Higher-Order Environments, Currying, and Introductory Recursion Solutions

Computer Science Mentors 61A

February 10 –February 14, 2025

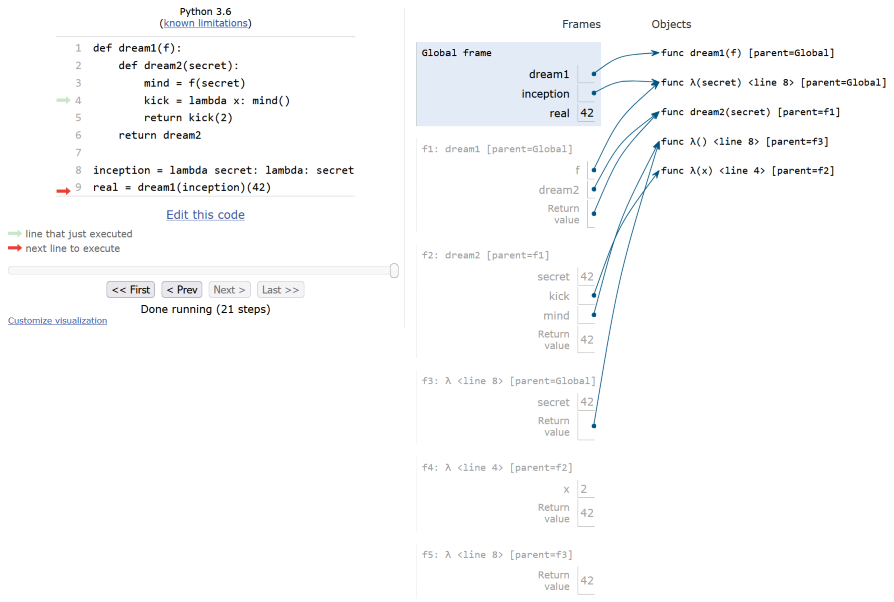
1 Higher-Order Functions in Environment Diagrams cont.

1. Draw the environment diagram that results from running the code below.

```
def dream1(f):
    def dream2(secret):
        mind = f(secret)
        kick = lambda x: mind()
        return kick(2)
    return dream2

inception = lambda secret: lambda: secret
real = dream1(inception)(42)
```

Output: 42



```
def dream1(f):
    def dream2(secret):
        mind = f(secret)
        kick = lambda x: mind()
        return kick(2)
    return dream2

inception = lambda secret: lambda: secret
real = dream1(inception)(42)
```

```
Global frame
        dream1  •  -> func dream1(f) [parent=Global]
     inception  •  -> func λ(secret) <line 8> [parent=Global]
          real  42

f1: dream1 [parent=Global]
             f  •  -> func λ(secret) <line 8> [parent=Global]
        dream2  •  -> func dream2(secret) [parent=f1]
  Return value  •  -> func dream2(secret) [parent=f1]

f2: dream2 [parent=f1]
        secret  42
          kick  •  -> func λ(x) <line 4> [parent=f2]
          mind  •  -> func λ() <line 8> [parent=f3]
  Return value  42

f3: λ <line 8> [parent=Global]
        secret  42
  Return value  •  -> func λ() <line 8> [parent=f3]

f4: λ <line 4> [parent=f2]
             x  2
  Return value  42

f5: λ <line 8> [parent=f3]
  Return value  42
```

https://imgur.com/a/ZKwZzdy

2. Draw the environment diagram that results from running the code.

```
def a(y):
    d = 1
    b = lambda x: y(x)
    e = lambda x: x(3)
    return e(b)
d = 5
a(lambda x: 4 - x + d)
```

https://goo.gl/9vxEwv

2 Recursion

There are three steps to writing a recursive function:

1. Create base case(s)
2. Reduce your problem to a smaller subproblem and call your function recursively to solve the smaller subproblem(s)
3. Use the subproblems' solutions as pieces to construct a larger problem's solution (This can happen in many layers!)

Real World Analogy for Recursion

Imagine that you're in line for boba, but the line is really long, so you want to know what position you're in.

You decide to ask the person in front of you how many people are in front of them. That way, you can take their response and add 1 to it to find your place. Now, the person in front of you is faced with the same problem that you were trying to solve, with one less person in front of them than you. They decide to take the same approach that you did by asking the person in front of them. This continues until the very first person in line is asked. At this point, the person at the front knows that there are 0 people in front of them, so they can tell the person behind them that there are 0 people in front. Now, the second person can figure out that there is 1 person in front of them, and can relay that back to the person behind them, and so on, until the answer reaches you.

Looking at this example, we see that we have broken down the problem of "how many people are there in front of me?" to 1 + "how many people are there in front of the person in front of me"? This problem will terminate with the person at the front of the line (with 0 people in front of them). Putting this into more formal terms, we are breaking down the problem into a **recurrence relationship**, and the termination case (when the question gets to the very first person in line) is called the **base case**.

As a program goes through recursion, it doesn't formally solve the problem until it reaches the base case, in which case it works its way up from the base case to the original input to construct your final answer. Just like your question as you stand in line, the program:

1. goes out (down the call stack), person by person (call by call) **(recursive case)**
2. until the person at the very front is reached **(base case)**
3. when the answers get added onto and eventually passed back to you, solving the problem. **(constructing the final solution with solutions to subparts)**

3. Here is a Python function that computes the nth Fibonnacci number. Identify the three parts of this recursive program.

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n - 1) + fib(n - 2)
```

The domain is in the integers and the range is in the integers. There are two base cases for checking if `n == 0` or if `n == 1`. There is one recursive case that makes two recursive calls, reducing the problem down to `fib(n - 1)` and `fib(n - 2)`, respectively.

The first part of recognizing what a recursive program is doing is by checking it's base cases, or, "ending cases." `fib` has two base cases: when we recurse down to either 0 or 1, to which we just return 0 or 1. You can liken this to a while loop's conditional, whereas the while loop loops until it reaches a certain criterion. The same logic applies to base cases: we keep opening new recursive frames until we get down to our base case.

The second part of this recursive program is how the recursive program breaks down the argument passed in into different subproblems. In the case of our problem here, we break down our argument by incrementing our arguments down linearly by 1 and 2. Continuing the while loop analogy, these act as "indices" telling us how much to step down in terms of value for our argument.

The third part of our program is seeing how we recursively call the method (aka, recognizing where our "recursive leap of faith" is). These recursive calls are acted upon when the base case is not fulfilled. In this case, we take a "recursive leap of faith" by calling fib(n - 1) and fib(n - 2), trusting that our fib method works as intended.

4. What is wrong with the following function? How can we fix it?

```
def factorial(n):
    return n * factorial(n)
```

There is no base case and the recursive call is made on the same `n`.

```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n - 1)
```

5. Complete the definition for `num_digits`, which takes in a number n and returns the number of digits it has . Implement the function recursively.

```
def num_digits(n):
    """Takes in an positive integer and returns the number of
    digits.

    >>> num_digits(0)
    1
    >>> num_digits(1)
    1
    >>> num_digits(7)
    1
    >>> num_digits(1093)
    4
    """

    if n < 10:
        return 1
    else:
        return 1 + num_digits(n // 10)
```