Logarithmic Quantization and Opportunistic Calibration for Physiological Signal Classifiers

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Abstract:

Physiological computing applications rely on time series sensor data, which vary widely between users and change over time. Application designers face a tradeoff between computational complexity, calibration time, and classification accuracy. We describe a novel quantization technique for electroencephalograph (EEG) signals, which we find increases the computational speed associated with training a machine-learning based classifier without significant detriment to the system's accuracy. We test this technique on a brain-computer interface (BCI) that uses a single EEG sensor, and find that an opportunistic calibration strategy can achieve high accuracy in 86.6% of subjects with under five minutes of training, and in 100% of subjects with under 15 minutes. We discuss implications for the design of consumer-ready BCI and other physiological computing applications.

1 INTRODUCTION

Physiological data do not carry universal meanings. Whereas the movement of a computer mouse can be mapped to the position of a cursor in a straightforward manner, the expression of bio-signals varies widely between individuals, and often changes within individuals over time. Brain-computer interface (BCI) serves as a good example of this phenomenon: regular calibration and re-calibration are essential due to the personal and non-stationary nature of neural signals (Dornhege, 2007; McFarland and Wolpaw, 2011).

Supervised learning algorithms have enabled systems that adapt to users' personal physiology after a calibration period. In BCI, this approach has yielded proof-of-concept systems ranging from braincontrolled keyboards and wheelchairs to prosthetic arms and hands (Blankertz et al., 2007; Millan et al., 2010; D. Mattia, 2011; Hill et al., 2014; Campbell et al., 2010).

Moving from laboratory settings into the real world, these systems will have fewer and less sensitive sensors due to cost, ergonomic and aesthetic considerations. They will also process noisier signals as data acquisition will occur while people are engaged in everyday activities, walking, talking, sleeping, and so on. As an additional challenge, computational complexity, measured both in terms of storage

and processing power requirements, may be limited by the mobile and wearable computing architectures on which these systems will be deployed.

In this paper, we study how the processing of physiological signals and the strategy for user calibration can impact the performance of a machine-learning based bio-signal classification system that uses signals acquired from a low-cost mobile electroencephalograph (EEG) device. Prior to classification, how can we operationalize the tradeoff between computational complexity and classification accuracy at the signal processing step? Given a well-tuned classifier, is it possible to realize user calibration on the order of minutes rather than hours or days?

We propose a novel signal quantization technique that applies logarithmic binning to power spectrum data from a single EEG electrode. We find that this technique can increase the computational speed of a classification-based BCI by 450% without a significant detriment to accuracy. In conjunction with an opportunistic user-calibration protocol, in which candidate mental gestures are tested only when necessary, we calibrate 86.6% of users to a threshold of BCI control in under five minutes of training data, and 100% of users in under 15 minutes.

This paper is organized as follows. We discuss related works in Section 2, and provide a summary of the dataset in Section 3. We describe our signal

quantization method in Section 4, and quantify its effect on classifier speed and accuracy in Section 5. We evaluate an opportunistic user calibration strategy in Section 6 before concluding.

2 RELATED WORK

2.1 Calibrating EEG-based BCI

BCI systems generally aim to recognize a user's mental gestures as one of a finite set of discrete symbols, a problem that can be thought of as a pattern recognition task (Lotte et al., 2007). The difficulty of this task stems primarily from the variable and non-stationary nature of neural signals: the "symbols" we wish to identify are expressed differently between individuals, and even vary within individuals from trial to trial (Vidaurre et al., 2011).

In order to compensate for variability in BCI signals, recent work has leveraged adaptive classification algorithms to distinguish between mental gestures (Lotte et al., 2007; Vidaurre et al., 2011). Automated calibration procedures have turned BCI novices into competent users over the course of hours instead of days or weeks, and without manual calibration by a researcher (Vidaurre et al., 2011; ?). During calibration, users perform "labeled" (that is, known) mental gestures in order to produce samples for the classifier. Meanwhile, the classifier attempts to determine which features of the data are most informative.

2.2 Statistical Signal Processing in EEG-based BCI

Calibrating a BCI requires an algorithm that can adapt to its inputs. Support vector machines (SVM) are a set of supervised machine learning methods that take labeled example data to create a model that can be used to predict the classes of unlabeled data. SVMs use a hyperplane (an *n*-dimensional construct in an *n*+1 dimensional space) to draw discriminatory boundaries between classes.

