A RELIABILITY EXPERIMENT USING DEGRADATION DATA

Oguz Cimenler

ESI 6247 Statistical Design Models

Final Project Report

Spring 2010

A RELIABILITY EXPERIMENT USING DEGRADATION DATA

Oguz Cimenler; Industrial and Management Systems Engineering

University of South Florida, Tampa, Florida, 33620, USA

Abstract

In today's world, most products functions for a long time. Since the reliability experiment is designed for a short period of time for practical reasons, the data censored during a reliability experiment will, therefore, not be able to show a real failure time of the product. A common practice is to select some product characteristics that can be related to the product reliability and observe its degradation over time [1]. In this project, a case study, which was already done previously by S.T. Tseng, M.S. Hamada, and C.H. Chiao, will be discussed and the results found in this paper will be regenerated by using MATLAB. This case study uses an experiment to improve the reliability (or lifetime) of florescent lamps [2]. For florescent lamps, failure is defined in terms of the amount of degradation in luminosity [1].

Keywords
Reliability Experiment, Degradation, Lognormal Distribution, Simple linear Regression, Fractional Factorial Design

INTRODUCTION

The paper proposes that design of experiment is broadly used in industries for improving quality, but less work was done for improving reliability. An experiment was conducted to find which factors associated with luminosity degradation affected the product performance and to identify their optimal settings in order to improve reliability [3]. In the florescent lamp experiment presented in the paper, unlike lifetime data, a degradation data was collected by choosing the most appropriate degradation characteristic, which is luminosity as given in this paper, throughout the life of the product [3]. A degradation data defines the failure as condition when the degradation characteristic crosses a certain threshold value. In florescent lamp experiment, fluorescent light bulb is considered as failed when the luminosity of the light bulb falls below a certain value such as 60% of its luminosity after 100 hours of use or aging [1, 3].

A MODEL FOR RELATIVE LUMINOSITY

A model for relative luminosity is given below

$$\ln[\Lambda(t)/\Lambda(100)] = -\lambda(t-100) \tag{1}$$

If the above equation is rewritten according to industry's standard definition for failure time, which is when a lamp's luminosity $\Lambda(t)$ falls below 60% of its luminosity after 100 hours of use or aging, i.e., $0.6\Lambda(100)$, the equation will be shown as

$$t = -\ln 0.6/\lambda + 100 \tag{2}$$

The parameter λ is known as the rate of degradation, which is the slope of the luminosity degradation path and a random variable. Figure 1 illustrates the luminosity degradation path for a single lamp. The lamp fails at the point where relative luminosity crosses 0.6. It is easily seen that ln(t-100) and $\ln\lambda$ are linearly related. Therefore, if λ has a lognormal distribution failure time will have a lognormal distribution. To see the failure times follow some distribution, degradation paths will be plotted for 100 lamps. Figure 2 illustrates the simulated degradation paths for 100 lamps. Figure 3 is a histogram which illustrates the failure times for 100 lamps. As noticed, the histogram is skewed to the left, meaning that this is the characteristic of lognormal distribution compared to normal distribution [1].

SELECTION OF EXPERIMENTAL DESIGN and CONTROL FACTORS

Three production factors which are expected to affect the luminosity of a lamp during manufacturing process were inquired by using fractional factorial design. Three factors proposed in the paper are (A) the amount of electric current in the exhaustive process (B) the concentration of the mercury dispenser in the mercury dispenser coating process, and (C) the concentration of argon in the argon filling process. These three factors were examined by using 2^{3-1} fractional design with I=ABC defining relation. Figure 4 illustrates the degradation paths for the four runs, which is compiled by utilizing the data in Table 1 that gives relative luminosities at the different inspection times for the five lamps from four production runs [1, 2]. Level for four runs were chosen like below matrix (X matrix)

$$\begin{array}{ccccc} & \textit{Intercept} & A & B & C \\ 1 & +1 & -1 & -1 & -1 \\ 2 & +1 & -1 & +1 & +1 \\ 3 & +1 & +1 & -1 & +1 \\ 4 & +1 & +1 & +1 & -1 \end{array}$$

