Answers

The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 3 MODULE, AUTUMN SEMESTER 2016-2017

KNOWLEDGE REPRESENTATION AND REASONING

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer all FOUR questions

Only silent, self contained calculators with a Single-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

1. This question is on expressing knowledge in first-order logic and logical entailment.

(a) Translate the sentences below from English into first-order logic. Use predicate symbols $Region$ for spatial regions, $Disjoint$ (where $Disjoint(x, y)$ means that x and y are disjoint), $Included$ (where $Included(x, y)$ means that x is included in y), $Overlap$ (where $Overlap(x, y)$ means that x and y overlap) and constants $dorset$, $fife$, $scotland$, $england$.

i. Any two regions are either disjoint, overlapping, or one of them is included in another. (2 marks)
ii. Every region is included in itself. (1 marks)
iii. If two regions are disjoint, they are not overlapping. (1 marks)
iv. If two regions are overlapping, none of them is included in another. (2 marks)
v. If one region is included in another, then they are not disjoint. (1 marks)
vi. If two regions are disjoint, then any region included in the first one is disjoint from the second one. (2 marks)
vii. Dorset and England are regions, Dorset is included in England. (1 marks)
viii. Fife and Scotland are regions, Fife is included in Scotland. (1 marks)
ix. Scotland and England are disjoint. (1 marks)

Answer. **Application**

i. $\forall x, y(Region(x) \wedge Region(y) \supset Disjoint(x, y) \vee Overlap(x, y) \vee Included(x, y) \vee Included(y, x))$ (award one mark if missing $Included(y, x)$)
ii. $\forall x(Region(x) \supset Included(x, x))$
iii. $\forall x \forall y(Region(x) \wedge Region(y) \supset (Disjoint(x, y) \supset \neg Overlap(x, y)))$
iv. $\forall x \forall y(Region(x) \wedge Region(y) \supset (Overlap(x, y) \supset \neg Included(x, y) \wedge \neg Included(y, x)))$ (award one mark if missing $Included(y, x)$)
v. $\forall x \forall y(Region(x) \wedge Region(y) \supset (Included(x, y) \supset \neg Disjoint(x, y)))$
vi. $\forall x \forall y(Region(x) \wedge Region(y) \supset (Disjoint(x, y) \supset \forall z(Region(z) \supset (Included(z, x) \supset Disjoint(z, y)))))$ (other correct answers possible, for example $\forall x \forall y \forall z(Region(x) \wedge Region(y) \wedge Region(z) \wedge Disjoint(x, y) \wedge Included(z, x) \supset Disjoint(z, y)))$
vii. $Region(dorset)$, $Region(england)$, $Included(dorset, england)$
viii. $Region(fife)$, $Region(scotland)$, $Included(fife, scotland)$
ix. $Disjoint(england, scotland)$

(b) Do the sentences from part (a) logically entail that Dorset does not overlap with Scotland? Justify your answer by giving a definition of logical entailment. Either show that entailment holds by reasoning about all possible interpretations (NOT by using resolution), or show that it does not hold by giving a counterexample interpretation. (7 marks)

Answer. **Application** The sentences in part (a) do entail $\neg Overlap(dorset, scotland)$. A set of sentences S logically entails sentence α if and only if any interpretation which makes all sentences in S true also makes α true. This particular entailment holds because any interpretation satisfying the sentences in part (a) satisfies $Disjoint(england, scotland) \supset (Included(dorset, england) \supset Disjoint(dorset, scotland))$ (because of the universally quantified sentence (vi)). Any such interpretation also satisfies $Disjoint(england, scotland)$ and $Included(dorset, england)$ (because of (vii) and (ix)). So, any such interpretation satisfies $Disjoint(dorset, scotland)$. Because of (iii), any such interpretation satisfies

$$Disjoint(dorset, scotland) \supset \neg Overlap(dorset, scotland).$$

So finally, it follows that any interpretation where sentences in part (a) are true also satisfies $\neg Overlap(dorset, scotland)$.

