INEQUALITIES, DISTRIBUTIONS, CONTINUOUS PROBABILITY, CONDITIONAL EXPECTATION 9

COMPUTER SCIENCE MENTORS 70

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1 LLSE

1.1 Introduction

Theorem: Consider two random variables, X, Y with a given distribution $\mathrm{P}[X = x, Y = y]$. Then

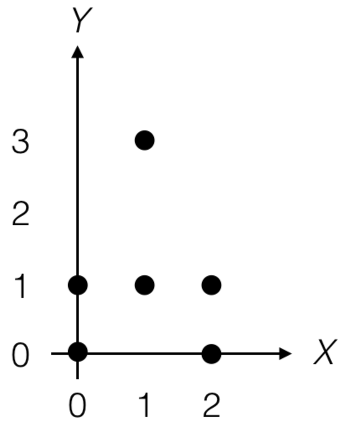
$$\mathrm{L}[Y|X] = \mathrm{E}(Y) + \frac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)}(X - \mathrm{E}(X))$$

1.2 Questions

1. Assume that
$$Y = \alpha X + Z$$
where X and Z are independent and $\mathrm{E}(X) = \mathrm{E}(Z) = 0$. Find $\mathrm{L}[X|Y]$.

2. The figure below shows the six equally likely values of the random pair (X, Y). Specify the functions of:

   - $L[Y \mid X]$
   - $E(X \mid Y)$
   - $L[X \mid Y]$
   - $E(Y \mid X)$



2 Conditional Expectation

2.1 Introduction

The **conditional expectation** of Y given X is defined by

$$\mathrm{E}[Y|X=x] = \sum_y y \cdot \mathrm{P}[Y=y|X=x] = \sum_y y \cdot \frac{\mathrm{P}[X=x, Y=y]}{\mathrm{P}[X=x]}$$

Properties of Conditional Expectation

$$\begin{aligned}
\mathrm{E}(a|Y)) &= a \\
\mathrm{E}(aX + bZ|Y) &= a \cdot \mathrm{E}(X|Y) + b \cdot \mathrm{E}(Z|Y) \\
\mathrm{E}(X|Y) &\geq 0 \text{ if } X \geq 0 \\
\mathrm{E}(X|Y) &= \mathrm{E}(X) \text{ if } X, Y \text{ independent} \\
\mathrm{E}(\mathrm{E}(X|Y)) &= \mathrm{E}(X)
\end{aligned}$$

2.2 Questions

1. Prove $\mathrm{E}(\mathrm{E}(Y|X)) = \mathrm{E}(Y)$

2. Prove $\mathrm{E}(h(X) \cdot Y|X) = h(X) \cdot \mathrm{E}(Y|X)$