The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 3 MODULE, AUTUMN SEMESTER 2011-2012

KNOWLEDGE REPRESENTATION AND REASONING

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR out of SIX questions

Only silent, self contained calculators with a Single-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

1. (a) Define what it means for a set of sentences Γ to logically entail a sentence α. (3 marks)

   Answer. $\Gamma \models \alpha$ if and only if α is true in every interpretation M which makes all sentences in Γ true.

   (b) Prove that the following set of sentences does not entail $\exists x(P(x) \wedge \neg Q(x))$: (12 marks)

   S1 $\forall x(P(x) \supset \neg Q(x))$

   S2 $\exists x \neg Q(x)$

   S3 $\exists x(P(x) \vee Q(x))$

   Answer. Consider an interpretation M with domain $D = \{d_1, d_2\}$ and interpretation mapping I such that $I(P) = \emptyset$, $I(Q) = \{d_1\}$. Then S1 is true in M because there is no element in D which assigned to x makes $P(x)$ true. S2 is true because d_2 assigned to x makes $\neg Q(x)$ true. S3 is true because d_1 assigned to x makes $Q(x)$ and hence $P(x) \vee Q(x)$ true. However, there is no value for x which satisfies $P(x)$, hence also no value which satisfies $P(x) \wedge \neg Q(x)$.

   (c) Translate the following sentences into first order logic and show that they cannot be satisfied in the same interpretation: (10 marks)

   S1 There is a person who is loved by everybody.

   S2 It is not true that everybody loves somebody.

   Answer.

   S1 $\exists y \forall x Loves(x, y)$

   S2 $\neg \forall x \exists y Loves(x, y)$

   $\neg \forall x \exists y Loves(x, y)$ is equivalent to $\exists x \neg \exists y Loves(x, y)$.

   Consider an arbitrary interpretation M. For S1 to be true in M, there should be an individual a in the domain of M, such that with y assigned a, $\forall x Loves(x, y)$ is true; for every element d in the domain, $\langle d, a \rangle \in I(Loves)$. For S2 to be true, there should be an individual b for which there is no d such that $\langle b, d \rangle \in I(Loves)$. But by S1 we should have $\langle b, a \rangle \in I(Loves)$. A contradiction.

2. (a) Reduce the following sentences to clausal form: (6 marks)

   S1 $\forall x \forall y(R(x, y) \supset (R(y, x) \wedge Q(y)))$

   S2 $\forall x \exists y \forall z(P(x, y, z) \supset \exists u R(x, u, z))$

   S3 $\forall x(\neg \exists y P(x, y) \wedge \neg(Q(x) \wedge \neg R(x)))$

   Answer.

   S1 $[\neg R(x, y), R(y, x)]$, $[\neg R(x, y], Q(y)]$

   S2 $[\neg P(x, f(x), z), R(x, g(x, z), z)]$

   S3 $[\neg P(x, y)]$, $[\neg Q(x), R(x)]$

   (b) Derive by resolution an empty clause from the following clauses: (9 marks)

   C1 $[\neg P(x_1), Q(x_1)]$

   C2 $[P(x_2), \neg Q(x_2)]$

   C3 $[\neg Q(x_3), Q(f(x_3))]$

   C4 $[\neg P(x_4), \neg P(f(x_4))]$

   C5 $[P(a)]$ where a is a constant.

Answer.

1. $[Q(a)]$ from C1, C5, x_1/a
2. $[Q(f(a))]$ from 1, C3, x_3/a
3. $[\neg P(f(a))]$ from C4, C5, x_4/a
4. $[\neg Q(f(a))]$ from 3, C2, $x_2/f(a)$
5. [] from 2,4.

(c) Use resolution with answer extraction to prove that S4 below follows from S1-S3 and to extract the answer (a substitution for x which makes $(Like(anne, x) \wedge Student(x)$ true). (10 marks)

S1 $\forall x \forall y(Friend(x, y) \supset Like(x, y))$

S2 $Friend(anne, ben)$

S3 $Student(ben)$

S4 $\exists x(Like(anne, x) \wedge Student(x))$

Answer. First we need to rewrite S1-S3 and the negation of S4 in clausal form:

C1 $[\neg Friend(x, y), Like(x, y)]$

C2 $[Friend(anne, ben)]$

C3 $[Student(ben)]$

C4 $[\neg Like(anne, x_1), \neg Student(x_1), A(x_1)]$ (this clause has an answer predicate added)

i. $[Like(anne, ben]$ from C1, C2, $x/anne, y/ben$

ii. $[\neg Student(ben), A(ben)]$ from 1, C4, x_1/ben

iii. $[A(ben)]$ from 2, C3.