(c) Do the sentences from part (a) logically entail that Fife does not overlap with Dorset? Either show that entailment holds by reasoning about all possible interpretations (NOT by using resolution), or show that it does not hold by giving a counterexample interpretation. (6 marks)

Answer. **Application**

The entailent holds. A special case of (vi) is

$$Disjoint(dorset, scotland) \supset (Included(dorset, dorset) \supset Included(fife, scotland) \supset$$

$$Disjoint(fife, dorset)))$$

2. This question is on clausal form, resolution and unification.

   (a) Reduce the following sentences to clausal form: (10 marks)

   S1 $\forall x \forall y(\neg(R(x,y) \wedge \neg R(y,x)))$

   S2 $\exists z \forall x \exists y P(x,y,z)$

   S3 $\forall x \forall y \exists z(\neg R(x,z) \vee \neg R(y,z))$

   S4 $\neg \forall x \exists y(R(x,y) \vee R(y,x))$

   S5 $\forall x \forall y \forall z(R(x,y) \supset (R(y,z) \supset R(x,z)))$

   Answer. **Application**

   S1 $[\neg R(x,y), R(y,x)]$

   S2 $[P(x, f(x), a]$

   S3 $[\neg R(x, f(x,y)), \neg R(y, f(x,y))]$

   S4 $[\neg R(c,y)], [\neg R(y,c)]$

   S5 $[\neg R(x,y), \neg R(y,z), R(x,z)]$

   (b) Derive by resolution an empty clause from the following clauses (where x, y, z are variables and a and b constants): (10 marks)

   C1 $[\neg R(x,x)]$

   C2 $[\neg R(x,y), R(y,x)]$

   C3 $[\neg R(x,y), \neg R(y,z), R(x,z)]$

   C4 $[R(a,b)]$

   Answer. **Application**

   C5 $[R(b,a)]$ from $C2, C4, x/a, y/b$

   C6 $[\neg R(b,z), R(a,z)]$ from $C3, C4, x/a, y/b$

   C7 $[R(a,a)]$ from $C6, C5, z/a$

   C8 $[]$ from $C1, C7, x/a$

   (c) For the pairs of literals below, state whether they unify, and if yes give a most general unifier. Note that x, y, z, u are variables and a, b constants.

   i. $P(x,x,y)$, $\neg P(a,b,z)$ (1 marks)

   ii. $R(x, f(x))$, $\neg R(y,z)$ (2 marks)

   iii. $R(x, f(x))$, $\neg R(f(y), y)$ (2 marks)

   Answer. **Application**

   i. no, x cannot be substituted for a as well as b

   ii. yes, $y/x, z/f(x)$

   iii. no, $x/f(y)$ does not work because then the next term $f(x)$ becomes $f(f(y)$, instead of y that we need. Substituting anything for y does not work because the first term is $f(y)$ and we cannot make it the same as x by substituting anything for y.

3. This question is on backward and forward chaining.

   (a) Give the backward chaining procedure for propositional Horn clauses. (5 marks)
   Answer. (Knowledge)

   Backward chaining .
   input: a finite set of atomic sentences $q_1, \ldots, q_n$
   output: YES if KB entails all of q_i, NO otherwise

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procedure: SOLVE[q1, ..., qn ]
if n = 0 then return YES
for each clause c in KB do
   if  c = [not p1,...,not pm, q1] and SOLVE [p1,...,pm,q2,...,qn]
   then return YES
end for
return NO
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   (b) Trace the backward chaining procedure on the following example:
   $KB = \{[\neg Elephant, Herbivore],$
   $[\neg Herbivore, \neg Large, EatsLots],$
   $[\neg Elephant, Large], [Elephant]\},$
   goal: $EatsLots$. (5 marks)
   Answer. (Application) $SOLVE[EatsLots]$ $c = [\neg Herbivore, \neg Large, EatsLots]$
   $SOLVE[Herbivore, Large]$ $c = [\neg Elephant, Herbivore]$
   $SOLVE[Elephant, Large]$ $c = [Elephant]$
   $SOLVE[Large]$ $c = [\neg Elephant, Large]$
   $SOLVE[Elephant]$ $c = [Elephant]$
   $SOLVE[]$
   YES

   (c) Give the forward chaining procedure for propositional Horn clauses. (5 marks)
   Answer. (Knowledge)

   Forward chaining .
   input: a positive atom q
   output: YES if KB entails q, NO otherwise

   0. FACTS = atoms p such that $[p]$ is in KB.
   1. if q is in FACTS, return YES
   2. if there is $[\neg p_1, \ldots, \neg p_n, p]$ in KB such that $p_1, \ldots, p_n$ are in FACTS and p is not in FACTS: add p to FACTS and go to 1; else return NO.

