G53KRR 2018 Answer to Informal Exercise 3

1. Rewrite the following propositional formulas to clausal form (conjunction of disjunctions of literals):

   (a) $p \vee q \supset r$

   (b) $r \supset s$

   (c) p

   Answer.

   The answer uses the following definitions:

   Definition of $\supset$ $\alpha \supset \beta$ is equivalent to $\neg\alpha \vee \beta$

   De Morgan law $\neg(\alpha \vee \beta)$ is equivalent to $\neg\alpha / an \wedge \neg\beta$
   (the second law: $\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$)

   Distributivity $\alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$
   (another instance of Distributivity is $\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$

   $p \vee q \supset r$ is by definition of $\supset$ equivalent to $\neg(p \vee q) \vee r$. $\neg(p \vee q) \vee r$ is by de Morgan's law equivalent to $(\neg p \wedge \neg q) \vee r$. By distributivity, $(\neg p \wedge \neg q) \vee r$ is equivalent to $(\neg p \vee r) \wedge (\neg q \vee r)$. The last formula is in clausal form.

   $r \supset s$ is equivalent to $\neg r \vee s$.

   p is already in clausal form.

   So we get: $\neg p \vee r$ (a1), $\neg q \vee r$ (a2), $\neg r \vee s$ (b), p (c).

2. Prove by resolution that the sentences above entail s.

   Answer. We derive a contradiction from the sentences above and *the negation of* s, $\neg s$ (d):

   1 $\neg p \vee r$ (a1)

   2 p (c)

   3 r resolution 1,2

   4 $\neg r \vee s$ (b)

   5 s resolution 3,4

   6 $\neg s$ (d)

   7 $\emptyset$

3. Show by resolution that the following set of clauses is unsatisfiable:

   (a) $p \vee q \vee r$,

   (b) $p \vee q \vee \neg r$,

   (c) $p \vee \neg q \vee r$,

   (d) $p \vee \neg q \vee \neg r$,

(e) $\neg p \vee q \vee r$,

(f) $\neg p \vee q \vee \neg r$,

(g) $\neg p \vee \neg q \vee r$,

(h) $\neg p \vee \neg q \vee \neg r$

Answer.

1 $p \vee q$ by resolution from (a) and (b) (we also use $\alpha \vee \alpha \equiv \alpha$; usually resolution is applied to sets rather than disjunstions)

2 $p \vee \neg q$ by resolution from (c) and (d)

3 p by resolution from 1 and 2

4 $\neg p \vee q$ by resolution from (e) and (f)

5 $\neg p \vee \neg q$ by resolution from (g) and (h)

6 $\neg p$ by resolution from 4 and 5

7 $\emptyset$ by resolution from 3 and 6