G53KRR 2018 Answer to Informal Exercise 4

1. Rewrite the following first order formulas to clausal form:

   (a) $\forall x(P(x) \wedge \neg\exists y R(x,y))$

   (b) $\forall z(\exists x \exists y R(x,y) \supset Q(x,y,z))$

   Answer.

   (a) $[P(x)]$, $[\neg R(x,y)]$

   (b) $[\neg R(x,y), Q(x,y,z)]$

   Explanation: (you don't have to provide it in formal exercise/exam, although in may be useful to show working in case you make a small mistake somewhere).

   (a) $\forall x(P(x) \wedge \neg\exists y R(x,y))$ is equivalent by $\neg\exists x\alpha \equiv \forall x\neg\alpha$ to $\forall x(P(x) \wedge \forall y \neg R(x,y))$, since y is not free in $P(x)$ it is equivalent to $\forall x \forall y(P(x) \wedge \neg R(x,y))$ which gives two clauses $[P(x)]$, $[\neg R(x,y)]$.

   (b) $\forall z(\exists x \exists y R(x,y) \supset Q(x,y,z))$ by the definition of implication is equivalent to $\forall z(\neg\exists x \exists y R(x,y) \vee Q(x,y,z))$ which is equivalent to $\forall z(\forall x \forall y \neg R(x,y) \vee Q(x,y,z))$ which gives a clause $[\neg R(x,y), Q(x,y,z)]$.

2. Derive by resolution an empty clause from the following clauses:

   C1 $[P(a), P(f(a))]$ where a is a constant

   C2 $[Q(f(x))]$

   C3 $[\neg P(x_1), R(x_1, f(x_1)]$

   C4 $[\neg R(x_2, y_2), \neg Q(y_2)]$

   Answer. the shortest proof (found in exam answers to exam 2013/14):

   C5 $[\neg R(x_2, f(x))]$ from C2, C4, $y_2/f(x)$

   C6 $[\neg P(x_1)]$ from C5, C3, x_2/x_1, x/x_1

   C7 $[P(a))]$ from C6, C1, $x_1/f(a)$

   C8 $[\ ]$ from C6, C7, x_1/a

   Another proof (one I came up with first for the model answers)

   C5 $[P(f(a)), R(a, f(a))]$ from C1, C3, x_1/a

   C6 $[P(f(a)), \neg Q(f(a)]$ from C5, C4, x_2/a, $y_2/f(a)$

   C7 $[P(f(a))]$ from C6, C2, x/a

   C8 $[R(f(a), f(f(a)))]$ from C7, C3, $x_1/f(a)$

   C9 $[\neg Q(f(f(a)))]$ from C8, C4, $x_2/f(a)$, $y_2/f(f(a))$

   C10 $[]$ from C9, C2, $x/f(f(a))$

3. For the pairs of literals below, state whether they unify, and if yes give a unifying substitution. Note that $x, y, z, z_1, z_2, z_3, u$ are variables and a a constant.

   (a) $R(x, f(a, x), g(y), y)$ and $R(a, z_1, g(z_2), z_3)$

   (b) $P(a, f(a), f(a))$ and $P(z, g(u), g(u))$

   Answer.

   (a) $R(x, f(a, x), g(y), y)$ and $R(a, z_1, g(z_2), z_3)$: $x/a, z_1/f(a, a), z_2/y, z_3/y$

   (b) $P(a, f(a), f(a))$ and $P(z, g(u), g(u))$: not unifiable.