G53KRR 2008-2009 answers

1. (a) Give an inductive truth definition for first order logic, that is, define when an interpretation $M = (D, I)$ and an assignment μ satisfy a formula ϕ. Assume that ϕ is defined by the following grammar:

$$\phi \quad = \quad P(t_1, \ldots, t_n) \mid \neg\phi \mid \phi \wedge \phi \mid \forall x\phi$$

where terms t_i are either variables or constants. (7 marks)

Answer. Let $[t]_{M,\mu}$ be the denotation of a term t in M under μ. For a variable x, $[x]_{M,\mu}$ is $\mu(x)$ and for a constant a, $[a]_{M,\mu}$ is $I(a)$. Then

$M, \mu \models P(t_1, \ldots, t_n)$ iff $\langle [t_1]_{M,\mu}, \ldots, [t_n]_{M,\mu} \rangle \in I(P)$

$M, \mu \models \neg\phi$ iff $M, \mu \not\models \phi$

$M, \mu \models \phi_1 \wedge \phi_2$ iff $M, \mu \models \phi_1$ and $M, \mu \models \phi_2$.

$M, \mu \models \forall x\phi$ iff for every assignment μ' which differs from μ at most in the value for x, $M, \mu' \models \phi$.

(b) Consider the following set of sentences:

S1 Andrew is the father of Bob.

S2 Bob is the father of Chris.

S3 Every grandfather is someone's father.

S4 Andrew is a grandfather of Chris.

Translate these sentences into first-order logic, using binary predicates $Father$ and $Grandfather$ and constants a, b, c for Andrew, Bob and Chris. Show semantically (by reasoning about interpretations) that **S1**–**S3** do not logically entail **S4**. (8 marks)

Answer. The sentences are:

S1 $Father(a, b)$

S2 $Father(b, c)$

S3 $\forall x(\exists y Grandfather(x, y) \supset \exists z Father(x, z))$

S4 $Grandfather(a, c)$

Consider an interpretation $M = (D, I)$ where $D = \{a, b, c\}$, the constants are interpreted in the obvious way, $I(Father) = \{(a, b), (b, c)\}$, and $I(Grandfather)$ is empty. Then in M, **S1**–**S3** are true and $Grandfather(a, c)$ is false.

(c) Write in first-order logic an additional sentence that defines a general property of grandfathers, and show that **S1**–**S3** together with this new sentence entail $Grandfather(a, c)$. (10 marks)

Answer. The sentence is:

$$\forall x \forall y \forall z(Father(x, y) \wedge Father(y, z) \supset Grandfather(x, z))$$

Now every interpretation which satisfies $Father(a, b)$, $Father(b, c)$, that is where $\{(a, b), (b, c)\} \subseteq I(Father)$, has to have $\{(a, c)\} \subseteq I(Grandfather)$, so has to satisfy $Grandfather(a, c)$.

2. (a) Reduce the following sentences to clausal form: (5 marks)

S1 $\exists x \forall y Less(x, y)$

S2 $\forall x \exists y Less(x, y)$

S3 $\forall x \forall y \forall z (Less(x, y) \wedge Less(y, z) \supset Less(x, z))$

Answer.

S1 $[Less(a, y)]$

S2 $[Less(x, f(x))]$

S3 $[\neg Less(x, y), \neg Less(y, z), Less(x, z)]$

(b) Show by resolution that clauses **C1**–**C3** below entail $\exists x Q(x)$. (10 marks)

C1 $[\neg P(x), \neg R(x, y), Q(x)]$

C2 $[\neg P(x), R(x, f(x))]$

C3 $[P(a)]$

Answer. We need to translate the negation of $\exists x Q(x)$ into clausal form, add it to **C1**–**C3** and derive the empty clause. The translation of $\neg \exists x Q(x)$ into clausal form is $\neg x \exists Q(x) = \forall x \neg Q(x) = [\neg Q(x)]$. So we add this clause to **C1**–**C3**:

C4 $[\neg Q(x)]$

Now the derivation:

1. $[\neg P(x), \neg R(x, y)]$ from C1, C4
2. $[R(a, f(a)]$ from C2, C3, x/a
3. $[\neg P(a)]$ from 2, C2, x/a
4. [] from 3, C3.

