Higher-Order Environments, Currying, and Function Scope Solutions

Computer Science Mentors 61A

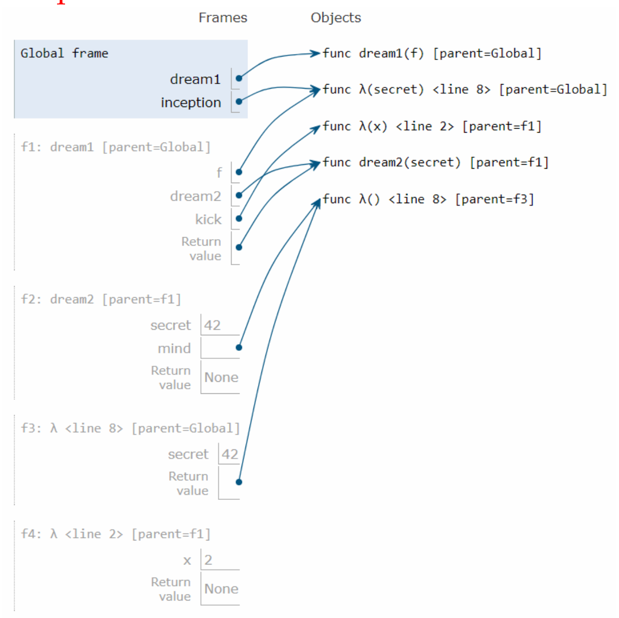
February 6–February 10, 2023

1. Draw the environment diagram that results from running the code.

```
def dream1(f):
    kick = lambda x: mind()
    def dream2(secret):
        mind = f(secret)
        kick(2)
    return dream2

inception = lambda secret: lambda: secret
real = dream1(inception)(42)
```

Output: 3


Frames
Objects
Global frame
dream1
inception
func dream1(f) [parent=Global]
func λ(secret) <line 8> [parent=Global]
func λ(x) <line 2> [parent=f1]
func dream2(secret) [parent=f1]
func λ() <line 8> [parent=f3]
f1: dream1 [parent=Global]
f
dream2
kick
Return value
f2: dream2 [parent=f1]
secret 42
mind
Return value None
f3: λ <line 8> [parent=Global]
secret 42
Return value
f4: λ <line 2> [parent=f1]
x 2
Return value None

https://goo.gl/q84uf4

2. Draw the environment diagram that results from running the code.

```
def a(y):
    d = 1
    b = lambda x: y(x)
    e = lambda x: x(3)
    return e(b)
d = 5
a(lambda x: 4 - x + d)
```

https://goo.gl/9vxEwv

3. Implement `compound`, which takes in a single-argument function `base_func` and returns a two-argument compounder function `g`. The function `g` takes in an integer `x` and positive integer `n`.

   Each call to `g` will print the result of calling `f` repeatedly 0,1,.., n-1 times on `x`. That is, `g(x, 2)` prints `x`, then `f(x)`. Then, `g` will return the next two-argument compounder function.

```
def compound(base_func, prev_compound=lambda x: x):
    """
    >>> add_one = lambda x: x + 1
    >>> adder = compound(adder)
    >>> adder = adder(3, 2)
    3       # 3
    4       # f(3)
    >>> adder = adder(4, 4)
    6       # f(f(4))
    7       # f(f(f(4)))
    8       # f(f(f(f(4))))
    9       # f(f(f(f(f(4)))))
    """
    def g(x, n):
      new_comp = ____________________________
      while n > 0:
        print(______________________________)
        new_comp = (lambda save_comp: \
                  _________________________)(_____________)
        ____________________________________________________
      return ______________________________________________
    return ___________________________________
```

```
def compound(base_func, prev_compound=lambda x : x):
  def g(x, n):
    new_comp = prev_compound
    while n > 0:
      print(new_comp(x))
      new_comp = (lambda save_comp: \
                  lambda x: base_func(save_comp(x)))(new_comp)
      n -= 1
    return compound(base_func, new_comp)
  return g
```

4. Write a function `partial_summer`, which takes in a list of integers `lst` and returns a function. The returned function takes in a non-negative integer `n`. It prints a sum derived from the first `n` elements of `lst`: if element `X` is even, divide `X` by `2` before adding it to the sum, and if `X` is odd. add `1` to `X` before adding it to the sum. If `n > len(lst)`, then sum as many elements of `lst` as you can. After printing the sum, the returned function returns another function, that when called, will perform the same procedure on the remaining `len(lst)- n` elements of `lst`.

```
def partial_summer(lst):
    """
    >>> lst = [1, 2, 3, 4, 5, 6, 7, 8, 9]
    >>> f = partial_summer(lst)(3)
    7 # 7 = (1+1) + (2/2) + (3+1)
    >>> g = f(4)
    19 # 19 = (4/2) + (5+1) + (6/2) + (7+1)
    >>> h = g(3)
    14 # 14 = (8/2) + (9+1)
    >>> i = h(1)
    0
    """
    def helper(n):

        total, i = ________, ________

        while ________________ and ___________________:

            if _______________________:

                total += ____________________________
            else:
                total += lst[i] + 1

            _________________________________________
        print(total)

        return ______________________________________
    return helper
```

```python
def partial_summer(lst):
    def helper(n):
        total, i = 0, 0
        while i < n and i < len(lst):
            if lst[i] % 2 == 0:
                total += lst[i] // 2
            else:
                total += lst[i] + 1
            i += 1
        print(total)
        return partial_summer(lst[n:])
    return helper
```

2 Recursion

There are three steps to writing a recursive function:

1. Create base case(s)
2. Reduce your problem to a smaller subproblem and call your function recursively to solve the smaller subproblem(s)
3. Use the subproblems' solutions as pieces to construct a larger problem's solution (This can happen in many layers!)

Real World Analogy for Recursion

Imagine that you're in line for boba, but the line is really long, so you want to know what position you're in. You decide to ask the person in front of you how many people are in front of them. That way, you can take their response and add 1 to it to find your place. Now, the person in front of you is faced with the same problem that you were trying to solve, with one less person in front of them than you. They decide to take the same approach that you did by asking the person in front of them. This continues until the very first person in line is asked. At this point, the person at the front knows that there are 0 people in front of them, so they can tell the person behind them that there are 0 people in front. Now, the second person can figure out that there is 1 person in front of them, and can relay that back to the person behind them, and so on, until the answer reaches you.

Looking at this example, we see that we have broken down the problem of "how many people are there in front of me?" to 1 + "how many people are there in front of the person in front of me"? This problem will terminate with the person at the front of the line (with 0 people in front of them). Putting this into more formal terms, we are breaking down the problem into a **recurrence relationship**, and the termination case (when the question gets to the very first person in line) is called the **base case**.

5. What is wrong with the following function? How can we fix it?

```
def factorial(n):
    return n * factorial(n)
```

There is no base case and the recursive call is made on the same `n`.

```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n - 1)
```

6. Complete the definition for `all_true`, which takes in a list `lst` and returns `True` if there are no False-y values in the list and `False` otherwise. Make sure that your implementation is recursive.

```
def all_true(lst):
    """
    >>> all_true([True, 1, "True"])
    True
    >>> all_true([1, 0, 1])
    False
    >>> all_true([])
    True
    """

    if not lst:
        return True
    elif not lst[0]:
        return False
    else:
        return all_true(lst[1:])
```