G53KRR 2018 answers to the first formal/assessed exercise ex1

1. Express the following sentences in first order logic using predicate symbols $Student$ (unary, $Student(a)$ means a is a student), $Tutor$ (binary, $Tutor(b, a)$ means b is a's tutor), $Lazy$ (unary), $Happy$ (unary):

   Note that there are infinitely many logically equivalent ways to express the sentences below. I am giving just one or two possible ways (that look simplest to me).

   S1 Every student has a tutor.

   $\forall x(Student(x) \supset \exists y Tutor(y, x))$
   or
   $\forall x \exists y(Student(x) \supset Tutor(y, x))$

   S2 There are no lazy students.

   $\neg\exists x(Lazy(x) \land Student(x))$
   or
   $\forall x(Student(x) \supset \neg Lazy(x))$

   S3 No student has two different tutors. *Hint: use =*

   $\neg\exists x(Student(x) \land \exists y \exists z(\neg(y = z) \land Tutor(y, x) \land Tutor(z, x)))$
   or
   $\forall x \forall y \forall z(Student(x) \land Tutor(y, x) \land Tutor(z, x) \supset (y = z))$

   S4 If a student is lazy, then the student's tutor is not happy.

   $\forall x \forall y(Student(x) \land Lazy(x) \land Tutor(y, x) \supset \neg Happy(y))$

   S5 There is a tutor all of whose tutees are lazy.

   $\exists x \forall y(Tutor(x, y) \supset Lazy(y))$

2. Consider an interpretation where the domain consists of 4 suitcases a, b, c, d where a and b are large and c and d are small. In other words, the predicate symbol $Large$ is interpreted as the set $\{a, b\}$ and $Small$ is interpreted as the set $\{c, d\}$. There is also a predicate symbol $FitsIn$ that is interpreted as the set of pairs $\{(c, a), (c, b), (d, a), (d, b)\}$ (small suitcases fit inside large ones). Are the following first order sentences true or false in this interpretation (and why):

   (a) $\forall x \forall y(Large(x) \land Small(y) \supset FitsIn(x, y))$

   False, because if x is large, y is small, then $FitsIn(x, y)$ requires that x fits into y, and this is false for all such pairs of values for x, y.

   (b) $\forall x \forall y(Large(x) \land Small(y) \supset FitsIn(y, x))$

   True, because for any pair of values for x and y, if x is large and y is small, then y fits in x.

   (c) $\exists x \forall y FitsIn(x, y)$

False: there is no value for x such as $FitsIn(x, y)$ is true for all y. For example, no suitcase fits into itself, so the same value for x as for y constitutes a counterexample.

(d) $\forall x \exists y \neg FitsIn(x, y)$

True for the same reason as above is false.

(e) $\forall x \forall y(\neg FitsIn(x, y) \vee \neg FitsIn(y, x))$

True: if x is a large suitcase, the $\neg FitsIn(x, y)$ is true for all y; if x is a small suitcase, $\neg FitsIn(y, x)$ is true for all y.