Introduction to Robust Parameter Design

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Abstract

It is possible to incorporate quality in the products from their design without increase their cost, the problems should be eliminated in the designing phase, not in the manufacture or in the field. According to this perspective, it is necessary to design robust products that tolerate variations in the production process and during servicing. The statistical methods should select the important factors affecting the design. In the design of the parameters determining the levels or values of the controllable factors (design parameters such as pressure applied to the Blister) to minimize the effect of uncontrollable factors, noise factors, on the characteristics of the finished product.

Keywords
Control factors, noise factors, location factors, dispersion factors, adjustment factor.

1. Introduction

To implement his concepts, Taguchi recommends methods that deviate part from those used in the classic experimental design; the terminology used is also somewhat different. First, Taguchi divides the factors of an experiment in controllable factors and uncontrollable factors, or noise. According to the methodology of design parameters, Taguchi recommends selecting two experimental designs, one for the controllable factors and one for the noise. In general, for conducting experiments it is apply fractional factorial design and also designs that are of the orthogonal type. The designs are combined in the layout of the design parameters, a sketch of two components:

- The settlement of the controllable factors (control factors).
- The settlement of uncontrollable factors (noise factors).

Control factors are those variables that once they are chosen their values remain fixed, while noise factors are variable that are uncontrollable during the process or use conditions. This paper will use one experiment strategies, which use cross array, and two modeling strategies, location and dispersion modeling and response modeling. Robust parameter design main focus is to improve the fundamental function of the process or product; as a result it is possible to have flexible designs and concurrent engineering. Robust design is said to be the most powerful methodology to reduce cost, improve quality and simultaneously reduce development interval.

2. Research problems

The problems that are going to be used to help to the understanding of the topic describe in this paper are the original layer growth experiments and the leaf spring experiment.

2.1 A robust Parameter design perspective of the Layer Growth Experiment

The experimental factors of the original layer growth experiments can be viewed in table 1. They are eight control factors (A-H) and two noise factor (L and M). On a susceptor there are four facets, this means that factor M has four levels. There are top and bottom position (factor L) on each facet. The original experiment, Kackar and Shoemaker (1986) reported result for six facets, for this paper it will only be consider only four facets.

Table 1: Factors and levels, Layer Growth Experiments

Control Factor	Level	
	-	+
A. Suspector-rotation method	Continuous	Oscillating
B. Code of wafers	668G4	678D4
C. Deposition temperature C	1210	1220
D. Deposition time	short	long
E. Arsenioc flow rate (%)	55	59
F. Hydrochloric acid etch temp C	1180	1215
G. Hydrochloric acid flow rate (%)	10	14
H. Nozzle position	2	6
Noise Factor	**Level**	
	-	+
L. location	Bottom	Top
M. facet	1 2	3 4

The experimental plan used a 2_{IV}^{8-4} design for the eight control factor with the following defining generators $D = -ABC, F = ABE, G = ACE, and\ H = BCE.$ As it can be viewed in table 2. A 2 X 4 design was used for the two noise factors. There are eight observations per control factors setting. The nominal value for thickness is 14.5 µm with specifications limits 14.5 ± 0.5 µm. The main objective is to minimize the epitaxial layer nonuniformity over facets 1-4 and top/bottom position while maintaining an average thickness of 14.5 µm.