Past work has used linear SVMs in BCI applications with great success (Garrett et al., 2003; Grierson and Kiefer, 2011). In contrast to linear discriminant analysis, which also have a long history of use in BCI, SVMs select the hyperplane that maximizes distance from the nearest training points, which has been shown to increase the model's generalizability (Burges, 1998).

In classification algorithms generally, larger feature vectors require an exponential increase in the amount of data needed to describe classes, a property known as "the curse of dimensionality" (Jain et al., 2000). Traditionally, BCI applications rely on dense, high-dimensional feature vectors produced by multi-electrode scanning caps with high temporal resolution, so dimensionality represents a major bottleneck in training classification algorithms. This bottleneck threatens the responsiveness of BCI from a user experience standpoint and places high requirements on end users' hardware.

2.3 Brain-Computer Interface "in the Wild"

In recent years, numerous inexpensive, mobile EEG devices have come to the consumer brainwave sensing market. Compared to their lab-based counterparts, these devices have significantly fewer electrodes and therefore much lower spatial resolution. Most of them also employ dry contact electrodes, which can produce significantly noisier signals (De Vos and Debener, 2014). Nonetheless, researchers have demonstrated several mobile-ready BCI systems that use these devices to detect emotional states, event-related potentials (ERP), and demonstrate the feasibility of brainwave-based biometric authentication (Crowley et al., 2010; Grierson and Kiefer, 2011; Chuang et al., 2013).

However, the use of consumer EEGs for the direct, real-time control of software interfaces has proven more difficult, as the number of electrodes on these headsets limit the spatial resolution required to discriminate between mental gestures (Carrino et al., 2012; Larsen and Hokl, 2011). While we expect continued improvements in successive generations of consumer-grade EEG devices, the signal from these devices will remain noisier than the lab-based counterparts, as users will be wearing and using them in everyday settings, with ambient electromagnetic signals interfering with endogenous bio-signals.

To transition BCI from the lab into naturalistic environments, we must squeeze more signal out of fewer, and less reliable, sensors. Furthermore, since BCIs are envisioned largely as always-available input devices, they will likely be deployed on mobile processors and perhaps even embedded processing systems. Thus, the available computational resources may be more comparable to that of a smartphone than of a desktop workstation. Furthermore, it is feasible that we may need to do some processing "in the cloud" (i.e., on a more powerful server to which the client sends data over the network, similar to the way Apple's Siri processes voice data). For effective BCI to occur in these environments, we must extract signal

in a maximally efficient way so as to limit our computational footprint, and perhaps even to minimize the size of data if we wish to ship it to an external server.

3 DATA

We did not acquire any experimental data for this study. Rather, we obtained an anonymized dataset from the Passthoughts study (Chuang et al., 2013). The dataset includes EEG recordings of 15 subjects, all students from UC Berkeley, performing seven mental tasks in a sitting position over two sessions. The signals were recorded using a consumer-grade EEG headset, the Neurosky MindSet, with a dry contact EEG sensor over the Fp2 position. In particular, we use the power spectrum time series data recorded using the Neuroview software.

For each of the two sessions, participants performed each of the seven mental tasks, enumerated below, ten times. Each of the ten trials lasted ten seconds.

The seven mental tasks were: (i) breathing with eyes closed; (ii) motor imagery of right index finger movement; (iii) motor imagery of subject's choice of repetitive sports motion; (iv) mentally sing a song or recite a passage; (v) listen for an audio tone with eyes closed; (vi) visual counting of rectangles of a chosen color on a computer screen; and (vii) any mental thought of subject's choice as their chosen "password".

The power spectrum time series data consists of one power spectrum every 0.5 seconds. Therefore, for a 10 second recording, we have a sequence of 20 power spectra. Each power spectrum contains frequency components from 0 Hz to 256 Hz at 0.25Hz intervals. Therefore there are 1024 values reported for each power spectrum.

The dataset was further cleaned by removing all readings marked as having suboptimal signal quality by the Neurosky SDK. The SDK delivers a signal quality value that is greater than zero when signal quality is suboptimal. Factors causing this value to be greater than zero include lack of contact between the electrode and skin, excessive non-EEG noise (e.g., EKG, EMG, EOG, electrostatic), and excessive motion.

