}) = \lambda \times \lambda' \tag{3}$$

- λ: the largest possible mapping error from S to $\tilde{S}$
- λ': the largest possible mapping error from $\tilde{S}$ to S

λ and λ' are extracted from the geometric parameters of S and $\tilde{S}$

Theorem (Distance Bound)

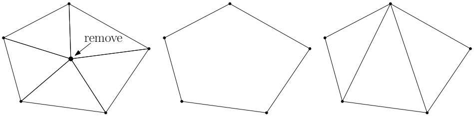
Let p_o be the SGP between a given pair of points on S and p be the path found by our algorithm. And let $\tilde{S}$ be the simplified surface. If $\Delta(S, \tilde{S}) \leq 1 + \epsilon$, then $|p| \leq (1 + \epsilon)|p_o|$.

Algorithm

Surface Simplifier

Given a terrain surface S, maximum slope θ_m and approximation ratio ϵ, the algorithm *Simplifier* simplifies S based on a greedy approach such that Theorem 1 is satisfied:

Sort the vertices of S by an error metric[6]
for all vertices v **do**
    Check whether v can be removed according to Theorem 1
    if yes **then**
        Removed v from S
        Triangulate the hole left by v's removal



(a) (b) (c)

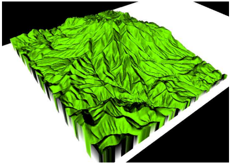
Figure: Triangulation

Experiment

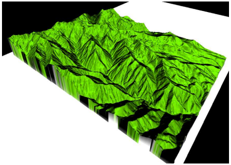
Settings

We did experiments on both real data sets and synthetic data sets. The real data set is available to the public for free at *Geo Community*, which was also used by previous research[7, 9, 8]

- CPU: 2xQuad Core 3GHz
- Memory: 32GB RAM
- Language: C++



(a) Eagle Peak

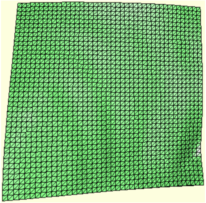


(b) Bear Head

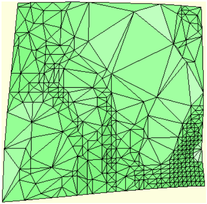
Figure: Real data sets

Experiment

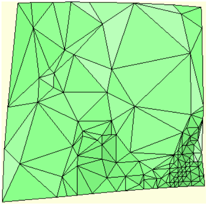
Illustration of the Results



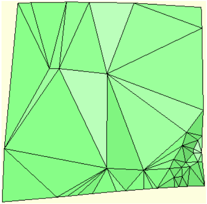
(a) Original surface



(b) $\epsilon = 0.1$



(c) $\epsilon = 0.25$



(d) $\epsilon = 0.5$

Figure: Result of surface simplification (Eagle Peak where $\theta_m = 0.3$)

Experiment

Effect of θ_m

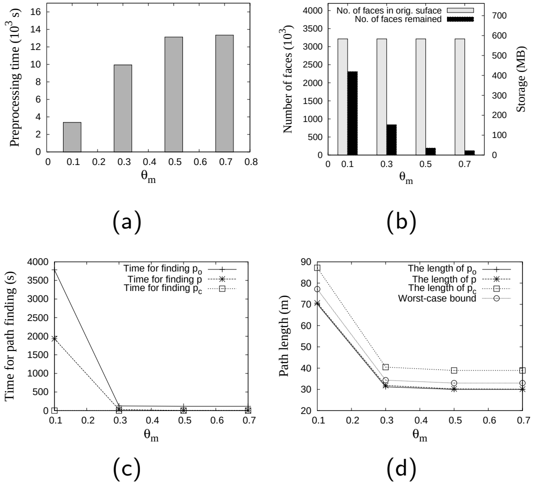


Figure: Effect of θ_m (Eagle Peak where $\epsilon = 0.1$)

