ANSWERS

The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 3 MODULE, AUTUMN SEMESTER 2010-2011

KNOWLEDGE REPRESENTATION AND REASONING

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR out of SIX questions

Only silent, self contained calculators with a Single-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

1. (a) Express the following sentences in first-order logic, using the binary predicates $StudyAt$, $WorkAt$, $LocatedIn$, $=$, the unary predicate $University$, and the constants uon (for the University of Nottingham) uob (for the University of Birmingham), and n (for Nottingham):

S1 Everybody who studies at the University of Nottingham, does not study at the University of Birmingham. (2 marks)
Answer: $\forall x(StudyAt(x, uon) \supset \neg StudyAt(x, uob))$

S2 There are exactly two universities in Nottingham. (2 marks)
Answer:
$\exists x \exists y(University(x) \wedge University(y) \wedge LocatedIn(x, n) \wedge LocatedIn(y, n) \wedge \neg(x = y) \wedge \forall z(University(z) \wedge LocatedIn(z, n) \supset (z = x \vee z = y)))$

S3 There are at least two people who both work at the University of Nottingham and study there. (2 marks)
Answer:
$\exists x \exists y(WorkAt(x, uon) \wedge StudyAt(x, uon) \wedge WorkAt(y, uon) \wedge StudyAt(y, uon) \wedge \neg(x = y))$

(b) Prove that the following argument is not logically sound (that the premises do not logically entail the conclusion). (10 marks)

Some people are clever.
Some people are greedy.
———————————————
Some people are clever and greedy.

Answer. The following interpretation is a counterexample (it satisfies the premises and does not satisfy the conclusion): the domain consists of two individuals a and b, where a is the only one who is clever and b is the only one who is greedy. So the sentences $\exists x Clever(x)$ and $\exists x Greedy(x)$ are true. However there is no individual who is both clever and greedy, so $\exists x(Clever(x) \wedge Greedy(x))$ is false.

(c) Prove that the propositional resolution rule below is sound (the premises logically entail the conclusion). (9 marks)

$$\frac{\alpha_1 \vee p \qquad \alpha_2 \vee \neg p}{\alpha_1 \vee \alpha_2}$$

Answer. Assume that the premises are true. We need to show that the conclusion is true. A disjunction is true if at least one of the disjuncts is true. In particular, if $\alpha_1 \vee p$ is true, then either α_1 or p must be true. If α_1 is true, then $\alpha_1 \vee \alpha_2$ is true. If p is true, then $\neg p$ is false, so α_2 is true. But then again $\alpha_1 \vee \alpha_2$ is true.

2. (a) Reduce the following sentences to clausal form:

S1 $\forall x \forall y (P(x, y) \supset \exists z R(x, y, z))$ (2 marks)

Answer. $[\neg P(x, y), R(x, y, f(x, y))]$

S2 $\forall x \exists y R(x, y) \lor \forall x \exists y \neg R(x, y)$ (2 marks)

Answer. $[R(x, f(x)), \neg R(z, g(z))]$

S3 $\exists x \exists y \forall z \neg (P(x, z) \lor P(y, z))$ (2 marks)

Answer. $[\neg P(a, z)], [\neg P(b, z)]$

(b) Show by resolution that clauses **C1**–**C3** below entail $\exists x Q(x, a)$.

C1 $[\neg P(x), R(x, f(x))]$

C2 $[\neg R(y, z), Q(z, y)]$

C3 $[P(a)]$ (10 marks)

Answer. First we need to negate $\exists x Q(x, a)$ and reduce the result to clausal form. It is $\neg \exists x Q(x, a) = \forall x \neg Q(x, a)$, which gives a clause $[\neg Q(x, a)]$. We call this clause **C4** and show that **C1-C4** derive the empty clause.

i. $[P(a)]$ C3
ii. $[\neg P(x), R(x, f(x))]$ C1
iii. $[R(a, f(a))]$ res. 1,2, x/a
iv. $[\neg R(y, z), Q(z, y)]$ C2
v. $[Q(f(a), a)]$ res. 3,4, $y/a, z/f(a)]$
vi. $[\neg Q(x, a)]$ C4
vii. $[]$ res. 5,6, $x/f(a)$

(c) Give a substitution which unifies $P(f(x), g(f(x), y), z)$ and $\neg P(x_1, g(x_1, f(y_1)), f(z_1))$. (3 marks)

Answer.
Consider $\theta = [x_1/f(x), y/f(y_1), z/f(z_1)]$;
$P(f(x), g(f(x), y), z)\theta = P(f(x), g(f(x), f(y_1)), f(z_1))$ and
$\neg P(x_1, g(x_1, f(y_1)), f(z_1))\theta = \neg P(f(x), g(f(x), f(y_1)), f(z_1))$.

