LINKED LISTS, MUTABLE TREES AND MIDTERM REVIEW Solutions

COMPUTER SCIENCE MENTORS 61A

October 28 – November 1, 2024

1 Linked Lists

For each of the following problems, assume linked lists are defined as follows:

```python
class Link:
    empty = ()
    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest is not Link.empty:
            rest_repr = ', ' + repr(self.rest)
        else:
            rest_repr = ''
        return 'Link(' + repr(self.first) + rest_repr + ')'

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

Linked lists are a recursive data structure for representing sequences. They consist of a series of "links," each of which has two attributes: `first` and `rest`. The `first` attribute contains the value of the link (which can hold any type of data, even another linked list!). The `rest` attribute, on the other hand, is a pointer to another link or `Link.empty`, which is just a "None" type value.

For example, `Link(1, Link(2, Link(3)))` is a linked list representation of the sequence $1, 2, 3$.

Created by Jim Fang, Bryan Adolfo Ventura Benitez. Worksheet feedback: links.jimfang.dev/feedback

Like trees, linked lists naturally lend themselves to recursive problem solving. Consider the following example, in which we double every value in linked list. We double the value of the current link and then recursively double the rest.

```
def double_values(link):
     if link is not Link.empty:
        link.first *= 2 # we mutate the value inside of the link
        double_val(link.rest) # we mutate the values in the rest
                              # of the linked list
    # if the link is empty then do nothing
```

1. What will Python output? Draw box-and-pointer diagrams along the way.

```
>>> a = Link(1, Link(2, Link(3)))
```

```
+---+---+  +---+---+  +---+---+
| 1 | --|->| 2 | --|->| 3 | / |
+---+---+  +---+---+  +---+---+
```

```
>>> a.first
```

1

```
>>> a.first = 5
```

```
+---+---+  +---+---+  +---+---+
| 5 | --|->| 2 | --|->| 3 | / |
+---+---+  +---+---+  +---+---+
```

```
>>> a.first
```

5

```
>>> a.rest.first
```

2

```
>>> a.rest.rest.rest.rest.first
```

Error: tuple object has no attribute rest (Link.empty has no rest)

```
>>> a.rest.rest.rest = a
```

```
   +---+---+  +---+---+  +---+---+
+->| 5 | --|->| 2 | --|->| 3 | --|--+
|  +---+---+  +---+---+  +---+---+  |
|                                   |
+-----------------------------------+
```

```
>>> a.rest.rest.rest.rest.first
```

```
2
```

```
>>> repr(Link(1, Link(2, Link(3, Link.empty))))
```

```
"Link(1, Link(2, Link(3)))"
```

```
>>> Link(1, Link(2, Link(3, Link.empty)))
```

```
Link(1, Link(2, Link(3)))
```

```
>>> str(Link(1, Link(2, Link(3))))
```

```
'<1 2 3>'
```

```
>>> print(Link(Link(1), Link(2, Link(3))))
```

```
<<1> 2 3>
```

2. Write a function `combine_two`, which takes in a linked list of integers `lnk` and a two-argument function `fn`. It returns a new linked list where every two elements of `lnk` have been combined using `fn`.

```
def combine_two(lnk, fn):
    """
    >>> lnk1 = Link(1, Link(2, Link(3, Link(4))))
    >>> combine_two(lnk1, add)
    Link(3, Link(7))
    >>> lnk2 = Link(2, Link(4, Link(6)))
    >>> combine_two(lnk2, mul)
    Link(8, Link(6))
    """
    if ______________________________________:

        return ______________________________

    elif ____________________________________

        return ______________________________

    combined = ______________________________

    return __________________________________
```

```
def combine_two(lnk, fn):
    if lnk is Link.empty:
        return Link.empty
    elif lnk.rest is Link.empty:
        return Link(lnk.first)
    combined = fn(lnk.first, lnk.rest.first)
    return Link(combined, combine_two(lnk.rest.rest, fn))
```

3. Write a function `middle_node` that takes as input a linked list `lst`. `middle_node` should return the middle node of the linked list. If there are two middle nodes, return the second middle node.

