LINKED LISTS, MUTABLE TREES AND MIDTERM REVIEW

COMPUTER SCIENCE MENTORS 61A

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1 Linked Lists

For each of the following problems, assume linked lists are defined as follows:

```
class Link:
    empty = ()
    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest is not Link.empty:
            rest_repr = ', ' + repr(self.rest)
        else:
            rest_repr = ''
        return 'Link(' + repr(self.first) + rest_repr + ')'

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

Linked lists are a recursive data structure for representing sequences. They consist of a series of "links," each of which has two attributes: `first` and `rest`. The `first` attribute contains the value of the link (which can hold any type of data, even another linked list!). The `rest` attribute, on the other hand, is a pointer to another link or `Link.empty`, which is just a "None" type value.

For example, `Link(1, Link(2, Link(3)))` is a linked list representation of the sequence $1, 2, 3$.

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Like trees, linked lists naturally lend themselves to recursive problem solving. Consider the following example, in which we double every value in linked list. We double the value of the current link and then recursively double the rest.

```
def double_values(link):
     if link is not Link.empty:
        link.first *= 2 # we mutate the value inside of the link
        double_val(link.rest) # we mutate the values in the rest
                              # of the linked list
    # if the link is empty then do nothing
```

1. What will Python output? Draw box-and-pointer diagrams along the way.

   ```
   >>> a = Link(1, Link(2, Link(3)))

   >>> a.first




   >>> a.first = 5

   >>> a.first




   >>> a.rest.first


   >>> a.rest.rest.rest.rest.first
   ```

```
>>> a.rest.rest.rest = a

>>> a.rest.rest.rest.rest.first


>>> repr(Link(1, Link(2, Link(3, Link.empty))))


>>> Link(1, Link(2, Link(3, Link.empty)))


>>> str(Link(1, Link(2, Link(3))))


>>> print(Link(Link(1), Link(2, Link(3))))
```

2. Write a function `combine_two`, which takes in a linked list of integers `lnk` and a two-argument function `fn`. It returns a new linked list where every two elements of `lnk` have been combined using `fn`.

```
def combine_two(lnk, fn):
    """
    >>> lnk1 = Link(1, Link(2, Link(3, Link(4))))
    >>> combine_two(lnk1, add)
    Link(3, Link(7))
    >>> lnk2 = Link(2, Link(4, Link(6)))
    >>> combine_two(lnk2, mul)
    Link(8, Link(6))
    """
    if _______________________________________:

        return ______________________________

    elif ____________________________________

        return ______________________________

    combined = ______________________________

    return __________________________________
```

3. Write a function `middle_node` that takes as input a linked list `lst`. `middle_node` should return the middle node of the linked list. If there are two middle nodes, return the second middle node.

```
def middle_node(lst):
    """
    >>> head = Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> middle_node(head)
    Link(3, Link(4, Link(5))) # The middle node of the list is node 3
    >>> head = Link(1, Link(2, Link(3, Link(4, Link(5, Link(6))))))
    Link(4, Link(5, Link(6))) # Since the list has two middle nodes with
       values 3 and 4, we return the second one
    """
    list_iter, middle = _______________, ________________

    length = _________________________________

    while _________________________________:

        length = _________________________________

        list_iter = _________________________________

    for _________________________________:

        middle = _________________________________

    if length % 2 == 1:

        middle = _________________________________

    return middle
```

Challenge version **(Optional)**:

```
def middle_node(lst):

    list_iter, middle = _______________, ________________

    while ___________________ and ___________________:

        list_iter = _________________________________

        middle = _________________________________

    return middle
```

4. Write a recursive function `insert_all` that takes as input two linked lists, `s` and `x`, and an index `index`. `insert_all` should return a new linked list with the contents of `x` inserted at index `index` of `s`.

```
def insert_all(s, x, index):
    """
    >>> insert = Link(3, Link(4))
    >>> original = Link(1, Link(2, Link(5)))
    >>> insert_all(original, insert, 2)
    Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> start = Link(1)
    >>> insert_all(original, start, 0)
    Link(1, Link(1, Link(2, Link(5))))
    """
    if ___________________ and ___________________:

        ____________________________________________

    if ___________________ and ___________________:

        ____________________________________________

    ________________________________________________
```

2 Trees

For the following problems, use this definition for the `Tree` class:

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = list(branches)

    def is_leaf(self):
        return self.branches == []

    # Implementation ommitted
```

Here are a few key differences between the `Tree` class and the Tree abstract data type, which we have previously encountered:

- Using the constructor: Capital T for the `Tree` class and lowercase t for tree ADT `t = Tree(1)` vs.

```
t = tree(1)
```

- In the class, `label` and `branches` are instance variables and `is_leaf()` is an instance method. In the ADT, all of these were globally defined functions.

  `t.label` vs. `label(t)`

  `t.branches` vs. `branches(t)`

  `t.is_leaf()` vs. `is_leaf(t)`

- A `Tree` object is mutable while the tree ADT is not mutable. This means we can change attributes of a `Tree` instance without making a new tree. In other words, we can solve tree class problems non-destructively and destructively, but can only solve tree ADT problems non-destructively.