(d) Define the notion of *most general unifier* (mgu). Then give an example of an mgu substitution and a non-mgu substitution. (3 marks)

Answer. θ is an mgu if for every other unifying substitution θ', there is a substitution θ^* such that $\theta' = \theta\theta^*$. The substitution above is an mgu. A non-mgu substitution would be, for example, $\theta' = [x_1/f(x), y/f(a), y_1/a, z/f(z_1)]$. $\theta = \theta'[y_1/a]$.

(e) Is it possible to unify $[P(g(x))]$ and $[\neg P(f(x))]$? Explain your answer. (3 marks)

Answer. No, whatever term t we substitute for x, the resulting term in the first literal will start with a g and in the second literal with an f.

3. Recall the description logic $\mathcal{DL}$ given in the textbook:
Concepts:

- atomic concept is a concept
- if r is a role and b is a concept, then [**ALL** r b] is a concept (e.g. [**ALL** $:Child\ Girl$] describes someone all of whose children are girls).
- if r is a role and n is a positive integer, then [**EXISTS** n r] is a concept (e.g. [**EXISTS** 2 $:Child$] describes someone who has at least 2 children)
- if r is a role and c is a constant, then [**FILLS** r c] is a concept (e.g. [**FILLS** $:Child\ john$] describes someone whose child is John).
- if $b_1, \ldots, b_n$ are concepts, [**AND** $b_1, \ldots, b_n$] is a concept.

Sentences:

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s, b_1 is *subsumed* by b_2)
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)
- if c is a constant and b a concept then $c \rightarrow b$ is a sentence (the individual denoted by c satisfies the description expressed by b).

(a) Express the following concepts in $\mathcal{DL}$ using the atomic concepts $Animal$, and $Fish$, and the roles $:Tail$, $:Leg$, and $:Eat$.

C1 An animal that has a tail (2 marks)
Answer. [**AND** $Animal$, **EXISTS** 1 $:Tail$]

C2 An animal that has a tail and four legs (2 marks)
Answer. [**AND** $Animal$, **EXISTS** 1 $:Tail$, **EXISTS** 4 $:Leg$]

C3 An animal that eats only fish (2 marks)
Answer. [**AND** $Animal$, **ALL** $:Eat\ Fish$]

C4 An animal that eats only things that themselves eat only fish (2 marks)
Answer. [**AND** $Animal$, **ALL** $:Eat$ [**ALL** $:Eat\ Fish$]]

(b) Express the following sentences in $\mathcal{DL}$ using the atomic concepts Cat, $Fish$, and $Animal$, the roles $:Leg$, and $:Eat$, and the constant $tiddles$:

S1 Tiddles is a cat who eats only fish (4 marks)
Answer. $tiddles \rightarrow$ [**AND** Cat, **ALL** $:Eat\ Fish$]

S2 Cats are animals that have four legs (3 marks)
Answer. $Cat \sqsubseteq$ [**AND** $Animal$, **EXISTS** 4 $:Leg$]

(c) Show that $john \rightarrow$ [**ALL** $:Child\ Girl$] and $john \rightarrow$ [**FILLS** $:Child\ mary$] entail $john \rightarrow$ [**EXISTS** 1 $:Child$] in $\mathcal{DL}$. (5 marks)

Answer. In order for the first sentence to be true, any object which is in relation $:Child$ to John has to be female. However, there may be no such objects. The second sentence implies that there is an object in relation $:Child$ to John, called Mary. Therefore since there is at least one object in relation $:Child$ to John (namely Mary), John is an instance of a concept of someone having at least one child.

(d) Show that the sentence $john \rightarrow [\mathbf{ALL}\ \ : Child\ Girl]$ does not entail $john \rightarrow [\mathbf{EXISTS}\ 1\ \ : Child]$ in description logic. (5 marks)

Answer. See above: it is possible to have an interpretation where John has no children, then $john \rightarrow [\mathbf{ALL}\ \ : Child\ Girl]$ is trivially true, but $john \rightarrow [\mathbf{EXISTS}\ 1\ \ : Child]$ is false.

4. (a) Give the backward chaining procedure for propositional Horn clauses. (5 marks)

Answer.