SIMPLE REGRESSION ANALYSIS FOR DEGRADATION DATA

As mentioned previously, there is an assumption that if λ is lognormally distributed failure times will be lognormally distributed. Based on this assumption, we will make two-stage analysis: (1) a fitted degradation rate $\hat{\lambda}$ for each lamp was calculated by using simple linear regression, and thereafter predicted failure times $\hat{t}$ will be calculated for each lamp. (2) Log of the predicted failure times found were analyzed in terms of the control factors. Using degradation paths in Figure 4, the below model is fitted to degradation path for each lamp at the level where ε is assumed to follow a normal distribution [1].

$$L(t) = \ln[\Lambda(t)/\Lambda(100)] = -\lambda(t-100) + \varepsilon \tag{3}$$

In order to find the least square estimates of degradation paths $\hat{\lambda}$ given in Table 2, simple linear regression principles were followed [4]. Related predicted failure times $\hat{t}$ for 20 lamps given in table 2 based on $\hat{\lambda}$ were found by below equation

$$\hat{t}_{ij} = -\ln 0.6 / \hat{\lambda}_{ij} + 100 \tag{4}$$

where i indicates the run number and j the lamp number [1].

IDENTFYING THE IMPORTANT FACTORS

Second step of two-stage analysis, as mentioned above, is to identify the important factors by using the log of the predicted failure times. The lognormal probability plot of $\hat{t}_{\text{ij}}$ in Figure 5 proves that the lognormal distribution visually fits the predicted failure times. Furthermore, least square estimates of normal location and scale parameters $\hat{\mu}_i$ and $\hat{\sigma}_i$ of the logged failure times for each run are found by using "logfit" MATLAB function. They are shown in Table 2 as well. By testing the variance homogeneity [5], the least square estimate of variances in Table 2 suggests no differences in the variances at the 0.05 level. Consequently, standard methods for analyzing replicated fractional designs can be applied to identify the important factors [1].

Least square estimates of the logged predicted failure times (replicates) are obtained by regression analysis based on factors with X matrix given above. Table 3 indicates that the only factors B and C are significant. Thus, the fitted model for the log predicted failure time is

$$\ln\hat{y} = 9.55 + 0.17x_B + 0.08x_C \quad (5)$$

To improve reliability if we set the factors B and C at the high levels (+, +), which are recommended levels

$$\hat{y} = \exp[9.55 + 0.17(+1) + 0.08(+1)] = 18{,}034 \text{ hours} \quad (6)$$

If compared with the estimated failure time at the original process settings (-, -)

$$\hat{y} = \exp[9.55 + 0.17(-1) + 0.08(-1)] = 10{,}938 \text{ hours} \quad (7)$$

Therefore, the relaibilty will be improved by 65% at the recommended settings [1, 2]

CONCLUSION

This case study showed that how degradation data and design of experiment whose results are from a simple four run can be used for improving reliability [2]. Additionally, the important factors determined by fractional factorial design were compared to the important factors determined by full factorial design. Figure 6 illustrates that full factorial experiment, which is made by 8 runs with the same 3 factors, shows that B and C are still important factors. Fractional factorial design is, therefore, chosen for run size economy [1]. A general degradation model [6] can be considered if the degradation data is remodeled.

