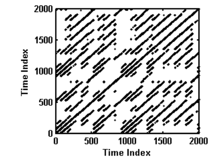

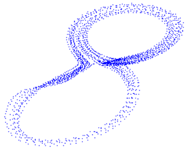

Self-organized Topology of Recurrence-based Complex Networks

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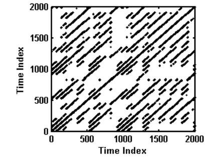
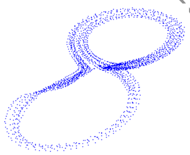
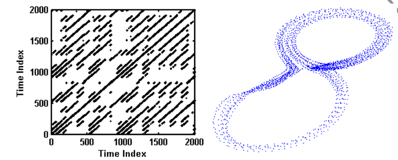




Relevant Publications

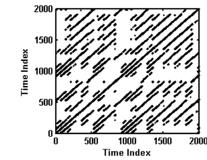
- **H. Yang*** and G. Liu†, "Self-organized topology of recurrence-based complex networks," ***Chaos: An Interdisciplinary Journal of Nonlinear Science,*** Vol. 23, No. 4, p043116, 2013, DOI: 10.1063/1.4829877
- G. Liu† and **H. Yang***, "Self-organizing network for group variable selection and predictive modeling," ***Annals of Operations Research***, 2017. DOI: 10.1007/s10479-017-2442-2
- C-B. Chen†, **H. Yang***, and S. Kumara, "Recurrence network modeling and analysis of spatial data," ***Chaos: An Interdisciplinary Journal of Nonlinear Science,*** Vol. 28, No. 8, p085714, 2018, DOI: 10.1063/1.5024917.
- G. Liu† and **H. Yang***, "Self-organized recurrence networks," *Proceedings of 2014 Industrial and Systems Engineering Research Conference (ISERC),* May 31, 2014, Montreal, Quebec, Canada. (**Best Paper Award in Computer and Information Systems Track**)

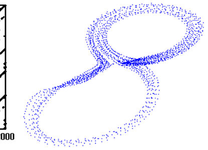


Outline

- ❑ Introduction
- ❑ Recurrence-based complex networks
- ❑ Force-directed recurrence networks
  - ➢ Spring-electrical model
  - ➢ Minimal-energy network
- ❑ Materials and experimental design
- ❑ Experimental results
- ❑ Conclusions

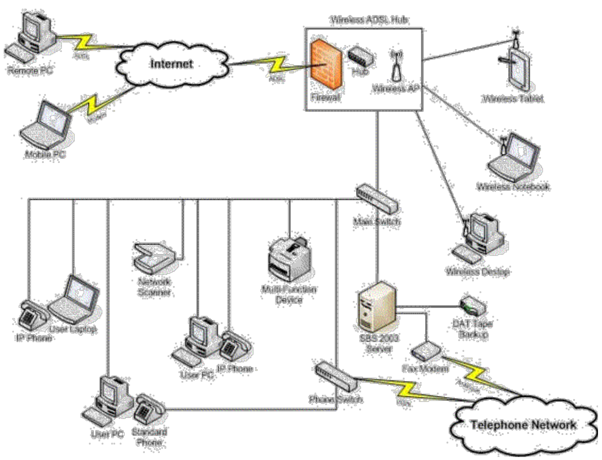
Introduction

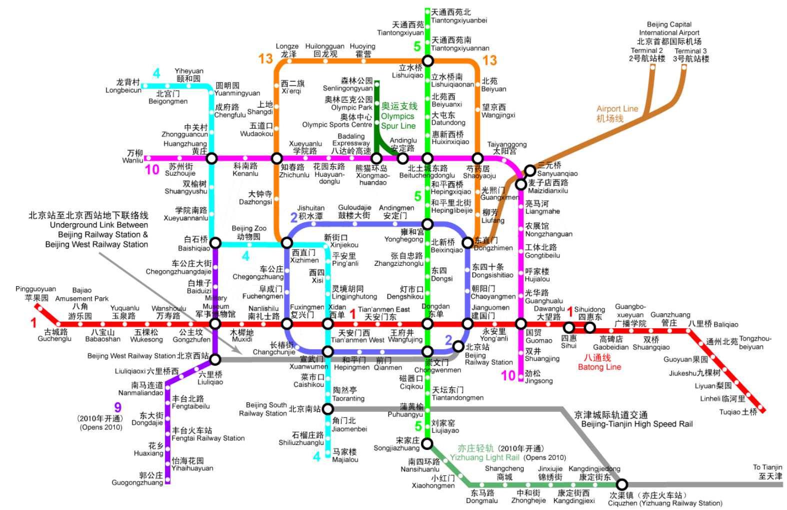




❑ Physical Network Analysis

- ➢ Known physical topologies





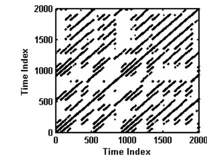
- ➢ **Network representation** – The adjacency matrix $\boldsymbol{A}$ is a means of representing which nodes of a network are adjacent to other nodes.

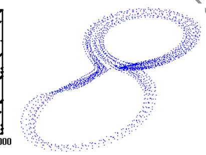
$$A_{ij} = \begin{cases} 1, & \text{node } i \text{ and node } j \text{ are linked} \\ 0, & \text{otherwise} \end{cases}$$

- ➢ **Network Statistics** – statistical quantification of physical networks.

Can we derive the physical topology from an adjacency matrix?

Introduction





❑ Background

- ➢ Recurrence-based networks – investigate recurrence characteristics of dynamical systems from the perspective of ***network theory***.
- ➢ Network-theoretic measures – new means to characterize the complexity of a nonlinear dynamical system.

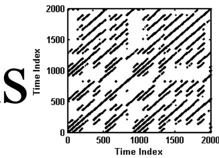
❑ Gaps

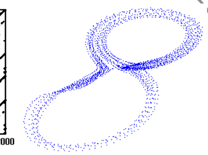
- ➢ Most previous works focus on deriving the adjacency matrix to represent the complex network and extract network-theoretic measures.
- ➢ From the same adjacency matrix, the geometry/topology of a complex network can take variable forms.

❑ Objectives

- ➢ To develop a self-organizing approach to *derive the unique and steady geometric structure of a network from the adjacency matrix*
- ➢ To investigate the *factors affecting this self-organizing process*

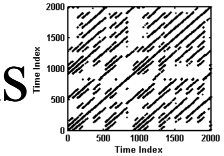
Recurrence-based Complex Networks

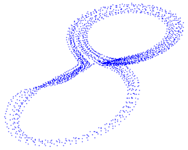




- *K*-nearest Neighbor Network
  - Directed network
  - Each node is connected to *k* nearest nodes in the network
  - A fixed number of neighbors
- Recurrence Network
  - Undirected network
  - Each node may have a different number of links in the network
  - A fixed size of the neighborhood
- Other Approaches
  - Transition networks [Nicolis, 2005], cycle networks [Zhang, 2006], correlation networks [Yang, 2008], Visibility graphs [Lacasa, 2008].
  - Donner, Marwan, *et. al* show that recurrence networks yield a unifying framework to transform nonlinear time series into complex networks.

