Answers G53KRR 2013-2014

BOOKWORK VS PROBLEM SOLVING: questions are problem solving unless stated otherwise. Students have done exercises of the same type (e.g. translating English into first-order logic, or resolution proofs) but sufficiently different, so that memorising a solution to the exercise is not useful.

1. This question is on first-order logic.

   (a) Translate the sentences below from English into first-order logic. Use a predicate symbol *Like* (where $Like(x, y)$ means that x likes y) and constants $jack$ and $jill$.

      i. Jack likes the same things Jill likes. (2 marks)

      ii. There is somebody who dislikes everything that Jack likes. (2 marks)

      iii. There is only one person apart from Jack who likes everything Jack likes, and this person is Jill. (3 marks)

   Answer.

      i. $\forall x(Like(jill, x) \supset Like(jack, x))$

      ii. $\exists x \forall y(Like(jack, y) \supset \neg Like(x, y))$

      iii. $\forall x \forall y((Like(jack, x) \supset Like(y, x)) \wedge \neg(y = jack) \supset y = jill)$

   (b) List any three logical symbols in the language of first-order logic. (3 marks)

   Answer. (BOOKWORK)
   Any three of, $\neg$, $\wedge$, other boolean connectives, $\exists$, $\forall$, variables and $=$.

   (c) What is an interpretation in first-order logic? State what an interpretation mapping assigns to non-logical symbols. (5 marks)

   Answer. (BOOKWORK)
   An interpretation is a pair (D, I) where D is a non-empty set and I is an interpretation mapping. I assigns to n-ary predicate symbols a subset of D^n, and an n-ary function on D to n-ary function symbols. [Constants are 0-ary function symbols, so it is not necessary to say that I assigns an element of D to a constant, but it is not a mistake and can be given an extra mark for if the answer omits general function symbols. They can also be given an extra compensation mark if they say that propositional variables are assigned either a set containing a 0-ary tuple (true) or an empty set (false).]

   (d) For the following pairs of sentences, show that the first sentence does not logically entail the second sentence by defining an interpretation where the first sentence is true and the second sentence is false:

      i. $\exists x P(x)$ and $P(a)$ (where a is a constant) (5 marks)

      ii. $\forall x \exists y R(x, y)$ and $\exists x \forall y R(x, y)$ (5 marks)

   Answer.

      i. For example, (D, I) where $D = \{d_1, d_2\}$ and $I(P) = \{d_1\}$ and $I(a) = d_2$

      ii. For example, (D, I) where $D = \{d_1, d_2\}$ and $I(R) = \{(d_1, d_2), (d_2, d_1)\}$.

2. This question is on clausal form and resolution.

(a) Reduce the following sentences to clausal form: (10 marks)

S1 $\forall x \forall y (P(x) \wedge \neg P(y) \wedge R(x,y) \supset \neg R(y,x))$

S2 $\forall x \forall y \exists z R(x,y,z)$

S3 $\neg \forall x \exists y (R(x,y) \supset R(y,x))$

S4 $\forall x (P(x) \supset R(x)) \wedge \exists y P(y)$

S5 $\forall x \forall y (P(x,y) \vee R(x,y) \supset \exists z Q(x,y,z))$

Answer.

S1 $[\neg P(x), P(y), \neg R(x,y), \neg R(y,x)]$

S2 $[R(x,y,f(x,y))]$

S3 $\neg \forall x \exists y (R(x,y) \supset R(y,x))$
$\exists x \forall y \neg (R(x,y) \supset R(y,x))$
$\exists x \forall y \neg (\neg R(x,y) \vee R(y,x))$
$\exists x \forall y (R(x,y) \wedge \neg R(y,x))$
$[R(c,y)]$, $[\neg R(y,c)]$

S4 $[\neg P(x), R(x)]$, $[P(c)]$

S5 $\forall x \forall y (P(x,y) \vee R(x,y) \supset \exists z Q(x,y,z))$
$\forall x \forall y (\neg (P(x,y) \vee R(x,y)) \vee \exists z Q(x,y,z))$
$\forall x \forall y (\neg P(x,y) \wedge \neg R(x,y)) \vee \exists z Q(x,y,z))$
$\forall x \forall y (\neg P(x,y) \vee \exists z Q(x,y,z)) \wedge (\neg R(x,y) \vee \exists z Q(x,y,z))$
$[\neg P(x,y), Q(x,y,f(x,y))]$, $[\neg R(x,y), Q(x,y,f(x,y))]$