```
def middle_node(lst):
    """
    >>> head = Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> middle_node(head)
    Link(3, Link(4, Link(5))) # The middle node of the list is node 3
    >>> head = Link(1, Link(2, Link(3, Link(4, Link(5, Link(6))))))
    Link(4, Link(5, Link(6))) # Since the list has two middle nodes with
       values 3 and 4, we return the second one
    """
    list_iter, middle = ________________, __________________

    length = _____________________________________

    while _____________________________________:

        length = _____________________________________

        list_iter = _____________________________________

    for _____________________________________:

        middle = _____________________________________

    if length % 2 == 1:

        middle = _____________________________________

    return middle
```

Challenge version **(Optional)**:

```
def middle_node(lst):

    list_iter, middle = ________________, __________________

    while _____________________ and _____________________:

        list_iter = _____________________________________

        middle = _____________________________________

    return middle
```

```python
def middle_node(lst):
    list_iter, middle = lst, lst
    length = 0

    while list_iter:
        length = length + 1
        list_iter = list_iter.rest

    for i in range(length // 2):
        middle = middle.rest

    if length % 2 == 1:
        middle = middle.rest

    return middle
```

In this solution, we first calculate the length of the linked list, and then finding the middle node based on that length.

Challenge version

```python
def middle_node(lst):
    list_iter, middle = lst, lst

    while list_iter and list_iter.rest:
        list_iter = list_iter.rest.rest
        middle = list_iter.rest

    return middle
```

In this solution, we iterate through the linked list with two pointers at different speeds. One pointer, `list_iter`, moves through the list one node at a time, while the other pointer, `middle`, moves through the list at half the speed of `list_iter`.

4. Write a recursive function `insert_all` that takes as input two linked lists, `s` and `x`, and an index `index`. `insert_all` should return a new linked list with the contents of `x` inserted at index `index` of `s`.

```
def insert_all(s, x, index):
    """
    >>> insert = Link(3, Link(4))
    >>> original = Link(1, Link(2, Link(5)))
    >>> insert_all(original, insert, 2)
    Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> start = Link(1)
    >>> insert_all(original, start, 0)
    Link(1, Link(1, Link(2, Link(5))))
    """
    if ____________________ and ____________________:

        ____________________________________________

    if ____________________ and ____________________:

        ____________________________________________

    ________________________________________________
```

```
def insert_all(s, x, index):
    """
    >>> insert = Link(3, Link(4))
    >>> original = Link(1, Link(2, Link(5)))
    >>> insert_all(original, insert, 2)
    Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> start = Link(1)
    >>> insert_all(original, start, 0)
    Link(1, Link(1, Link(2, Link(5))))
    >>> insert_all(original, insert, 3)
    Link(1, Link(2, Link(5, Link(3, Link(4)))))
    """
    if s is Link.empty and x is Link.empty:
        return Link.empty
    if x is not Link.empty and index == 0:
        return Link(x.first, insert_all(s, x.rest, 0))
    return Link(s.first, insert_all(s.rest, x, index - 1))
```

All of our return statements should return a new linked list.

Our base case should be the simplest possible version of the problem: when both `x` and `s` are empty, clearly the result is just the empty list.

We can now think of ways to break down this problem even further. Note that when the index to be inserted at is `0`, the problem is relatively easy: we just have to put all of the elements of `x` followed by all the elements of `s`. So the first element of the new list should `x.first`, and the rest of the new list should be `x.rest` concatenated with `s`, or `insert_all(s, x.rest, 0)`. Since we are using `x.first` and `x.rest`, we must check that `x` is nonempty to ensure that we do not error.

Finally, when the index to be inserted at is nonzero, we know that we're going to have some elements of `s`, then the elements of `x`, and then the rest of the elements from `s`. So the first element of the new list should be `s.first`. Then we can get the rest of the new list by inserting the contents of `x` at index `index - 1` of `s.rest`, reducing the index by 1 to account for the fact that we have removed the first element of `s`.

There's one issue we glossed over here: what if `x` is empty but `s` is not? Then we want to return the contents of `s`. But because the problem requires that we return a new linked list, we must recursively reconstruct `s` instead of simply returning it. You could add another base case to handle this, but as it turns out the second recursive case will handle this just fine since `Link(s.first, insert_all(s.rest, x, index - 1))` is just equivalent to `Link(s.first, s.rest)` when `x` is empty. Since the `x` **`is not`** `Link.empty` condition for the first recursive case will direct all situations where `x` is empty but `s` is not to the second recursive case, it turns out that we do not need to add anything else to this solution.

Convincing yourself that this problem works requires that you eventually reach a base case. Note that in either recursive call, we either reduce `s` or `x` by one element. So the base case will always eventually be reached, and the solution is valid.

2 Trees

For the following problems, use this definition for the `Tree` class:

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = list(branches)

    def is_leaf(self):
        return self.branches == []

    # Implementation ommitted
```