Recurrence-based Complex Networks



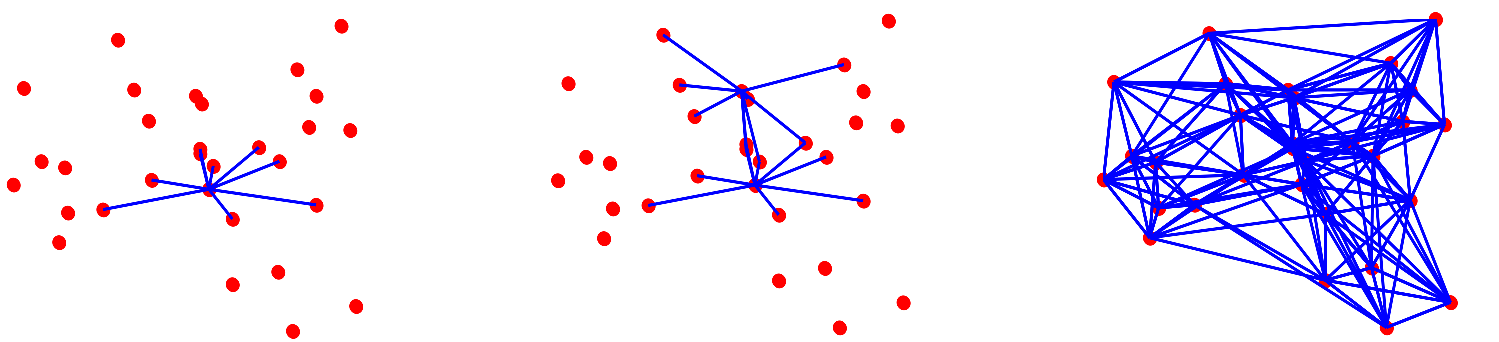


❑ **K-nearest Neighbor Networks** [Small, 2008]

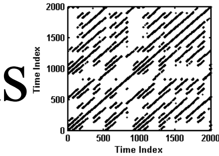
- Given a time series: $\boldsymbol{X} = \{x_1, x_2, \dots, x_N\}^{\boldsymbol{T}}$
- Sate space reconstruction: $\boldsymbol{x}(i) = (x_i, x_{i+\tau}, \dots, x_{i+\tau(M-1)})$

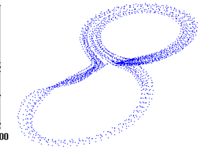
  M is embedding dimension and τ is time delay
- A node $\boldsymbol{x}(i)$ is connected to its k nearest neighbors, but excluding the nodes in the same strand of the trajectory.

$$A_{ij} = \begin{cases} 1, & |j-i| > \Delta t \;\&\; j \in \{k \text{ nearest neighbors of } i\} \\ 0, & \text{otherwise} \end{cases}$$



Recurrence-based Complex Networks

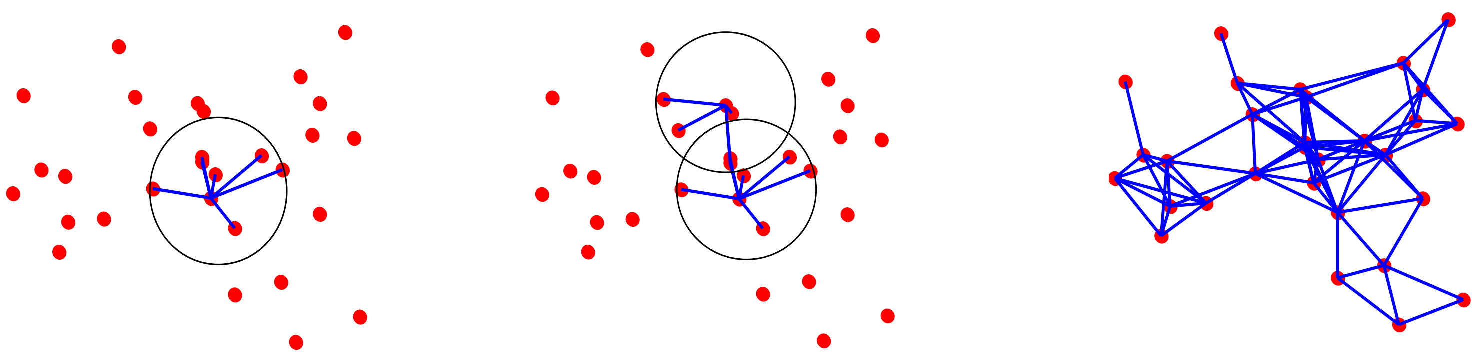


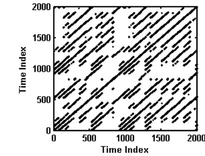


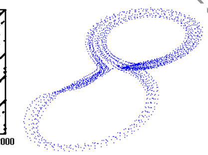
- **Recurrence Networks** [Marwan, 2008]
  - Given a time series: $\boldsymbol{X} = \{x_1, x_2, \dots, x_N\}^T$
  - Sate space reconstruction: $\boldsymbol{x}(i) = (x_i, x_{i+\tau}, \dots, x_{i+\tau(M-1)})$

    M is embedding dimension and τ is time delay
  - The recurrences are treated as links in the network
  - The adjacency matrix A is obtained from the recurrence matrix by removing the diagonal identities:

$$A_{i,j} = R_{i,j} - I_{i,j}$$







Network-theoretic Measures

- Node degree k_i – the number of neighboring nodes of node i

$$k_i = \sum_{j=1}^{n} A_{ij}, \; n \text{ is the number of nodes}$$

- Link density ρ – the ratio of the number of edges to the number of possible edges

$$\rho = \frac{1}{n(n-1)} \sum_{i,j=1}^{n} A_{ij}$$

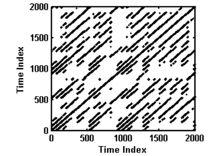
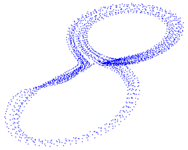
- Distance $d_{i,j}$ – the minimal number of edges to travel from node i to node j
- Average path length L – the average of all paired distances

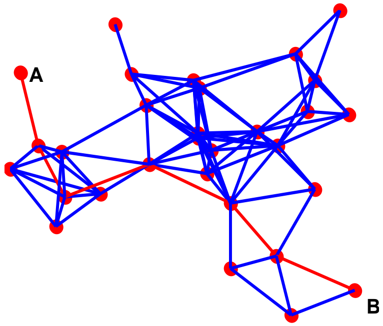
$$L = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{i,j}$$

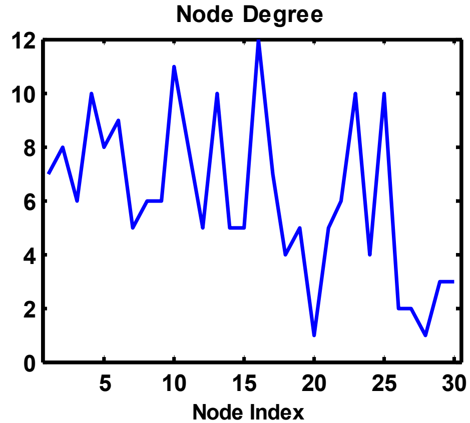
- Diameter D – the longest of all shortest paths, $D = Max\{d_{i,j}\}$
- Clustering coefficient of a node – the probability that two neighbors of a node i are also neighbors.