Table 2: Cross Array and Thickness Data, Layer Growth Experiment

Control Factor								Noise Factor							
								L-Bottom				L-Top			
A	B	C	D	E	F	G	H	M-1	M-2	M-3	M-4	M-1	M-2	M-3	M-4
-	-	-	+	-	-	-	-	14,2908	14,1924	14,2714	14,1876	15,3182	15,4279	15,2657	15,4056
-	-	-	+	+	+	+	+	14,803	14,7193	14,696	14,7635	14,9306	14,8954	14,921	15,1349
-	-	+	-	-	-	+	+	13,8793	13,9213	13,8532	14,0849	14,0121	13,9386	14,2118	14,0789
-	-	+	-	+	+	-	-	13,4054	13,4788	13,5878	13,5167	14,2444	14,2573	14,3951	14,3724
-	+	-	-	-	+	-	+	14,1736	14,0306	14,1398	14,0796	14,1492	14,1654	14,1487	14,2765
-	+	-	-	+	-	+	-	13,2539	13,3338	13,192	13,443	14,2204	14,3028	14,2689	14,4104
-	+	+	+	-	+	+	-	14,0623	14,0888	14,1766	14,0528	15,2969	15,5209	15,42	15,2077
-	+	+	+	+	-	-	+	14,3068	14,4055	14,678	14,5811	15,01	15,0618	15,5724	15,4668
+	-	-	-	-	+	+	-	13,7259	13,2934	12,6502	13,2666	14,9039	14,7952	14,1886	14,6254
+	-	-	-	+	-	-	+	13,8953	14,5597	14,4492	13,7064	13,7546	14,3299	14,2224	13,8209
+	-	+	+	-	+	-	+	14,2201	14,3974	15,2757	15,0363	14,1936	14,4295	15,5537	15,22
+	-	+	+	+	-	+	-	13,5228	13,5828	14,2822	13,8449	14,564	14,467	15,2293	15,1099
+	+	-	+	-	-	+	+	14,5335	14,2492	14,6701	15,2799	14,7437	14,1827	14,9695	15,5484
+	+	-	+	+	+	-	-	14,5676	14,031	13,7099	14,6375	15,8717	15,2239	14,97	16,0001
+	+	+	-	-	-	-	-	12,9012	12,7071	13,1484	13,894	14,2537	13,8368	14,1332	15,1681
+	+	+	-	+	+	+	+	13,9532	14,083	14,1119	13,5963	13,8136	14,0745	14,4313	13,6862

2.2 A robust Parameter design perspective of the Leaf Spring Experiment

The factors for the leaf spring experiment are listed in the table 3. The quench oil temperature Q is not controllable in normal production. Q can be chosen in two ranges of values, 130-150 F and 150-170 F, and it is treated as a noise factor. There are four control factors (B-E) and one noise factor (Q).the design and the free height data can be viewed in table 4. A 2_{IV}^{4-1} design with *I=BCDE* was chosen for the control factors. Two noise factor were chosen for each control factor each control and noise factor is replicated three times. The main goal is to minimize the variation about the target that is 8 inches.

Table 3: Factors and Levels, Leaf Spring

Control Factor	Level -	Level +
B. High heat temperature F	1840	1880
C. Heating time sec.	23	25
D. Transfer time sec.	10	12
E. Hold down time sec.	2	3
Noise Factor	**-**	**+**
Q. Quench oil Temperature F	130-150	150-170

Table 4: Cross Array and Height Data, Leaf Spring Experiment

Control Factor				Noise Factor					
B	C	D	E	Q-			Q+		
-	+	+	-	7.78	7.78	7.81	7.50	7.25	7.12
+	+	+	+	8.15	8.18	7.88	7.88	7.88	7.44
-	-	+	+	7.50	7.56	7.50	7.50	7.56	7.50
+	-	+	-	7.59	7.56	7.75	7.63	7.75	7.56
-	+	-	+	7.94	8.00	7.88	7.32	7.44	7.44
+	+	-	-	7.69	8.09	8.06	7.56	7.69	7.62
-	-	-	-	7.56	7.62	7.44	7.18	7.18	7.25
+	-	-	+	7.56	7.81	7.69	7.81	7.50	7.59

3. Why use Robust Parameter Design method.

During the last years companies have tried to reduce waste during the manufacturing and operations, these companies have invested in Six Sigma approach, making an impact on the cost and hence in the bottom line of those companies. Numerous of them have achieved the maximum potential of the traditional Six Sigma approach. Then the question that has to be answer is what would be the engine for the next wave of productivity improvement?

Robust Design method is essential to improving engineering productivity. Established by Dr. Genichi Taguchi past the Second World War, the technique has evolved over the last five decades. There are many companies around the world have make significant saves of hundreds of millions of dollars because they have used the technique, some of the industries that have implement the method are: automobiles, xerography, telecommunications, electronics, software, etc.

Table 5: Comparison of Strengths and Weaknesses of Two Approaches

COMPARISON OF STRENGTHS AND WEAKNESSES OF TWO APPROACHES	
Strenghs	
Classical experimental design	RD
You can explore all the interactions between factors at the same time. No need for a deep knowledge of the functioning of the processes.	The "philosophy" Taguchi as a whole is highly recommended. The concept of loss function is useful and innovative.
Weaknesses	
You can not take or use prior knowledge about the process. There is no way to streamline the processes really thinking about how interacting inputs.	Leads to complex experimental designs. Promotes experiments sometimes inefficient. there are some problems with the methods of data analysis.

