G53KRR answer to the exercise on situation calculus

Suppose we have a robotic arm which can move between several bins, grab a thing from the bin if it is over the bin and not holding anything, and drop a thing into the bin if it is holding the thing and is over the bin. Moving is always possible, it does not require any preconditions.

The actions are: $drop(x, y)$ (drop x into y), $move(y)$ (move to be over the bin y), $grab(x, y)$ (grab x from y).

The fluents are: $Holding(x, s)$ (the arm is holding x in situation s), $Over(y, s)$ (the arm is over bin y in situation s), $In(x, y, s)$ (thing x is in bin y in situation s).

1. Write possibility axioms for move, drop and grab actions.
2. Write successor state axioms for all the fluents.

Answer

1. Possibility axioms:
$$\forall y \forall s Poss(move(y), s)$$
$$\forall x \forall y \forall s (Poss(drop(x, y), s) \equiv (Over(y, s) \wedge Holding(x, s)))$$
$$\forall x \forall y \forall s (Poss(grab(x, y, s) \equiv (Over(y, s) \wedge \neg \exists z Holding(z, s) \wedge In(x, y, s)))$$
2. Successor state axioms: (universal quantifiers dropped for readability)
$Holding(x, do(a, s)) \equiv (\exists y (a = grab(x, y) \wedge In(x, y, s) \wedge Over(y, s)) \vee$
. $(Holding(x, s) \wedge \neg \exists y (a = drop(x, y))))$

$Over(y, do(a, s)) \equiv (a = move(y) \vee (Over(y, s) \wedge \neg \exists z (\neg (y = z) \wedge a = move(z))))$

$In(x, y, do(a, s)) \equiv ((Over(y, s) \wedge Holding(x, s) \wedge a = drop(x, y)) \vee$
. $(In(x, y, s) \wedge \neg (Over(y, s) \wedge a = grab(x, y))))$