4 SIGNAL QUANTIZATION FOR RAPID CLASSIFICATION

Our objective is to maximize the accuracy of the classifier while minimizing its computational expense. One way to reduce the computational requirements of a classifier is to reduce the size of the feature vectors on which it is trained and tested. We propose a signal quantization method that allows us to directly adjust the size of feature vectors by changing the signal's resolution. This allows us to operationalize a tradeoff between the running time and accuracy of the classifier

First, we compress the power spectrum time series in the temporal dimension. Given a sequence of 20 power spectra (from a 10 second trial), with 1024 frequency components per spectrum, we compute a discrete probability density function (pdf) in which each component is the mean of its corresponding frequency components through time. This results in a discrete pdf with 1024 components for each trial.

4.1 Logarithmic Binning

Now, we apply the quantization step by performing data binning on the probability density function. Data binning offers a simple way to "quantize" the information contained in the full signal. By taking the mean of several adjacent points in the pdf, we are left with a single bin that compresses the information contained in its local area of frequencies. For instance, four contiguous frequencies (1Hz, 1.25Hz, 1.5Hz, 1.75Hz) of the values (4, 4, 5, 5) could be combined into a single bin with the value 4.5. The number of bins can be chosen to provide a desired level of resolution on the signal.

Since EEG activity is associated with frequencies from 1-40Hz, we presume this range contains the majority of relevant signal. However, we do not rule out the possibility that useful signal exists in other frequency ranges. Muscular activity, for example, might be correlated with mental gestures in some cases. In order to exploit the entire frequency spectrum while preserving our bias toward known sources of useful signal, we select a logarithmic spacing of the data bins through the pdf. Figure 1 shows an example of logarithmic binning with 100 bins. It offers a 10x compression ratio while still preserving the structure of the original 1024-point pdf.

The output of the logarithmic binning step is a single feature vector, whose size is controlled by the number of bins. This vector can now be used as an input into the classifier.

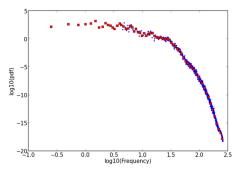


Figure 1: In double logarithmic scale, the original 1024 bins (blue dots) of the probability density function (pdf) obtained from averaging the n power spectra of one recording, and the resulting "quantized" pdf with a resolution of 100 logbins (red dots). The quantized pdf preserves very well the structure of the original, 1024-point pdf.

4.2 Binary BCI Classifier

To test the performance of the quantization method, we build a binary BCI using a support vector machine (SVM) classifier, which we train individually on each subject's recordings while varying the bin size. We use LinearSVC (Fan et al., 2008), a wrapper for LibLinear exposed in Python through the scikitlearn library (Pedregosa et al., 2011). We chose LinearSVC because BCI classification problems are generally presumed to be linear (Garrett et al., 2003; Lotte et al., 2007), and because LibLinear's underlying C implementation boasts among the fastest trainand test-time performance among state-of-the-art solutions (Fan et al., 2008). We use a hyperparameter of 100, found through a "grid search", or an exhaustive search through a randomly-selected sample of our dataset. We use scikit-learn's built-in cross-validation toolkit, which performs seven cross-validation steps using different splits of data in each round.

Out of the seven mental gestures in the dataset, we want to identify and select, for each individual subject, the two best gestures for that subject. This will result in a personalized binary (two-class) classifier, where the SVM can discriminate between two mental gestures performed by the subject with the highest classification accuracy. The gesture-pairs may vary from subject to subject. For example, one subject's best-case pair may be *song* and *sport* while another's may be *color* and *finger*.

In order to simulate a binary BCI with our dataset, we splice all mental task recordings into 0.5-second chunks, each one representing a single power spectrum reading. In Section 5, we simulate calibrating the BCI to a task-pair by cross-validating an SVM trained on all task-pair data. In Section 6, we use a more realistic approach in which we train the SVM on the first 80 seconds of data for both tasks, then test

the classifier on the remaining 40 seconds of data.

5 THE EFFECT OF QUANTIZATION TECHNIQUE ON CLASSIFIER SPEED AND ACCURACY

Generally, we seek to maxmize our system's classification accuracy while minimizing its computational expense. One way to reduce the computational requirements of a SVM classifier is to reduce the size of the feature vectors on which it is trained and tested. Our signal quantization method allows us to directly adjust the size of feature vectors by changing the signal's resolution (see 3.1), though lowering the resolution of feature vectors could negatively effect the classifier's performance.