4. How the Problem was solved.

The used of Matlab was to replicate all the half/normal plots and iterations plots to compare this plots with the ones that were in the book, and then apply the methodologies that are going to be describe further to analyze this plots and make conclusions about the problems.

4.1 Experimentation and modeling strategies Cross Array

The noise factors have to be varied in the experiment to show the noise variation that happens in a normal process or use conditions, this happens when these noise factors are identified. As mentioned before these factors are difficult to control or uncontrollable, because of this it is needed extra effort to vary them during the parameter experiment. For a better understanding we can use the Leaf Spring experiment to describe the robustness to the temperature variation. As it is mentioned before the quench oil temperature is not controllable which make this factor a noise factor, then it is necessary to use two or more levels or more levels of the temperature for the experiment. Trying to control the temperature in two different ranges will make the experiment longer and it will require an extra effort.

A control array is the design matrix for the control factors and a noise array is the design matrix for the noise factors. Each level combinations of the control array are cross with all the level combinations of the noise array. A control array is the one that consist of all the level combination between those in the noise array and those in the control array. Frequently orthogonal are used for the control array and for the noise array. This is because orthogonal array represent points on the region of the noise factor running an experiment based on a cross array can be understood as taking a sample of noise variation. When there are more than three levels of the noise factors the orthogonal array for the noise factor is going to be large.

4.1.1 Location and Dispersion

The location and dispersion modeling approach constructs models for measures of location and dispersion separately in terms of the control factor main effects and interactions. For each control factor at each setting, $\bar{y}_i$ and ln S_i^2(the sample mean and log sample variance over the noise replicates) are used as the measures of location and dispersion and are computed as:

$$\bar{y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij}, \qquad S_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_i)^2, \qquad (1)$$

Where n_i is the number of noise replicates for the *i*th control factor setting. The location factors are defined as those factors that appear at the location model and the dispersion factors are those factors that appear at the dispersion model. For those factors that are location factors but are not dispersion factors are called adjustment factors. Following is the procedure of nominal the best and it is helpful to obtain settings of control factors.

Two-Step Procedure for Nominal-the-Best Problem

i. Select the levels of the dispersion factors to minimize dispersion.
ii. Select the level of the adjustment factor to bring the location on target.

If the factor use in step two also is a dispersion factor this could affect both the location and dispersion. As a consequence of the problem mentioned before would be that while the location is going to be adjusted on target the dispersion is going to increase and this will require to the dispersion factors and the iterations between the two steps. If one of the adjustment factor is not able to bring the location on target it may required to use two or more adjustment factor in step ii to do this.

Two-Step Procedure for Larger-the-Better and Smaller-the-Better Problems

i. Select the levels of the location factors to maximize (or minimize) the location
ii. Select the levels of the dispersion factors that are not location factors to minimize dispersion.

This location-dispersion modeling approach is really simple to understand and also to put into practice. This is used where the experiment is based on cross array.

Layer growth experiment

The $y_i$and ln S_i^2 for the 16 control factor combinations of the layer growth experiment can be observed in table 6. Half-normal of the location and dispersion were use to identify the important location and dispersion effects (this was base on the eight main effects and seven two-factor iterations involving factor A), using $\overline{y_i}$ and $z_i = lnS_i^2$ as responses, this half-normal plots are presented in figure 1. From the plots we can conclude that D is an important location factor and A and H are important dispersion factors.

Table 6: Means, Log Variances, and Signal to noise Ratios, Layer Growth Experiment

Control Factor											
A	B	C	D	E	F	G	H	yi	lnsi2	lnyi	ni
-	-	-	+	-	-	-	-	14.795	-1.018	5.389	6.407
-	-	-	+	+	+	+	+	14.858	-3.879	5.397	9.276
-	-	+	-	-	-	+	+	13.998	-4.205	5.278	9.483
-	-	+	-	+	+	-	-	13.907	-1.623	5.265	6.888
-	+	-	-	-	+	-	+	14.145	-5.272	5.299	10.570
-	+	-	-	+	-	+	-	13.803	-1.236	5.250	6.485
-	+	+	+	-	+	+	-	14.728	-0.760	5.380	6.140
-	+	+	+	+	-	-	+	14.885	-1.503	5.401	6.904
+	-	-	-	-	+	+	-	13.931	-0.383	5.268	5.651
+	-	-	-	+	-	-	+	14.092	-2.176	5.291	7.467
+	-	+	+	-	+	-	+	14.791	-1.238	5.388	6.626
+	-	+	+	+	-	+	-	14.325	-0.868	5.324	6.192
+	+	-	+	-	-	+	+	14.772	-1.483	5.385	6.869
+	+	-	+	+	+	-	-	14.876	-0.418	5.400	5.817
+	+	+	-	-	-	-	-	13.755	-0.418	5.243	5.661
+	+	+	-	+	+	+	+	13.969	-2.636	5.274	7.910

