G53KRR 2018 Answer to Informal Exercise 2

1. Prove by reasoning about interpretations that sentences A1-A5 from the answer to the informal exercise 1 logically entail A6 $\exists x(Member(x) \wedge Climber(x) \wedge \neg Skier(x))$. For your convenience, here they are:

   A1a $Member(tony)$

   A1b $Member(mike)$

   A1c $Member(john)$

   A2 $\forall x(Member(x) \wedge \neg Skier(x) \supset Climber(x))$

   A3a $\forall x(Climber(x) \supset \neg Like(x, rain))$

   A3b $\forall x(\neg Like(x, snow) \supset \neg Skier(x))$

   A4a $\forall x(Like(tony, x) \supset \neg Like(mike, x))$

   A4b $\forall x(\neg(Like(tony, x) \supset Like(mike, x))$

   A5a $Like(tony, rain)$

   A5b $Like(tony, snow)$

   Answer. Suppose some interpretation $J = (D, I)$ satisfies all sentences A1a-A5b. We need to show that there is some object $d \in D$ such that $d \in I(Member)$, $d \in I(Climber)$ and $d \notin I(Skier)$ (then d substituted for x will satisfy $Member(x) \wedge Climber(x) \wedge \neg Skier(x)$).

   We can only rely on there being 3 objects in $I(Member)$, by A1a-A1c, which are $I(tony)$, $I(mike)$ and $I(john)$. Lets us call them Tony, Mike and John. John is a non-starter for d because we don't know anything about him. Tony likes rain so he cannot be a climber by A3a. Let us try Mike. Mike is in $I(Member)$. Mike does not like snow because by A5b, Tony likes snow, and so by A4a, Mike does not like snow. By A3b, since Mike does not like snow, he is not a skier. So $I(mike) \notin I(Skier)$. But by A2, if someone is a member of the club and not a skier, they are a climber: $I(mike) \in I(Climber)$. So we got $d = mike$ is a substitution for x which makes $\exists x(Member(x) \wedge Climber(x) \wedge \neg Skier(x))$ true.

2. Prove that A1-A5 do not logically entail $Climber(john)$.

   Answer. To prove that $Climber(john)$ is not entailed, we need to find an interpretation $J = (D, I)$ where A1a-A5b are true and $Climber(john)$ is false; or in other words, $\neg Climber(john)$ is true. Let $D = \{Tony, Mike, John, Rain, Snow\}$ and $I(Member) = \{Tony, Mike, John\}$. So A1a-A1c are true. Then let $I(Skier) = \{Tony, John\}$, and $I(Like) = \{(Tony, Rain), (Tony, Snow), (John, Snow), (Mike, Tony), (Mike, Mike), (Mike, John)\}$. Then A5a,A5b are true because Tony likes rain and snow, A3b is true because all skiers like snow, and A4a and A4b are true because Tony likes rain and snow and Mike everything else. Let $I(Climber) = \{mike\}$. A3a is true because the only climber does not like rain. And finally every member of the club who is not a skier (Mike) is a climber. Finally, since John is not a climber, $I(john) \notin I(Climber)$ and $\neg Climber(john)$ is true in J.