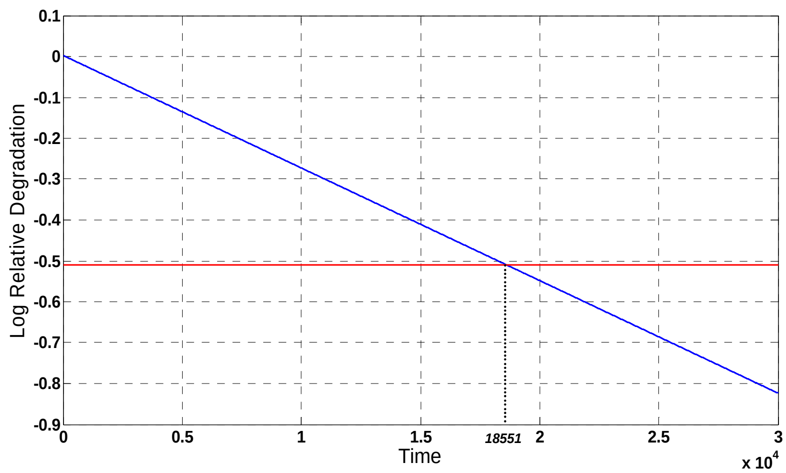

Figure 1: Failure Time for a Single Lamp

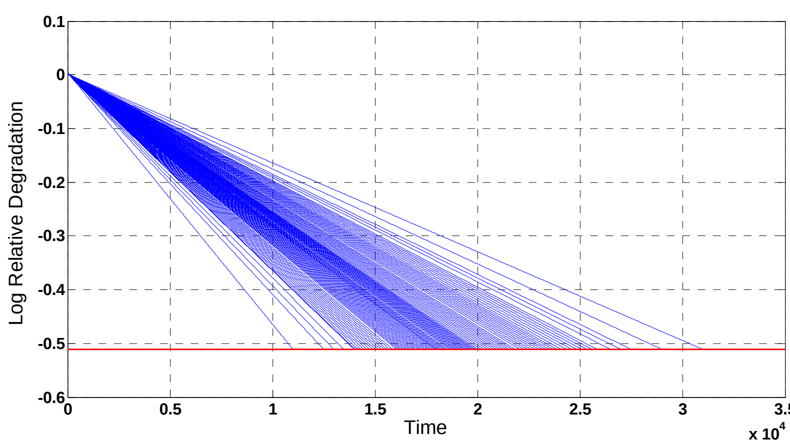

Figure 2: Failure Times for 100 Lamps

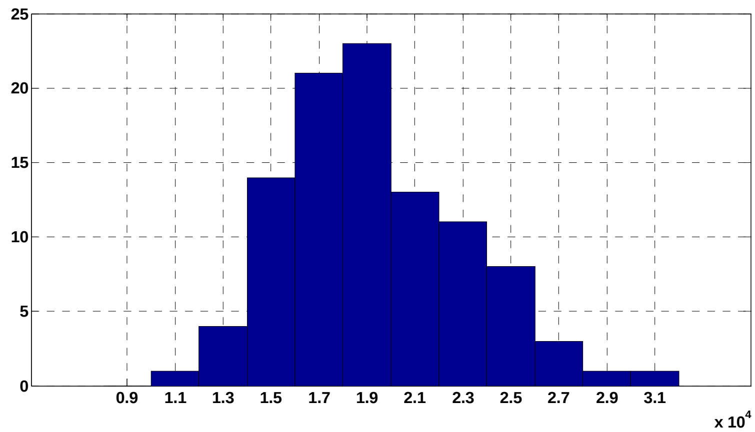

Figure 3: Histogram of 100 Lamp Failure Times