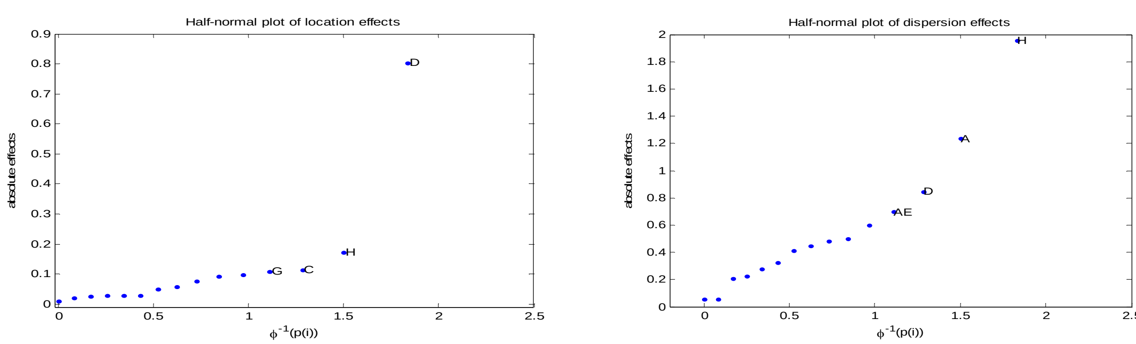

Figure 1: Half-normal plots of location and dispersion effects, layer growth experiment.

The corresponding location and dispersion models are as follow:

$$\hat{y} = 14.352 + 0.402\ X_D$$

And

$$\hat{z} = -1.822 + 0.619X_A - 0.982X_H \qquad (2)$$

Using the two step procedure for nominal-the best, the recommended levels to reduce variation are A at the low level (-) and H at the high level (+). Then D can be used to bring the mean on target. By solving we have that:

$$\hat{y} = 14.352 + 0.402\, X_D = 14.5$$
$$0.402X_D = 14.5 - 14.352$$
$$X_D = 0.368$$

This means that the deposition time D should be equal to:

$$short\ time + \frac{0.368—(-1)}{1-(-1)}(long\ time - shorttime)$$
$$0.316\ short\ time + 0.684\ long\ time,$$

It can be appreciated that is approximately longer than the short time by two-thirds of the time span between the short and long times.

Leaf spring experiment

The $y_i$and ln S_i^2 for the 8 control factor combinations of the leaf spring experiment can be observed in table 7. This was obtain using the six observations , which consist of three replicates each at two levels of noise factor Q. Half-normal plots of location and dispersion effects are used to identify the important location and dispersion effect, this plots are in figure 2. From figure 2 it can be observed that B, C, and E are important location effects and C is an important dispersion factor.

Table 7: means and Log Variances, Leaf Spring Experiment.

Control Factor					
B	C	D	E	yi	lnsi2
-	+	+	-	7.540	-2.408
+	+	+	+	7.902	-2.649
-	-	+	+	7.520	-6.949
+	-	+	-	7.640	-4.838
-	+	-	+	7.670	-2.399
+	+	-	-	7.785	-2.939
-	-	-	-	7.372	-3.270
+	-	-	+	7.660	-4.058

