RECURSION Solutions

COMPUTER SCIENCE MENTORS 61A

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There are three steps to writing a recursive function:

1. Create base case(s)
2. Reduce your problem to a smaller subproblem and call your function recursively to solve the smaller subproblem
3. Figure out how to use the smaller subproblem's solution in the larger problem's solution

Real World Analogy for Recursion

Imagine that you're in line for boba, but the line is really long, so you want to know what position you're in. You decide to ask the person in front of you how many people are in front of them. That way, you can take their response and add 1 to it. Now, the person in front of you is faced with the same problem that you were trying to solve, with one less person in front of them than you. They decide to take the same approach that you did by asking the person in front of them. This continues until the very first person in line is asked. At this point, the person at the front knows that there are 0 people in front of them, so they can tell the person behind them that there are 0 people in front. Now, the second person can figure out that there is 1 person in front of them, and can relay that back to the person behind them, and so on, until the answer reaches you.

Looking at this example, we see that we have broken down the problem of "how many people are there in front of me?" to 1 + "how many people are there in front of the person in front of me"? This problem will terminate with the person at the front of the line (with 0 people in front of them). Putting this into more formal terms, we are breaking down the problem into a **recurrence relationship**, and the termination case is called the **base case**.

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1. What is wrong with the following function? How can we fix it?

```
def factorial(n):
    return n * factorial(n)
```

There is no base case and the recursive call is made on the same `n`.

```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n - 1)
```

2. Write a function `selective_sum`, which takes in an integer `n` and a predicate function `cond`. `selective_sum` returns the sum of all positive integers up to `n` for which `cond(n)` is true.

```
def selective_sum(n, cond):
    """
    >>> is_odd = lambda x: x % 2 == 1
    >>> selective_sum(5, is_odd) # 5 + 3 + 1 = 9
    9
    >>> bigger_than_10 = lambda x: x > 10
    >>> selective_sum(13, bigger_than_10) # 13 + 12 + 11 = 36
    36
    >>> selective_sum(-1, is_odd) # no positive integers <= 1
    0
    """
    if _____:

        return _____

    if _____:

        return _____

    return _____


def selective_sum(n, cond):
    if n == 0:
        return 0
    if cond(n):
        return n + selective_sum(n - 1, cond)
    return selective_sum(n - 1, cond)
```

3. Write a function `is_sorted` that takes in an integer `n` and returns true if the digits of that number are nondecreasing from right to left.

```
def is_sorted(n):
    """
    >>> is_sorted(2)
    True
    >>> is_sorted(22222)
    True
    >>> is_sorted(9876543210)
    True
    >>> is_sorted(9087654321)
    False
    """


    right_digit = n % 10
    rest =  n // 10
    if rest == 0:
        return True
    elif right_digit > rest % 10:
        return False
    else:
        return is_sorted(rest)
```

First, let's look into the base case. At what point will you know a number is sorted/not sorted immediately?

1. If `n` only has 1 digit or is 0, we know it is definitely sorted with itself. This corresponds to the first if condition, `rest == 0`.
2. If the 2nd-to-last and last digits are not in sorted order, we know the number is not sorted. To do this, we need at least 2 digits in `n` to compare, which is why we check this in **`elif`** after ensuring `n` is not 0.

Next, let's go into the recursive step. We build off of the base cases: if the base cases fail, then we can now work off of the assumption that `n` has at least 2 digits and the last 2 digits of n are in sorted order. Next, notice that after chopping off the last digit, to check that the rest of `n` is sorted, we can use our function `is_sorted` on the number `rest`. So finally, we make the recursive call with `rest` as the argument.

4. Fill in `collapse`, which takes in a non-negative integer `n` and returns the number resulting from removing all digits that are equal to an adjacent digit, i.e. the number has no adjacent digits that are the same.

```
def collapse(n):
    """
    >>> collapse(12234441)
    12341
    >>> collapse(11200000013333)
    12013
    """
    rest, last = n // 10, n % 10

    if _____________________________________:

        _____________________________________

    elif _____________________________________:

        _____________________________________
    else:

        _____________________________________
```

```
def collapse(n):
    rest, last = n // 10, n % 10
    if rest == 0:
        return last
    elif last == rest % 10:
        return collapse(rest)
    else:
        return collapse(rest) * 10 + last
```

5. The *Mandelbrot sequence starting at* (a, b) is a sequence of points in the plane recursively defined by the following:

   - The first term of the sequence is (a, b).
   - If a term in the sequence is (x, y), then the following term is $(x^2 - y^2 + a, 2xy + b)$.

   For example, the first three terms of the Mandelbrot sequence starting at $(1, -1)$ are as follows:

$$(1, -1)$$
$$(1^2 - (-1)^2 + 1, 2(1)(-1) + -1) = (1, -3)$$
$$(1^2 - (-3)^2 + 1, 2(1)(-3) - 1) = (-7, -7)$$

   Write a higher order function `mandelbrot_seq` that accepts two numbers, `start_x` and `start_y`. `mandelbrot_seq` returns a function that takes two numbers `x` and `y` and returns the next term after `(x, y)` in the Mandelbrot sequence starting at `(start_x, start_y)`.

```
def mandelbrot_seq(start_x, start_y):
    """
    >>> seq = mandelbrot_seq(1, -1)
    >>> seq(1, -1)
    (1, -3)
    >>> seq(1, -3)
    (-7, -7)
    """
    def mandelbrot_next(x, y):

        return _____________________, _____________________________

    return ____________________________________


def mandelbrot_seq(start_x, start_y):
    def mandelbrot_next(x, y):
        return x ** 2 - y ** 2 + start_x, 2 * x * y + start_y
    return mandelbrot_next
```

6. Write a function `in_or_out`, which returns `False` if any of the first `limit` terms of the Mandelbrot sequence starting at `(start_x, start_y)` is a distance of more than 2 away from the point $(0, 0)$, and `True` otherwise.

```
def in_or_out(start_x, start_y, limit):
    """
    >>> in_or_out(1, -1, 1) # (1, -1) dist is sqrt(2) < 2
    True
    >>> in_or_out(1, -1, 3) # (1, -3) dist is sqrt(10) > 2
    False
    >>> in_or_out(100, 100, 0) # no terms to consider
    True
    """
    next_term = ____________________________________________________
    def helper(x, y, limit):

        if ____________________________________________________:
            return True

        elif ____________________________________________________:
            return False
        else:
            next_x, next_y = next_term(x, y)

            return ____________________________________________

    return ____________________________________________________


def in_or_out(start_x, start_y, limit):
    next_term = mandelbrot_seq(start_x, start_y)

    def helper(x, y, limit):
        if limit <= 0:
            return True
        elif x ** 2 + y ** 2 > 4:
            return False
        else:
            next_x, next_y = next_term(x, y)
            return helper(next_x, next_y, limit - 1)

    return helper(start_x, start_y, limit)
```

The *Mandelbrot set* is the set of all points (x, y) in the plane for which the Mandelbrot sequence starting at (x, y) does not escape to infinity. An approximate picture of this set can be seen by plotting all the points `(x,y)` where `in_or_out(x, y, limit)` returns `True` (don't worry if you don't understand this code):

```
for y in range(50, -50, -1):
    for x in range(-100, 25):
        if in_or_out(x/50, y/50, 20):
            print('#', end='')
        else:
            print(' ', end='')
    print()
```