3. Recall the description logic $\mathcal{DL}$ given in the textbook:
Concepts:

- atomic concept is a concept
- if r is a role and b is a concept, then $[\textbf{ALL}\ r\ b]$ is a concept (e.g. $[\textbf{ALL}\ :Child\ Girl]$ describes someone all of whose children are girls).
- if r is a role and n is a positive integer, then $[\textbf{EXISTS}\ n\ r]$ is a concept (e.g. $[\textbf{EXISTS}\ 2\ :Child]$ describes someone who has at least 2 children)
- if r is a role and c is a constant, then $[\textbf{FILLS}\ r\ c]$ is a concept (e.g. $[\textbf{FILLS}\ :Child\ john]$ describes someone whose child is John).
- if $b_1, \ldots, b_n$ are concepts, $[\textbf{AND}\ b_1, \ldots, b_n]$ is a concept.

Sentences:

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s)
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)
- if c is a constant and b a concept then $c \rightarrow b$ is a sentence (the individual denoted by c satisfies the description expressed by b).

(a) Express the following concepts in $\mathcal{DL}$ using atomic concepts *School* and *Female*, roles $:Pupil$ and $:Employee$, and a constant *anne*:

i. A school which has at least 30 pupils. (2 marks)
Answer. [**AND** *School*, **EXISTS** 30 : *Pupil*]

ii. A school which has at least 30 pupils and 5 employees. (2 marks)
Answer. [**AND** *School*, **EXISTS** 30 : *Pupil*, **EXISTS** 5 : *Employee*]

iii. A school where all the pupils are girls. (2 marks)
Answer. [**AND** *School*, **ALL** : *Pupil Female*]

iv. A school where one of the pupils is Anne. (2 marks)
Answer. [**AND** *School*, **Fills** : *Pupil anne*]

(b) Express the following sentences in $\mathcal{DL}$ using the atomic concepts *School*, *Female*, *GirlsSchool*, the roles : *Pupil* and : *Employee*:

i. A girls school is defined as a school where all pupils are girls. (3 marks)
Answer. $GirlsSchool \doteq$ [**AND** *School*, **ALL** : *Pupil Female*]

ii. In girls schools all employees are female. (4 marks)
Answer. $GirlsSchool \sqsubseteq$ [**ALL** : *Employee Female*]

(c) Prove that sentences S1-S3 below do not entail S4. (10 marks)

S1 $a \rightarrow$ [**EXISTS** 1 : *Friend*]

S2 $a \rightarrow$ [**ALL** : *Friend Female*]

S3 [**ALL** : *Friend Female*] $\sqsubseteq$ [**ALL** : *Friend* [**ALL** : *Friend Female*]]

S4 $a \rightarrow$ [*Female*]

Answer. Consider an interpretation $M = (D, I)$ where $D = \{john, mary\}$ and $I(a) = john$, $I(Friend) = \{(john, mary)\}$, $I(Female) = \{mary\}$. Then S1 is true n M because the individual denoted by a (john) is an element of a set of people who have a friend I([**EXISTS** 1 : *Friend*]) (which happens to be just $\{john\}$). All John's friends are female, so S2 is true in M. The set I([**ALL** : *Friend Female*]) in addition to John contains Mary, because she does not have any friends and any statement about her friends is trivially true. The set of individuals in I([**ALL** : *Friend* [**ALL** : *Friend Female*]]) contains John (because all of his friends are in I([**ALL** : *Friend Female*]) and Mary (because she has no friends). Since $\{john, mary\} \subseteq \{john, mary\}$, S3 is true in M. Finally, $john \notin I(Female)$, so S4 is false.

So we found an interpretation where S1-S3 are true and S4 is false, so S4 is not true in all intepretations which make S1-S3 true (hence the entailment does not hold).