(c) Show by resolution that the following set of clauses is inconsistent. (8 marks)

$$[A, B, C],\ [A, B, \neg C],\ [A, \neg B, C],\ [A, \neg B, \neg C],$$

$$[\neg A, B, C],\ [\neg A, B, \neg C], [\neg A, \neg B, C],\ [\neg A, \neg B, \neg C]$$

Answer.

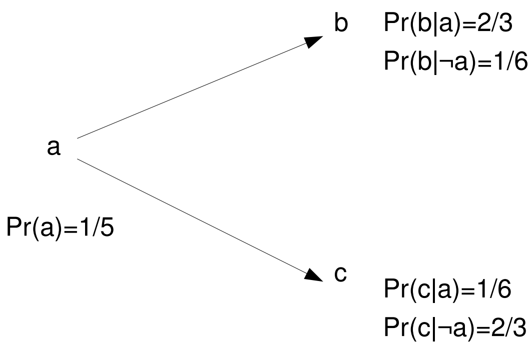
1. $[A, B]$ from $[A, B, C]$, $[A, B, \neg C]$.
2. $[A, \neg B]$ from $[A, \neg B, C]$, $[A, \neg B, \neg C]$.
3. $[A]$ from 1,2.
4. $[\neg A, B]$ from $[\neg A, B, C]$, $[\neg A, B, \neg C]$.
5. $[\neg A, \neg B]$ from $[\neg A, \neg B, C]$, $[\neg A, \neg B, \neg C]$.
6. $[\neg A]$ from 4,5.
7. [] from 3,6.

(d) Say whether your proof in part (c) is an SLD resolution proof. Explain your answer. (2 marks) *Answer.* This is not an SLD derivation (in fact there is no SLD derivation of [] from these clauses). The reason it is not an SLD derivation is because there is a step which involves two derived clauses ($[A]$ and $[\neg A]$), instead of the last derived clause and one of the clauses from the set of premises.

3. (a) Explain what the nodes arcs represent in a belief network or Bayesian network, and how to compute a joint probability distribution given a belief network. What is the advantage of using belief networks compared to explicitly giving a joint probability distribution? (5 marks)

Answer. Nodes are boolean variables, arcs correspond to causal relationships. Each propositional variable in the belief network is conditionally independent from the nonparent variables given the parent variables. This is a more compact way of representing probabilistic information than joint probability distribution.

(b) Given the following belief network:



i. Give an example of an independence assumption that is implicit in this network. (3 marks)
ii. What is the probability that a, b and c are all true? (4 marks)
iii. What is the probability that a, b and c are all false? (4 marks)
iv. What is the probablity of $b \wedge c$? (4 marks)
v. Is $b \wedge c$ more probable when a is true or when a is false? (5 marks)

*A*nswer.

i. $Pr(c|a) = Pr(c|a \wedge b)$
ii. $Pr(a \wedge b \wedge c) = Pr(a) \times Pr(b|a) \times Pr(c|a) = 1/5 \times 2/3 \times 1/6 = 2/90 = 1/45$.
iii. $Pr(\neg a \wedge \neg b \wedge \neg c) = (1 - Pr(a)) \times (1 - Pr(b|\neg a)) \times (1 - Pr(c|\neg a)) = 4/5 \times 5/6 \times \times 1/3 = 20/90 = 2/9$.
iv. $Pr(b \wedge c) = Pr(a \wedge b \wedge c) + Pr(\neg a \wedge b \wedge c) = 1/45 + (1 - Pr(a)) \times Pr(b|\neg a) \times Pr(c|\neg a) = 1/45 + 4/5 \times 1/6 \times 2/3 = 1/45 + 8/90 = 1/45 + 4/45 = 5/45 = 1/9$.
v. $Pr(b \wedge c|a) = Pr(a \wedge b \wedge c)/Pr(a) = 1/45 \times 5 = 1/9$. $Pr(b \wedge c|\neg a) = Pr(\neg a \wedge b \wedge c)/Pr(\neg a) = 4/45 \ / \ 4/5 = 4 \times 5/45 \times 4 = 1/9$. No difference.