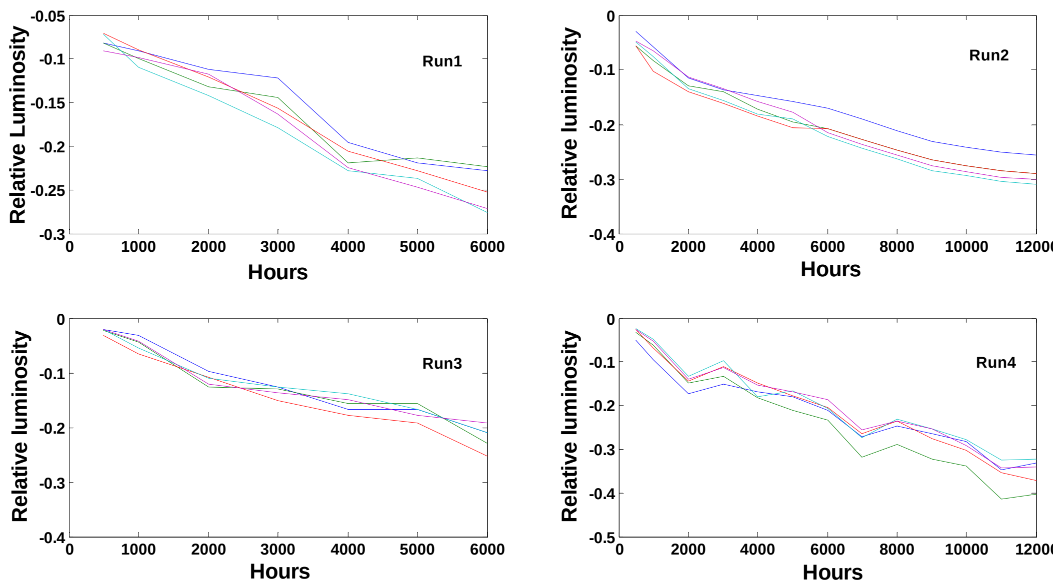

Figure 4: Degradation Paths, compiled by data in Table 1

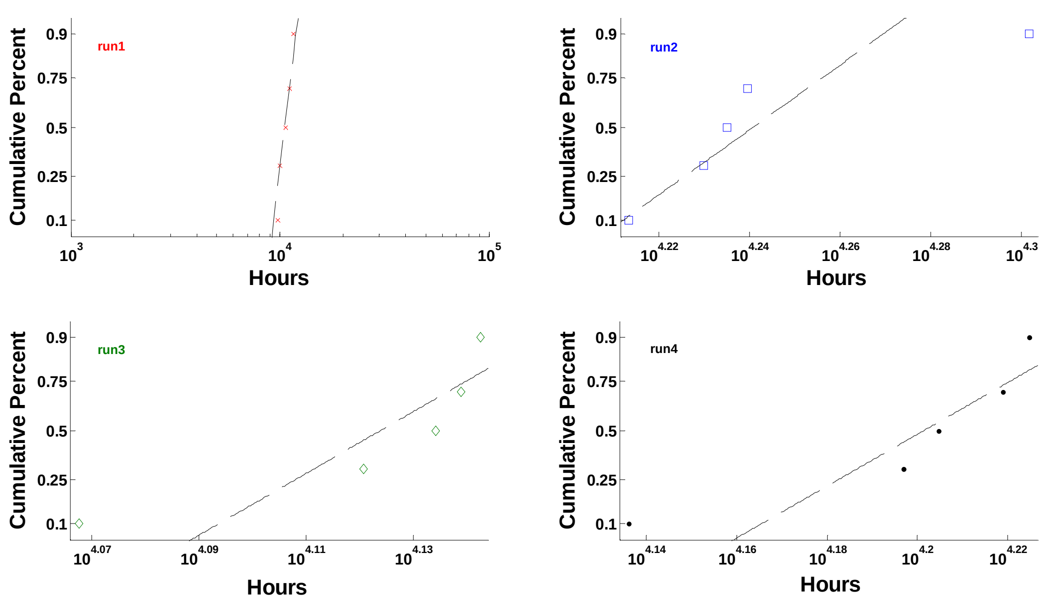

Figure 5: Lognormal Plot of Predicted Failure Times

Table 1: Relative Luminosity Data, 4 Production Runs, and 5 Lamps at different Inspection times