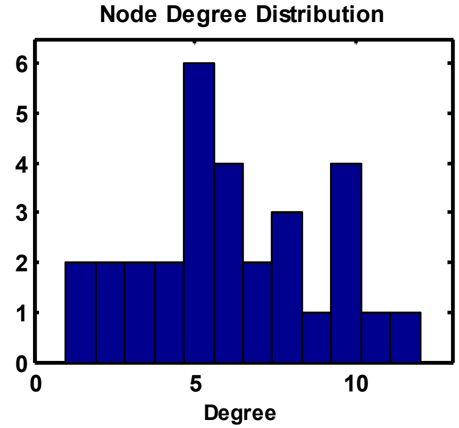
Note that most of the metrics are calculated from the adjacency matrix

Network-theoretic Measures



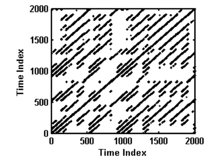


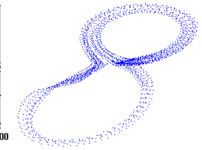


- The distance between node A and B is 6 - **# of links**.
- The average path length is 2.3210
- The clustering coefficients: 90.48% (node 1), 0 (node 20), 100% (node 27)
- The average clustering coefficient for the network: 62.71%

Network-theoretic measures – actual node-to-node distances?

How to derive the network topology from an adjacency matrix?

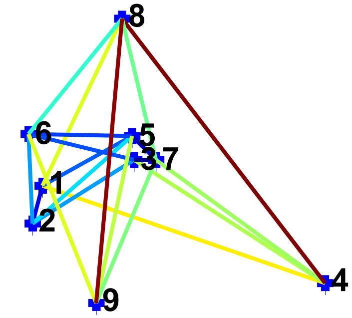


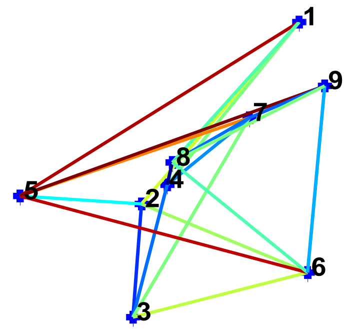


❑ Adjacency Matrix ⇨ Network Topology

- From a recurrence adjacency matrix, we have a variety of possible topological structures for the network.

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

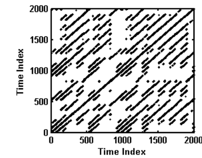


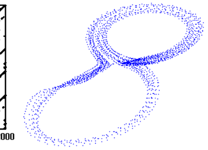


?

- Is there a unique and stable topological structure for a recurrence-based network?

Force-directed Recurrence Networks





❑ Spring-electrical Model

- Nodes – electrically charged particles
- Edges – springs between nodes
- Note that the repulsive force exists between any pair of nodes
  the attractive force exists only between neighboring nodes

Repulsive force: $f_r(i,j) = -\frac{CK^{1+p}}{\|\boldsymbol{x}(i)-\boldsymbol{x}(j)\|^p}, i \neq j$

Attractive force: $f_a(i,j) = \frac{\|\boldsymbol{x}(i)-\boldsymbol{x}(j)\|^2}{K}, i \leftrightarrow j$

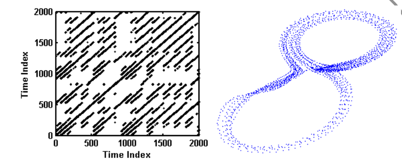
- Combined force at a node i:

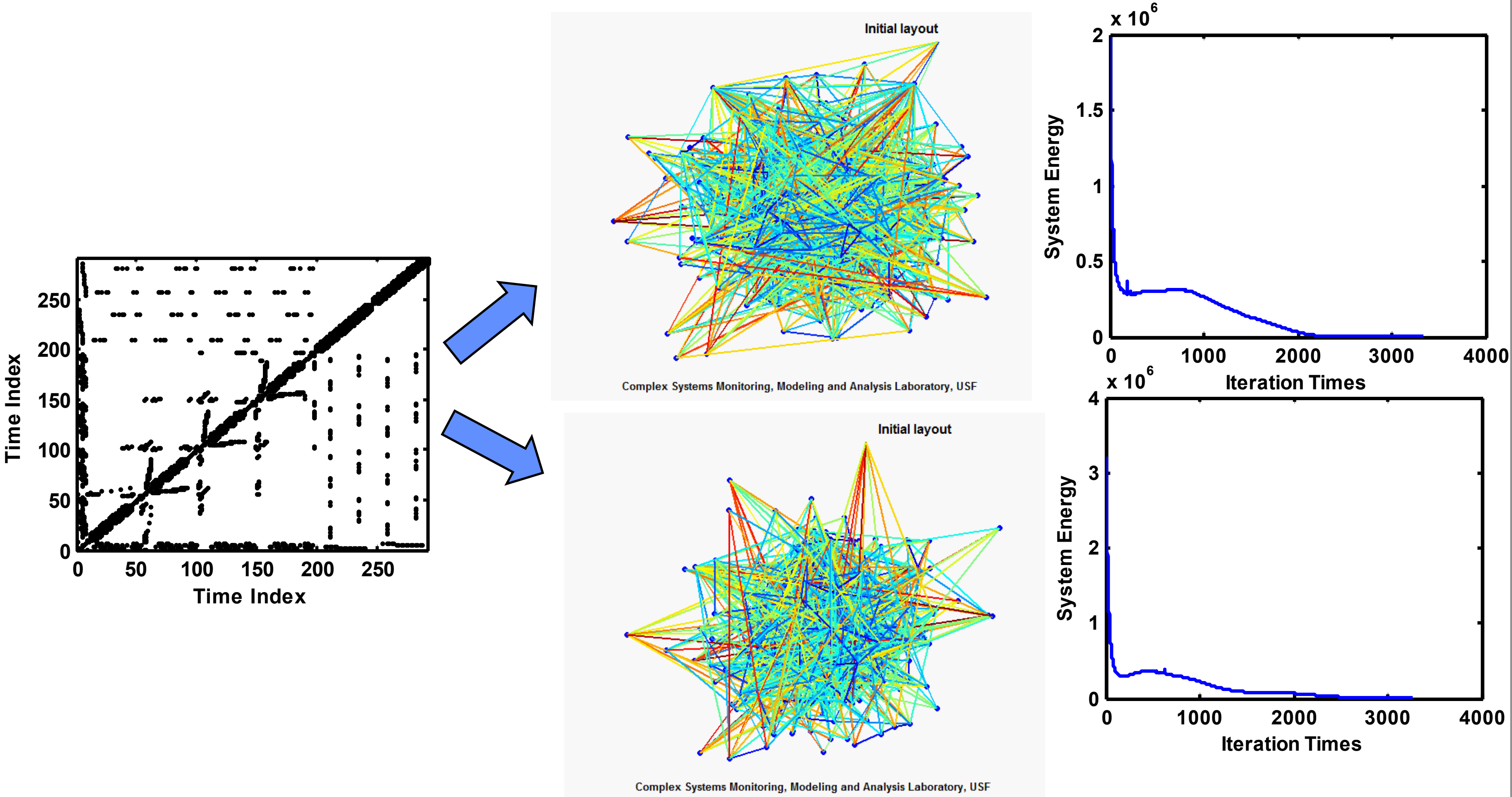
$$f(i,\boldsymbol{x},K,C) = \sum_{i\neq j} -\frac{CK^{1+p}}{\|\boldsymbol{x}(i)-\boldsymbol{x}(j)\|^{p+1}}(\boldsymbol{x}(i)-\boldsymbol{x}(j)) + \sum_{i\leftrightarrow j}\frac{\|\boldsymbol{x}(i)-\boldsymbol{x}(j)\|}{K}(\boldsymbol{x}(i)-\boldsymbol{x}(j))$$

