**G53KRR 2018 first formal/assessed exercise ex1**

Your name: Junsong Yang
Your user name: psyjy3

1. Express the following sentences in first order logic using predicate symbols *Student* (unary, $Student(a)$ means $a$ is a student), $Tutor$ (binary, $Tutor(b, a)$ means $b$ is $a$'s tutor), $Lazy$ (unary), $Happy$ (unary):

   **S1** Every student has a tutor.

   **ANS 1** $\forall x \exists y(Student(x) \supset Tutor(y, x))$

   **S2** There are no lazy students.

   **ANS 2** $\forall x(Student(x) \supset \neg Lazy(x))$

   **S3** No student has two different tutors. *Hint: use* $=$

   **ANS 3** $\forall x \forall y \forall z(Student(x) \supset (Tutor(y, x) \wedge Tutor(z, x) \supset y = z))$

   **S4** If a student is lazy, then the student's tutor is not happy.

   **ANS 4** $\exists x \forall y(Student(x) \wedge Lazy(x) \supset Tutor(y, x) \wedge \neg Happy(y))$

   **S5** There is a tutor all of whose tutees are lazy.

   **ANS 5** $\exists x \forall y(Student(y) \supset (Tutor(x, y) \supset Lazy(y)))$

2. Consider an interpretation where the domain consists of 4 suitcases $a, b, c, d$ where $a$ and $b$ are large and $c$ and $d$ are small. In other words, the predicate symbol $Large$ is interpreted as the set $\{a, b\}$ and $Small$ is interpreted as the set $\{c, d\}$. There is also a predicate symbol $FitsIn$ that is interpreted as the set of pairs $\{(c, a), (c, b), (d, a), (d, b)\}$ (small suitcases fit inside large ones). Are the following first order sentences true or false in this interpretation (and why):

   **(a)** $\forall x \forall y(Large(x) \wedge Small(y) \supset FitsIn(x, y))$
   **False.** Only small suitcases can fit in large suitcases, since x is large suitcases, it cannot fit in small suitcases. FitsIn(x,y) doesnot hold.

   **(b)** $\forall x \forall y(Large(x) \wedge Small(y) \supset FitsIn(y, x))$
   **True.** Small suitcases can always fit in large suitcases.

   **(c)** $\exists x \forall y FitsIn(x, y)$
   **False.** for x = c or x = d, then not every y such that FitsIn(x,y) holds (it doesnot hold for y = c or y = d). For x = a or x = b, there is no y such that FitsIn(x,y) holds.

   **(d)** $\forall x \exists y \neg FitsIn(x, y)$
   **True.** For x = a or x = b, there is no value of y such that FitsIn(x,y) holds. For x = c or x = d, the value of y is c or d such that FitsIn(x,y) doesnot hold.

   **(e)** $\forall x \forall y(\neg FitsIn(x, y) \vee \neg FitsIn(y, x))$
   **True.** For x = a or x = b, there is no y such that FitsIn(x,y) holds. For x = c or x = d, there is no y such that FitsIn(y,x) holds.