Answers G53KRR 2012-2013

BOOKWORK VS PROBLEM SOLVING: questions are problem solving unless stated otherwise. Students have done exercises of the same type (e.g. translating English into first-order logic, or resolution proofs) but sufficiently different, so that memorising a solution to the exercise is not useful.

1. This question is on first-order logic.

   (a) Translate the sentences below from English into first-order logic. Use the following predicates: *Lecturer* (unary predicate), *Student* (unary predicate), *Teaches* (binary predicate, $Teaches(x, y)$ means lecturer x teaches module y).

      i. Nobody is both a lecturer and a student (2 marks)
      ii. Every lecturer teaches some module (2 marks)
      iii. No two lecturers teach the same module (3 marks)

   Answer.

      i. $\neg\exists x(Lecturer(x) \wedge Student(x))$
      ii. $\forall x(Lecturer(x) \supset \exists y Teaches(x, y))$
      iii. $\neg\exists x\exists y\exists z(Teaches(x, z) \wedge Teaches(y, z) \wedge \neg(x = y))$
      Other versions are possible, for example

$$\forall x\forall y\forall z(Teaches(x, z) \wedge Teaches(y, z) \supset x = y))$$

      or

$$\forall x\forall y\forall z\forall u(Teaches(x, z) \wedge Teaches(y, u) \wedge \neg(x = y) \supset \neg(z = u))$$

      etc. All will get full marks.

   (b) Which variables (if any) are free in the formulas below:

      i. $\forall x(\exists y R(x, y) \vee R(x, f(z)))$ (1 marks)
      ii. $(\exists x P(x)) \wedge (\exists y R(x, y))$ (1 marks)
      iii. $\exists x(P(x) \wedge (\exists y R(x, y)))$ (1 marks)

   Answer.

      i. z
      ii. x (in $\exists y R(x, y)$)
      iii. none

   (c) For each of the first-order sentences below, state whether it is true or false in the following interpretation, *and explain why*. The interpretation is (D, I) where $D = \{d_1, d_2, d_3\}$ and $I(P) = \{d_1, d_2\}$, $I(Q) = \{d_3\}$, $I(R) = \{\langle d_1, d_2\rangle, \langle d_2, d_3\rangle, \langle d_3, d_3\rangle\}$.

      i. $\forall x(P(x) \supset \neg Q(x))$ (5 marks)
      ii. $\forall x\exists y(R(x, y) \wedge P(y))$ (5 marks)
      iii. $\forall x(P(x) \vee \exists y\neg R(x, y))$ (5 marks)

   Answer.

      i. $\forall x(P(x) \supset \neg Q(x))$ is true, because for all assignments to x satisfying $P(x)$ (for x assigned d_1 or d_2), $Q(x)$ is false, hence $\neg Q(x)$ is true.
      ii. $\forall x\exists y(R(x, y) \wedge P(y))$ is false, because there is an assignment to x (of d_2 or d_3) such that there is no assignment to y which makes $R(x, y) \wedge P(y)$ true (the only value for y for which $R(x, y)$ holds is d_3, but then $P(y)$ is false).
      iii. $\forall x(P(x) \vee \exists y\neg R(x, y))$ is true because for all possible assignments to x either $P(x)$ is true (for d_1, d_2) or $\exists y\neg R(x, y)$ holds (if x is d_3, then a suitable value for y would be for example d_1).

2. This question is on clausal form and resolution.

(a) Reduce the following sentences to clausal form: (10 marks)

S1 $\forall x \forall y(R(x, y) \wedge P(y) \supset Q(x))$

S2 $\forall x((\exists z R(x, z)) \supset Q(x))$

S3 $\forall x \exists y(R(x, y) \vee R(y, x))$

S4 $\exists x P(x) \wedge \forall y \exists z R(y, z)$

S5 $\neg \exists x(P(x) \wedge \forall y \neg R(x, y))$

Answer.

S1 $[\neg R(x, y), \neg P(y), Q(x)]$

S2 $[\neg R(x, z), Q(x)]$

S3 $[R(x, f(x)), R(f(x), x)]$

S4 $[P(c)], [R(y, g(y)]$

S5 $[\neg P(x), R(x, h(x))]$

(b) Derive by resolution an empty clause from the following clauses: (10 marks)

C1 $[R(x_0, f(x_0))]$

C2 $[\neg R(x, y), \neg R(y, z), R(x, z)]$

C3 $[\neg R(x_1, f(f(f(x_1))))]$

Answer.

