

# QUANTIFIERS, METHODS OF PROOF 0

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COMPUTER SCIENCE MENTORS 70

Independent review

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## 1 Quantifiers

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### 1.1 Questions

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1. Let  $P(x, y)$  denote some proposition involving  $x$  and  $y$ . For each statement below, either prove that the statement is correct or provide a counterexample if it is false.

a.  $\forall x \forall y P(x, y) \implies \forall y \forall x P(x, y)$ .

b.  $\exists x \exists y P(x, y) \implies \exists y \exists x P(x, y)$ .

c.  $\forall x \exists y P(x, y) \implies \exists y \forall x P(x, y)$ .

d.  $\exists x \forall y P(x, y) \rightarrow \forall y \exists x P(x, y)$ .

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## 2 Contrapositive and Contradiction

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### 2.1 Questions

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1. Write the contrapositive of the following statements and, if applicable, the statement in mathematical notation. (Using quantifiers, etc.)

a If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides. (Skip mathematical notation for this problem, just write the contrapositive)

b For all natural numbers  $a$  where  $a^2$  is even,  $a$  is even.

c Negate this statement: For all integers  $x$ , there exists an integer  $y$  such that  $x^2 + y = 16$ .

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2. Prove or disprove: If  $P \implies Q$  and  $R \implies \neg Q$ , then  $P \implies \neg R$ .

### 3 Proof by Cases

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#### 3.1 Questions

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1. For any integer  $x$ ,  $x^2$  has remainder 1 or 0 when divided by 3.

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## 4 Induction

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### 4.1 Questions

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1. What are the three steps of induction?

2. Prove that  $\sum_{i=0}^n i * i! = (n + 1)! - 1$  for  $n \geq 1$  where  $n \in \mathbb{N}$ .

Use any method of proof to answer the following questions.

1. Let  $x$  be a positive real number. Prove that if  $x$  is irrational (i.e., not a rational number), then  $\sqrt{x}$  is also irrational.
2. McDonalds sells chicken McNuggets only in 6, 9, and 20 piece packages. This means that you cannot purchase exactly 8 pieces, but can purchase 15. The Chicken McNugget Theorem states that the largest number of pieces you cannot purchase is 43. Formally state the Chicken McNugget Theorem using quantifiers.

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