Here are a few key differences between the `Tree` class and the Tree abstract data type, which we have previously encountered:

- Using the constructor: Capital T for the `Tree` class and lowercase t for tree ADT `t = Tree(1)` vs. `t = tree(1)`
- In the class, `label` and `branches` are instance variables and `is_leaf()` is an instance method. In the ADT, all of these were globally defined functions.

  `t.label` vs. `label(t)`

  `t.branches` vs. `branches(t)`

  `t.is_leaf()` vs. `is_leaf(t)`
- A `Tree` object is mutable while the tree ADT is not mutable. This means we can change attributes of a `Tree` instance without making a new tree. In other words, we can solve tree class problems non-destructively and destructively, but can only solve tree ADT problems non-destructively.

  `t.label = 2` is allowed but `label(t)= 2` would error.

Apart from these differences, we can take the same general approaches we used for the tree ADT and apply them to the `Tree` class!

1. Define `delete_path_duplicates`, which takes in `t`, a tree with non-negative labels. If there are any duplicate labels on any path from root to leaf, the function should mutate the label of the occurrences deeper in the tree (i.e. farther from the root) to be the value `-1`.

```
def delete_path_duplicates(t):
    """
    >>> t = Tree(1, [Tree(2, [Tree(1), Tree(1)])])
    >>> delete_path_duplicates(t)
    >>> t
    Tree(1, [Tree(2, [Tree(-1), Tree(-1)])])
    >>> t2 = Tree(1, [Tree(2), Tree(2, [Tree(2, [Tree(1, [Tree(5)])])])])
    >>> delete_path_duplicates(t2)
    >>> t2
    Tree(1, [Tree(2), Tree(2, [Tree(-1, [Tree(-1, [Tree(5)])])])])
    """
    def helper(_______________, _______________):

        if ________________________________:

            ________________________________

        else:

            ________________________________

        for _______ in ____________________:

            ________________________________

    ________________________________________
```

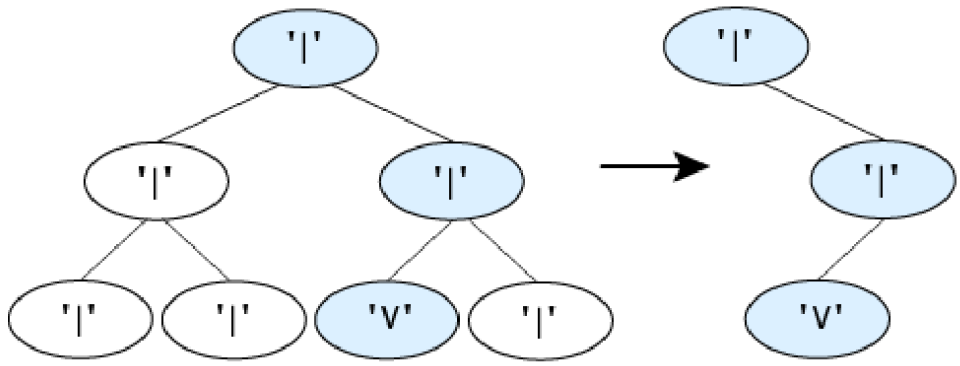
```
    def helper(t, seen_so_far):
        if t.label in seen_so_far:
          t.label = -1
        else:
            seen_so_far = seen_so_far + [t.label]
        for b in t.branches:
            helper(b, seen_so_far)
    return helper(t, [])
```

2. Given a tree t, mutate the tree so that each leaf's label becomes the sum of the labels of all nodes in the path from the leaf node to the root node.