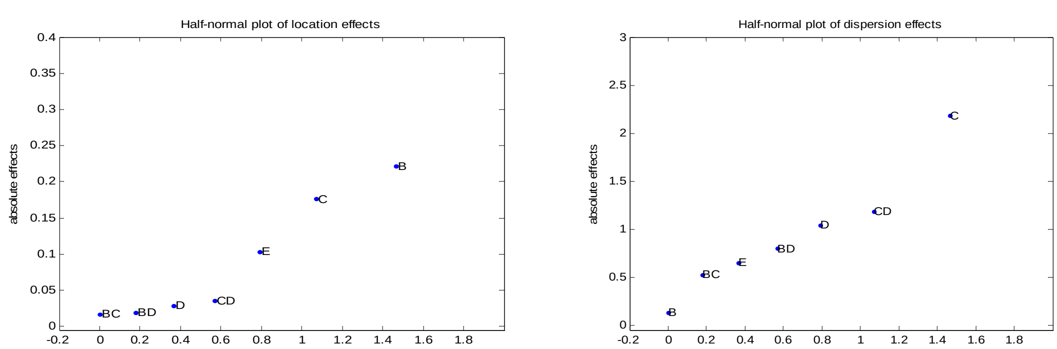

Figure 2: Half-normal plots of location and dispersion effects, leaf spring experiment.

The corresponding location and dispersion models are as follow:

$$\hat{y} = 7.6360 + 0.1106\ X_B + 0.0881 X_C + 0.0519 X_E \qquad (3)$$

And

$$\hat{z} = -3.6886 + 1.0901 X_C \qquad (4)$$

Using the two step procedure for nominal-the best, set C at a low level (-) to reduce dispersion and then use factors b and E to bring the location on target. With $X_C = -1$the equation for $\hat{y}$ becomes

$$\hat{y} = 7.5479 + 0.1106\ X_B + 0.0519 X_E \qquad (5)$$

We can notice that the target 8 inches cannot be achieve unless X_B and X_E can be chosen beyond +1. The several options can be discuss.

First option cannot be increase B and E beyond +1. Then for E and B at their high level (+) and C at his low level (-) using (3) and (4), it can be obtain,

$$\hat{y} = 7.6360 + 0.1106\ (1) + 0.0881(-1) + 0.0519(1) = 7.71$$

And

$$\hat{\sigma}^2 = exp\hat{z} = \exp\big(-3.6886 + 1.0901(-1)\big) = 0.0084$$

And the mean square error is

$$MSE = (7.71 - 8)^2 + 0.0084 = 0.0925$$

This has the adverse of moving the mean away from the target.

Second option cannot be increase B and E beyond +1. Then for E and B at their high level (+) and C at his high level (+) using (3) and (4), it can be obtain,

$$\hat{y} = 7.6360 + 0.1106\ (1) + 0.0881(1) + 0.0519(1) = 7.89$$

And

$$\hat{\sigma}^2 = exp\hat{z} = \exp\big(-3.6886 + 1.0901(1)\big) = 0.0744$$

And the mean square error is

$$MSE = (7.89 - 8)^2 + 0.0744 = 0.0865$$

This is smaller than 0.0925 obtained by the two step procedure because decrease in squared bias dominates the increase in variance.

Third option is to increase B and E beyond +1. B and E can be chosen to 2.78 this by putting (5) =8 and C at a low level (-). Then the MSE is going to be

$$MSE = (8 - 8)^2 + 0.0084 = 0.0084$$

It can be viewed that this option has the smaller MSE. Among the three options discuss previously, the first one, where the two step procedure is implemented is the worst option, this shows that the two step procedure sometimes fail to find optimal control settings if its second step cannot bring the location on target. When the deviation of the location from target is significant it may overwhelm the gain made in the first step by reducing dispersion.

4.1.2 Response Modeling

One disadvantage of the location-dispersion modeling is that it would mask some important relationship between controls and noise factor when the location and dispersion are in terms of the control factors. A response model treats the response $\hat{y}$ as a function of both the control and noise factors. The analysis then consists of two parts:

i. Make control-by-noise interaction plots for the significant interaction effects in the response model. From these plots, control factor settings, at which y has a flatter relationship with the noise factor, are chosen as robust settings.
ii. Based on the fitted model$\hat{y}$, compute Var ($\hat{y}$) with respect to the variation among the noise factors in $\hat{y}$. Call Var ($\hat{y}$) the predicted variance model. Because Var ($\hat{y}$) is a function of the control factors, we can use it to identify control factor settings with small predicted variance.

The layer growth experiment will be use to illustrate the response model approach.

