GRAPHS - EXTRA PRACTICE 2

COMPUTER SCIENCE MENTORS 70

September 10 to September 14, 2018

1 Eulerian Tour

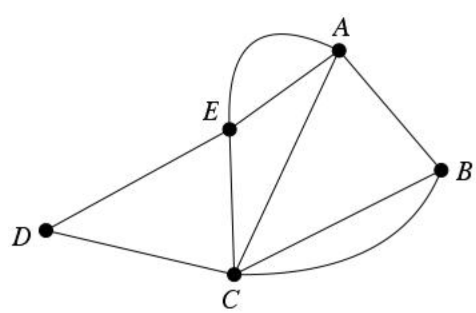
1.1 Introduction

An **Eulerian path** is a path that uses every edge exactly once.

An **Eulerian tour** is a path that uses each edge exactly once and starts and ends at the same vertex.

Euler's Theorem: An undirected graph $G = (V, E)$ has an Eulerian tour if and only if G is even degree and connected (except possibly for isolated vertices).

1.2 Questions



1. Is there an Eulerian Tour? If so, find one. Repeat for an Eulerian Path.

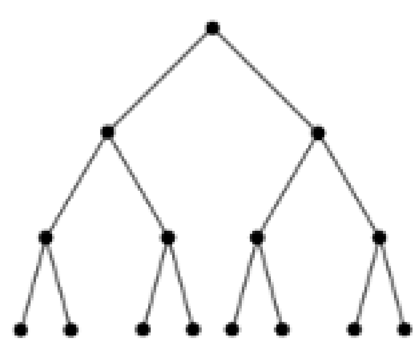
2. If every node has even degree except two nodes that have odd degree, prove that the graph has a Eulerian path.

1.3 Assorted Graph Questions

1. Prove that every undirected finite graph where every vertex has degree of at least 2 has a cycle.

2. Prove that every undirected finite graph where every vertex has degree of at least 3 has a cycle of even length.

3. Recall from the notes that a **rooted tree** is a tree with a particular node designated as the root, and the other nodes arranged in levels, growing down from the root. An alternative, recursive, definition of rooted tree is the following: A rooted tree consists of a single node, the root, together with zero or more branches, each of which is itself a rooted tree. The root of the larger tree is connected to the root of each branch.



Prove that given any tree, selecting any node to be the root produces a rooted tree according to the definition above.

4. Show that the edges of a complete graph on n vertices for even n can be partitioned into $\frac{n}{2}$ edge disjoint spanning trees.
Hint: Recall that a complete graph is an undirected graph with an edge between every pair of vertices. The complete graph has $\frac{n*(n-1)}{2}$ edges. A spanning tree is a tree on all n vertices – so it has $n-1$ edges. So the complete graph has enough edges (for even n) to create exactly $\frac{n}{2}$ edge disjoint spanning trees (i.e. each edge participates in exactly one spanning tree). You have to show that this is always possible.

5. **Coloring Hypercubes**
Let $G = (V, E)$ be an undirected graph. G is said to be k-vertex-colorable if it is possible to assign one of k colors to each vertex of G so that no two adjacent vertices receive the same color. G is k-edge-colorable if it is possible to assign one of k colors to each edge of G so that no two edges incident on the same vertex receive the same color.
Show that the n-dimensional hypercube is 2-vertex-colorable for every n.