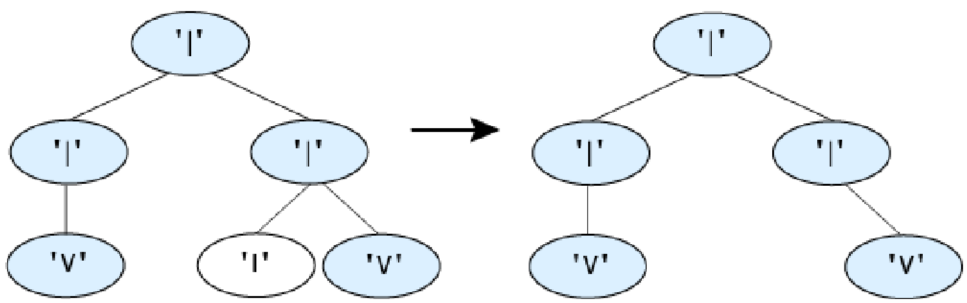
```
def replace_leaves_sum(t):
    """
    >>> t = Tree(1, [Tree(3, [Tree(2), Tree(8)]), Tree(5)])
    >>> replace_leaves_sum(t)
    >>> t
    Tree(1, [Tree(3, [Tree(6), Tree(12)]), Tree(6)])
    """
    def helper(______________ , _________________):

        if t.is_leaf():

            _____________________________________

        for b in t.branches:

            _________________________________

    _____________________________________________
```

```
def replace_leaves_sum(t):
    def helper(t, total):
        if t.is_leaf():
            t.label = total + t.label
        else:
            for b in t.branches:
                helper(b, total + t.label)
    helper(t, 0)
```

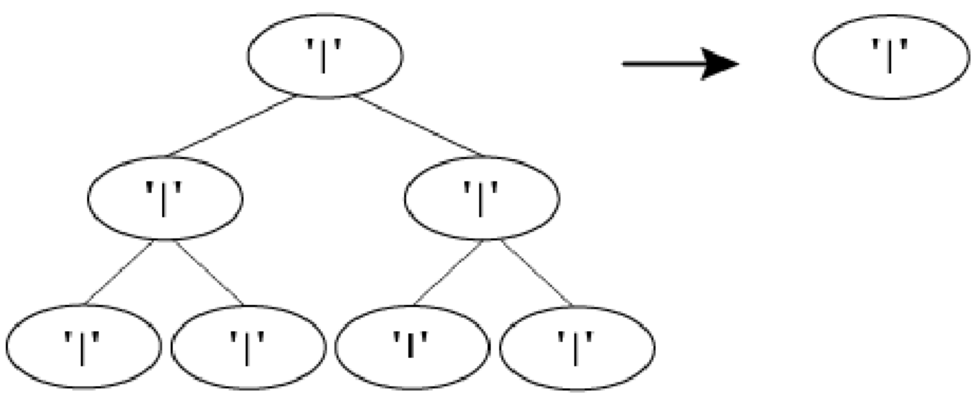
3. *From Sp'22 MT2:*
Implement $flower\_keeper$, a function that mutates a tree t so that the only paths that remain are ones which end in a leaf node with a Tulip flower ('V'). For example, consider this tree where only one path ends in a flower. After calling $flower\_keeper$, the tree has only three nodes left, the ones that lead to the flower:



The shaded nodes in the diagram indicate paths that end in flowers. For this tree where two paths end in flowers, the tree keeps both paths that lead to flowers.



For a tree where none of the nodes are flowers, the function removes every branch except the root node.



