MUTABLE TREES AND MIDTERM REVIEW Meta

COMPUTER SCIENCE MENTORS 61A

October 24, 2022–October 28, 2022

Recommended Timeline

- Trees
  - Trees minilecture: 5 min
  - Tree Sum: 4 min
  - Delete Path Duplicates: 13 min
  - Replace Leaves Sum: 11 min
  - Contains N: 15 min
  - Efficiency Problems - 5 min
- Midterm review for the rest of the time.

The second midterm is on Thursday, October 27, 2022. Therefore, sections are optional this week. Mentors are strongly encouraged to hold section, and students are encouraged to come, but it's flexible. All students get marked present regardless.

This worksheet includes a very large number of problems because we essentially intend it to be a problem bank for any review activities you decide to do in section. You, of course, are free to ignore any and all of the problems we provide you and simply do exam problems, which is honestly what I would probably do if I were teaching a section this week.

Note that students have not yet been exposed to mutable trees in CSM (though they have, of course, been exposed to this content in lecture). We have elected to include content on mutable trees in this week's worksheet. (Sorry.) While the timing on this is not ideal, students have already been exposed to immutable trees, so it shouldn't be the end of the world if they don't get to do it on this worksheet.

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1 Trees

For the following problems, use this definition for the `Tree` class:

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = list(branches)

    def is_leaf(self):
        return self.branches == []

    # Implementation ommitted
```

Here are a few key differences between the `Tree` class and the Tree abstract data type, which we have previously encountered:

- Using the constructor: Capital T for the `Tree` class and lowercase t for tree ADT `t = Tree(1)` vs. `t = tree(1)`
- In the class, `label` and `branches` are instance variables and `is_leaf()` is an instance method. In the ADT, all of these were globally defined functions.

  `t.label` vs. `label(t)`

  `t.branches` vs. `branches(t)`

  `t.is_leaf()` vs. `is_leaf(t)`
- A `Tree` object is mutable while the tree ADT is not mutable. This means we can change attributes of a `Tree` instance without making a new tree. In other words, we can solve tree class problems non-destructively and destructively, but can only solve tree ADT problems non-destructively.

  `t.label = 2` is allowed but `label(t)= 2` would error.

Apart from these differences, we can largely take the approaches we used for the tree ADT and apply them to the `Tree` class!

Feel free to not spend too much time on this section! Your students already covered immutable trees when practicing ADTs.

1. Implement `tree_sum`, which takes in a `Tree` object and replaces the label of the tree with the sum of all the values in the tree. `tree_sum` should also return the new label.

```
def tree_sum(t):
    """
    >>> t = Tree(1, [Tree(2, [Tree(3)]), Tree(4)])
    >>> tree_sum(t)
    10
    >>> t.label
    10
    >>> t.branches[0].label
    5
    >>> t.branches[1].label
    4
    """

    for b in t.branches:
        t.label += tree_sum(b)
    return t.label
```

Teaching Tips

- Make sure students understand why an explicit is_leaf() base case is unnecessary. If the function is called on a leaf, the for loop does not run, and it simply returns the label.
- The recursion occurs as part of the expression updating the label, which may confuse students at first. Explain how the returning of the label makes this work.
  - It may also help to show how the code would be written without tree_sum(b) on the right hand side of the expression to make the recursion clearer.
- Consider first drawing the Tree out and running through a doctest, showing how you would sum the labels in subtrees first before updating the root label.

2. Define `delete_path_duplicates`, which takes in `t`, a tree with non-negative labels. If there are any duplicate labels on any path from root to leaf, the function should mutate the label of the occurrences deeper in the tree (i.e. farther from the root) to be the value `-1`.