- Minimal Energy Network:

$$\min_{\boldsymbol{x}}\{Energy_{se}(\boldsymbol{x},K,C)\} = \min_{\boldsymbol{x}}\left\{\sum_i f^2(i,\boldsymbol{x},K,C)\right\}$$

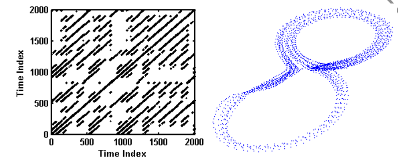
An Example





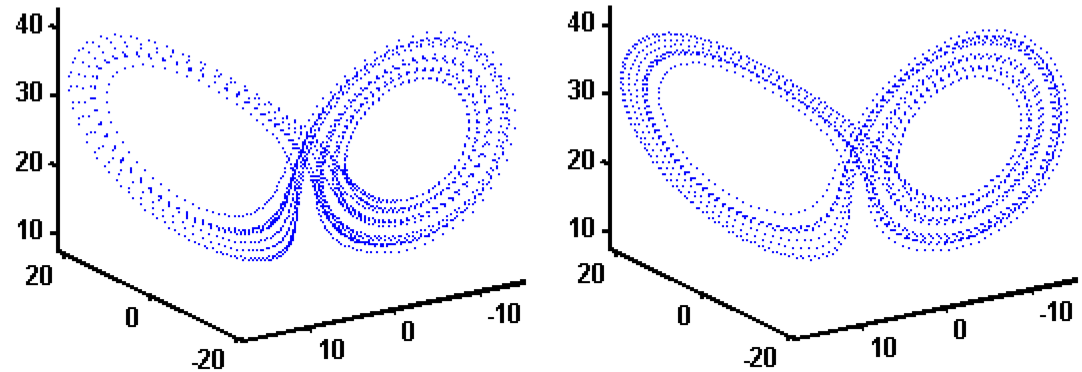
- For an adjacency matrix, the self-organizing approach yields a unique and stable network topological structure by minimizing the system energy, even from different initial settings.

Nonlinear Dynamical Systems



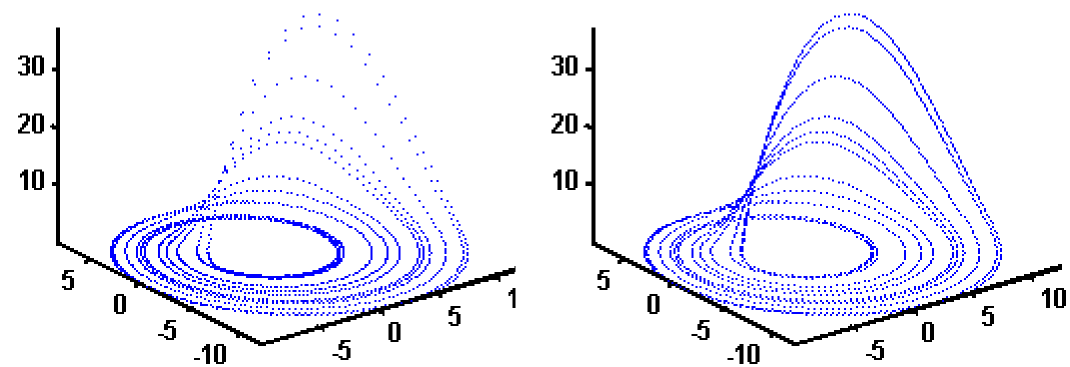
- Lorenz system:

$$\begin{cases} x' = 10(y-x) \\ y' = x(28-z)-y \\ z' = xy-8z/3 \end{cases}$$



- Rossler system:

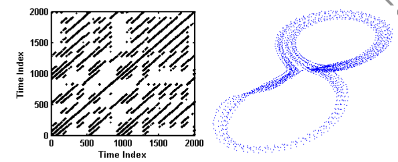
$$\begin{cases} x' = -y-z \\ y' = x+0.2y \\ z' = 0.2+z(x-5.7) \end{cases}$$

