Quantifiers, Methods of Proof 0

Computer Science Mentors 70

Independent review

1 Quantifiers

1.1 Questions

1. Let $P(x, y)$ denote some proposition involving x and y. For each statement below, either prove that the statement is correct or provide a counterexample if it is false.
   a. $\forall x \forall y P(x, y) \implies \forall y \forall x P(x, y)$.

   b. $\exists x \exists y P(x, y) \implies \exists y \exists x P(x, y)$.

   c. $\forall x \exists y P(x, y) \implies \exists y \forall x P(x, y)$.

d. $\exists x \forall y P(x,y) \to \forall y \exists x P(x,y)$.

2 Contrapositive and Contradiction

2.1 Questions

1. Write the contrapositive of the following statements and, if applicable, the statement in mathematical notation. (Using quantifiers, etc.)

   a If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides. (Skip mathematical notation for this problem, just write the contrapositive)

   b For all natural numbers a where a^2 is even, a is even.

   c Negate this statement: For all integers x, there exists an integer y such that $x^2+y=16$.

2. Prove or disprove: If $P \implies Q$ and $R \implies \neg Q$, then $P \implies \neg R$.

3 Proof by Cases

3.1 Questions

1. For any integer x, x^2 has remainder 1 or 0 when divided by 3.

4 Induction

4.1 Questions

1. What are the three steps of induction?

2. Prove that $\Sigma_{i=0}^{n} i * i! = (n+1)! - 1$ for $n \geq 1$ where $n \in N$.

5 More Practice

Use any method of proof to answer the following questions.

1. Let x be a positive real number. Prove that if x is irrational (i.e., not a rational number), then $\sqrt{x}$ is also irrational.

2. McDonalds sells chicken McNuggets only in 6, 9, and 20 piece packages. This means that you cannot purchase exactly 8 pieces, but can purchase 15. The Chicken McNugget Theorem states that the largest number of pieces you cannot purchase is 43. Formally state the Chicken McNugget Theorem using quantifiers.

3. Prove or disprove the following statement: If n is a positive integer such that $\frac{n}{3}$ leaves a remainder of 2, then n is not a perfect square.

4. Suppose that there are $2n + 1$ airports where n is a positive integer. The distances between any two airports are all different. For each airport, there is exactly one airplane departing from it, and heading towards the closest airport. Prove by induction that there is an airport which none of the airplanes are heading towards.