```
def delete_path_duplicates(t):
    """
    >>> t = Tree(1, [Tree(2, [Tree(1), Tree(1)])])
    >>> delete_path_duplicates(t)
    >>> t
    Tree(1, [Tree(2, [Tree(-1), Tree(-1)])])
    >>> t2 = Tree(1, [Tree(2), Tree(2, [Tree(2, [Tree(1,
       [Tree(5)])])])])
    >>> delete_path_duplicates(t2)
    >>> t2
    Tree(1, [Tree(2), Tree(2, [Tree(-1, [Tree(-1,
       [Tree(5)])])])])
    """
    def helper(________________, ________________):

        if __________________________________:

            __________________________________

        else:

            __________________________________

        for _______ in _____________________:

            __________________________________

    __________________________________________
```

```
def helper(t, seen_so_far):
    if t.label in seen_so_far:
      t.label = -1
    else:
        seen_so_far = seen_so_far + [t.label]
    for b in t.branches:
        helper(b, seen_so_far)
helper(t, [])
```

Teaching Tips

- To clarify, the problem is asking to delete *path* duplicates, and not *tree* duplicates. As illustrated in the last doctest, it is acceptable to keep two identical labels if they appear on different branches.
- Draw out the doctest Tree and walk through how you would delete path duplicates by hand. Then, ask your students, "how would we write this in code?"
- Recap with your students the core properties for trees such as label and branches.
- We don't need to use the is_leaf() function because our for loop will not run if there are no branches (which only occurs if the tree is a leaf). But, you can write in this base case to start with.
- Make sure to point out the reason why we can't use `seen_so_far.append(t.label)` in the else case. (The reason is that we need to create a new list in each frame, rather than mutating the same one. If append is used, `seen_so_far` would contain everything seen in the tree so far, not just the current branch.)

3. Given a tree `t`, mutate the tree so that each leaf's label becomes the sum of the labels of all nodes in the path from the leaf node to the root node.

```
def replace_leaves_sum(t):
    """
    >>> t = Tree(1, [Tree(3, [Tree(2), Tree(8)]), Tree(5)])
    >>> replace_leaves_sum(t)
    >>> t
    Tree(1, [Tree(3, [Tree(6), Tree(12)]), Tree(6)])
    """
    def helper(______________ , _________________):

        if t.is_leaf():

            _____________________________________

        for b in t.branches:

            _________________________________

    _____________________________________________
```

```
def replace_leaves_sum(t):
    def helper(t, total):
        if t.is_leaf():
            t.label = total + t.label
        else:
            for b in t.branches:
                helper(b, total + t.label)
    helper(t, 0)
```

4. Write a function that returns `True` if there exists a path from root to leaf that contains at least `n` instances of `elem` in a tree `t`.

   Hint: recall that the built-in function **`any`** *takes in an iterable and returns* `True` *if any of the iterable's elements are truthy.*

```
def contains_n(elem, n, t):
    """
    >>> t1 = Tree(1, [Tree(1, [Tree(2)])])
    >>> contains_n(1, 2, t1)
    True
    >>> contains_n(2, 2, t1)
    False
    >>> contains_n(2, 1, t1)
    True
    >>> t2 = Tree(1, [Tree(2), Tree(1, [Tree(1), Tree(2)])])
    >>> contains_n(1, 3, t2)
    True
    >>> contains_n(2, 2, t2) # Not on a path
    False
    """
    if n == 0:

        return True

    elif ________________________________________:

        return ___________________________________

    elif ________________________________________:

        return ___________________________________

    else:

        return ___________________________________
```

```
    if n == 0:
        return True
    elif t.is_leaf():
        return n == 1 and t.label == elem
    elif t.label == elem:
        return any([contains_n(elem, n - 1, b) for b in
          t.branches])
    else:
        return any([contains_n(elem, n, b) for b in
          t.branches])
```

Base cases: The simplest case we have is when `n == 0`, or when we want at least 0 instances of `elem` in `t`. In this case, we always return `True`. The other simple case we consider is when the tree is only a leaf — there is nothing left to recurse on. In that case, we simply check to see that both `n == 1` and that `t.label == elem`, meaning that we have one element left to satisfy, and the leaf label satisfies the final element we are looking for. If we have more elements to search for (ie. n ¿ 1), then we will not satisfy that many elements at the leaf node; conversely, if we have fewer (ie. `n == 0`), then the case would already be covered by the first base case.

Recursive cases: If the current node isn't a leaf, then there's two different cases we should consider. Either the label of the current node is equal to `elem` or the label is not equal to `elem`. For the former, we would have to search for `n` more `elems` in each branch of `t` and return `True` if any of the branches contain `n` elems. For the latter, we would have `(n - 1)` elements remaining, so we would search for `(n - 1)` more `elems` in each branch of `t` and return `True` if any of the branches contain `(n - 1) elems`. Since there is not room to do a for loop, we can use a list comprehension to recursively call the function on each branch. Thus, our two list comprehension statements would be `[contains_n(elem, n, b) for b in t.branches]` and `[contains_n(elem, n - 1, b) for b in t.branches]`. To determine if any of the branches contain either `n` elems or `(n - 1)` elems, we can check if there's a `True` element in the respective lists.

Teaching Tips

- To understand what the function is doing, try walking through a test case on `t2`.
- Ask students if there are any more base cases (lot of **`elif`** statements).
- If we are at some node with label `elem`, what does this mean about the amount of instances of `elem` we have yet to search for?
- Where could we find more instances of `elem`? The branches; so we should recurse on them.
- Remind students about the **`any`** and **`all`** functions for dealing with lists of booleans.

2 Efficiency

1. What is the order of growth for `foo`?

   (a)
   ```
   def foo(n):
       for i in range(n):
           print('hello')
   ```

   Linear. This is a for loop that will run `n` times.

   (b) What's the order of growth of `foo` if we change `range(n)` to

      i. `range(n/2)`?

      Linear. The loop runs `n/2` times, but the runtime still scales linearly proportionally to `n`.

      ii. `range(n**2 + 5)`?

      Quadratic. The number of times the loop runs is proportional to n^2.

      iii. `range(10000000)`?

      Constant. No matter the size of `n`, we will run the loop the same number of times.

Teaching Tips

- When measuring orders of growth, note that we are measuring in terms of the growth in relationship to n (as a function of n).
- One thing that might help students is to draw upon math knowledge. For example, if `f(n)= 10000000`, this is a "constant" function, i.e. a slope 0 line which will eventually be outpaced.

2. What is the order of growth for `belgian_waffle`?

```
def belgian_waffle(n):
    total = 0
    while n > 0:
        total += 1
        n = n // 2
    return total
```

Logarithmic. Notice that with each pass through the while loop, the value of `n` is halved. Since we are halving till 0, this would be a logarithmic runtime.

Teaching Tips

- Try to draw out a diagram for this problem:
  - We can tally up how many times the "while" loop runs.
  - For belgian_waffle(16):
    * Before running while loop: `n` = 16, total = 0. Runs = 0.
    * During first iteration: `n` = 8, total = 1. Runs = 1.
    * During second iteration: `n` = 4, total = 2. Runs = 2.
    * During third iteration: `n` = 2, total = 3. Runs = 3.
    * During fourth iteration: `n` = 1, total = 4. Runs = 4.
    * During fifth iteration: `n` = 0, total = 5. Runs = 5.
  - Note that $16 = 2^4 = 2^{5-1}$ and we are halving `n` every time. How many times can you halve `n`? `log(n)` times.

3 Higher Order Functions

1. Write a function, `make_digit_remover`, which takes in a single digit `i`. It returns another function that takes in an integer and, scanning from right to left, removes all digits from the integer up to and including the first occurrence of `i`. If `i` does not occur in the integer, the original number is returned.

```
def make_digit_remover(i):
    """
    >>> remove_two = make_digit_remover(2)
    >>> remove_two(232018)
    23
    >>> remove_two(23)
    0
    >>> remove_two(99)
    99
    """
    def remove(_______):

        removed = _______________________

        while _______________________ > 0:

            _____________________________

            removed = removed // 10

            if __________________________:

                _________________________

        return __________________________

    return __________________________
```

```
def make_digit_remover(i):
    def remove(n):
        removed = n
        while removed > 0:
            digit = removed % 10
            removed = removed // 10
            if digit == i:
                return removed
        return n
    return remove
```

1. Write a function that takes as input a number `n` and a list of numbers `lst` and returns `True` if we can find a subset of `lst` that sums to `n`.