		500	**1000**	**2000**	**3000**	**4000**	**5000**	**6000**						
RUN1	**Lamp1**	-0.0822	-0.0903	-0.1122	-0.1225	-0.1958	-0.2187	-0.2285						
	Lamp2	-0.0817	-0.0999	-0.1322	-0.1444	-0.2186	-0.2136	-0.2237						
	Lamp3	-0.0702	-0.0898	-0.1209	-0.1564	-0.2054	-0.2279	-0.2522						
	Lamp4	-0.0719	-0.1094	-0.1417	-0.1785	-0.2282	-0.2374	-0.2761						
	Lamp5	-0.0912	-0.0983	-0.1172	-0.1634	-0.2244	-0.2468	-0.2712						
		500	**1000**	**2000**	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9000**	**10000**	**11000**	**12000**
RUN2	**Lamp1**	-0.0302	-0.0575	-0.1152	-0.1362	-0.1475	-0.1585	-0.1705	-0.1905	-0.2109	-0.2312	-0.2414	-0.2516	-0.2567
	Lamp2	-0.0556	-0.0829	-0.1297	-0.1403	-0.173	-0.1955	-0.2076	-0.2278	-0.248	-0.2649	-0.2751	-0.285	-0.2902
	Lamp3	-0.0556	-0.1031	-0.1407	-0.1621	-0.184	-0.2067	-0.2076	-0.2278	-0.248	-0.2649	-0.2751	-0.285	-0.2902
	Lamp4	-0.0486	-0.0762	-0.1346	-0.1562	-0.1809	-0.19	-0.2231	-0.2436	-0.264	-0.2843	-0.2944	-0.3046	-0.3095
	Lamp5	-0.0473	-0.0654	-0.1131	-0.1345	-0.1572	-0.1783	-0.2159	-0.236	-0.2559	-0.2764	-0.2865	-0.2967	-0.3016
		500	**1000**	**2000**	**3000**	**4000**	**5000**	**6000**						
RUN3	**Lamp1**	-0.0205	-0.0304	-0.0968	-0.1257	-0.1663	-0.1667	-0.2099						
	Lamp2	-0.0215	-0.0442	-0.1263	-0.1293	-0.1555	-0.156	-0.2302						
	Lamp3	-0.0315	-0.066	-0.1078	-0.1511	-0.1773	-0.1927	-0.2519						
	Lamp4	-0.0205	-0.055	-0.1103	-0.1257	-0.1379	-0.1667	-0.2099						
	Lamp5	-0.0203	-0.0414	-0.1213	-0.1367	-0.1489	-0.1777	-0.1913						
		500	**1000**	**2000**	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9000**	**10000**	**11000**	**12000**
RUN4	**Lamp1**	-0.0496	-0.0938	-0.1721	-0.1504	-0.1692	-0.1796	-0.2105	-0.2712	-0.246	-0.2648	-0.2827	-0.3481	-0.332
	Lamp2	-0.0319	-0.0621	-0.1486	-0.1333	-0.1817	-0.2104	-0.2325	-0.3185	-0.2881	-0.3231	-0.3385	-0.4149	-0.4038
	Lamp3	-0.0261	-0.0676	-0.1429	-0.1098	-0.148	-0.1775	-0.2041	-0.2653	-0.2358	-0.2751	-0.3032	-0.354	-0.371
	Lamp4	-0.0229	-0.047	-0.132	-0.0963	-0.1804	-0.1666	-0.2062	-0.2727	-0.2313	-0.254	-0.2781	-0.3254	-0.3235
	Lamp5	-0.0264	-0.0528	-0.1396	-0.112	-0.1527	-0.168	-0.1856	-0.2562	-0.2364	-0.2526	-0.2909	-0.3417	-0.3403

Table 2: Estimates for Degradation Model, Predicted Failure Times and Location-Scale Parameters of the Lognormal Distribution

RUN1	$\hat{\lambda}_{ij}$ **10^{-5}**	$\hat{t}_{ij}$	$\hat{\mu}_i$	$\hat{\sigma}_i$	**RUN2**	$\hat{\lambda}_{ij}$ **10^{-5}**	$\hat{t}_{ij}$	$\hat{\mu}_i$	$\hat{\sigma}_i$
Lamp1	4.44	11598.26	9.27	0.0732	Lamp1	2.56	20038.17	9.77	0.0780
Lamp2	4.61	11181.04			Lamp2	2.96	17363.95		
Lamp3	4.82	10689.06			Lamp3	2.99	17181.96		
Lamp4	5.28	9771.20			Lamp4	3.15	16339.77		
Lamp5	5.17	9664.97			Lamp5	3.03	16974.84		
RUN3	$\hat{\lambda}_{ij}$ **10^{-5}**	$\hat{t}_{ij}$	$\hat{\mu}_i$	$\hat{\sigma}_i$	**RUN4**	$\hat{\lambda}_{ij}$ **10^{-5}**	$\hat{t}_{ij}$	$\hat{\mu}_i$	$\hat{\sigma}_i$
Lamp1	3.78	16320.22	9.49	0.0710	Lamp1	3.21	16020.39	9.66	0.0816
Lamp2	3.90	13203.61			Lamp2	3.76	13678.42		
Lamp3	4.41	11685.35			Lamp3	3.27	15740.21		
Lamp4	3.71	13884.22			Lamp4	3.06	16779.90		
Lamp5	3.74	13767.50			Lamp5	3.10	16557.92		