For a tree with only a single node that is a flower, the function does not remove anything.


```
def flower_keeper(t):
    """
    Mutates the tree T to keep only paths that end in flowers ('V').
    If a path consists entirely of stems ('|'), it must be pruned.
    If T has no paths that end in flowers, the root node is still kept.
    You can assume that a node with a flower will have no branches.
    >>> one_f = Tree('|', [Tree('|', [Tree('|'), Tree('|')]), Tree('|',
       [Tree('V'), Tree('|')])])
    >>> print(one_f)
    |
        |
            |
            |
        |
            V
            |
    >>> flower_keeper(one_f)
    >>> one_f
    Tree('|', [Tree('|', [Tree('V')])])
    >>> print(one_f)
    |
        |
        V
    >>> no_f = Tree('|', [Tree('|', [Tree('|'), Tree('|')]), Tree('|',
       [Tree('|'), Tree('|')])])
    >>> flower_keeper(no_f)
    >>> no_f
    Tree('|')
    >>> just_f = Tree('V')
    >>> flower_keeper(just_f)
    >>> just_f
    Tree('V')
    >>> two_f = Tree('|', [Tree('|', [Tree('V')]), Tree('|', [Tree('|'),
       Tree('V')])])
    >>> flower_keeper(two_f)
    >>> two_f
    Tree('|', [Tree('|', [Tree('V')]), Tree('|', [Tree('V')])])
    """
    for b in __________:

        __________

    __________ = [__________ for b in __________ if __________]
```

```
for b in t.branches:
    flower_keeper(b)
t.branches = [b for b in t.branches if b.label == 'V' or not
   b.is_leaf()]
```

3 Higher Order Functions

1. Write a function, `make_digit_remover`, which takes in a single digit `i`. It returns another function that takes in an integer and, scanning from right to left, removes all digits from the integer up to and including the first occurrence of `i`, starting from the ones place. If `i` does not occur in the integer, the original number is returned.

```
def make_digit_remover(i):
    """
    >>> remove_two = make_digit_remover(2)
    >>> remove_two(232018)
    23
    >>> remove_two(23)
    0
    >>> remove_two(99)
    99
    """
    def remove(________):

        removed = ______________________________

        while ______________________________ > 0:

            ______________________________________

            removed = removed // 10

            if __________________________________:

                _________________________________

        return __________________________________

    return ______________________________
```

```
def make_digit_remover(i):
    def remove(n):
        removed = n
        while removed > 0:
            digit = removed % 10
            removed = removed // 10
            if digit == i:
                return removed
        return n
    return remove
```

4 Lists

1. Draw the environment diagram that results from running the following code.

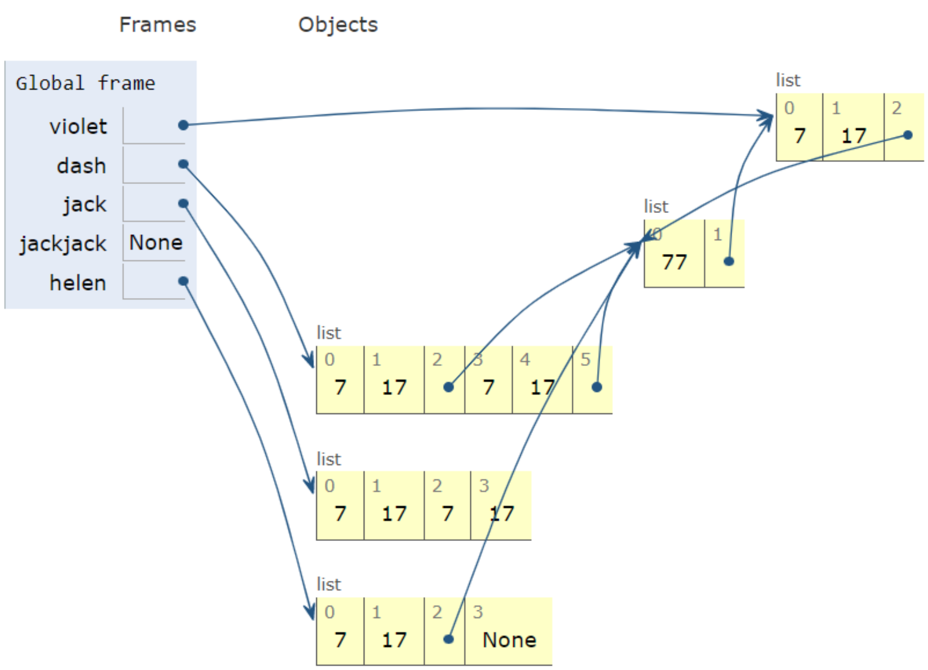
```
bless, up = 3, 5
another = [1, 2, 3, 4]
one = another[1:]

another[bless] = up
another.append(one.remove(2))
another[another[0]] = one
one[another[0]] = another[1]
one = one + [another.pop(3)]
another[1] = one[1][1][0]
one.append([one.pop(1)])
```