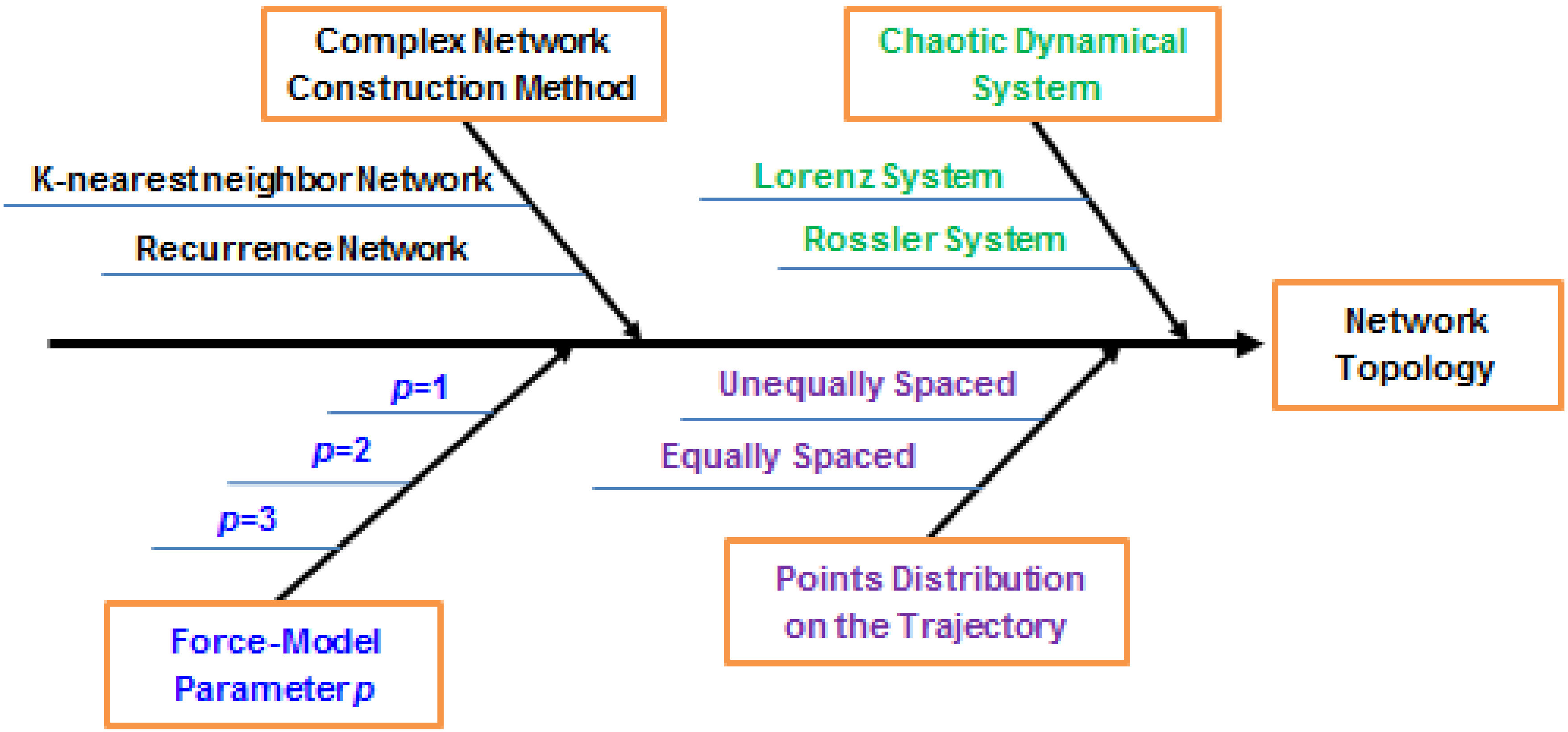


- Continuous dynamical systems
- Discrete sampling

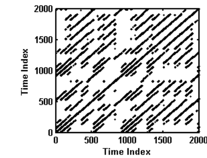
Attractors with states unequally and equally spaced

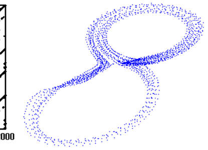
Experimental Design



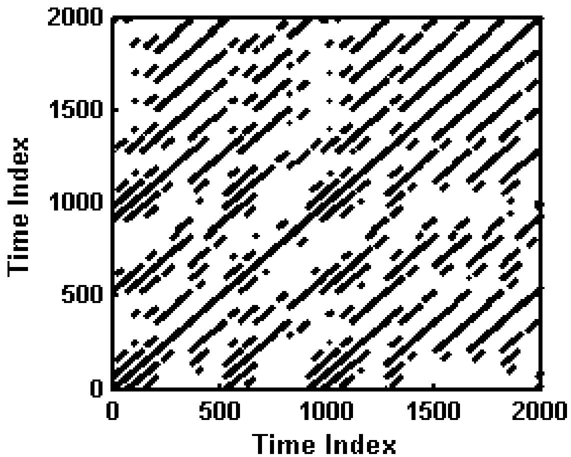


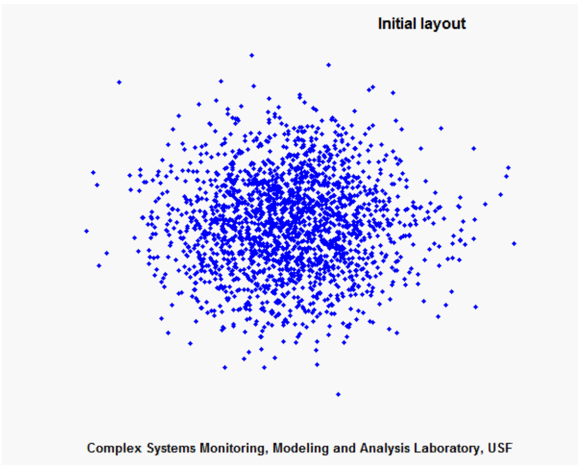
Experimental Results

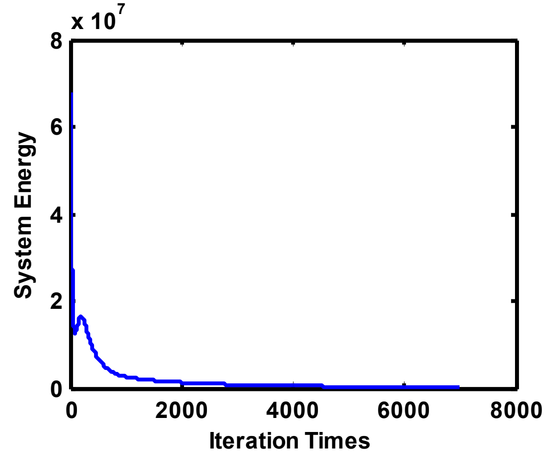


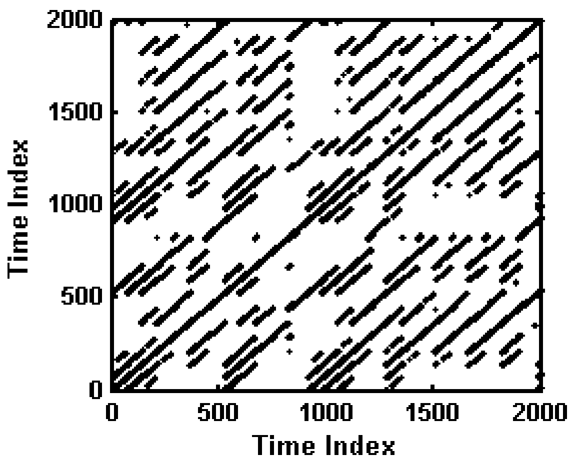


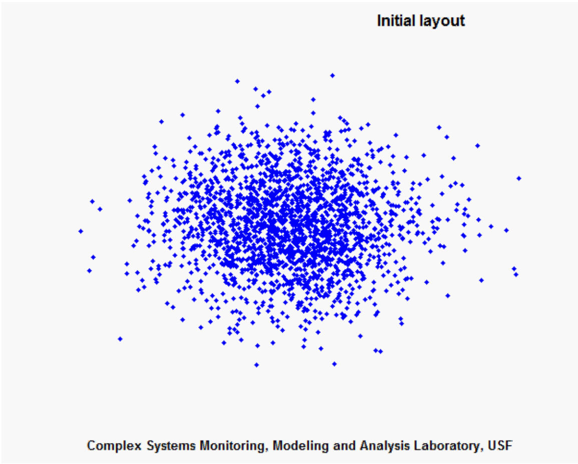
❑ ***KNN Network vs. Recurrence Network*** (Lorenz system)