Table 3: Least Square Estimates, Standard Errors, t Statistics, and p value from Predicted Failure Times

Effect	Estimate	Standard Error	t	P Value
Intercept	9.5483	0.0152	627.4485	0.0000
A	0.0271	0.0152	1.7822	0.0899
B	0.1689	0.0152	11.0987	0.0000
C	0.0819	0.0152	5.3849	0.0000

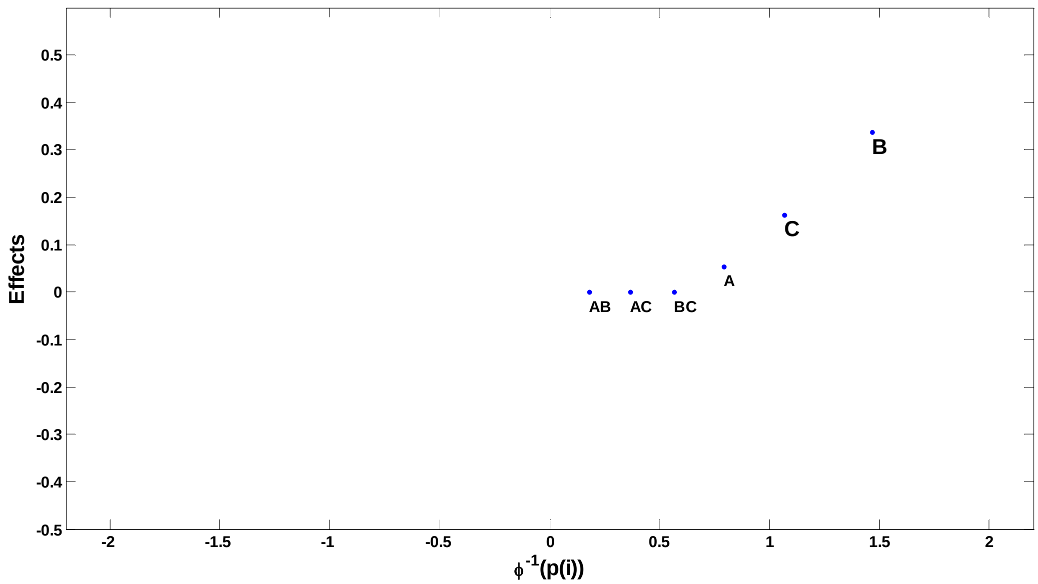

Figure 6: Half-Normal Plot of Factors, Full Factorial Design

References

1. Wu, C.F.J. and Hamada, M., 2006, Chapter 12.7: Reliability Experiments with Degradation Data, 2rd Edition, John Wiley & Sons Inc., New York.
2. Tseng, S.T., Hamada, M., and Chiao, C.H., 1995, “Using Degradation Data to Improve Florescent Lamp Reliability,” Journal of Quality Control, 27, 363-369.
3. Joseph, V.R. and Yu, I., 2006, “Reliability Improvement Experiments with Degradation Data,” IEEE Transactions on Reliability, 5, 149-157.
4. Montgomery, C.D. and Runger, G.C.,2007, Chapter 11.2: Simple Linear Regression and Correlation, 4rd Edition, John Wiley & Sons Inc., New York.
5. Wu, C.F.J. and Hamada, M., 2006, Chapter 3.14: Testing Variance Homogeneity, 2rd Edition, John Wiley & Sons Inc., New York.
6. Gorjian, N., Ma, L., Mittinty, M., Yarlagadda, P., and Sun, Y., 2009, “A review on Degradation Models in Reliability Analysis,” Proceedings of the 4th World Congress on Engineering Asset Management, 28-30, September 2009, Marriott Athens Ledra Hotel, Athens.