Layer growth experiment

A half-normal plot was use to identify the important effects, it can be viewed in figure 3. Because the noise factor M has 4 levels, it is going to be using the following three orthogonal contrasts for the M main effects:

$$Ml = (M1 + M2) - (M3 + M4),$$
$$Mq = (M1 + M4) - (M2 + M3),$$
$$Mc = (M1 + M3) - (M2 + M4),$$

Where M_i stands for the effect of facet i. (Each of Ml, Mq and Mc can be viewed as a two-level factor with − on two facets and + on the other two facets.). A response model includes:

- 15 control effects (A,B,C,D,E, F, G,H,AB,AC,AD,AE,AF,AG,AH)
- 7 noise effects (L, Ml, Mq, Mc, LMl, LMq, LMc)
- 15 × 7 control-by-noise effects: xy, where x is a control effect and y a noise effect.

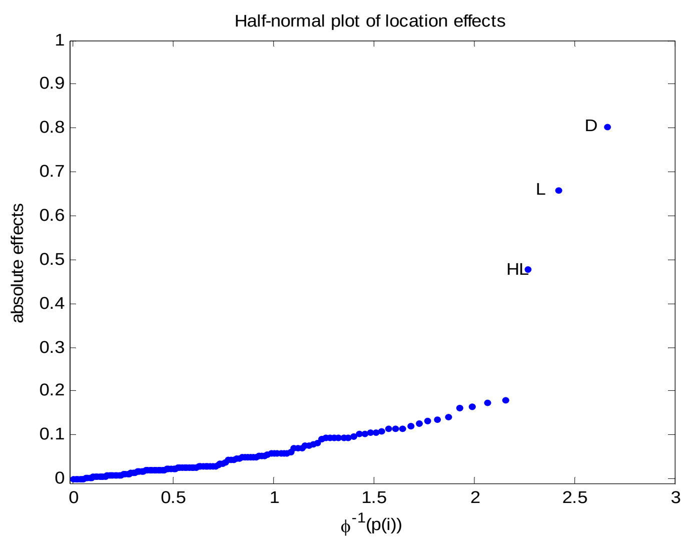

Figure 3: Half-normal plot of response model effects, layer growth experiment.

From figure 3 we can identify that the important effects are D, L and HL. The next four effects are H, Ml, CMl and AHMq, once the the D, L and HL effects are remove we can see that the effects mentioned before can be viewed in the half-normal plot, figure 4.

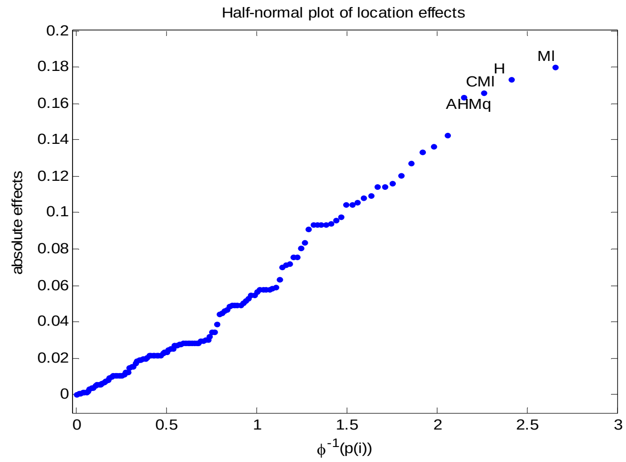

Figure 4: Half-normal plot of response model effects (D, L, HL, removed), layer growth experiment.

The model including these seven effects is as follow:

$$\hat{y} = 14.352 + 0.402\, X_D + 0.087 X_H + 0.330 X_L - 0.090 X_{Ml} - 0.239 X_H X_L - 0.083 X_C X_{Ml} - 0.082 X_A X_H X_{Mq} \quad (6)$$

This model contains three control-by-noise iterations effects. These control-by-noise plots can be used to choose control factors settings to reduce the response variation due to noise. From figure 5 the recommended levels are high level for H and low level for C. also it suggest to set A at a low level since the two middle response curve are more al A- are more flatter than the other two curves at A+, this can be seen at figure 6. D does not interact with the noise factors M (and L) then D is an adjustment factor this is show in figure 6. The recommended settings H = + and A = − are consistent with those based on the dispersion model previously. The response model provides the additional information that H interacts with the noise factor L and that A interacts with the noise factor M.