```
def add_up(n, lst):
    """
    >>> add_up(10, [1, 2, 3, 4, 5])
    True
    >>> add_up(8, [2, 1, 5, 4, 3])
    True
    >>> add_up(-1, [1, 2, 3, 4, 5])
    False
    >>> add_up(100, [1, 2, 3, 4, 5])
    False
    """
    if n == 0:
        return True
    if lst == []:
         return False
    else:
        first, rest = lst[0], lst[1:]
        return add_up(n - first, rest) or add_up(n, rest)
```

5 Lists

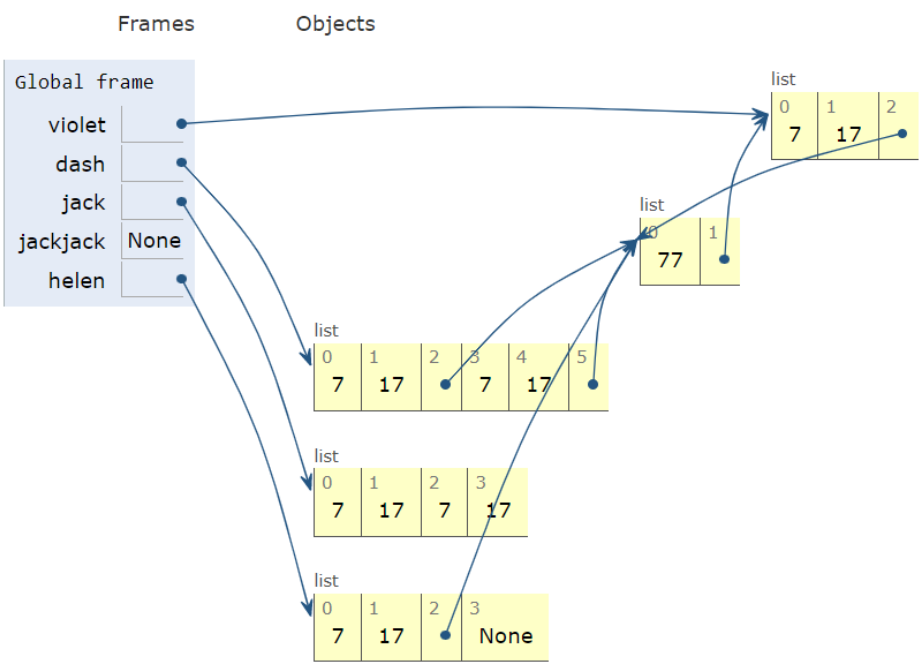
1. Draw the box-and-pointer diagram.

```
>>> violet = [7, 77, 17]
>>> violet.append([violet.pop(1)])

>>> dash = violet * 2
>>> jack = dash[3:5]
>>> jackjack = jack.extend(jack)

>>> helen = list(violet)
>>> helen += [jackjack]
>>> helen[2].append(violet)
```

https://goo.gl/EAmZBW



Teaching Tips

- Draw out the box and pointer diagram for each part.
- Try to highlight when pointers change vs. when the values a pointer points to change.
- If students are very confused about this problem, try going over the PythonTutor!

2. Implement `subsets`, which takes in a list of values and an integer `n` and returns all subsets of the list of size exactly `n` in any order. You may not need to use all the lines provided.

```
def subsets(lst, n):
    """
    >>> three_subsets = subsets(list(range(5)), 3)
    >>> for subset in sorted(three_subsets):
    ...     print(subset)
    [0, 1, 2]
    [0, 1, 3]
    [0, 1, 4]
    [0, 2, 3]
    [0, 2, 4]
    [0, 3, 4]
    [1, 2, 3]
    [1, 2, 4]
    [1, 3, 4]
    [2, 3, 4]
    """
    if n == 0:
        ________________________________________

    if ________________________________________:

        ________________________________________

    ________________________________________________________

    ________________________________________________________

    return ________________________________________________
```

```
if n == 0:
    return [[]]
if len(lst) == n:
    return [lst]
with_first = [[lst[0]] + x for x in subsets(lst[1:], n -
   1)]
without_first = subsets(lst[1:], n)
return with_first + without_first
```

Teaching Tips

- Keep in mind the return type is a list of lists - students might mistakenly think it's just one list given the doctest.
- The base case might not be intuitive to all students:
  - If you start with the base case, keep in mind the return type is a list of lists, and use that to motivate the base case being [[]] instead of []
- Recursive call:
  - Ask the students if we can separate into different possibilities based on the first element (either it's in the subset or it isn't)
  - In the recursive calls, ask how `n` changes and how `lst` changes. You can draw similarities to the "count partitions" question.

1. Write a generator function `num_elems` that takes in a possibly nested list of numbers `lst` and yields the number of elements in each nested list before finally yielding the total number of elements (including the elements of nested lists) in `lst`. For a nested list, yield the size of the inner list before the outer, and if you have multiple nested lists, yield their sizes from left to right.