https://goo.gl/FyMmbJ

2. Draw the box-and-pointer diagram.

```
>>> violet = [7, 77, 17]
>>> violet.append([violet.pop(1)])

>>> dash = violet * 2
>>> jack = dash[3:5]
>>> jackjack = jack.extend(jack)

>>> helen = list(violet)
>>> helen += [jackjack]
>>> helen[2].append(violet)
```

https://goo.gl/EAmZBW



3. Write a function duplicate_list, which takes in a list of positive integers and returns a new list with each element x in the original list duplicated x times.

```
def duplicate_list(lst):
    """
    >>> duplicate_list([1, 2, 3])
    [1, 2, 2, 3, 3, 3]
    >>> duplicate_list([5])
    [5, 5, 5, 5, 5]
    """
    ________________________________

    for ____________________________:

        for ____________________________:

            ___________________________________

    ________________________________
```

```
new_list = []
for x in lst:
     for i in range(x):
          new_list = new_list + [x]
return new_list
```

4. Write a function that takes as input a number n and a list of numbers lst and returns True if we can find a subset of lst that sums to n.

```
def add_up(n, lst):
    """
    >>> add_up(10, [1, 2, 3, 4, 5])
    True
    >>> add_up(8, [2, 1, 5, 4, 3])
    True
    >>> add_up(-1, [1, 2, 3, 4, 5])
    False
    >>> add_up(100, [1, 2, 3, 4, 5])
    False
    """
    if n == 0:
        return True
    if lst == []:
         return False
    else:
        first, rest = lst[0], lst[1:]
        return add_up(n - first, rest) or add_up(n, rest)
```

5 Iterators and Generators

1. Write a generator function `num_elems` that takes in a possibly nested list of numbers `lst` and yields the number of elements in each nested list before finally yielding the total number of elements (including the elements of nested lists) in `lst`. For a nested list, yield the size of the inner list before the outer, and if you have multiple nested lists, yield their sizes from left to right.

```
def num_elems(lst):
    """
    >>> list(num_elems([3, 3, 2, 1]))
    [4]
    >>> list(num_elems([1, 3, 5, [1, [3, 5, [5, 7]]]]))
    [2, 4, 5, 8]
    """

    count = ________________

    for _____________________________:

        if _________________________:

            for __________________________________:

                yield ____________________________

            ________________________________

        else:

            ________________________________

    yield ____________________________
```

```
def num_elems(lst):
    count = 0
    for elem in lst:
        if isinstance(elem, list):
            for c in num_elems(elem):
                yield c
            count += c
        else:
            count += 1
    yield count
```

`count` refers to the number of elements in the current list `lst` (including the number of elements inside any nested list). Determine the value of `count` by looping through each element of the current list `lst`. If we have an element `elem` which is of type **`list`**, we want to yield the number of elements in each nested list of `elem` before finally yielding the total number of elements in `elem`. We can do this with a recursive call to `num_elems`. Thus, we yield all the values that need to be yielded using the inner for loop. The last number yielded by this inner loop is the total number of elements in `elem`, which we want to increase `count` by. Otherwise, if `elem` is not a list, then we can simply increase `count` by 1. Finally, yield the total count of the list.