1 $[\neg R(f(x), z), R(x, z)]$ from C1, C2, substitution x_0/x, $y/f(x)$

2 $[R(x, f(f(x))]$ from 1, C1, substitution $x_0/f(x)$, z/f(f(x))

3 $[\neg R(f(f(x)), f(f(f(x)))), R(x, f(f(f(x))))]$ from 2, C2, substitution $y/f(f(x)), z/f(f(f(x)))$

4 $[R(x, f(f(f(x)))]$ from C1,3, substitution $x_0/f(f(x))$

5 [ ] from C3, 4, x/x_1

(c) Derive by resolution an empty clause from the clauses below. Use equality axioms. Note that a, b are constants. (5 marks)

C1 $[P(a)]$

C2 $[\neg P(b)]$

C3 $[x = b]$

Answer.

1 $[\neg(x = y), \neg P(x), P(y)]$ equality axiom

2 $[\neg P(a), P(b)]$ from 1, C3, substitution x/a, y/b

3 $[P(b)]$ from C1, 2

4 [ ] from C2, 3

3. This question is on unification.

   (a) What is a most general unifier for two literals? (3 marks)

   Answer. (BOOKWORK) A unifier of two literals ρ_1 and ρ_2 is a substitution θ such that $\rho_1\theta = \rho_2\theta$. A most general unifier (mgu) for ρ_1 and ρ_2 is a unifier θ such that for any other unifier θ', there is a further substitution $\theta*$ such that $\theta' = \theta\theta*$.

   (b) Give an algorithm for finding a most general unifier for two literals ρ_1 and ρ_2. (7 marks)

   Answer. (BOOKWORK)

   1 start with $\theta = \{\ \}$

   2 exit if $\rho_1\theta = \rho_2\theta$

   3 set DS to be the pair of terms at the first place where $\rho_1\theta$ and $\rho_2\theta$ disagree

   4 find a variable v in DS and a term t in DS not containing v; if none exist, fail

   5 otherwise set θ to $\theta\{v/t\}$ and go to step 2.

   (c) For the pairs of literals below, state whether they unify, and if yes give a unifying substitution. Note that x, y, z, u are variables and a a constant.

      i. $P(x, f(x, g(y)), y)$ and $P(a, z, u)$ (5 marks)

      ii. $P(x, g(y), x)$ and $P(z, f(u, u), z)$ (5 marks)

      iii. $P(g(x), g(y), g(y))$ and $P(z, u, u)$ (5 marks)

   Answer. Unifiers are given below, other answers are possible and if correct will get full marks.

      i. $P(x, f(x, g(y)), y)$ and $P(a, z, u)$ a unifier is $x/a, z/f(a, g(y)), u/y$

      ii. $P(x, g(y), x)$ and $P(z, f(u, u), z)$ do not unify (mgu algorithm will fail trying to unify $g(y)$ and $f(u, u)$)

      iii. $P(g(x), g(y), g(y))$ and $P(z, u, u)$ a unifier is $z/g(x), u/g(y)$

4. This question is on Horn clauses.

(a) Consider the following clauses:

C1 $[P(x), Q(y)]$
C2 $[P(x)]$
C3 $[\neg Q(f(y))]$
C4 $[\,]$
C5 $[P(x), \neg Q(y)]$

i. Which (if any) of the clauses above are positive Horn clauses? (2 marks)
ii. Which (if any) of the clauses above are negative Horn clauses? (2 marks)
iii. Which (if any) of the clauses above are unit Horn clauses? (2 marks)
iv. Which (if any) of the clauses above are *not* Horn clauses? (2 marks)

Answer.

i. positive Horn clauses: C2, C5
ii. negative Horn clauses: C3, C4
iii. unit Horn clauses: C2, C3
iv. not Horn: C1

(b) Give forward chaining procedure for *first-order* Horn clauses. Assume that the input is a finite KB and a ground unit clause A, and the procedure should return yes if A follows from KB and no otherwise. Assume that KB does not contain functions, and consists of facts (ground unit clauses) and rules (positive Horn clauses for which an additional condition holds: all free variables of the positive literal in the clause also occur in one of the negative literals of the clause. For example, $[P(x), \neg Q(x)]$ satisfies this condition, and $[R(x, y), \neg Q(x)]$ does not, because y only occurs in the positive literal). (10 marks)

Answer. Approximately 50% BOOKWORK (the textbook has propositional forward chaining).

Forward chaining .
input: a ground unit clause A
output: YES if KB entails A, NO otherwise

0. FACTS = ground unit clauses in KB.
1. if A is in FACTS, return YES
2. if there is $[\neg\rho_1, \ldots, \neg\rho_n, \rho]$ in KB such that for some substitution θ, $[\rho_1]\theta, \ldots, [\rho_n]\theta$ are in FACTS and $[\rho]\theta$ is not in FACTS: add $[\rho]\theta$ to FACTS and go to 1; else return NO.