```
def num_elems(lst):
    """
    >>> list(num_elems([3, 3, 2, 1]))
    [4]
    >>> list(num_elems([1, 3, 5, [1, [3, 5, [5, 7]]]]))
    [2, 4, 5, 8]
    """

    count = ________________

    for ______________________________:

        if ___________________________:

            for _____________________________________:

                yield ______________________________

            ___________________________________
        else:

            ___________________________________

    yield ______________________________
```

```
def num_elems(lst):
    count = 0
    for elem in lst:
        if isinstance(elem, list):
            for c in num_elems(elem):
                yield c
            count += c
        else:
```

```
            count += 1
    yield count
```

`count` refers to the number of elements in the current list `lst` (including the number of elements inside any nested list). Determine the value of `count` by looping through each element of the current list `lst`. If we have an element `elem` which is of type **`list`**, we want to yield the number of elements in each nested list of `elem` before finally yielding the total number of elements in `elem`. We can do this with a recursive call to `num_elems`. Thus, we yield all the values that need to be yielded using the inner for loop. The last number yielded by this inner loop is the total number of elements in `elem`, which we want to increase `count` by. Otherwise, if `elem` is not a list, then we can simply increase `count` by 1. Finally, yield the total count of the list.

Teaching Tips

- Double check with your students to make sure they understand the differences between iterables and iterators.
- When we call next(), we pick up from where the last yield statement ran.
- The `+= c` line may be tricky to get. It could be useful to tell students beforehand that the variable in a **`for`** loop persists after iteration as the last value it took on.
- Try walking through one of the doctests if students are confused by what the problem is asking for.
- Make sure they understand that nested lists are processed first; this implies some kind of recursion.

7 Object Oriented Programming

1. Let's use OOP to help us implement our good friend, the ping-pong sequence!

   As a reminder, the ping-pong sequence counts up starting from 1 and is always either counting up or counting down.

   At element `k`, the direction switches if `k` is a multiple of 7 or contains the digit 7.

   The first 30 elements of the ping-pong sequence are listed below, with direction swaps marked using brackets at the 7th, 14th, 17th, 21st, 27th, and 28th elements:

   ```
   1 2 3 4 5 6 [7] 6 5 4 3 2 1 [0] 1 2 [3] 2 1 0 [-1] 0 1 2 3 4
   [5] [4] 5 6
   ```

   Assume you have a function `has_seven(k)` that returns True if k contains the digit 7.

   ```
   >>> tracker1 = PingPongTracker()
   >>> tracker2 = PingPongTracker()
   >>> tracker1.next()
   1
   >>> tracker1.next()
   2
   >>> tracker2.next()
   1

   class PingPongTracker:
       def __init__(self):




       def next(self):
   ```

```python
class PingPongTracker:
    def __init__(self):
        self.current = 0
        self.index = 1
        self.add = True

    def next(self):
        if self.add:
            self.current += 1
        else:
            self.current -= 1
        if has_seven(self.index) or self.index % 7 == 0:
            self.add = not self.add
        self.index += 1
        return self.current
```

Teaching Tips

- Emphasize the fact that the important part of such sequence problems are *keeping track of state at a given time step*. With OOP, this state is inherently saved as object attributes.
- Make sure the difference between `self.current` and `self.index` is clear: index always increases by 1 each step, while current is the actual pingpong sequence number we want.
- Remember that the **index** denotes the the progress we've made along the pingpong sequence- current is just the current number of our sequence we happen to be on.
- Students may have seen a version of pingpong that uses -1 and 1 as a direction variable instead of a boolean "add" variable. Make sure to clarify how the add variable operates here, and how it differs from the -1/1 version.

1. Write a function `combine_two`, which takes in a linked list of integers `lnk` and a two-argument function `fn`. It returns a new linked list where every two elements of `lnk` have been combined using `fn`.