4. (a) Give a forward-chaining procedure for propositional Horn clauses. (5 marks)

Answer.

input: a finite list of atomic sentences, $q_1, \ldots, q_n$

output: YES if KB entails all of q_i, NO otherwise

1. if all goals q_i are marked as solved, return YES
2. check if there is a clause $[p, \neg p_1, \ldots, \neg p_m]$ in KB, such that all of $p_1, \ldots, p_m$ are marked as solved and p is not marked as solved
3. if there is such a clause, then mark p as solved and go to step 1.
4. otherwise, return NO.

(b) Explain the notions of rule matching, rule instance, conflict set, and conflict resolution strategy in rule-based systems. Give two examples of common conflict resolution strategies. Illustrate your answers on the following example of rules and working memory elements. State what the conflict set is for the current state of the working memory and which rules will be fired first under each conflict resolution strategy. You can also refer to the conflict set at the next cycle, after the selected rules are fired. (10 marks)

F1 $animal(tiger)$

F2 $animal(cat)$

F3 $large(tiger)$

F4 $eatsMeat(tiger)$

F4 $eatsMeat(cat)$

R1 $\forall x(animal(x) \land large(x) \land eatsMeat(x) \supset dangerous(x))$

R2 $\forall x(animal(x) \supset breathesOxygen(x))$

R3 $\forall x(dangerous(x) \supset runAwayNow)$

Answer.

In rule-based systems, patterns in the body of each rule are matched against workging memory elements. Each successful matching (a unification which makes the patterns identical with the working memory elements) is a rule instance. The conflict set contains all rule instances applicable for the current state of working memory. The conflict resolution strategy is used to determine which of the rule instances in the conflict set will actually be fired. Most conflict resolution strategies pick a single rule instance based on specificity of the rules (which rule has a more specific pattern in the body), the order in which rules appear in the program, the order in which facts were added to working memory (for example, depth first where rule instances involving more recent facts are preferred) etc.

The conflict set is: **R1** with $x/tiger$ (matching **F1, F3, F4**), **R2** with $x/tiger$ (matching **F1**) and x/cat (matching **F2**).

Under the rule order and most specific rule first, the first rule instance will be selected to assert $dangerous(tiger)$ into the working memory.

This adds a new matching rule instance and the conflict set is **R2** with $x/tiger$ (matching **F1**) and x/cat (matching **F2**), **R3** with $x/tiger$. Now under the order of rules strategy the next fact to be fired will be an instance of **R2**, for example and under more recent first strategy, the instance of **R3**, because it matches more recent fact.

(c) Explain how decision tables can be used for knowledge elicitation and designing an expert system. (3 marks)

Answer. Decision tables can be used to structure the knowledge before producing a rule-based expert system. First we need to decide on all conditions relevant for making a decision, and all possible actions. A decision table consists of a list of condition stubs (possible relevant conditions), action stubs, condition entries (yes, no, and - for not relevant, for each condition) and action entries (which action is to be taken if specified conditions hold). Each column in the table corresponds to a rule (what to do if the pattern of conditions applies).

(d) Suppose all you have to work with in designing an expert system for recognising spam email is the following set of correctly classified messages. Produce a decision table based on this set of examples. For full marks, do not include irrelevant checks in the rules. (7 marks)

Message 1 Properties: has an attachment, does not contain images, sender is in the receiver's address book, subject line contains "Prize". **Decision**: spam.

Message 2 Properties: no attachments, contains images, sender is not in the receiver's address book, subject line contains "Goods". **Decision**: spam.

Message 3 Properties: has an attachment, contains images, sender is in the receiver's address book, subject line contains "Prize". **Decision**: spam.

Message 4 Properties: no attachments, does not contain images, sender is not in the receiver's address book, subject line does not contain "Prize" or "Goods". **Decision**: not spam.

Message 5 Properties: has an attachment, does not contain images, sender is not in the receiver's address book, subject line contains "Prize". **Decision**: spam.

Message 6 Properties: has no attachments, contains images, sender is in the receiver's address book, subject line contains "Goods". **Decision**: not spam.

Message 7 Properties: has no attachments, does not contain images, sender is not in the receiver's address book, subject line contains "Goods". **Decision**: spam.