(b) Derive by resolution an empty clause from the following clauses: (10 marks)

C1 $[P(a), P(f(a))]$ where a is a constant

C2 $[Q(f(x))]$

C3 $[\neg P(x_1), R(x_1, f(x_1)]$

C4 $[\neg R(x_2, y_2), \neg Q(y_2)]$

Answer.

the shortest proof (found in exam answers):

C5 $[\neg R(x_2, f(x))]$ from C2, C4, $y_2/f(x)$

C6 $[\neg P(x_1)]$ from C5, C3, x_2/x_1, x/x_1

C7 $[P(a))]$ from C6, C1, $x_1/f(a)$

C8 $[\,]$ from C6, C7, x_1/a

Another proof (the one I came up with first for the model answers)

C5 $[P(f(a)), R(a, f(a))]$ from C1, C3, x_1/a

C6 $[P(f(a)), \neg Q(f(a)]$ from C5, C4, x_2/a, $y_2/f(a)$

C7 $[P(f(a))]$ from C6, C2, x/a

C8 $[R(f(a), f(f(a)))]$ from C7, C3, $x_1/f(a)$

C9 $[\neg Q(f(f(a)))]$ from C8, C4, $x_2/f(a)$, $y_2/f(f(a))$

C10 $[]$ from C9, C2, $x/f(f(a))$

(c) Explain why resolution is not a complete inference procedure, and what does it mean that it is complete for refutation. (5 marks)

Answer. (BOOKWORK)

Resolution is not complete because it is not the case that for every S and α such that $S \models \alpha$ there is a resolution derivation of α from S: for example, $p \models p \vee q$ and there is no way to apply resolution rule to p and no resolution derivation of $p \vee q$ from p. However, if S is unsatisfiable ($S \models \bot$) then there is a resolution derivation of an empty clause from S: resolution is complete for refutation.

3. This question is on unification.

(a) For the following pairs of literals and a substitution, state whether the substitution unifies the two literals and whether it is a most general unifier: (3 marks)

i. $R(x, b)$ and $R(a, y)$, substitution x/a, y/b
ii. $R(x, f(x))$ and $R(y, f(y))$, substitution x/a, y/a
iii. $R(x, f(a))$ and $R(y, g(y))$, substitution x/a, y/a

Answer.

i. $R(x, b)$ and $R(a, y)$, substitution x/a, y/b: unification, mgu
ii. $R(x, f(x))$ and $R(y, f(y))$, substitution x/a, y/a: unification, not mgu
iii. $R(x, f(a))$ and $R(y, g(y))$, substitution x/a, y/a: not unification

(b) Give an algorithm for finding a most general unifier for two literals ρ_1 and ρ_2. (7 marks)

Answer. (BOOKWORK)

1 start with $\theta = \{ \}$
2 exit if $\rho_1\theta = \rho_2\theta$
3 set DS to be the pair of terms at the first place where $\rho_1\theta$ and $\rho_2\theta$ disagree
4 find a variable v in DS and a term t in DS not containing v; if none exist, fail
5 otherwise set θ to $\theta\{v/t\}$ and go to step 2.

(c) For the pairs of literals below, state whether they unify, and if yes give a most general unifying substitution. Note that $x, y, z, z_1, z_2, z_3, u$ are variables and a a constant.

i. $R(x, f(a, x), g(y), y)$ and $R(a, z_1, g(z_2), z_3)$ (5 marks)
ii. $P(x, g(y), y)$ and $P(z, g(f(u)), z)$ (5 marks)
iii. $P(a, f(a), f(a))$ and $P(z, g(u), g(u))$ (5 marks)

Answer.

i. $R(x, f(a, x), g(y), y)$ and $R(a, z_1, g(z_2), z_3)$: $x/a, z_1/f(a, a), z_2/y, z_3/y$
ii. $P(x, g(y), y)$ and $P(z, g(f(u)), z)$: $x/z, y/f(u), z/f(u)$ which is the same as $x/f(u), y(f(u), z/f(u)$.
iii. $P(a, f(a), f(a))$ and $P(z, g(u), g(u))$: not unifiable.