In this experiment, we examine the effect of resolution, operationalized by the number of bins used in the quantization step, on our BCI's performance, operationalized by the SVM's training time and by the SVM's estimated accuracy.

H1: SVM accuracy will decrease with resolution. H2: SVM training time will decrease with resolution.

5.1 Protocol

For each subject, we generate every pair of two tasks and cross-validate our SVM seven times on the recordings for this pair of tasks. We vary the resolution of the samples we feed to the SVM. For every task pair processed, we record mean classification accuracy across all rounds of cross-validation. For each subject, we record the best-performing taskpair, which corresponds to our estimation of optimal performance of the BCI for that subject.

As an additional performance audit, we measure the time needed to fit an SVM to the data for two randomly selected taskpairs across all subjects. We repeat this process ten thousand times at different resolutions, collecting the minimum time observed in each series of attempts.

5.2 Results

We find support for H1. Although resolution was positively correlated with classifier accuracy (slope = .0013 R-squared = .773, p < .001), this effect appears only at resolutions lower than 100 points. We find no significant increase in SVM accuracy at resolutions over 100 bins.

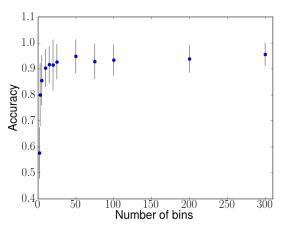


Figure 2: Mean best-case accuracy among all subjects compared to data resolution. At reoslutions of 100 points and greater, we find no evidence of an increase in classification accuracy.

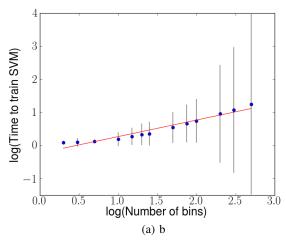


Figure 3: Log of mean classifier training time compared to log of data resolution. The time needed to train the classifier increases logarithmically with resolution.

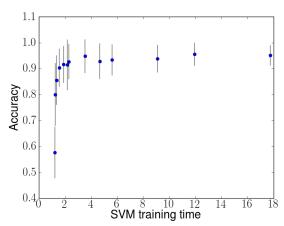


Figure 4: Best-case accuracy compared to the time needed to train the classifier. By decreasing the number of bins in the EEG data, we can decrease the time needed to train the support vector machine up to nine times without without significant detriment to classifier accuracy.

Resolution was also positively correlated with time to train classifier (slope = 0.5 R-squared = .947, p < .001). We compare accuracy and SVM training time directly in 4. Thus, we find support for H2.

Overall, we find that relatively small feature vectors produced with our method (100 values) yield classifiers as accurate as full-resolution samples (1024 values), and that reducing vector size in this way can dramatically increase the computational speed of training an SVM.

6 OPPORTUNISTIC STRATEGY FOR CALIBRATING A BINARY BCI

In the previous experiment, we found that our compression technique can speed up an SVM classifier without significant detriment to BCI accuracy. However, for the technique to be useful in real-world applications, it must also allow users to quickly calibrate the system to their personal physiological signals.

In this experiment, we evaluate an opportunistic strategy for user calibration. Using a resolution of 100 points identified as optimal in the previous experiment, we measure user calibration time (the time it takes a user to achieve a threshold accuracy with the BCI) and the classification accuracy each user achieves after calibration. We hypothesize that this technique will allow for faster calibration than an exhaustive search (720 seconds) while maintaining sufficient system accuracy across users.

6.1 Protocol

As a baseline, we perform an exhaustive search of SVM accuracy on taskpairs identified each subject's best-performing taskpair. We recorded the frequency of each task's occurrence in a best-case taskpair 1. We used these datato inform the order in which our opportunistic calibration strategy would prompt the user to record tasks.