Backward chaining .
input: a finite set of atomic sentences $q_1, \ldots, q_n$
output: YES if KB entails all of q_i, NO otherwise

```
procedure: SOLVE[q1, ..., qn ]
if n = 0 then return YES
for each clause c in KB do
   if  c = [not p1,...,not pm, q1] and SOLVE [p1,...,pm,q2,...,qn]
   then return YES
end for
return NO
```

(b) Trace it on the following example: $KB = \{[r, \neg p, \neg q], [t, \neg r, \neg s], [p], [q], [s]\}$, goal: t. (5 marks)

Answer. $SOLVE[t]$ $c = [t, \neg r, \neg s]$
$SOLVE[r, s]$ $c = [r, \neg p, \neg q]$
$SOLVE[p, q, s]$ $c = [p]$
$SOLVE[q, s]$ $c = [q]$
$SOLVE[s]$ $c = [s]$
$SOLVE[]$
YES

(c) Define SLD derivation and explain why backward chaining is a special case of SLD derivation. (5 marks)

Answer. An SLD derivation of clause c from a set of clauses S is a sequence of steps $c_1, \ldots, c_n = c$ such that $c_1 \in S$ and c_{i+1} is a resolvent of c_i and a clause in S. Backward chaining produces an SLD resolution because we start with a negative clause (negation of the goal) and at each step resolve a negative clause from the previous step (current goals) with a positive clause from the KB.

(d) Show that there is an SLD derivation of [] from the KB in part(b) and $[\neg t]$. (5 marks)

Answer.

$c_1 = [\neg t]$

i. $[\neg t]$
ii. $[\neg r, \neg s]$ from 1 and $[t, \neg r, \neg s]$
iii. $[\neg r]$ from 2 and $[s]$
iv. $[\neg p, \neg q]$ from 3 and $[r, \neg p, \neg q]$
v. $[\neg p]$ from 4 and $[q]$
vi. [] from 5 and $[p]$.

(e) Is backward chaining for propositional Horn clauses guaranteed to terminate? If not, give an example of a case where backward chaining does not terminate and explain why. (5 marks)

Answer. No, it is not guaranteed to terminate. A simple example where it is not going to terminate is a clause $[p, \neg p]$ and goal p.

5. (a) Define the closed world assumption and the entailment relation $\models_{CWA}$. (3 marks)

Answer. Closed-world assumption (CWA): if an atomic sentence is not classically entailed by the knowledge base, then its negation is entailed. The corresponding entailment $\models_{CWA}$:

$$KB \models_{CWA} \phi \;\Leftrightarrow\; KB^+ \models \phi$$

where $KB^+ = KB \cup \{\neg p : p \text{ is atomic and } KB \not\models p\}$.

(b) Why is the closed world assumption used in knowledge representation (i.e., why not store negative data in the knowledge base)? (2 marks)

Answer. There are too many (potentially infinitely many) facts that *do not* hold; it is inefficient or impossible to store all of them.

(c) What does monotonicity of an entailment relation mean? Give an example to show that $\models_{CWA}$ is non-monotonic. (5 marks)

Answer. An entailment relation is monotonic if for any two sets of premises Γ and Γ', if $\Gamma \subseteq \Gamma'$, then the set of consequences of Γ is included in the set of consequences of Γ'; in other words, if we learn more information, we can derive more consequences.

$\models_{CWA}$ is non-monotonic; for example, $\{\ \} \models_{CWA} \neg P(a)$, $\{\ \} \subseteq \{P(a)\}$, but $\{P(a)\} \not\models_{CWA} \neg P(a)$.

(d) How are rules of the form 'Normally, As are Bs' formalised in circumscription theory? Formalise 'Normally, scientists are intelligent'. (3 marks)

Answer. These rules are formalised using a 'not abnormal' extra condition, introducing a predicate Ab (different for each rule). The example is formalised as $\forall x(\mathit{Scientist}(x) \wedge \neg \mathit{Ab}(x) \supset \mathit{Intelligent}(x))$.

(e) Give definitions of minimal models and minimal entailment in circumscription theory. (5 marks)

Answer.

Minimal entailment does not consider all models of the knowledge base, but only those where the set of exceptions (the extension of Ab) is made as small as possible. Let A be the set of Ab predicates we want to minimise. Let $M_1 = (D, I_1)$ and $M_2 = (D, I_2)$ be two interpretations over the same domain such that every constant and function are interpreted the same way.

$$M_1 \leq M_2 \;\Leftrightarrow\; \forall Ab \in A\ (I_1(Ab) \subseteq I_2(Ab))$$

$M_1 < M_2$ if $M_1 \leq M_2$ but not $M_2 \leq M_1$. (There are strictly fewer abnormal things in M_1). *Minimal entailment:* $KB \models_{\leq} \phi$ iff for all interpretations M which make KB true, either $M \models \phi$ or M is not minimal (there exists an M' such that $M' < M$ and $M' \models KB$).

(f) Show that 'John is intelligent' follows by circumscription from a knowledge base which contains sentences 'John is a scientist' and 'Normally, scientists are intelligent'. (5 marks)

Answer. Consider a model M which satisfies 'Normally, scientists are intelligent' and 'John is a scientist'. Either M satisfies 'John is intelligent', or $Ab(John)$. However, in the latter case M is not minimal, since the denotation of John can be removed from the extension of Ab with M still satisfying the two sentences (provided John is made intelligent).

