General Errors, Uncountability, Self Reference, Counting 5

Computer Science Mentors 70

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1 General Errors (Berlekamp and Welch)

1.1 Introduction

Now instead of losing packets, we know that k packets are corrupted. Furthermore, we do not know which k packets are changed. Instead of sending k additional packets, we will send an additional $2k$.

$$\boxed{3}\ \boxed{1}\ \boxed{5}\ \boxed{0} \rightarrow \boxed{4}\ \boxed{1}\ \boxed{5}\ \boxed{1}$$

Solomon-Reed Codes

1. Identical to erasure errors: Alice creates $n-1$ degree polynomial $P(x)$.

$$P(x) = p_0 + p_1 x + \ldots + p_{n-1} x^{n-1}$$

2. Alice sends $P(1), \ldots, P(n+2k)$

3. Bob receives $R(1), \ldots, R(n+2k)$

For how many points does $R(x) = P(x)$?

True or false: $P(x)$ is the unique degree $n-1$ polynomial that goes through at least $n+k$ of the received points.

Write the matrix view of encoding the points $P(1), \ldots, P(n+2k)$

Berlekamp Welch

How do we find the original polynomial $P(x)$?

Suppose that $m_1, \ldots, m_k$ are the corrupted packets. Let $E(x) = (x - m_1)\ldots(x - m_k)$

Then $P(i) * E(i) = r_i * E(i)$ for any i greater than 1 and less than $n + 2k$. Why?

Let $Q(i) = P(i)E(i)$ So we have $Q(i) = P(i)E(i) = r_i * E(i)$ where $1 \leq i \leq 2k + n$ What degree is $Q(i)$?

How many coefficients do we need to describe $Q(i)$?

What degree is $P(i)$?

How many unknown coefficients do we need to describe $E(i)$?

We can write $Q(i) = r_iE(i)$ for every i that is $1 \leq i \leq 2k + n$.

How many equations do we have? How many unknowns?

Once we have the above described equations, how do we determine what $P(i)$ is?

1.2 Questions

1. (a) Alice sends Bob a message of length 3 on the Galois Field of 5 (modular space of mod 5). Bob receives the following message: (3, 2, 1, 1, 1). Assuming that Alice is sending messages using the proper general error message sending scheme, set up the linear equations that, when solved, give you the $Q(x)$ and $E(x)$ needed to find the original $P(x)$.

   (b) What is the encoded message that Alice actually sent? What was the original message? Which packet(s) were corrupted?

2 Secret Sharing

2.1 Questions

1. You want to send a super secret message consisting of 10 packets to the space station through some astronauts. You are afraid that some malicious spy people are going to

tell the wrong message and make the space station go spiraling out of orbit. Assuming that up to 5 of the astronauts are malicious, design a scheme so that the group of astronauts (including the malicious ones) still find the correct message that you want to send. You can send any number of astronauts, but try to make the number that you have to send as small as possible. Astronauts can only carry one packet with them.

3 Uncountability

3.1 Introduction

1. (a.) What does it mean for a set to be countably infinite?

   (b.) Do $\mathbb{N}$ and $\mathbb{Z}^+$ have the same cardinality? Does adding one element change cardinality?

   (c.) Cantor-Bernstein Theorem: Suppose there is an injective function from set A to set B and there is an injective function from set B to set A. Then there is a bijection between A and B. Use this theorem to prove that Q is countable.

3.2 Questions

1. Are these sets countably infinite/uncountably infinite/finite? If finite, what is the order of the set?

   (a) Finite bit strings of length n.

   (b) All finite bit strings of length 1 to n.

   (c) All finite bit strings

   (d) All infinite bit strings

   (e) All finite or infinite bit strings.

2. Find a bijection between N and the set of all integers congruent to $1 \mod n$, for a fixed n.

3. **True/False**

   (a) Every infinite subset of a countable set is countable

   (b) If A and B are both countable, then $A \times B$ is countable

   (c) Every infinite set that contains an uncountable set is uncountable.

4 Self Reference

4.1 Introduction

The Halting Problem: Does a given program ever halt when executed on a given input?

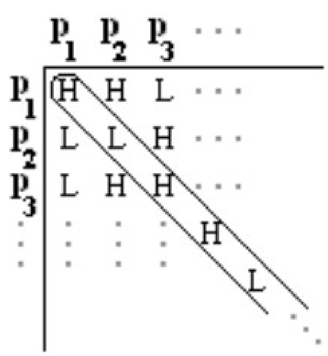
$$\texttt{TestHalt(P,x)} = \begin{cases} \text{"yes",} & \text{if program } P \text{ halts on input } x \\ \text{"no",} & \text{if program } P \text{ loops on input } x \end{cases}$$

How do we prove that `TestHalt` does not exist? Lets assume that it does, and hope we reach a contradiction.
Define another program:

```
Turing(P)
  if TestHalt(P,P) = "yes" then loop forever
  else halt
```

What happens when we call `Turing(Turing)`?

How is this just a reformulation of proof by diagonalization?



Therefore the Halting Problem is unsolvable. We can use this to prove that other problems are also unsolvable. Say we are asked if program M is solvable. To prove it is not, we just need to prove the following claim: If we can compute program M, then we could also compute the halting problem.
This would then prove that M can not exist, since the halting problem is not computable. This amounts to proof by contradiction.

4.2 Questions

1. Say that we have a program M that decides whether any input program halts as long as it prints out the string ABC as the first operation that it carries out. Can such a program exist? Prove your answer.

5 Intro to Counting

5.1 Introduction

Counting:
In this class, the basic premise of counting is determining the total number of possible ways something can be done. Reaching a particular outcome requires a number of specific choices to be made. In counting, we determine the number of ways we can make these choices and reach different outcomes. To figure out the total number of outcomes, we multiply together the number of potential choices at each step.

5.2 When Order Matters

1. (a) You have 15 chairs in a room and there are 9 people. How many different ways can everyone sit down?

   (b) How many ways are there to fill 9 of the 15 chairs? (We do not care who sits in them.)

2. The numbers 1447, 1005, and 1231 have something in common. Each of them is a four digit number that begins with 1 and has two identical digits. How many numbers like this are there?

5.3 More Practice

1. At Starbucks, you can choose either a Tall, a Grande, or a Venti drink. Further, you can choose whether you want an extra shot of espresso or not. Furthermore, you can choose whether you want a Latte, a Cappuccino, an Americano, or a Frappuccino.

   How many different drink combinations can you order?

2. We grab a deck of cards and it is poker time. Remember, in poker, order does not matter.

   (a) How many ways can we have a hand with exactly one pair? This means a hand with ranks (a, a, b, c, d).

   (b) How many ways can we have a hand with four of a kind? This means a hand with ranks (a, a, a, a, b).

   (c) How many ways can we have a straight? A straight is 5 consecutive cards, that do not all necessarily have the same suit.

   (d) How many ways can we have a hand of all of the same suit?

   (e) How many ways can we have a straight flush? This means we have a consecutive-rank hand of the same suit. For example, (2, 3, 4, 5, 6), all of spades, is a straight flush, while (2, 3, 5, 7, 8), all of spades, is NOT, as the ranks are not consecutive.