

# SCHEME LISTS

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## COMPUTER SCIENCE MENTORS 61A

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### 1 Scheme Review

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1. Define `apply-multiple` which takes in a single argument function `f`, a nonnegative integer `n`, and a value `x` and returns the result of applying `f` to `x` a total of `n` times.

```
;doctests
scm> (apply-multiple (lambda (x) (* x x)) 3 2)
256
scm> (apply-multiple (lambda (x) (+ x 1)) 10 1)
11
scm> (apply-multiple (lambda (x) (* 1000 x)) 0 5)
5
```

```
(define (apply-multiple f n x)
```

```
)
```

## 2 Scheme Lists

Unlike Python, all Scheme lists are linked lists. Recall that, in Python, a linked list is made up of `Links` that each have a `first` and a `rest`, where the `rest` is another `Link`. Similarly, each Scheme list is a “pair” where the first element of the pair is the first element of the list, and the second element of the pair is the rest of the list (also a pair).

We use the `cons` procedure to construct Scheme lists, and `nil` to represent empty lists. The sequence 1, 2, 3 may then be represented as follows:

```
scm> (cons 1 (cons 2 (cons 3 nil)))  
(1 2 3)
```

The `car` and `cdr` procedures are used to access the elements of a Scheme list. `car` gets the first element of a list, while `cdr` gets the rest of the list:

```
scm> (define lst (cons 1 (cons 2 (cons 3 nil))))  
lst  
scm> (car lst)  
1  
scm> (cdr lst)  
(2 3)
```

You can make the following analogy between linked lists in Python and Scheme:

<code>Link(1, Link.empty)</code>	<code>(cons 1 nil)</code>
<code>a = Link(1, Link(2, Link.empty))</code>	<code>(define a (cons 1 (cons 2 nil)))</code>
<code>a.first</code>	<code>(car a)</code>
<code>a.rest</code>	<code>(cdr a)</code>

The `list` procedure and quotation give us additional convenient ways to construct lists:

```
scm> (list 1 2 3)  
(1 2 3)  
scm> '(1 2 3)  
(1 2 3)  
scm> (list 1 (+ 1 1) 3)  
(1 2 3)  
scm> '(1 (+ 1 1) 3)  
(1 (+ 1 1) 3)
```

Note that quotation will prevent any of the list items from being evaluated, which can occasionally be inconvenient.

### 2.1 Useful procedures

In addition to the procedures mentioned above, the following procedures are often useful when dealing with Scheme lists:

- `(null? s)`: returns true if `s` is `nil`.
- `(length s)`: returns the length of `s`.
- `(append s1 ... sn)`: returns the result of concatenating lists `s1`, ..., `sn`.
- `(map f s)`: returns the result of applying the procedure `f` to each element of `s`.

- `(filter pred s)`: returns a list containing the elements of `s` for which the single-argument procedure `pred` returns true.
- `(reduce comb s)`: combines the elements of `s` into a single value using the two-argument procedure `comb`.

## 2.2 Equality testing

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Equality testing in Scheme is a bit confusing as it is handled by three separate procedures:

- `(= a b)`: returns true if `a` equals `b`. Both must be numbers.
- `(eq? a b)`: returns true if `a` and `b` are equivalent primitive values. For two objects, `eq?` returns true if both refer to the exactly same object in memory (like `is` in Python).
- `(equal? a b)`: returns true if `a` and `b` are equivalent. Two lists are equivalent if their elements are equivalent.

1. What will Scheme output? Draw box-and-pointer diagrams to help determine this. (Ask your mentor if you're unsure what's going on. You aren't expected to understand this completely on your own.)

```
scm> (cons 1 (cons 2 nil))
```

```
scm> (cons 1 '(2 3 4 5))
```

```
scm> (cons 1 '(2 (cons 3 nil)))
```

```
scm> (cons 1 (2 (cons 3 nil)))
```

```
scm> (cons 3 (cons (cons 4 nil) nil))
```

```
scm> (define a '(1 2 3))
```

```
scm> a
```

```
scm> (car a)
```

```
scm> (cdr a)
```

```
scm> (car (cdr a))
```

How can we get the 3 out of a?

2. Define `sixty-ones`. Return the number of times that 1 follows 6 in the list.

```
> (sixty-ones '(4 6 1 6 0 1))
1
> (sixty-ones '(1 6 1 4 6 1 6 0 1))
2
> (sixty-ones '(6 1 6 1 4 6 1 6 0 1))
3
```

3. You are creating a computer from scratch. In their rawest form, computers use 0s and 1s to compose commands and data. Fill in a function that takes a list of boolean values representing an **unsigned binary number** and returns its **decimal representation**. Each `#t` in the list represents a 1 and each `#f` represents a 0, with the **first** element in the list being the **rightmost** (smallest) binary digit and the **last** element being the **leftmost** (largest) binary digit.

```
;Doctests
scm> (binary (list #f #t)) ; 10
2
scm> (binary (list #t #f #t #t)) ; 1101
13
scm> (binary (list #t #t #f #f #t)) ; 10011
19
scm> (binary (list #f)) ; 0
0
```

```
(define (binary bin-list)
  (cond
    ((null? _____)
     _____)
    ((_____ )
     _____)
    (else
     _____)
  )
)
```

4. Now, write the binary to decimal function, but in tail recursive form. Note that the `expt` function takes in a base and an exponent. For example, `(expt 2 3)` raises 2 to the third power, returning 8.

```
;Doctests
scm> (binary-tail (list #f #t)) ; 10
2
scm> (binary-tail (list #t #f #t #t)) ; 1101
13
scm> (binary-tail (list #t #t #f #f #t)) ; 10011
19
scm> (binary-tail (list #f)) ; 0
0
```

```
(define (binary-tail bin-list)
  (define (helper bin-list i sum)
    (cond
      ((null? _____)
       _____
      )
      ((_____ )
       _____
      )
      (else
       _____
      )
    )
  )
  (helper _____)
)
```

5. Define `is-prefix`, which takes in a list `p` and a list `lst` and determines if `p` is a prefix of `lst`. That is, it determines if `lst` starts with all the elements in `p`.

```
; Doctests:
scm> (is-prefix '() '())
#t
scm> (is-prefix '() '(1 2))
#t
scm> (is-prefix '(1) '(1 2))
#t
scm> (is-prefix '(2) '(1 2))
#f
; Note here p is longer than lst
scm> (is-prefix '(1 2) '(1))
#f

(define (is-prefix p lst)
```

```
)
```

6. Implement `argmax`, a function that takes in a list, `lst`, and returns the index of the largest element in `lst`. If there are two or more elements that are the largest element, return the index of the one that appears first in `lst`.

You can assume all elements of `lst` are non-negative integers, and `lst` has at least 1 element and no nested lists.

```
(define (argmax lst)
  (define (max-helper lst max-so-far max-index curr-index)
    (cond
```

```

    ( ( _____ ) _____ )
    ( ( _____ ) _____
      _____ )
    (else _____ )
  )
)
(max-helper _____ )
)

```