(c) Show that $[Q(b)]$ is derivable by the forward chaining procedure from the following set of Horn clauses: (3 marks)

H1 $[\neg P(x), \neg R(x, y), Q(y)]$
H2 $[P(a)]$
H3 $[\neg P(x), \neg S(x, y), R(x, y)]$
H4 $[S(a, b)]$

Answer. FACTS = $\{[P(a)], [S(a,b)]\}$
$[Q(b)]$ is not in FACTS
add $[R(a,b)]$ to FACTS ($\theta = x/a, y/b$, H3)
$[Q(b)]$ is not in FACTS
add $[Q(b)]$ to FACTS ($\theta = x/a, y/b$, H1).

(d) Is the forward chaining procedure guaranteed to terminate if KB does contain functions? If yes, explain why. If not, give an example when it would not terminate. (4 marks)

Answer. Approximately 50% BOOKWORK (explained Herbrand theorem for general clauses, but did not dwell on it for Horn clauses).

The answer is no, and a simple example is $[P(a)]$, $[\neg P(x), P(f(x))]$. This will assert $[P(f(a))]$, $[P(f(f(a)))]$, $[P(f(f(f(a))))]$,...

5. This question is on non-monotonic reasoning.

(a) Define closed world assumption and the entailment relation $\models_{CWA}$. (3 marks)
Answer BOOKWORK
Closed-world assumption (CWA): if an atomic sentence is not classically entailed by the knowledge base, it is assumed to be false. The corresponding entailment $\models_{CWA}$:

$$KB \models_{CWA} \phi \;\Leftrightarrow\; KB^{+} \models \phi$$

where $KB^{+} = \{\neg p : p \text{ is atomic and } KB \not\models p\}$.

(b) Consider the following knowledge base:

$KB = \{R(a,b), R(c,d), R(a,e), R(c,a), \forall x \forall y(R(x,y) \supset R(y,x))\}$

i. Does it hold that $KB \models_{CWA} R(b,a)$? Explain why. (3 marks)
ii. Does it hold that $KB \models_{CWA} \neg R(b,a)$? Explain why. (3 marks)
iii. Does it hold that $KB \models_{CWA} R(a,d)$? Explain why. (3 marks)
iv. Does it hold that $KB \models_{CWA} \neg R(a,d)$? Explain why. (3 marks)

Answer.

i. yes, because $R(a,b)$ and $\forall x \forall y(R(x,y) \supset R(y,x))$ classically entail $R(b,a)$.
ii. no, because $KB \models R(b,a)$, so KB^{+} does not contain $\neg R(b,a)$.
iii. no, because $KB \not\models R(a,d)$, so KB^{+} contains $\neg R(a,d)$.
iv. yes, because $KB \not\models R(a,d)$, so KB^{+} contains $\neg R(a,d)$.

(c) Give the definitions of minimal models and minimal entailment in circumscription theory. (5 marks)
Answer. BOOKWORK
Minimal entailment considers not all models of the knowledge base but only those where the set of exceptions is made as small as possible. Namely, consider a predicate Ab (for abnormal) and the formulation of a default rule as

$$\forall x(Bird(x) \wedge \neg Ab(x) \supset Flies(x))$$

and say that a conclusion follows by default if it is entailed on all interpretations where the extension of Ab is as small as possible. (This is called *circumscribing* Ab and the approach is called *circumscription*.) We need one Ab for every default rule, because a bird which is abnormal with respect to flying may be normal with respect to having two legs etc.

Let A be the set of Ab predicates we want to minimise. Let $M_1 = (D, I_1)$ and $M_2 = (D, I_2)$ be two interpretations over the same domain such that every constant and function are interpreted the same way.

$$M_1 \leq M_2 \;\Leftrightarrow\; \forall Ab \in A \; (I_1(Ab) \subseteq I_2(Ab))$$

$M_1 < M_2$ if $M_1 \leq M_2$ but not $M_2 \leq M_1$. (There are strictly fewer abnormal things in M_1). *Minimal entailment:* $KB \models_{\leq} \phi$ iff for all interpretations M which make KB true, either $M \models \phi$ or M is not minimal (there exists an M' such that $M' < M$ and $M' \models KB$).