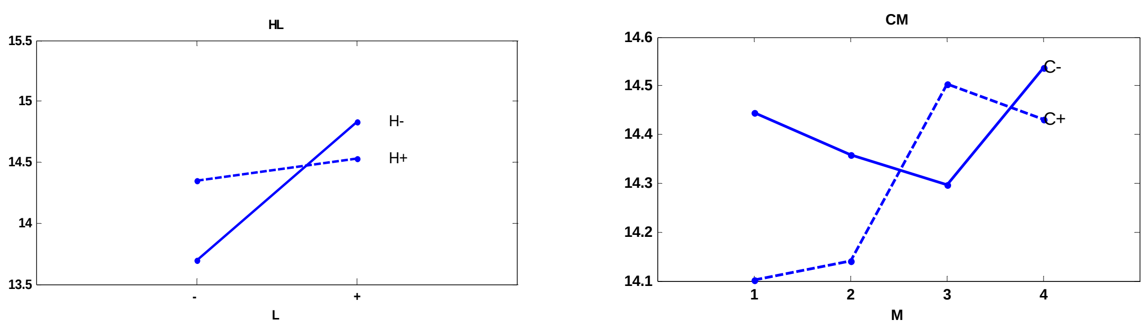

Figure 5:H x L and C x M iterations plot, layer growth experiments

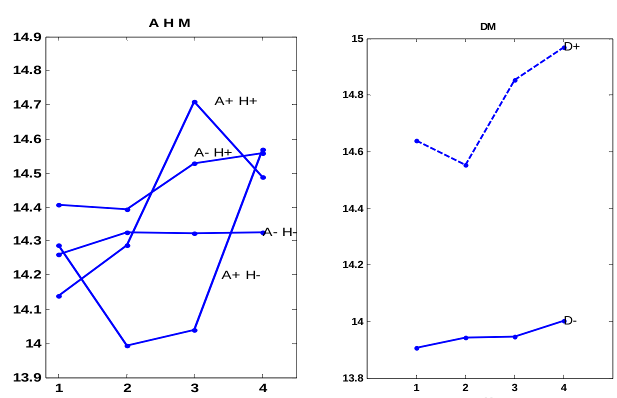

Figure 6: A x H x M and D x M iterations plot, layer growth experiments

It was used the response model in (6) to illustrated the computation of the transmitted variance model. It is assume each control or noise factor in takes two values, −1 and +1, and also it is assume that L, Ml and Mq are independent random variables, then:

$$X_L^2 = X_{Ml}^2 = X_{Mq}^2 = X_A^2 = X_C^2 = X_H^2 = 1,$$
$$E(X_L) = E(X_{Ml}) = E\left(X_{Mq}\right) = 0,$$
$$Cov(X_L, X_{Ml}) = Cov\left(X_L, X_{Mq}\right) = Cov\left(X_{Ml}, X_{Mq}\right) = 0 \quad (7)$$

Since the first three terms of the model in (6) are not random, the variance of $\hat{y}$ is same as the sum of the last five terms in (6), and it can be group as,

$$(0.330 - 0.239X_H)X_L - (0.090 + 0.083X_C)X_{Ml} - (0.082X_AX_H)X_{Mq}$$

From (7),

$$\begin{aligned} Var(\hat{y}) &= (0.330 - 0.239X_H)^2\, Var(X_L) + (0.090 + 0.083X_C)^2 Var(X_{Ml}) + (0.082X_AX_H)^2 Var(X_{Mq}) \\ &= CONSTANT + (0.330 - 0.239X_H)^2 + (0.090 + 0.083X_C)^2 \\ &= CONSTANT - (2(0.330 - 0.239)X_H) \quad + (2(0.090 + 0.083)X_C) \\ &= CONSTANT - 0.158X_H + 0.015X_C \quad (8) \end{aligned}$$

To minimize the predicted variance choose H = + and C = −, factor A effect is significant in the dispersion model and also according to the A × H ×Mq interaction plot. However, factor A does not appear in (8) since $X_A^2 = 1$ for X_A = ±1 this is a possible disadvantage of the predicted variance approach.

5. Conclusions

Definitely recommend the use of these methods, each of them in the best conditions of implementation. Here raises the question "What method should be used: the experimental design or robust parameter design." If you do not know the fundamental processes of the system under study or the interactions between different factors, or whether the experiments should be done are so long that must necessarily be either input, the design of experiments is the recommended tool. If, however, know the rudiments of the processes underlying the studied system, you cannot test all possible combinations of factors and the robustness and consistency of results is as important as the outcome itself, then you should use the this methods.

6. References

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