**G53KRR 2018 Answer to the Informal Exercise 1**

1. Express the following sentences in first order logic:

   **S1** Tony, Mike and John belong to the Alpine Club.

   **A1** $Member(tony)$, $Member(mike)$, $Member(john)$
   (or $Member(tony) \wedge Member(mike) \wedge Member(john)$)

   **S2** Every member of the Alpine Club who is not a skier is a mountain climber.

   **A2** $\forall x(Member(x) \wedge \neg Skier(x) \supset Climber(x))$

   **S3** Mountain climbers do not like rain, and anyone who does not like snow is not a skier.

   **A3** $\forall x(Climber(x) \supset \neg Like(x, rain))$, $\forall x(\neg Like(x, snow) \supset \neg Skier(x))$

   **S4** Mike dislikes whatever Tony likes, and likes whatever Tony dislikes.

   **A4** $\forall x(Like(tony, x) \supset \neg Like(mike, x))$, $\forall x(\neg Like(tony, x) \supset Like(mike, x))$

   **S5** Tony likes rain and snow.

   **A5** $Like(tony, rain)$, $Like(tony, snow)$

   **S6** There is a member of Alpine Club who is a mountain climber but not a skier.

   **A6** $\exists x(Member(x) \wedge Climber(x) \wedge \neg Skier(x))$

2. Consider an interpretation where the domain consists of Ann, Bob, and Carol, and there is a relation Older, which holds exactly for the set of pairs $\{(Ann, Bob), (Ann, Carol), (Bob, Carol)\}$ (Ann is older than Bob and Carol, and Bob is older than Carol; the relation does not hold for any other pairs). Are the following first order sentences true or false in this interpretation (and why):

   (a) $\forall x \exists y Older(x, y)$: false because for $x = Carol$ there is no $y$ such that $Older(carol, y)$ holds

   (b) $\forall x \neg Older(x, x)$: true because none of the pairs $(Ann, Ann)$, $(Bob, Bob)$, $(Carol, Carol)$ is in the relation Older.

   (c) $\exists x \forall y(\neg(x = y) \supset Older(x, y))$: true because for $x = Ann$, any value of $y$ which is not $Ann$ satisfies $Older(Ann, y)$.

   (d) $\forall x \exists y(Older(x, y) \vee Older(y, x))$: true because for every $x$ there is a $y$ such that either $(x, y)$ or $(y, x)$ is in Older relation.