  `t.label = 2` is allowed but `label(t)= 2` would error.

Apart from these differences, we can take the same general approaches we used for the tree ADT and apply them to the `Tree` class!

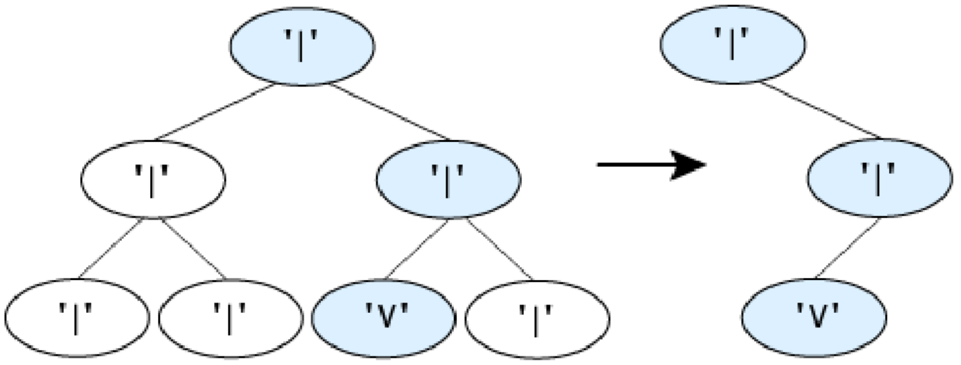
1. Define `delete_path_duplicates`, which takes in `t`, a tree with non-negative labels. If there are any duplicate labels on any path from root to leaf, the function should mutate the label of the occurrences deeper in the tree (i.e. farther from the root) to be the value `-1`.

```
def delete_path_duplicates(t):
    """
    >>> t = Tree(1, [Tree(2, [Tree(1), Tree(1)])])
    >>> delete_path_duplicates(t)
    >>> t
    Tree(1, [Tree(2, [Tree(-1), Tree(-1)])])
    >>> t2 = Tree(1, [Tree(2), Tree(2, [Tree(2, [Tree(1, [Tree(5)])])])])
    >>> delete_path_duplicates(t2)
    >>> t2
    Tree(1, [Tree(2), Tree(2, [Tree(-1, [Tree(-1, [Tree(5)])])])])
    """
    def helper(_______________, _______________):

        if _________________________________:

            _________________________________

        else:

            _________________________________

        for _______ in ____________________:

            _________________________________

    _________________________________________
```

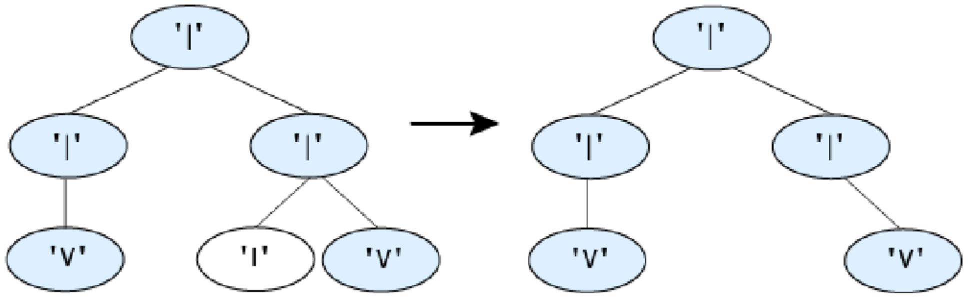
2. Given a tree `t`, mutate the tree so that each leaf's label becomes the sum of the labels of all nodes in the path from the leaf node to the root node.

```
def replace_leaves_sum(t):
    """
    >>> t = Tree(1, [Tree(3, [Tree(2), Tree(8)]), Tree(5)])
    >>> replace_leaves_sum(t)
    >>> t
    Tree(1, [Tree(3, [Tree(6), Tree(12)]), Tree(6)])
    """
    def helper(______________ , _________________):

        if t.is_leaf():

            _____________________________________

        for b in t.branches:

            _________________________________

    _____________________________________________
```

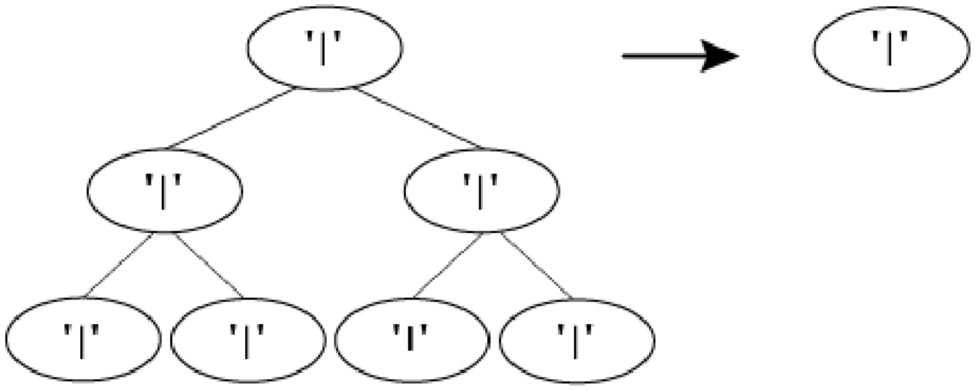
3. *From Sp'22 MT2:*
Implement $flower\_keeper$, a function that mutates a tree t so that the only paths that remain are ones which end in a leaf node with a Tulip flower ('V'). For example, consider this tree where only one path ends in a flower. After calling $flower\_keeper$, the tree has only three nodes left, the ones that lead to the flower:



The shaded nodes in the diagram indicate paths that end in flowers. For this tree where two paths end in flowers, the tree keeps both paths that lead to flowers.