We begin with examples of the three tasks most commonly associated with best-case performance (base, pass, color) for an initial user calibration time of 120 seconds. We then perform a seven-fold cross-validation on every permutation of two of these tasks (base versus task, pass versus task, pass versus color, etc). The taskpair with the highest mean score across cross-validation rounds is selected for an additional testing session, in which the reamining 80 seconds of

recordings for both tasks are used to generate an estimate of the classifier's accuracy on new EEG signals.

| Task | Freq Bestcase |
|--------|---------------|
| Color | 10 |
| Base | 5 |
| Pass | 4 |
| Sport | 3 |
| Finger | 2 |
| Song | 2 |
| Eye | 2 |

Table 1: An exhaustive search of SVM accuracy on taskpairs identified each subject's best-performing taskpair. We recorded the frequency of each task's occurrence in a best-case taskpair, shown here. We used these datato inform the order in which our opportunistic calibration strategy would prompt the user to record tasks.

If the score on this additional testing round is below 75% (a threshold for BCI literacy) (Vidaurre and Blankertz, 2010), the user records sixty seconds of the taskpair next most correlated with bestcase accuracy across users. We repeat the above process on unexplored taskpairs repeated until a taskpair acheives over 75% accuracy on post-calibration data, or until all taskpairs have been evaluated.

H1: We hypothesize that the opportunistic calibration strategy will reach threshold accuracy in less time than will the exhaustive search.

H2: We hypothesize that the opportunistic strategy will acheive lower accuracy than will the exhaustive method, as it could find local optima.

6.2 Results

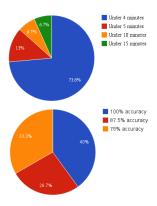


Figure 5: Calibration time across subjects (top) and classifier accuracy (bottom). The vast majority of subjects acheive acceptable accuracy in under five minutes of training, and all subjects acheive BCI literacy in under 15 minutes.

A baseline exhaustive search required 2520 sec-

onds of calibration time and produced an average accuracy of 92.5% (σ = .09) across subjects . Our opportunistic strategy took an average of 225.3 seconds of calibration time (σ = 52.2) and produced an average accuracy of 88.3% (σ = .11).

We find that our strategy calibrates users to BCI control significantly more quickly than an exhaustive search (p < .001). Thus, we find support for H1. Further, we do not a statistically significant difference in per-user accuracy between an opportunistic strategy and an exhaustive search (p = .264). Thus, we find no support for H2.

7 CONCLUSION

In this study, we investigate the effect of a signal quantization technique on the performance of a binary BCI that uses a low-cost, single-channel EEG headset as input. We find that our technique allows for a computationally efficient BCI that achieves good simulated accuracy for all subjects in our dataset and boasts quick user calibration times overall.

Specifically, we find that our quantization method decreases the computational expense of EEG-based calibration (from 18 ms to 2ms for SVM training time) without a significant detriment to accuracy and, using quantized data, our opportunistic user calibration strategy achieves an average of 88.3% accuracy across all subjects. All subjects require under fifteen minutes of calibration time, and 86.6% of these subjects require five minutes or fewer.

The conclusions to be drawn from this study are limited in a few regards. First, calibration and classification are performed offline, so factors involving the user interface (such as feedback) are not taken into account. We cannot be sure, for instance, that our findings based on the splicing of 10-second-long recorded data will persist when a system solicits recordings of only a second or under. Furthermore, a few of the tasks (e.g., the color task) relied on exogenous stimuli, which may be impractical in naturalistic settings for ergonomic reasons.

Our study implies that practical BCI can be achieved with as few as one inexpensive EEG sensor, minimal processing power, and a only a few minutes of user calibration. Future work could build usable, online BCI systems to test this claim more rigorously, especially in mobile and out-of-lab environments.

Since many types of physiological data can be represented as power spectra (e.g., electrocardiography, electromyography), future work could test our quantization technique in other classification-based physiological applications, such as heart sensing, gesture

recognition, or systems with heterogeneous sensors.

Logarithmic binning dramatically decreases the size of physiological data in memory. This technique could allow developers to more easily ship bio-signals to remote servers. Future work could explore the design space associated with the storage and transmission of physiological signals. BCI calibration, for example, could occur "in the cloud" where the client would quantize power spectra data using our method before shipping these compressed data to a more powerful server in the network.

Alternatively, the small size of quantized feature vectors could enable long-term, pervasive recording of mental states. The data would be small enough to ship and store on a centralized server, or to store locally in a decentralized fashion. Monitoring mental and other biophysical activity continuously in every-day settings could yield novel observations about human activity & physiology.

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