Message 8 Properties: has no attachments, contains images, sender is not in the receiver's address book, subject line does not contain "Prize" or "Goods". **Decision**: not spam.

Answer.

	Rule 1	Rule 2	Rule 3	Rule 4
Subject contains "Prize"	Y	N	N	N
Subject contains "Goods"	-	N	Y	Y
Sender in address book	-	-	N	Y
Spam	X		X	
Not spam		X		X

5. Consider a description logic with the following definition of a concept (note that it is slightly different from the one in the textbook, namely the first concept constructor is new and the forth concept constructor is different from [**EXISTS** n r]):

   - $\top$ is a special atomic concept which describes any object (it is a property which is trivially true for everything)
   - an atomic concept is a concept
   - if r is a role and b is a concept, then [**ALL** r b] is a concept (describing objects all of whose r-successors are described by b)
   - if r is a role and b is a concept, then [**EXISTS** r b] is a concept (describing objects which have at least one r-successor which is described by b)
   - if r is a role and c is a constant, then [**FILLS** r c] is a concept (describing objects which have an r-successor denoted by c)
   - if $b_1, \ldots, b_n$ are concepts, [**AND** $b_1 \ldots b_n$] is a concept (describing objects which are described by all of $b_1, \ldots, b_n$)

   and the following definition of a sentence:

   - if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s)
   - if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)
   - if c is a constant and b a concept then $c \rightarrow b$ is a sentence (the individual denoted by c satisfies the description expressed by b).

   (a) Define interpretations (D, I) for this description logic and give an inductive definition of the meaning of concepts (extend the interpretation mapping I to complex concepts). Give conditions for the truth of sentences in an interpretation. (10 marks)

   Answer.
   Interpretations for DL consist of a set of individuals D and an interpretation mapping I such that

   - for a constant c, $I(c) \in D$
   - for an atomic concept a, $I(a) \subseteq D$
   - $I(\top) = D$
   - for a role r, $I(r) \subseteq D \times D$
   - $I([\mathbf{A}LL\ r\ b]) = \{x \in D : \text{for any } y, \text{ if } (x, y) \in I(r), \text{ then } y \in I(b)\}$.
   - $I([\textbf{EXISTS}\ r\ b]) = \{x \in D : \text{there is a } y \text{ such that } (x, y) \in I(r) \text{ and } y \in I(b)\}$.
   - $I([\textbf{FILLS}\ r\ c]) = \{x \in D :\ (x, I(c)) \in I(r)\}$.
   - $I([\textbf{AND}\ b_1 \ldots b_n]) = I(b_1) \cap \ldots \cap I(b_n)$.

   Finally, for sentences:

   - $(D, I) \models c \rightarrow b$ iff $I(c) \in I(b)$.
   - $(D, I) \models b_1 \sqsubseteq b_2$ iff $I(b_1) \subseteq I(b_2)$.
   - $(D, I) \models b_1 \doteq b_2$ iff $I(b_1) = I(b_2)$.

   (b) Given the atomic concepts $Female$, $Male$, roles $:Child$, $:Sibling$ and constant $alice$, define in the description logic above the following concepts:

i. "Mother of Alice" (someone female whose child is Alice) (3 marks)
ii. "Parent" (someone who has a child) [Hint: use $\top$ to describe the child] (3 marks)
iii. "Uncle" (someone male who has a sibling who has a child) (3 marks)

Answer.

i. $[\textbf{AND}\ Female,\ \textbf{FILLS}\ :Child\ alice]]$
ii. $[\textbf{EXISTS}\ :Child\ \top]$
iii. $[\textbf{AND}\ Male,\ [\textbf{EXISTS}\ :Sibling\ [\textbf{EXISTS}\ :Child\ \top]]]$

(c) Using the same atomic concepts as in part (b), translate the following sentences in description logic:

i. Every grandparent is a parent (3 marks)
ii. Alice is a grandmother (3 marks)

Answer.

i. $[\textbf{EXISTS}\ :Child\ [\textbf{EXISTS}\ :Child\ \top]] \sqsubseteq [\textbf{EXISTS}\ :Child\ \top]$
ii. $alice \rightarrow [\textbf{AND}\ Female, [\textbf{EXISTS}\ :Child\ [\textbf{EXISTS}\ :Child\ \top]]]$

6. (a) Give the definitions of minimal models and minimal entailment in circumscription theory. (5 marks)

Answer.