Experiment

Effect of ϵ

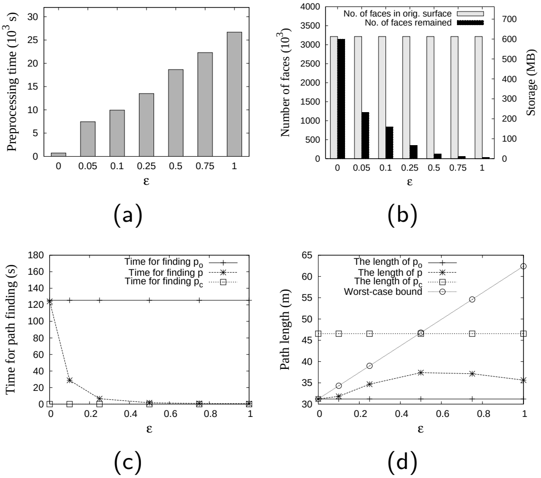


Figure: Effect of ϵ (Eagle Peak where $\theta_m = 0.3$)

Experiment

Results for the Traditional Problem

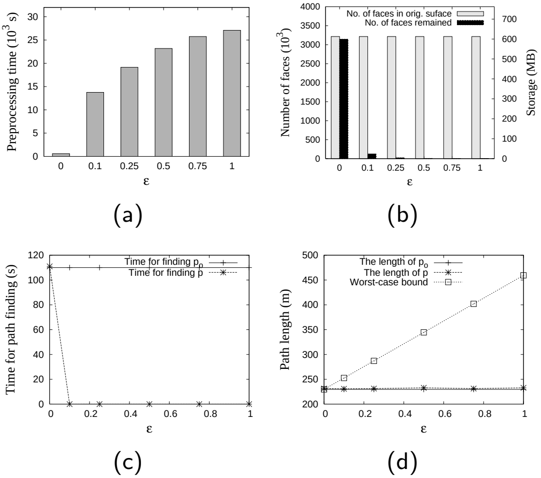


Figure: Effect of ϵ (Eagle Peak where $\theta_m = \pi/2$)

Reference I

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Q & A

Frequently Asked Questions

- **Q1**: The details of distance bound?

Q & A I

Q1: Distance Bound

We introduce the following term called *MRS* to denote the overall length ratio between p and $\tilde{p}$ when mapping a *specific* path segment $\tilde{p}$ from $\tilde{f}$ to f, which is an immediate result from Eq. 1 and 2:

$$MRS(< p, f >, < \tilde{p}, \tilde{f} >) = \max\{1, \frac{\sin\theta'}{\sin\theta_m}\} \cdot \frac{\cos\tilde{\theta}}{\cos\theta'} \qquad (4)$$

Let $MR(f, \tilde{f})$ be the greatest possible mapping ratio of an arbitrary $\tilde{p}$. That is,

$$MR(f, \tilde{f}) = \max_{\forall\tilde{p}\in\tilde{f}} MRS(< p, f >, < \tilde{p}, \tilde{f} >) \qquad (5)$$

$MR(f, \tilde{f})$ can be written as a function of $\vec{n}$ and $\vec{\tilde{n}}$, the normal vectors of f and $\tilde{f}$. And $MR(\tilde{f}, f)$ can be similarly defined

Q & A II

We say that face f *overlaps* $\tilde{f}$ if there exists a vertical line that intersects both f and $\tilde{f}$
We then define a face pair set $\mathcal{CS}(S, \tilde{S})$ that contains all overlapping face pairs from S and $\tilde{S}$ as follows:

$$\mathcal{CS}(S, \tilde{S}) = \{(f, \tilde{f}) | f \in F, \tilde{f} \in \tilde{F} \text{ and } f \text{ overlaps } \tilde{f}\} \quad (6)$$

Let λ be the largest possible $MR(f, \tilde{f})$ for all face pairs in $\mathcal{CS}(S, \tilde{S})$, so

$$\lambda = \max_{(f,\tilde{f}) \in \mathcal{CS}(S,\tilde{S})} MR(f, \tilde{f}) \quad (7)$$

And λ' is similarly defined as

$$\lambda' = \max_{(f,\tilde{f}) \in \mathcal{CS}(S,\tilde{S})} MR(\tilde{f}, f) \quad (8)$$