```
def combine_two(lnk, fn):
    """
    >>> lnk1 = Link(1, Link(2, Link(3, Link(4))))
    >>> combine_two(lnk1, add)
    Link(3, Link(7))
    >>> lnk2 = Link(2, Link(4, Link(6)))
    >>> combine_two(lnk2, mul)
    Link(8, Link(6))
    """
    if ______________________________________:

        return ______________________________

    elif ____________________________________

        return ______________________________

    combined = ______________________________

    return __________________________________
```

```
def combine_two(lnk, fn):
    if lnk is Link.empty:
        return Link.empty
    elif lnk.rest is Link.empty:
        return Link(lnk.first)
    combined = fn(lnk.first, lnk.rest.first)
    return Link(combined, combine_two(lnk.rest.rest, fn))
```

2. Write a recursive function `insert_all` that takes as input two linked lists, `s` and `x`, and an index `index`. `insert_all` should return a new linked list with the contents of `x` inserted at index `index` of `s`.

```
def insert_all(s, x, index):
    """
    >>> insert = Link(3, Link(4))
    >>> original = Link(1, Link(2, Link(5)))
    >>> insert_all(original, insert, 2)
    Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> start = Link(1)
    >>> insert_all(original, start, 0)
    Link(1, Link(1, Link(2, Link(5))))
    """
    if ____________________ and ____________________:

        ____________________________________________

    if ____________________ and ____________________:

        ____________________________________________

    ________________________________________________
```

```
def insert_all(s, x, index):
    """
    >>> insert = Link(3, Link(4))
    >>> original = Link(1, Link(2, Link(5)))
    >>> insert_all(original, insert, 2)
    Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> start = Link(1)
    >>> insert_all(original, start, 0)
    Link(1, Link(1, Link(2, Link(5))))
    >>> insert_all(original, insert, 3)
    Link(1, Link(2, Link(5, Link(3, Link(4)))))
    """
    if s is Link.empty and x is Link.empty:
        return Link.empty
    if x is not Link.empty and index == 0:
        return Link(x.first, insert_all(s, x.rest, 0))
    return Link(s.first, insert_all(s.rest, x, index - 1))
```

All of our return statements should return a new linked list.

Our base case should be the simplest possible version of the problem: when both `x` and `s` are empty, clearly the result is just the empty list.

We can now think of ways to break down this problem even further. Note that when the index to be inserted at is `0`, the problem is relatively easy: we just have to put all of the elements of `x` followed by all the elements of `s`. So the first element of the new list should `x.first`, and the rest of the new list should be `x.rest` concatenated with `s`, or `insert_all(s, x.rest, 0)`. Since we are using `x.first` and `x.rest`, we must check that `x` is nonempty to ensure that we do not error.

Finally, when the index to be inserted at is nonzero, we know that we're going to have some elements of `s`, then the elements of `x`, and then the rest of the elements from `s`. So the first element of the new list should be `s.first`. Then we can get the rest of the new list by inserting the contents of `x` at index `index - 1` of `s.rest`, reducing the index by 1 to account for the fact that we have removed the first element of `s`.

There's one issue we glossed over here: what if `x` is empty but `s` is not? Then we want to return the contents of `s`. But because the problem requires that we return a new linked list, we must recursively reconstruct `s` instead of simply returning it. You could add another base case to handle this, but as it turns out the second recursive case will handle this just fine since `Link(s.first, insert_all(s.rest, x, index - 1))` is just equivalent to `Link(s.first, s.rest)` when `x` is empty. Since the `x` **`is not`** `Link.empty` condition for the first recursive case will direct all situations where `x` is empty but `s` is not to the second recursive case, it turns out that we do not need to add anything else to this solution.

Convincing yourself that this problem works requires that you eventually reach a base case. Note that in either recursive call, we either reduce `s` or `x` by one element. So the base case will always eventually be reached, and the solution is valid.

Despite being just a few lines and exercising a familiar concepts with lists, I've found that this problem is quite difficult, so one thing I would emphasize is to draw out this problem with a box-and-pointer diagram and illustrate the different steps of our function. Illustrate how the function works for the doctests, which should cover all possible cases of inserting a new linked list into the beginning, middle, and end of the original linked list

If students are lost, which they most likely will be, here are some leading questions you could ask:

- When do we know that we are done inserting items into the list?
- What should the parameters be equal to if we are going to start inserting `x`, what if we are not currently inserting `x`?

- How do ensure to add all elements of `x` into `x`?

You should tell your students that they should feel free to disregard the provided skeleton, because it is quite difficult to think of the solution to this problem when you are trying to fit everything into the skeleton.