Approximative Optimal Control Solution

Tomas Ukkonen

We have time-series $\boldsymbol{x}(t)$ from municipalies data. Assume simple linear dif.eq. model for the time-series and solve for some kind of optimal control to have wanted changes.

$$\frac{d\boldsymbol{x}}{d\,t} = \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{f}(t)$$

We need to solve for optimal control $\boldsymbol{f}(t)$ which is assumed to be constant $\boldsymbol{c} = \boldsymbol{A}\,\boldsymbol{\Delta}$ for simplicity.

Given time-series, each per year so $\Delta t = 1$, we have linear equation $\Delta\boldsymbol{x} = \boldsymbol{A}\,\boldsymbol{x}$ from datapoints which we can solve using linear optimization from the time-series.

After solving for $\boldsymbol{A}$. We have a target change to Mikkeli's parameters $\boldsymbol{y} = \Delta\boldsymbol{x}$ which we want to maximize within one year from target values $\boldsymbol{x}$. We minimize $e(\boldsymbol{\Delta}) = \frac{1}{2}\|\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{\Delta}) - \boldsymbol{y}\|^2$, by derivating

$$\frac{d\,e(\boldsymbol{\Delta})}{d\,\boldsymbol{\Delta}} = (\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{\Delta}) - \boldsymbol{y})^T\boldsymbol{A} = \boldsymbol{0} \Rightarrow \boldsymbol{\Delta} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}(\boldsymbol{A}^T\boldsymbol{y} - \boldsymbol{A}^T\boldsymbol{A}\,\boldsymbol{x}).$$

In pratice, $\boldsymbol{y}$ is selected to increase työllisyysaste (employment rate) by 10%.

TODO: Write *Python* script to calculate this all. Generate 2nd derivates from $\boldsymbol{x}(t)$ time-series variables in order to vector in order to possible have sinusoidal complex eigenvalue solutions in a solution set.