Approximative Optimal Control Solution

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We have time-series $\boldsymbol{x}(t)$ from municipalies data. Assume simple linear dif.eq. model for the time-series and solve for some kind of optimal control to have wanted changes.

$$\frac{d\boldsymbol{x}}{d\,t} = \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{f}(t)$$

We need to solve for optimal control $\boldsymbol{f}(t)$ which is assumed to be constant $\boldsymbol{c} = \boldsymbol{A}\,\boldsymbol{\Delta}$ for simplicity.

Given time-series, we have value for each per year so $\Delta t = 1$ and we have linear equation $\Delta\boldsymbol{x} = \boldsymbol{A}\,\boldsymbol{x}$ from datapoints which we can solve using linear optimization.

After solving for $\boldsymbol{A}$. Target change to Mikkeli's parameters needs to be solved. We have $\boldsymbol{y} = \Delta\boldsymbol{x}$ which we want to maximize within one year from target values $\boldsymbol{x}$. We minimize $e(\boldsymbol{\Delta}) = \frac{1}{2}\|\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{\Delta}) - \boldsymbol{y}\|^2$, by derivating

$\frac{d\,e(\boldsymbol{\Delta})}{d\,\boldsymbol{\Delta}} = (\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{\Delta}) - \boldsymbol{y})^T\boldsymbol{A} = \boldsymbol{0} \Rightarrow \boldsymbol{\Delta} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}(\boldsymbol{A}^T\boldsymbol{y} - \boldsymbol{A}^T\boldsymbol{A}\,\boldsymbol{x})$.

In pratice, $\boldsymbol{y}$ is selected to increase työllisyysaste (employment rate) by 5%.

Update: our target is $\boldsymbol{y} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{\Delta x}$ so we also use predicted next step parameters as target values. With this choice of $\boldsymbol{y}$, we have $\boldsymbol{\Delta} = \boldsymbol{A}^{-1}\boldsymbol{\Delta x}$. This gives slightly different results.

TODO: Generate 2nd derivates from $\boldsymbol{x}(t)$ time-series variables in order to vector in order to possible have sinusoidal complex eigenvalue solutions in a solution set.