4. (a) Consider the following Horn clauses. Draw a goal tree for the goal *HasFootballClub*. (5 marks)

C1 $[\neg Nottingham, InEngland]$

C2 $[\neg Nottingham, City]$

C3 $[\neg InEngland, \neg City, EnglishCity]$

C4 $[\neg EnglishCity, HasFootballClub]$

C5 $[Nottingham]$

Answer.

```
           HasFootballClub
                  |
             EnglishCity
                /and\
               /     \
       InEngland     City
           |           |
       Nottingham    Nottingham
```

(b) Give the backward chaining procedure for propositional Horn clauses. (5 marks)

Answer.

Backward chaining .
input: a finite set of atomic sentences $q_1, \ldots, q_n$
output: YES if KB entails all of q_i, NO otherwise

```
procedure: SOLVE[q1, ..., qn ]
if n = 0 then return YES
for each clause c in KB do
   if  c = [not p1,...,not pm, q1] and SOLVE [p1,...,pm,q2,...,qn]
   then return YES
end for
return NO
```

(c) Trace this procedure for the example in part (a). (5 marks)

Answer.

SOLVE(HasFootballClub)
$c = [\neg EnglishCity, HasFootballClub]$
SOLVE(EnglishCity)
$c = [\neg InEngland, \neg City, EnglishCity]$
SOLVE(InEngland, City)
$c = [\neg Nottingham, InEngland]$
SOLVE(Nottingham, City)
$c = [Nottingham]$
SOLVE(City)
$c = [\neg Nottingham, City]$
SOLVE(Nottingham)
$c = [Nottingham]$
SOLVE()

(d) Define an SLD resolution derivation of a clause c from a set of clauses S. (4 marks)

Answer.

An SLD resolution derivation of a clause c from a set of clauses S is a sequence of clauses $c_1, \ldots, c_n$ such that $c_1 \in S$, $c_n = c$, and each c_{i+1} is the result of resolving c_i with one of the clauses from S.

(e) Give an SLD resolution derivation of an empty clause from the clauses C1–C5 in part (a) and $[\neg HasFootballClub]$. (6 marks)

Answer.

1. $[\neg HasFootballClub]$
2. $[\neg EnglishCity]$ (from 1 and C4)
3. $[\neg InEngland, \neg City]$ (from 2, C3)
4. $[\neg Nottingham, \neg City]$ (from 3, C1)
5. $[\neg City]$ (from 4, C5)
6. $[\neg Nottingham]$ (from 5, C2)
7. [] (from 6, C5)

5. (a) Describe the process of knowledge acquisition for a production rule system using decision tables. Show a decision table for a production rule system making medical insurance reimbursement decisions based on the following policy:

Charges below \$10 are not reimbursed. If the charges are above \$10, the amount to be reimbursed depends on whether or not the doctor or hospital is approved by the insurance company. Visits to an approved doctor are reimbursed at 70 % and visits to approved hospitals are reimbursed at 90%. Otherwise, doctor's visits are reimbursed at 60% and hospital visits at 80 %. (15 marks)

Answer First we need to decide on all conditions relevant for making a decision, and all possible actions. A decision table consists of a list of condition stubs (possible relevant conditions), action stubs (progress), condition entries ("yes", "no", and "–" for not relevant, for each condition) and action entries (which action is to be taken if specified conditions hold).

The relevant conditions are:

i. value of the claim (at least \$10) (possible values Y,N)
ii. whether the provider is approved (possible values Y,N)
iii. type of visit (D for doctor, H for hospital).

The possible actions are:

i. reimburse 70%
ii. reimburse 90%
iii. reimburse 60%
iv. reimburse 80%
v. no reimbursement

The resulting table is

```
Conditions
Claim at least $10?    Y   Y   Y   Y  N
Approved Provider?     Y   Y   N   N  -
Type of visit          D   H   D   H  -
Actions
Reimburse 70/%         X
Reimburse 90/%             X
Reimburse 60/%                 X
```

```
Reimburse 80/%                     X
No reimbursement                      X
```

The rules correspond to columns in the table, for example (if the claim is at least \$10 and approved provider and doctor, then reimburse 70%).

[Approximately 7 marks for the general description of the procedure, and 8 for the decision table.]

(b) Give the forward chaining procedure for propositional Horn clauses. (5 marks)

Answer.

input: a finite list of atomic sentences, $q_1, \ldots, q_n$

output: YES if KB entails all of q_i, NO otherwise

1. if all goals q_i are marked as solved, return YES
2. check if there is a clause $[p, \neg p_1, \ldots, \neg p_m]$ in KB, such that all of $p_1, \ldots, p_m$ are marked as solved and p is not marked as solved
3. if there is such a clause, then mark p as solved and go to step 1.
4. otherwise, return NO.