   (d) Trace the forward chaining procedure on the example from part (b) (show that $EatsLots$ is derivable by forward chaining). (5 marks)
   Answer. (Application)
   $FACTS = \{Elephant\}$
   $FACTS = \{Elephant, Herbivore\}$ (from $[\neg Elephant, Herbivore]$)
   $FACTS = \{Elephant, Herbivore, Large\}$ (from $[\neg Elephant, Large]$)
   $FACTS = \{Elephant, Herbivore, Large, EatsLots\}$ (from $[\neg Herbivore, \neg Large, EatsLots]$)
   YES

(e) Which of the two procedures is guaranteed to terminate and why? (5 marks)

Answer. (Knowledge)

Backward chaining is not guaranteed to terminate. A simple example where it is not going to terminate is a clause $[p, \neg p]$ and goal p. Forward chaining for propositional case is guaranteed to terminate because KB is finite, hence contains finitely many atoms, and the procedure will terminate when all derivable atoms are added to FACTS.

4. This question is on non-monotonic reasoning.

   (a) What is non-monotonic reasoning? Why is it used in knowledge-based systems? Give an example of how non-monotonic reasoning may be used in a knowledge-based system. (9 marks)

   Answer. (Knowledge, Comprehension)

   Classical logical entailment is monotonic – the more we know, the more consequences we can derive. Non-monotonic reasoning corresponds to 'jumping to conclusions': consequences may be retracted if additional information can be obtained. The simplest example is closed world assumption; if we do not explicitly state negative information and instead derive that there is e.g. no flight between Nottingham and London because no flight is explicitly listed in the database. This is done to avoid storing potentially infinitely many negative facts. More advanced example is defeasible inference: where we explicitly state rules which in fact may have exceptions, and use them if there is no explicit information to the contrary.

   (b) Explain what a default rule $\dfrac{\alpha : \beta}{\delta}$ in Reiter's default logic means. (3 marks)

   Answer. (Knowledge)

   A *default rule* consists of a *prerequisite* α, *justification* β, *conclusion* δ and says 'if α holds and it is consistent to believe β, then believe δ': $\dfrac{\alpha : \beta}{\delta}$

   (c) What is an extension of a default theory (F, D) (where F is a set of first order sentences and D a set of default rules)? (5 marks)

   Answer. (Knowledge)

   E is an *extension* of (F, D) iff for every sentence π,

$$\pi \in E \;\Leftrightarrow\; F \cup \{\delta \mid \frac{\alpha : \beta}{\delta} \in D, \alpha \in E, \neg\beta \notin E\} \models \pi$$

   where $\{\delta \mid \frac{\alpha:\beta}{\delta} \in D, \alpha \in E, \neg\beta \notin E\}$ is a set of *applicable assumptions.*

   (d) Is $P(c)$ a logical consequence of the following default theory (F, D)? List all possible extensions of this theory and explain your working.

$$F = \{R(a),\ R(b),\ R(c),\ \neg(a = b),$$

$$\neg(b = c),\ \neg(a = c),\ \neg P(a) \vee \neg P(b)\}$$

   and

$$D = \{\frac{R(x) : P(x)}{P(x)}\}$$

   (8 marks)

   Answer. (Application) There are two possible extensions. One of them contains applicable assumptions $P(a), P(c)$, and another $P(b), P(c)$. A skeptical reasoner will only believe sentences which belong to all extensions of the default theory; a credulous reasoner will choose an arbitrary extension. In either case, $P(c)$ is a consequence.