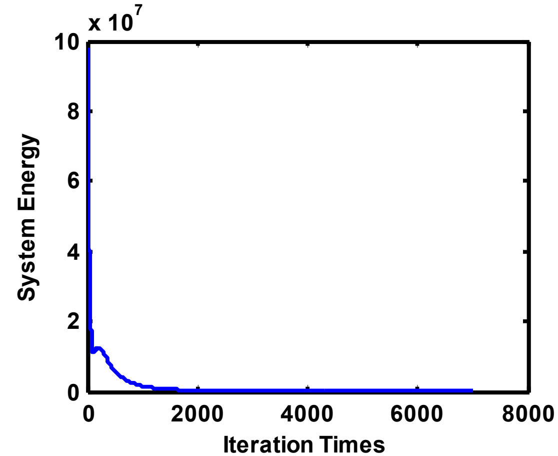




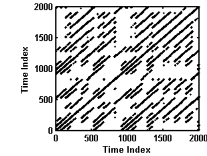


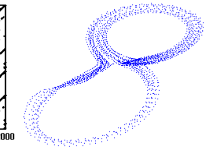




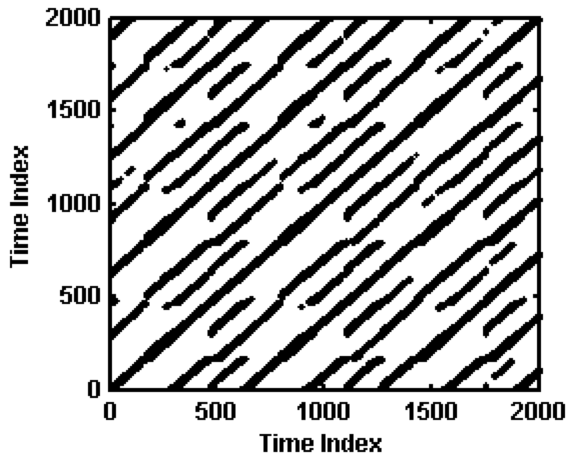


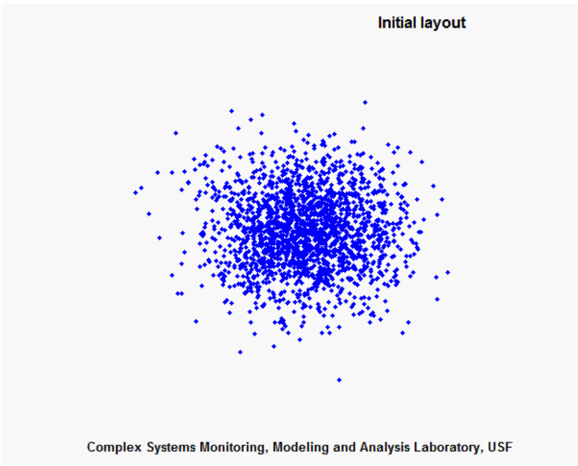
Experimental Results

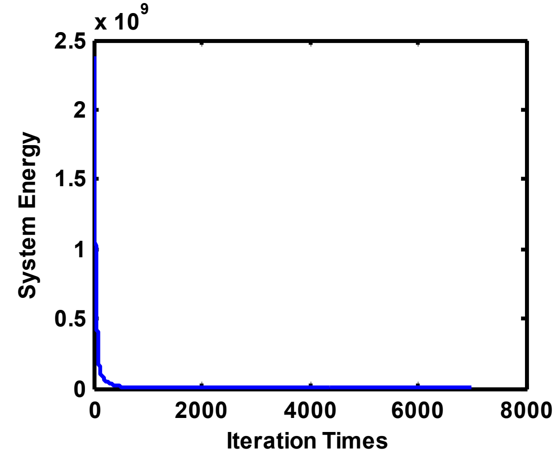


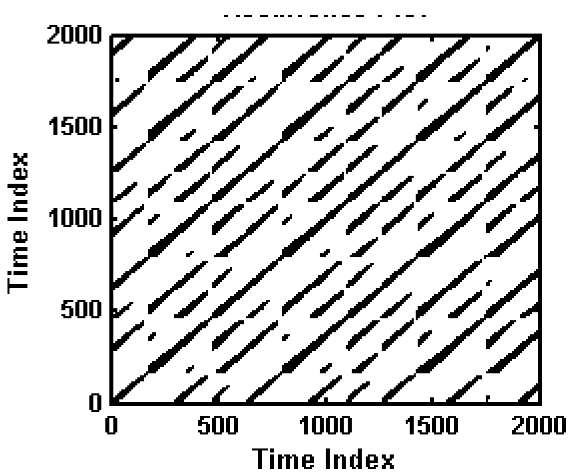


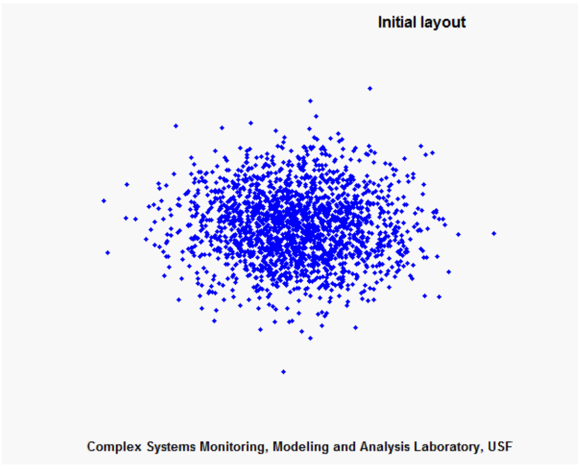
❑ *KNN Network vs. Recurrence Network* (Rossler system)

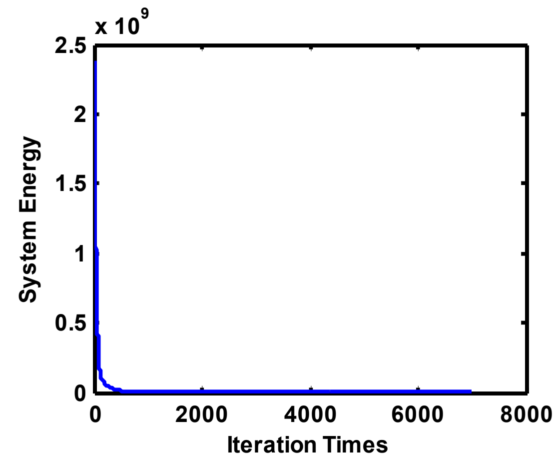




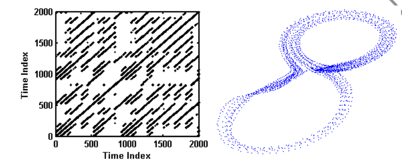






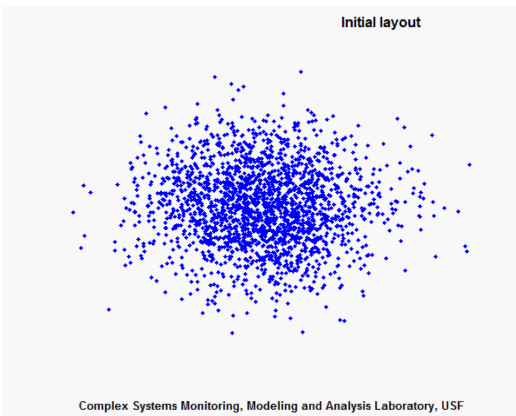


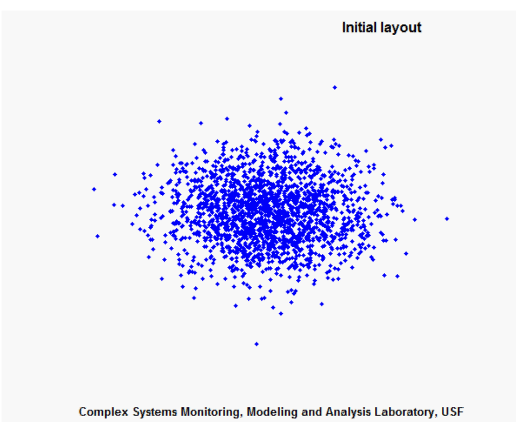
Experimental Results



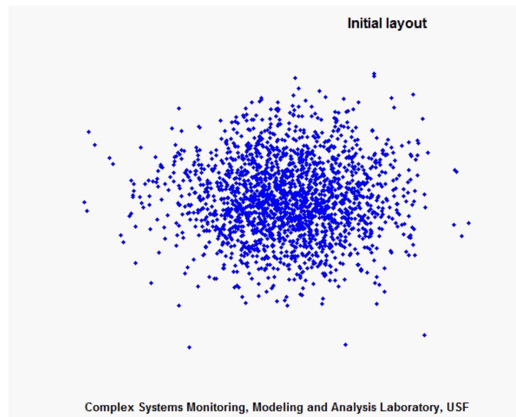
❑ ***Force-model parameter*** $p = 1, 2, 3$?

