EE254-2F Exam #5 Spring 2020 Name:___Connor McGarty_________

Instructions:

Question{01}: Briefly explain three approached we have discuss during this semester to improve the accuracy of your numerical integration. Include an example for each.

a. Increase number of segments/decrease xdelta

One method of improving accuracy of numerical integrations is to increase the number of segments in the integration interval of Newton-Cotes formula. For instance, employing the trapezoidal rule and integrating from point a to point b with one segment is less accurate than using the composite trapezoidal rule which uses multiple segments; for example, 100 segments in the range point a to point b.
Example: function $y = x^2$, x = 2 to x = 8
Trapezoidal results, 1 segment = 204, e_t = 36
Composite Trapezoidal, 10 segments = 168.36, e_t = 0.36

b. Increase the order of the approximation

Another way to improve accuracy is to utilize the higher-order Newton-Cotes formulas, such as Simpson's 1/3 or 3/8 rule, or Boole's rule. These formulas consider more points per segment. For example, the trapezoidal formula considers two points per segment, while Simpon's 1/3 considers 3, 3/8 considers 4, and Boole's 5. These points are weighted differently on an individual segment and provides a more accurate result that better responds to curves in graphs.

c. Use Romberg Integration

Romberg Integration is a technique for efficient integration which uses successive application of the trapezoidal rule to obtain a superior result with less computation. To do this, it requires two trapezoidal integrations, a more accurate and a less accurate one, for instance a 1 segment calculation and a 2 segment. The two are them combined with weighted to produce a better approximation.
Example:

$$f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$$

From a = 0 to b = 0.8
1 segment: 0.1728, e_t = 89.5%
2 segment: 1.0688, e_t = 34.9%
Richardson Extrapolation:

$$I = \frac{4}{3}(I_m) - \frac{1}{3}(I_l)$$

Therefore

$$I = \frac{4}{3}(1.0688) - \frac{1}{3}(0.1728) = 1.367467$$

Which is an e_t = 16.6%, which is a much better approximation than the 1 or 2 segment but without any additional computation.