(d) For the following KB:

$KB = \{P(a), P(b), P(c), a \neq b, a \neq c, b \neq c, \forall x(P(x) \wedge \neg Ab(x) \supset Q(x)), \neg Q(c)\}$
state whether the sentence $Q(a)$ is minimally entailed, and explain why. (5 marks)
Answer. The sentence is minimally entailed. The minimal models of KB are those where the extension $I(Ab)$ of Ab is $I(c)$. In all of them $I(a)$ is not in $I(Ab)$ and in $I(P)$, so in all of them it is in $I(Q)$, so $Q(a)$ holds.

6. This question is on the description logic DL given in the textbook. A summary of its syntax and semantics is given after this question.

   (a) For each of the following expressions state whether it is a correctly formed concept description in this logic and if not, why (assuming an atomic concept $Male$ and a role $:Friend$)

   i. [**ALL** $:Friend$ [**ALL** $:Friend$ $Male$]] (1 marks)

   ii. [**EXISTS** $:Friend$ $Male$] (1 marks)

   iii. [**EXISTS** 3 $:Friend$] (1 marks)

   Answer.

   i. [**ALL** $:Friend$ $Male$] matches the basic syntax definition, and this expression is just [**ALL** $:Friend$ c] where $c =$ [**ALL** $:Friend$ $Male$].

   ii. not correct, **EXISTS** requires a number and a role, not a role and a concept description.

   iii. correct, matches the syntax definition ($n = 3$).

   (b) Given roles $:Sister$ and $:Brother$, define the following concepts:

   i. Someone who has 7 sisters (2 marks)

   ii. Someone who has 7 sisters and 7 brothers (2 marks)

   iii. Someone all of whose sisters have 7 brothers (3 marks)

   Answer.

   i. [**EXISTS** 7 $:Sister$]

   ii. [**AND** [**EXISTS** 7 $:Sister$] [**EXISTS** 7 $:Brother$]]

   iii. [**ALL** $:Sister$[**EXISTS** 7 $:Brother$]]

   (c) Consider the following interpretation (D, I): $D = \{d_1, d_2, d_3\}$, $I(r) = \{\langle d_1, d_2\rangle, \langle d_1, d_3\rangle\}$, $I(a) = d_1$ (a is a constant), $I(b) = \{d_2, d_3\}$ (b is an atomic concept). Which of the following sentences are true in this interpretation and why?

   i. $a \rightarrow$ [**ALL** r b] (5 marks)

   ii. $a \rightarrow$ [**EXISTS** 1 r] (5 marks)

   iii. [**EXISTS** 2 r] $\sqsubseteq$ [**EXISTS** 1 r] (5 marks)

   Answer.

   i. yes, $I(a)$ is d_1, and all objects related to d_1 by $I(r)$ (d_2, d_3) are in $I(b)$

   ii. yes, $I(a)$ is d_1, and it has at least one (in fact 2) objects related to it by $I(r)$

   iii. yes, this holds in general (objects which have at least 2 r-successors are included in the set of objects which have at least one r-successor).

   Summary of the syntax and semantics of DL :

   - an atomic concept is a concept
   - if r is a role and b is a concept, then [**ALL** r b] is a concept
   - if r is a role and n is a positive integer, then [**EXISTS** n r] is a concept
   - if r is a role and c is a constant, then [**FILLS** r c] is a concept
   - if $b_1, \ldots, b_n$ are concepts, [**AND** $b_1, \ldots, b_n$] is a concept.

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence
- if c is a constant and b a concept then $c \to b$ is a sentence

Interpretations for DL are similar to FOL: a set of individuals D and an interpretation mapping I such that

- for a constant c, $I(c) \in D$
- for an atomic concept a, $I(a) \subseteq D$
- for a role r, $I(r) \subseteq D \times D$
- $I([\textbf{ALL}\ r\ b]) = \{x \in D : \text{for any } y, \text{ if } (x,y) \in I(r), \text{ then } y \in I(b)\}$.
- $I([\textbf{EXISTS}\ n\ r]) = \{x \in D : \text{there are at least } n \text{ distinct } y \text{ such that } (x,y) \in I(r)\}$.
- $I([\textbf{FILLS}\ r\ c]) = \{x \in D :\ (x, I(c)) \in I(r)\}$.
- $I([\textbf{AND}\ b_1 \ldots b_n]) = I(b_1) \cap \ldots \cap I(b_n)$.

Finally, the truth definition for DL is:

- $(D, I) \models c \to b$ iff $I(c) \in I(b)$.
- $(D, I) \models b_1 \sqsubseteq b_2$ iff $I(b_1) \subseteq I(b_2)$.
- $(D, I) \models b_1 \doteq b_2$ iff $I(b_1) = I(b_2)$.