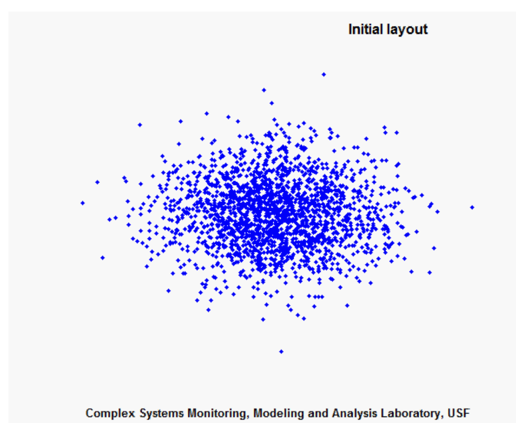
$p = 1$



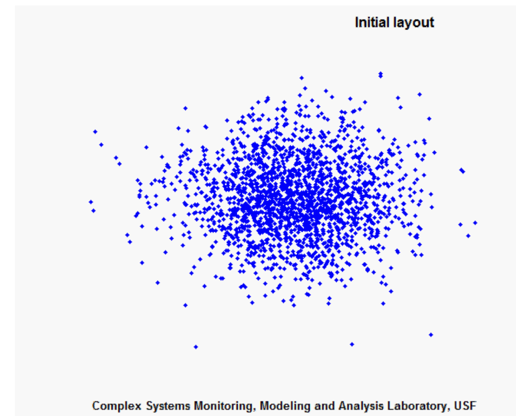


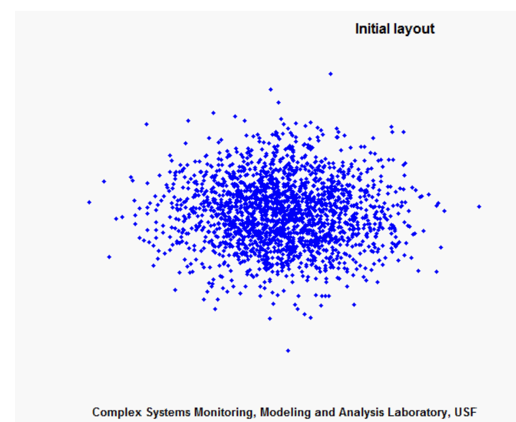
$p = 2$

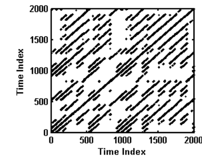
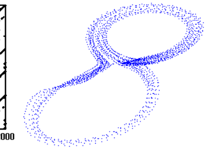




$p = 3$

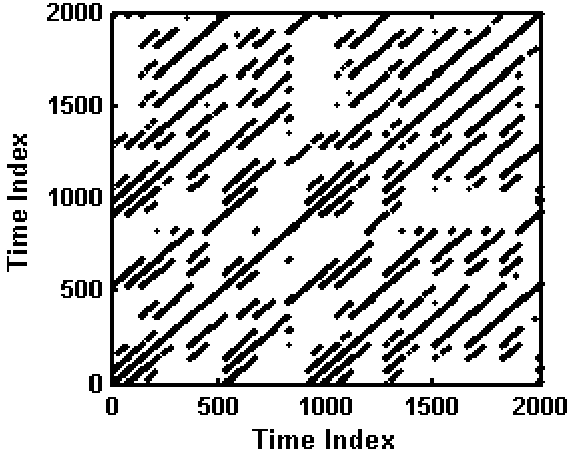


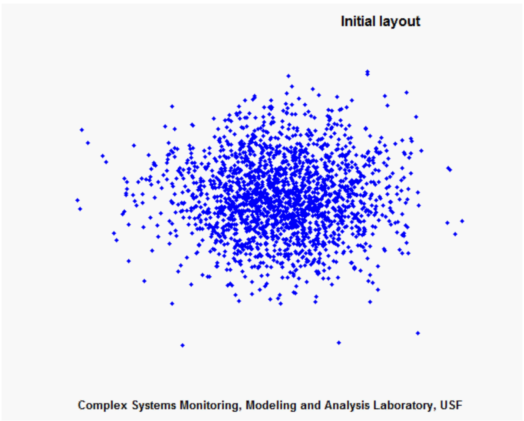


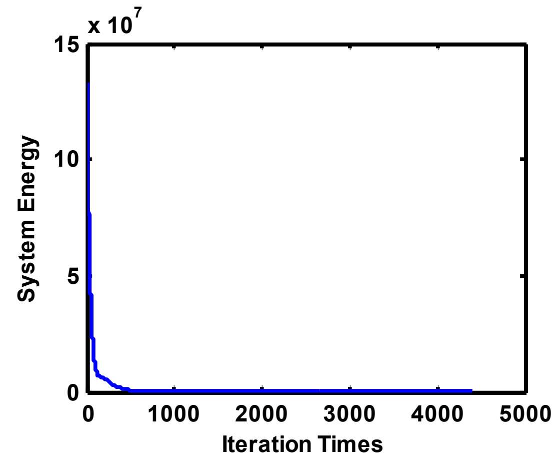



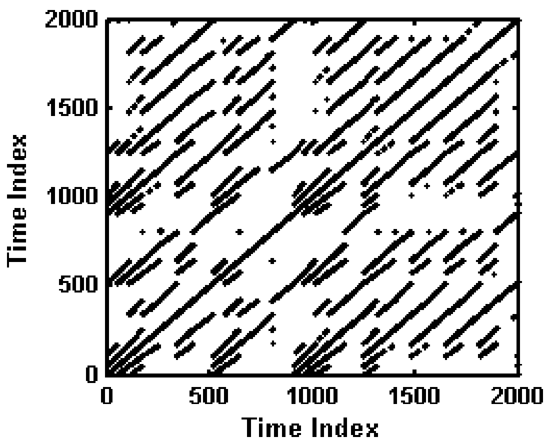
Experimental Results

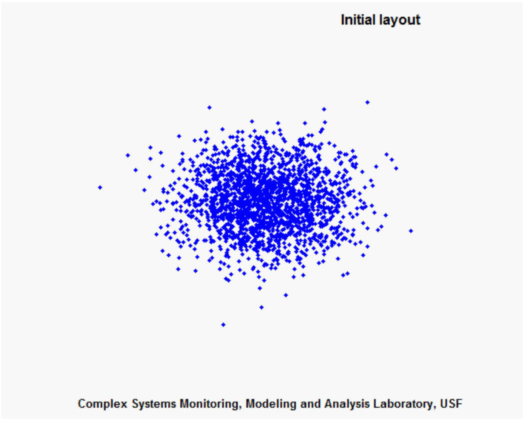
- ❑ ***Unequally-spaced vs. equally-spaced*** (Lorenz)