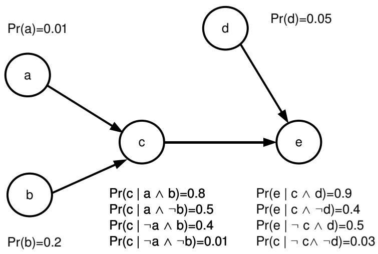
(g) Show that minimal entailment is not monotonic. (2 marks)

Answer. If the knowledge base is extended with $\neg Intelligent(John)$, the previous conclusion does not any longer follow (since we now have $Ab(John)$).

6. (a) What is a Bayesian network? What kind of information does it represent and how? Explain what an independence assumption is. (5 marks)

   Answer. A Bayesian network is a directed graph. Its nodes are boolean variables and arcs correspond to causal relationships. Each propositional variable in the belief network is conditionally independent from the nonparent variables given the parent variables. Conditional probabilities given the parent variables are given.

   (b) Give an example of an independence assumption implicit in the following network:



(3 marks)

   Answer. For example, $Pr(b|a) = Pr(b)$, $Pr(e|a \wedge b \wedge c \wedge d) = Pr(e|c \wedge d)$

   (c) What is the advantage of using belief networks compared to explicitly giving a joint probability distribution? (2 marks)

   Answer. Belief networks are a more compact way of representing probabilistic information than the joint probability distribution (the joint probability distribution requires recording 2^n probabilities where n is the number of nodes. There are usually many fewer parents for each node than n, so although the number of probabilities to record is exponential in the number of parents, it is smaller than 2^n).

   (d) Give Bayes's conditional probability rule (i.e., define $Pr(a|b)$ in terms of $Pr(a \wedge b)$ and $Pr(b)$). (1 marks)

   Answer. $Pr(a|b) = Pr(a \wedge b)/Pr(b)$.

   (e) Give the chain rule for computing the probability of the conjunction $Pr(a_1 \wedge \ldots \wedge a_n)$. (1 marks)

   Answer. $Pr(a_1 \wedge \ldots \wedge a_n) = Pr(a_1) \cdot Pr(a_2|a_1) \cdot \ldots \cdot Pr(a_n|a_1 \wedge \ldots \wedge a_{n-1})$.

   (f) Suppose that we have events a, b and c, and we know that $Pr(a)$ is $1/3$, $Pr(b|\neg a)$ is $1/2$ and $Pr(c)$ is $1/2$. We also know that c is conditionally independent of a and b. Compute $Pr(\neg a \wedge b \wedge c)$. (3 marks)

   Answer. $Pr(\neg a \wedge b \wedge c) = Pr(\neg a) \cdot Pr(b|\neg a) \cdot Pr(c|a \wedge b) = (1 - Pr(a)) \cdot Pr(b|\neg a) \cdot Pr(c) = (1 - 1/3) \cdot 1/2 \cdot 1/2) = 2/12 = 1/6$.

   (g) Represent the following scenario as a Bayesian network: (5 marks)

   > Disease d is caused by exposure to chemical c. The probability of c is 0.03. The probability of having d after exposure to c is 0.8. d almost never

occurs without exposure to c (probability of d given no exposure is 0.001). The disease d may cause complication a. However a may also be caused by another disease b. The probability of b is 0.1. The probability of a given d but not b is 0.6, which is the same as the probability of a given b but not d. The probability of a given both b and d is 0.9, the probability of a without either b or d is 0.02.

Answer.

```
                 b
                 |
                 |
                 v
c ----> d ------> a
```

Where $Pr(c) = 0.03$, $Pr(d|c) = 0.8$, $Pr(d|\neg c) = 0.001$, $Pr(a|b \wedge d) = 0.9$, $Pr(a|b \wedge \neg d) = 0.6$, $Pr(a|\neg b \wedge d) = 0.6$, $Pr(a|\neg b \wedge \neg d) = 0.02$.

(h) What is the probability of $d \wedge \neg c \wedge b$ given the scenario in part (g) above? (5 marks)

Answer. $Pr(d \wedge \neg c \wedge b) = Pr(d \wedge \neg c) \cdot Pr(b|d \wedge \neg c) = Pr(d \wedge \neg c) \cdot Pr(b)$.

$Pr(d \wedge \neg c) = Pr(\neg c) \cdot Pr(d|\neg c) = (1 - Pr(c)) \cdot Pr(d|\neg c) = 0.97 \cdot 0.001 = 0.00097$.

$Pr(d \wedge \neg c \wedge b) = 0.00097 \cdot 0.1 = 0.000097$.