(c) Trace it on the following example, for the input $\{HasFootballClub\}$: (5 marks)

C1 $[\neg Nottingham, InEngland]$

C2 $[\neg Nottingham, City]$

C3 $[\neg InEngland, \neg City, EnglishCity]$

C4 $[\neg EnglishCity, HasFootballClub]$

C5 $[Nottingham]$

Answer. solved = $\{\ \}$

clause: $[Nottingham]$

solved = $\{Nottingham\}$

clause: $[\neg Nottingham, InEngland]$

solved = $\{Nottingham, InEngland\}$

clause: $[\neg Nottingham, City]$

solved = $\{Nottingham, InEngland, City\}$

clause: $[\neg InEngland, \neg City, EnglishCity]$

solved: $\{Nottingham, InEngland, City, EnglishCity\}$

clause: $[\neg EnglishCity, HasFootballClub]$

solved: $\{Nottingham, InEngland, City, EnglishCity, HasFootballClub\}$

6. (a) What are the main differences between subjective probabilities (degrees of belief) and fuzzy truth values? Give examples of typical uses of subjective probabilities and fuzzy truth values. (5 marks)

Answer. The two main differences are in the meaning of the numbers, and in operations for combining them. Meaning: degree of belief is about how likely you think that something will happen/is true, for example if if the symptoms displayed by a patient A are very often associated with a given disease D and are not very likely to occur otherwise, we may assign a high degree of belief, say 0.9, to the proposition that A has D. Fuzzy truth value is how applicable a property is to an object; for example, how appropriate it is to classify a 1.9m tall man as 'tall' (say 0.95 appropriate) and opposed to classifying a 1.75m tall man as 'tall' (say 0.6). Operations: subjective probabilities are combined in accordance to the rules of probability theory (so for example $Pr(a \vee b) = Pr(a) + Pr(b) - Pr(a \wedge b)$), but operations on fuzzy truth values are truth-functional (usually $T(a \vee b) = max(T(a), T(b))$).

(b) Consider the following example. Denote the proposition that a patient A has a symptom S by s, and the proposition that patient A has disease D by d. Suppose we want to use subjective probabilities to calculate the degree of belief that A has D given that A has S ($Pr(d|s)$): (6 marks)

i. which formula do we use?
Answer. Bayes's rule $Pr(d|s) = Pr(d) \times Pr(s|d)/Pr(s)$. [Approx. 2 marks]

ii. Assume that D always causes S. Which a priori probabilities do we need?
Answer. In this case, $Pr(s|d) = 1$, so we need $Pr(d)$ and $Pr(s)$. [Approx. 2 marks]

iii. How can we arrive at those a priori probabilities?
Answer. We can use objective statistical data on D and S. For example, if 1% of the population has D, we will set $Pr(d)$ to be 0.01. [Approx. 2 marks.]

(c) Consider the following Bayesian network:

```
fire ---> smoke
 \
  \
   \
    v
   fire engine
```

$Pr(fire) = 0.01$
$Pr(smoke|fire) = 0.9$
$Pr(smoke|\neg fire) = 0$
$Pr(engine|fire) = 1$
$Pr(engine|\neg fire) = 0.01$

i. What is the probability that there is a fire without smoke? (3 marks)

ii. What is the probabilility that there is no fire but a fire engine turns up? (3 marks)

iii. What is the probability that there is a fire given that there is smoke? (4 marks)

iv. What is the probability that there is a fire given that there is smoke and a fire engine has arrived? (4 marks)

Answer.

i. $Pr(fire \wedge \neg smoke) = 0.01 \times (1 - 0.9) = 0.01 \times 0.1 = 0.001$

ii. $Pr(\neg fire \wedge engine) = 0.99 \times 0.01 = 0.0099$

iii. $Pr(fire|smoke) = Pr(fire \wedge smoke)/Pr(smoke) = Pr(fire \wedge smoke)/(Pr(fire \wedge smoke) + Pr(\neg fire \wedge smoke))$ Since $Pr(\neg fire \wedge smoke) = 0$, we have $Pr(fire|smoke) = 1$.

iv. similarly 1.