For a tree where none of the nodes are flowers, the function removes every branch except the root node.



For a tree with only a single node that is a flower, the function does not remove anything.

```
def flower_keeper(t):
    """
    Mutates the tree T to keep only paths that end in flowers ('V').
    If a path consists entirely of stems ('|'), it must be pruned.
    If T has no paths that end in flowers, the root node is still kept.
    You can assume that a node with a flower will have no branches.
    >>> one_f = Tree('|', [Tree('|', [Tree('|'), Tree('|')]), Tree('|',
       [Tree('V'), Tree('|')])])
    >>> print(one_f)
    |
        |
            |
            |
        |
            V
            |
    >>> flower_keeper(one_f)
    >>> one_f
    Tree('|', [Tree('|', [Tree('V')])])
    >>> print(one_f)
    |
        |
        V
    >>> no_f = Tree('|', [Tree('|', [Tree('|'), Tree('|')]), Tree('|',
       [Tree('|'), Tree('|')])])
    >>> flower_keeper(no_f)
    >>> no_f
    Tree('|')
    >>> just_f = Tree('V')
    >>> flower_keeper(just_f)
    >>> just_f
    Tree('V')
    >>> two_f = Tree('|', [Tree('|', [Tree('V')]), Tree('|', [Tree('|'),
       Tree('V')])])
    >>> flower_keeper(two_f)
    >>> two_f
    Tree('|', [Tree('|', [Tree('V')]), Tree('|', [Tree('V')])])
    """
    for b in __________:

        __________

    __________ = [__________ for b in __________ if __________]
```

3 Higher Order Functions

1. Write a function, `make_digit_remover`, which takes in a single digit `i`. It returns another function that takes in an integer and, scanning from right to left, removes all digits from the integer up to and including the first occurrence of `i`, starting from the ones place. If `i` does not occur in the integer, the original number is returned.

```
def make_digit_remover(i):
    """
    >>> remove_two = make_digit_remover(2)
    >>> remove_two(232018)
    23
    >>> remove_two(23)
    0
    >>> remove_two(99)
    99
    """
    def remove(________):

        removed = ______________________________

        while ______________________________ > 0:

            ______________________________________

            removed = removed // 10

            if _________________________________:

                ________________________________

        return _________________________________

    return _________________________________
```

1. Draw the box-and-pointer diagram.

```
>>> violet = [7, 77, 17]
>>> violet.append([violet.pop(1)])

>>> dash = violet * 2
>>> jack = dash[3:5]
>>> jackjack = jack.extend(jack)

>>> helen = list(violet)
>>> helen += [jackjack]
>>> helen[2].append(violet)
```

2. Write a function `duplicate_list`, which takes in a list of positive integers and returns a new list with each element `x` in the original list duplicated `x` times.

```
def duplicate_list(lst):
    """
    >>> duplicate_list([1, 2, 3])
    [1, 2, 2, 3, 3, 3]
    >>> duplicate_list([5])
    [5, 5, 5, 5, 5]
    """
    ________________________________

    for ____________________________:

         for ____________________________:

              ___________________________________

    ________________________________
```

3. Write a function that takes as input a number `n` and a list of numbers `lst` and returns `True` if we can find a subset of `lst` that sums to `n`.

```
def add_up(n, lst):
    """
    >>> add_up(10, [1, 2, 3, 4, 5])
    True
    >>> add_up(8, [2, 1, 5, 4, 3])
    True
    >>> add_up(-1, [1, 2, 3, 4, 5])
    False
    >>> add_up(100, [1, 2, 3, 4, 5])
    False
    """
    if ___________________________________________:

        return True

    if lst == []:

        _____________________________________________________

    else:

        first, rest = ___________________, ___________________

        return ___________________________________________
```

5 Iterators and Generators

1. Write a generator function `num_elems` that takes in a possibly nested list of numbers `lst` and yields the number of elements in each nested list before finally yielding the total number of elements (including the elements of nested lists) in `lst`. For a nested list, yield the size of the inner list before the outer, and if you have multiple nested lists, yield their sizes from left to right.

```
def num_elems(lst):
    """
    >>> list(num_elems([3, 3, 2, 1]))
    [4]
    >>> list(num_elems([1, 3, 5, [1, [3, 5, [5, 7]]]]))
    [2, 4, 5, 8]
    """

    count = _______________

    for ___________________________:

        if ________________________:

            for _________________________________:
```

```
            yield ______________________________
        ___________________________________
    else:
        ___________________________________
    yield ______________________________
```