4. This question is on Horn clauses.

(a) Consider the following clauses:

C1 $[\neg R(x,y), \neg Q(x), Q(y)]$
C2 $[R(x,y), \neg Q(x), Q(y)]$
C3 $[\neg R(x,y), \neg Q(x), \neg Q(f(y))]$
C4 $[\neg R(x,y)]$
C5 $[Q(y)]$

i. Are any of the clauses above positive Horn clauses? If yes, list all of them. (2 marks)

ii. Are any of the clauses above negative Horn clauses? If yes, list all of them. (2 marks)

iii. Are any of the clauses above unit Horn clauses? If yes, list all of them. (2 marks)

iv. Are any of the clauses above *not* Horn clauses? If yes, list all non-Horn clauses. (2 marks)

Answer.

i. C1, C5
ii. C3, C4
iii. C4, C5
iv. C2

(b) Give backward chaining procedure for first-order Horn clauses (*not* for propositional Horn clauses). (10 marks)

Answer. Approximately 50% BOOKWORK (the textbook has propositional backward chaining).

Answer.

Backward chaining .
input: a finite set of atomic goals $Q_1, \ldots, Q_n$
output: YES if KB entails all of Q_i, NO otherwise

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procedure: SOLVE[Q1, ..., Qn]
if n = 0 then return YES
for each clause c in KB do
   if  c = [not P1,...,not Pm, Q1'], there exists a substitution S such that
Q1'S = Q1S and SOLVE [P1S,...,PmS,Q2S,...,QnS]
   then return YES
end for
return NO
```

(c) Does this procedure terminate? If not, give an example when it does not. (2 marks)

Answer. (BOOKWORK) This procedure does not terminate (even in propositional case). An example would be $KB = \{[\neg P(x), P(x)]\}$ and goal $P(a)$ (or simply $KB = \{[\neg p, p]\}$ and goal p.)

(d) Show that the backward chaining procedure answers YES with goal $P(a)$ and a knowledge base consisting of the following clauses:

C1 $[\neg A_1(x), \neg A_2(x), P(x)]$
C2 $[\neg B_1(x), \neg B_2(x), A_1(x)]$
C3 $[\neg B_3(x), \neg B_4(x), A_2(x)]$
C4 $[B_1(a)]$
C5 $[B_2(a)]$
C6 $[B_3(a)]$
C7 $[B_4(a)]$

(5 marks)

Answer. The following calls to $SOLVE$ will eventually result in YES:
$SOLVE[P(a)]$
$SOLVE[A_1(a), A_2(a)]$ (there is a clause $[\neg A_1(x), \neg A_2, P(x)]$, $S = x/a$)
$SOLVE[B_1(a), B_2(a), A_2(a)]$ (there is a clause $[\neg B_1(x), \neg B_2(x), A_1(x)]$, $S = x/a$)
$SOLVE[B_2(a), A_2(a)]$ (there is a clause $[B_1(a)]$)
$SOLVE[A_2(a)]$ (there is a clause $[B_2(a)]$)
$SOLVE[B_3(a), B_4(a)]$ (there is a clause $[\neg B_3(x), \neg B_4(x), A_2(x)]$, $S = x/a$)
$SOLVE[B_4(a)]$ (there is a clause $[B_3(a)]$)
$SOLVE[]$ (there is a clause $[B_4(a)]$)
return YES

5. This question is on defeasible reasoning.

(a) Explain what are α, β, γ and what is the meaning of a default rule $\dfrac{\alpha : \beta}{\gamma}$ (3 marks)

Answer (BOOKWORK) A default rule consists of a *prerequisite* α, *justification* β, *conclusion* γ and says 'if α holds and it is consistent to believe β, then believe γ'.

(b) What is a default theory and how is an extension of a default theory defined? (10 marks)

Answer (BOOKWORK)

A *default theory* $KB = \{F, D\}$, where F is a finite set of first order sentences and D is a finite set of default rules. E is an *extension* of (F, D) iff for every sentence π,

$$\pi \in E \;\Leftrightarrow\; F \cup \{\gamma \mid \frac{\alpha : \beta}{\gamma} \in D, \alpha \in E, \neg\beta \notin E\} \models \pi$$

(c) List all extensions of the following default theory and explain your working: (10 marks)

First-order theory: $F = \{Dutch(chris), Jockey(chris), \forall x(Tall(x) \supset \neg Short(x))\}$

Default rules: $\dfrac{Dutch(x) : Tall(x)}{Tall(x)}$ $\dfrac{Jockey(x) : Short(x)}{Short(x)}$

Answer

With respect to F and its consequences, both rules match with $x/chris$ and both $Tall(chris)$ and $Short(chris)$ are applicable assumptions. However, if we add $Tall(chris)$ by the first rule, $\neg Short(chris)$ becomes derivable from the result and F. So $Short(chris)$ stops being an applicable assumption. Similarly, if we use the second rule first, we add $Short(chris)$ and then the first rule is not applicable. Hence we have two extensions:

E_1 contains F, $\{Tall(chris), \neg Short(chris)\}$ and their first-order consequences.

E_2 contains F, $\{\neg Tall(chris), Short(chris)\}$ and their first-order consequences.

(d) How are logical consequences of a default theory defined? (2 marks)

Answer (BOOKWORK) A skeptical reasoner will only believe sentences which belong to all extensions of the default theory; a credulous reasoner will choose an arbitrary extension.

