G53KRR 2018 Answers to remedial first formal/assessed exercise r-ex1

1. Express the following sentences in first order logic using predicate symbols $Student$ (unary, $Student(a)$ means a is a student), $Tutor$ (binary, $Tutor(b, a)$ means b is a's tutor), $Lazy$ (unary), $Happy$ (unary):

   S1 All students are happy.

   $\forall x(Student(x) \supset Happy(x))$

   S2 Tutors of happy students are happy.

   $\forall x(Tutor(x) \land Student(y) \land Tutor(x, y) \land Happy(y) \supset Happy(x))$

   S3 Some tutor has at least two students. *Hint: use = and* $\neg$

   $\exists x \exists y \exists z(Tutor(x, y) \land Tutor(x, z) \land Student(y) \land Student(z) \land \neg(y = z))$

   S4 There is a student whose tutor is lazy.

   $\exists x \exists y(Student(x) \land Tutor(y, x) \land Lazy(y))$

   S5 Exactly one student is lazy. *Hint: use = and* $\neg$

   $\exists x(Student(x) \land Lazy(x) \land \forall y(Student(y) \land Lazy(y) \supset x = y))$

2. Consider an interpretation where the domain consists of 4 suitcases a, b, c, d where a and b are large and c and d are small. In other words, the predicate symbol $Large$ is interpreted as the set $\{a, b\}$ and $Small$ is interpreted as the set $\{c, d\}$. There is also a predicate symbol $FitsIn$ that is interpreted as the set of pairs $\{(c, a), (c, b), (d, a), (d, b)\}$ (small suitcases fit inside large ones). Are the following first order sentences true or false in this interpretation (and why):

   (a) $\forall x \forall y(Large(x) \land Small(y) \supset \neg FitsIn(x, y))$

   True because for every value of x, if it is Large, then it does not fit into anything.

   (b) $\forall x \exists y(Small(x) \supset FitsIn(x, y))$

   True because for every small x there is a (large) suitcase into which it fits.

   (c) $\neg \exists x \exists y FitsIn(x, y)$

   False because there exists a pair of values for x and y where x fits into y, for example $x = c$ and $y = a$

   (d) $\exists x \forall y \neg FitsIn(x, y)$

   True because for example $x = a$ does not fit into any possible value for y.

   (e) $\forall x \exists y(Small(x) \land \neg FitsIn(y, x))$

   False because it says that all values for x are small (and something about y, but this additional stuff cannot make it true).