Minimal entailment considers not all models of the knowledge base but only those where the set of exceptions is made as small as possible. Namely, consider a predicate Ab (for abnormal) and the formulation of a default rule as

$$\forall x(Bird(x) \land \neg Ab(x) \supset Flies(x))$$

and say that a conclusion follows by default if it is entailed on all interpretations where the extension of Ab is as small as possible. (This is called *circumscribing* Ab and the approach is called *circumscription.*) We need one Ab for every default rule, because a bird which is abnormal with respect to flying may be normal with respect to having two legs etc.

Let A be the set of Ab predicates we want to minimise. Let $M_1 = (D, I_1)$ and $M_2 = (D, I_2)$ be two interpretations over the same domain such that every constant and function are interpreted the same way.

$$M_1 \leq M_2 \ \Leftrightarrow\ \forall Ab \in A\ (I_1(Ab) \subseteq I_2(Ab))$$

$M_1 < M_2$ if $M_1 \leq M_2$ but not $M_2 \leq M_1$. (There are strictly fewer abnormal things in M_1).

Minimal entailment: $KB \models_{\leq} \phi$ iff for all interpretations M which make KB true, either $M \models \phi$ or M is not minimal (there exists an M' such that $M' < M$ and $M' \models KB$).

(b) For the following KB:

$$KB = \{Dutchman(peter),\ Dutchman(hans),\ Dutchman(johan),$$
$$peter \neq hans,\ hans \neq johan,\ peter \neq johan,$$
$$\neg Tall(peter) \lor \neg Tall(hans),\ \forall x(Dutchman(x) \land \neg Ab(x) \supset Tall(x))\}$$

state whether the following sentences are minimally entailed, and explain why:

i. $Tall(johan)$ (4 marks)

ii. $Tall(peter) \vee Tall(hans)$ (4 marks)

Answer.

i. $Tall(johan)$ is minimally entailed. The minimal models of KB are those where the extension $I(Ab)$ of Ab is either $\{I(peter)\}$ or $\{I(hans)\}$, but not both. In all of them $I(johan)$ is in $I(Dutchman)$ and not in $I(Ab)$, so it is in $I(Tall)$.

ii. $Tall(peter) \vee Tall(hans)$ is also minimally entailed, because if $I(Ab)$ includes $I(peter)$ then in the minimal model it does not include $I(hans)$, and vice versa.

(c) Explain what a default rule $\frac{\alpha : \beta}{\delta}$ in Reiter's default logic means. (3 marks)

Answer.

A *default rule* consists of a *prerequisite* α, *justification* β, *conclusion* δ and says 'if α holds and it is consistent to believe β, then believe δ': $\frac{\alpha : \beta}{\delta}$

(d) What is an extension of a default theory (F, D) (where F is a set of first order sentences and D a set of default rules)? (5 marks)

Answer.

E is an *extension* of (F, D) iff for every sentence π,

$$\pi \in E \ \Leftrightarrow\ F \cup \{\delta \mid \frac{\alpha : \beta}{\delta} \in D, \alpha \in E, \neg\beta \notin E\} \models \pi$$

where $\{\delta \mid \frac{\alpha:\beta}{\delta} \in D, \alpha \in E, \neg\beta \notin E\}$ is a set of *applicable assumptions.*

(e) Is there an extension of the following default theory (F, D) which contains $Tall(peter)$:

$$F = \{Dutchman(peter),\ Dutchman(hans),\ Dutchman(johan),\ peter \neq hans,$$

$$hans \neq johan,\ peter \neq johan,\ \neg Tall(peter) \vee \neg Tall(hans)\}$$

and

$$D = \{\frac{Dutchman(x) : Tall(x)}{Tall(x)}\}$$

If yes, list applicable assumptions for this extension. (4 marks)

Answer.

Yes, and the applicable assumptions are $Tall(johan), Tall(peter)$.