6. This question is on knowledge based systems in general.

(a) What is the point of description logics and other ontology languages? Why don't knowledge representation professionals use first-order logic for everything? (5 marks)
Answer (MOSTLY BOOKWORK)
The main reason for using resticted ontology languages is that restriction in expressive power makes reasoning in them much more efficient than in first order logic. Another reason is that hierarchies of concepts, and entity-relationship diagrams are familiar to many users and are considered more intutitive. [Full marks can be obtained by making only the first point properly.]

(b) Describe the development cycle of a knowledge-based system. (10 marks)
Answer (BOOKWORK) Development cycle of a knowledge based system:

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                         1. Plan knowledge base
                                |
                                v
                         2. Select domain experts
                         and knowledge sources
                                |
                                v
                         3. Elicit knowledge
                                |
                                v
                         4. Formulate and represent
                         knowledge
Continue                        |
knowledge                       v
acquisition              5. Implement knowledge base
                                |
    ^                           v
    |                    6. Test knowledge base
    |                           |
    |       no                  v
    -------------------- OK?
                                | yes
                                v
                         7. Systems test
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In 1, the content of the knowledge base, relevant inputs and outputs, strategy for testing, knowledge dictionary, concepts etc. are identified.
In 2, sources of knowledge are selected (domain experts, corpora, etc.). In 3, the process of knowledge elicitation takes place.
In 4, the knowledge is formulated in the form suitable for inference.
In 5, it is encoded in machine-readable form. [A complete answer (listing all relevant stages) gets full marks, pro-rata for partial answers.]

(c) Show how to use a decision table to design a production rule system to make student progression decisions using the regulations below. It is sufficient to give conditions for the 'progress' action. (10 marks)

The pass mark for a module is 40%.
A student who passes all the modules (120 credits) in a given stage of their course will complete that stage and be awarded the total credit for that stage.
A student who fails one or more modules will still complete that stage, and so be awarded the total credit for that stage provided that they have:
(i) passed modules worth at least 80 credits and have a weighted average for the stage of at least 40% with no module marks of less than 30%
or
(ii) passed modules worth at least 100 credits and have a weighted average for the stage of at least 50%
or
(iii) passed modules worth at least 90 credits, have marks of 30% or more in modules worth at least 110 credits, and have a weighted average for the stage of at least 45%.

Answer. First we need to decide on all conditions relevant for making a decision, and all possible actions. A decision table consists of a list of condition stubs (possible relevant conditions), action stubs (progress), condition entries (yes, no, and - for not relevant, for each condition) and action entries (which action is to be taken if specified conditions hold).
The relevant conditions are:

i. whether the value of av (weighted average) is above 40, above 45, above 50
ii. how many credits have marks above 40 (C40)
iii. how many credits have marks above 30 (C30)

The possible actions are:

i. progress
ii. not progress

The resulting table is

Conditions	(normal)	(a)	(b)	(c)
C40 >= 120	Y	N	N	N
C40 >= 100	(Y)	-	Y	-
C40 >= 90	(Y)	-	(Y)	Y
C40 >= 80	(Y)	Y	(Y)	(Y)
C30 >= 120	Y	Y	-	-
c30 >= 110	(Y)	(Y)	-	Y
av >= 40	-	Y	(Y)	(Y)
av >= 45	-	-	(Y)	Y
av >= 50	-	-	Y	-
Actions				
progress	Y	Y	Y	Y

(Y) means that this condition is automatically true because another condition is true, but would be redundant as a condition in the rule.