Efficiency, and Midterm Review

CSM 61A

October 25, 2021 - October 29, 2021

1 Efficiency

An order of growth (OOG) characterizes the runtime **efficiency** of a program as its input becomes extremely large. Since we care about rate of growth, we ignore constant coefficients and exclusively consider the fastest growing term. For example, on very large inputs, $2n^2 + 3n - 20$ behaves the same as n^2. Common runtimes, in increasing order of time, are: constant, logarithmic, linear, quadratic, and exponential.

Examples:

Constant time means that no matter the size of the input, the runtime of your program is consistent. In the function `f` below, no matter what you pass in for `n`, the runtime is the same.

```
def f(n):
    return 1 + 2
```

A common example of a linear OOG involves a single for/while loop. In the example below, as `n` gets larger, the amount of time to run the function grows proportionally.

```
def f(n):
    while n > 0:
        print(n)
        n -= 1
```

We can modify this while loop to get an example of logarithmic OOG. Suppose that, instead of subtracting 1 each time, we halve the size of `n`. For `n = 1000`, the program would take 10 iterations to terminate (since $2^10 = 1024$). The runtime is propotional to $\log(n)$.

```
def f(n):
    while n > 0:
```

```
        print(n)
        n /= 2
```

An example of a quadratic runtime involves nested for loops. For every one of the `n` iterations of the outer loop, there is `n` work done in the inner loop. This means that the runtime is proportional to n^2.

```
def f(n):
    for i in range(n):
        for j in range(n):
            print(i*j)
```

1. What is the order of growth for `foo`?

   (a)
   ```
   def foo(n):
       for i in range(n):
           print('hello')
   ```

   (b) What's the order of growth of `foo` if we change `range(n)`:

      i. To `range(n/2)`?

      ii. To `range(n**2 + 5)`?

      iii. To `range(10000000)`?

2. What is the order of growth for `belgian_waffle`?

```
def belgian_waffle(n):
    total = 0
    while n > 0:
        total += 1
        n = n // 2
    return total
```

2 Midterm Review

1. Draw the box-and-pointer diagram.

```
>>> violet = [7, 77, 17]
>>> violet.append([violet.pop(1)])

>>> dash = violet * 2
>>> jack = dash[3:5]
>>> jackjack = jack.extend(jack)

>>> helen = list(violet)
>>> helen += [jackjack]
>>> helen[2].append(violet)
```

2. Implement `subsets`, which takes in a list of values and an integer `n` and returns all subsets of the list of size exactly `n` in any order. You may not need to use all the lines provided.

```
def subsets(lst, n):
    """
    >>> three_subsets = subsets(list(range(5)), 3)
    >>> for subset in sorted(three_subsets):
    ...     print(subset)
    [0, 1, 2]
    [0, 1, 3]
    [0, 1, 4]
    [0, 2, 3]
    [0, 2, 4]
    [0, 3, 4]
    [1, 2, 3]
    [1, 2, 4]
    [1, 3, 4]
    [2, 3, 4]
    """
    if n == 0:
        ________________________________

    if ________________________________:

        ________________________________

    ______________________________________________

    ______________________________________________

    return ________________________________________
```

3. Write a generator function `num_elems` that takes in a possibly nested list of numbers `lst` and yields the number of elements in each nested list before finally yielding the total number of elements (including the elements of nested lists) in `lst`. For a nested list, yield the size of the inner list before the outer, and if you have multiple nested lists, yield their sizes from left to right.

```
def num_elems(lst):
    """
    >>> list(num_elems([3, 3, 2, 1]))
    [4]
    >>> list(num_elems([1, 3, 5, [1, [3, 5, [5, 7]]]]))
    [2, 4, 5, 8]
    """

    count = ______________

    for ________________________:

        if _____________________:

            for ______________________________:

                yield ________________________

            ____________________________
        else:

            ____________________________

    yield ________________________
```

4. Define `delete_path_duplicates`, which takes in `t`, a tree with non-negative labels. If there are any duplicate labels on any path from root to leaf, the function should mutate the label of the occurrences deeper in the tree (i.e. farther from the root) to be the value `-1`.

```
def delete_path_duplicates(t):
    """
    >>> t = Tree(1, [Tree(2, [Tree(1), Tree(1)])])
    >>> delete_path_duplicates(t)
    >>> t
    Tree(1, [Tree(2, [Tree(-1), Tree(-1)])])
    >>> t2 = Tree(1, [Tree(2), Tree(2, [Tree(2, [Tree(1, [Tree
       (5)])])])])
    >>> delete_path_duplicates(t2)
    >>> t2
    Tree(1, [Tree(2), Tree(2, [Tree(-1, [Tree(-1, [Tree(5)])])
       ])])
    """
    def helper(________________, ________________):

        if ________________________________:

            ________________________________

        else:

            ________________________________

        for _______ in ____________________:

            ________________________________

    ________________________________________
```

5. Write a function that returns true only if there exists a path from root to leaf that contains at least `n` instances of `elem` in a tree `t`.

```
def contains_n(elem, n, t):
    """
    >>> t1 = Tree(1, [Tree(1, [Tree(2)])])
    >>> contains_n(1, 2, t1)
    True
    >>> contains_n(2, 2, t1)
    False
    >>> contains_n(2, 1, t1)
    True
    >>> t2 = Tree(1, [Tree(2), Tree(1, [Tree(1), Tree(2)])])
    >>> contains_n(1, 3, t2)
    True
    >>> contains_n(2, 2, t2) # Not on a path
    False
    """
    if n == 0:

        return True

    elif ____________________________________________:

        return _____________________________________

    elif ____________________________________________:

        return _____________________________________

    else:

        return _____________________________________
```