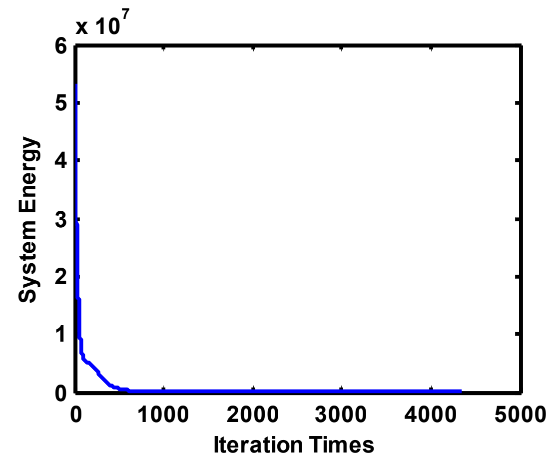




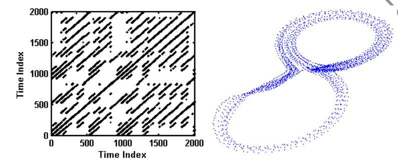




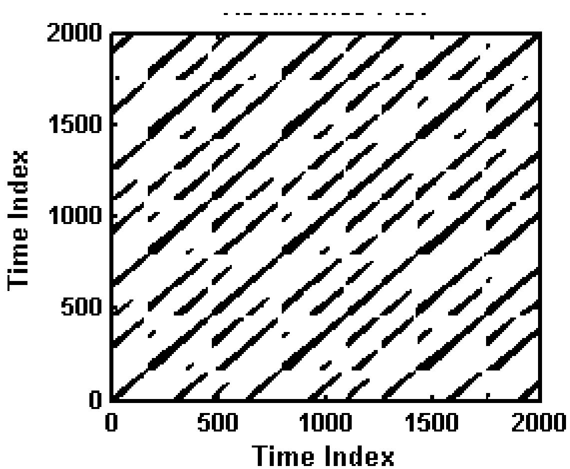


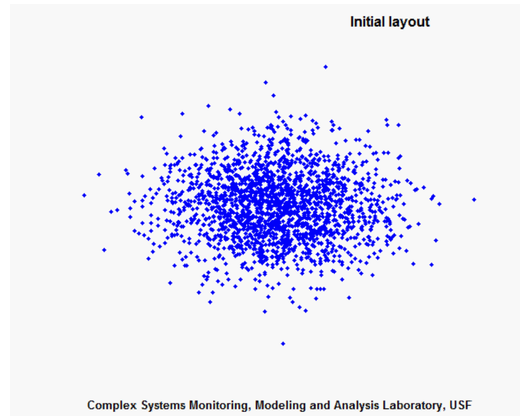


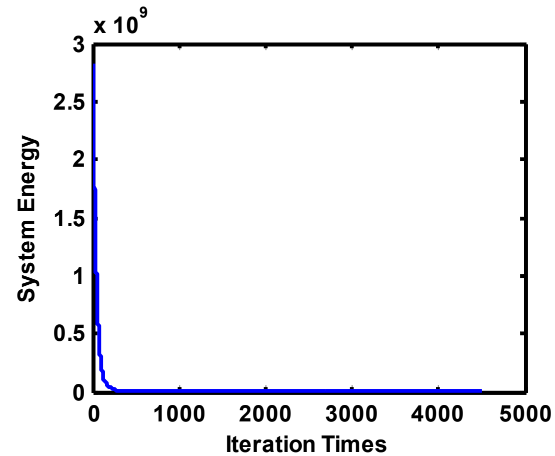
Experimental Results

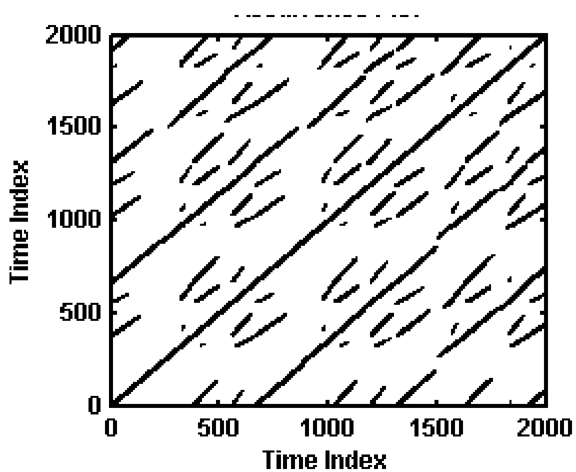


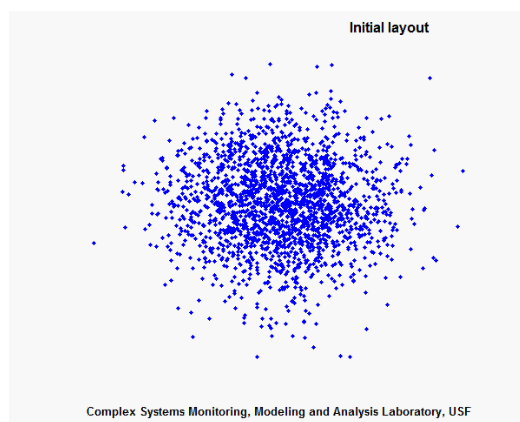
❑ ***Unequally-spaced vs. equally-spaced*** (Rossler)

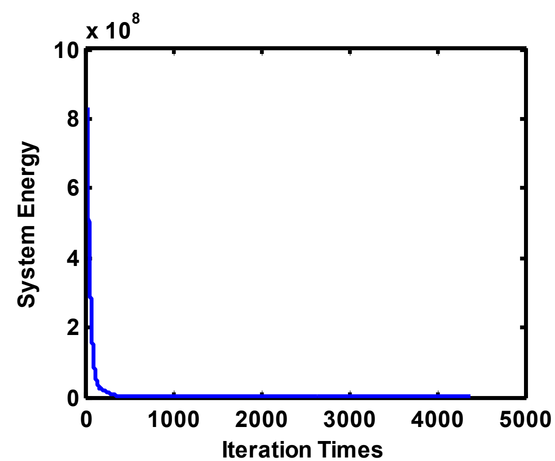




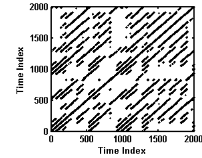


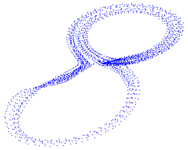




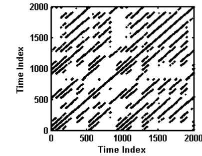


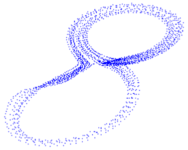
Conclusions





- Interestingly, the self-organizing geometry of a recurrence network recovers the attractor of the dynamical system.
  - Disclose the geometry of a recurrence network.
  - Provide a new way to reproduce the attractor or time series from the recurrence-based adjacency matrix.
- Important factors affecting the self-organizing process
  - ***Network construction method*** - The recurrence network shows a better performance to reconstruct the system dynamics.
  - ***Force-model parameter p*** - If the force-model parameter *p* is too small, the peripheral nodes tend to tightly cluster. Otherwise, the peripheral nodes are loosely distributed.
  - ***Discrete Sampling*** - Aliasing effects in the self-organized topology due to the distribution of sparse states in some local regions (e.g., the outer parts of the Rossler attractor). The self-organizing process performs better than the unequally-spaced attractors





END
Questions?