G53KRR answers to the 4th formal/assessed exercise ex4.

1. Translate the following statements into ALC, using atomic concepts $Actor$, $FilmActor$, $TheatreActor$, $Film$, $TheatrePerformance$, roles $actIn$ ($actIn(x, y)$ means x acts in y), $friend$ ($friend(x, y)$ means x has friend y), and constant $john$:

   S1 Film actor is an actor who acts in some film.

   $FilmActor \doteq Actor \sqcap \exists\ actIn.Film$

   S2 Theatre actor is an actor who only acts in theatre performances.

   $TheatreActor \doteq Actor \sqcap \forall\ actIn.TheatrePerformance$

   S3 All actors are either film actors or theatre actors (there are no other kinds of actors).

   $Actor \doteq FilmActor \sqcup TheatreActor$ (correct logically, but circular definitions are not approved of in description logic).
   Can say $Actor \sqcap \neg(FilmActor \sqcup TheatreActor) \sqsubseteq \bot$, and separately $Actor \sqsubseteq FilmActor \sqcup TheatreActor$.

   S4 John is a film actor who has a friend who is a theatre actor.

   $(FilmActor \sqcap \exists\ friend.TheatreActor)(john)$

   S5 John is someone all of whose friends are actors.

   $\forall\ friend.Actor(john)$

   S6 John has a friend who has a friend who is not an actor.

   $\exists\ friend.\exists\ friend.\neg Actor(john)$

2. Is the following sentence valid:

   $$\exists r.C_1 \sqcap \exists r.C_2 \sqsubseteq \exists r.(C_1 \sqcap C_2)?$$

   Explain your answer. If the subsumption does not hold, give a interpretation where $I(\exists r.C_1 \sqcap \exists r.C_2)$ is not included in $I(\exists r.(C_1 \sqcap C_2))$.

   The sentence is not valid. A counterexample interpretation is $J = (D, I)$, where $D = \{d_1, d_2, d_3\}$, $I(r) = \{(d_1, d_2), (d_1, d_3)\}$, $I(C_1) = \{d_2\}$ and $I(C_2) = \{d_3\}$. Then $d_1 \in I(\exists r.C_1 \sqcap \exists r.C_2)$, and $I(\exists r.(C_1 \sqcap C_2))$ is empty.

3. Does $A \doteq \forall r.(C_1 \sqcap C_2)$ entail $A \sqsubseteq \forall r.(C_1 \sqcup C_2)$? Explain your answer. If the entailment does not hold, give a interpretation where $A \doteq \forall r.(C_1 \sqcap C_2)$ is true and $A \sqsubseteq \forall r.(C_1 \sqcup C_2)$ is false.

   Yes, if $I(A) = I(\forall r.(C_1 \sqcap C_2))$ then for objects in $I(A)$, all r edges should be to objects in $I(C_1 \sqcap C_2)$. Then since $I(C_1) \cap I(C_2) \subseteq I(C_1) \subseteq I(C_1) \cup I(C_2)$, all r edges are to $I(C_1 \sqcup C_2)$.