

COMP4075/G54RFP: Lecture 10

Monad Transformers

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For example: State and Error/Partiality?

We could implement a suitable monad from scratch:

$$\text{newtype } SE \ s \ a = SE \ (s \rightarrow Maybe \ (a, s))$$

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- Duplication of effort: similar patterns related to specific effects are going to be repeated over and over in the various combinations.

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- A form of **aspect-oriented programming**.
- MTL is one example of such a library.

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- Monad transformer libraries can be developed, each transformer each adding a specific effect (state, error, ...).
- A form of *aspect-oriented programming*.
- MTL is one example of such a library.

Will consider the general idea of monad transformers first; specific libraries discussed later.

Monad Transformers in Haskell (1)

- A *monad transformer* maps monads to monads. Represented by a type constructor T of the following kind:

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- A **monad transformer** maps monads to monads. Represented by a type constructor T of the following kind:

$$T :: (* \rightarrow *) \rightarrow (* \rightarrow *)$$

- Additionally, a monad transformer **adds** computational effects.
- A mapping **lift** maps a computation in the underlying monad to one in the transformed monad:

$$\text{lift} :: M\ a \rightarrow T\ M\ a$$

Monad Transformers in Haskell (2)

- These requirements are captured by the following (multi-parameter) type class:

```
class (Monad m, Monad (t m))  
      ⇒ MonadTransformer t m where  
    lift :: m a → t m a
```

Classes for Specific Effects

A monad transformer adds specific effects to *any* monad. Thus the effect-specific operations needs to be overloaded. For example:

class *Monad* *m* \Rightarrow *E m* **where**

eFail :: *m a*

eHandle :: *m a* \rightarrow *m a* \rightarrow *m a*

class *Monad* *m* \Rightarrow *S m s* | *m* \rightarrow *s* **where**

sSet :: *s* \rightarrow *m* ()

sGet :: *m s*

The Identity Monad

We are going to construct monads by successive transformations of the identity monad:

newtype $I\ a = I\ a$

$unI\ (I\ a) = a$

instance *Monad* I **where**

$return\ a = I\ a$

$m \gg= f = f\ (unI\ m)$

$runI :: I\ a \rightarrow a$

$runI = unI$

The Error Monad Transformer (1)

newtype $ET\ m\ a = ET\ (m\ (Maybe\ a))$

$unET\ (ET\ m) = m$

The Error Monad Transformer (2)

Any monad transformed by ET is a monad:

instance $Monad\ m \Rightarrow Monad\ (ET\ m)$ **where**

$return\ a = ET\ (return\ (Just\ a))$

$m \gg= f = ET\ \$\ \mathbf{do}$

$ma \leftarrow unET\ m$

case ma **of**

$Nothing \rightarrow return\ Nothing$

$Just\ a \rightarrow unET\ (f\ a)$

The Error Monad Transformer (3)

We need the ability to run transformed monads:

$$\text{runET} :: \text{Monad } m \Rightarrow \text{ET } m \ a \rightarrow m \ a$$
$$\text{runET } etm = \text{do}$$
$$ma \leftarrow \text{unET } etm$$
$$\text{case } ma \text{ of}$$
$$\text{Just } a \rightarrow \text{return } a$$
$$\text{Nothing} \rightarrow \text{error "Should not happen"}$$

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$$\text{case } ma \text{ of}$$
$$\text{Just } a \rightarrow \text{return } a$$
$$\text{Nothing} \rightarrow \text{error "Should not happen"}$$

(Note: To simplify use, we discarded information about the effect, but as a result, we get a partial function. Returning *Maybe a* better in general.)

The Error Monad Transformer (4)

ET is a monad transformer:

instance *Monad* *m* \Rightarrow
 MonadTransformer ET m **where**
 lift m = *ET* (*m* $\gg=$ $\lambda a \rightarrow \text{return (Just a)}$)

The Error Monad Transformer (5)

Any monad transformed by ET is an instance of E :

instance *Monad* $m \Rightarrow E (ET\ m)$ **where**

$eFail = ET\ (return\ Nothing)$

$m1\ 'eHandle'\ m2 = ET\ \$\ \mathbf{do}$

$ma \leftarrow unET\ m1$

case ma **of**

$Nothing \rightarrow unET\ m2$

$Just\ _ \rightarrow return\ ma$

The Error Monad Transformer (6)

A state monad transformed by ET is a state monad:

instance $S\ m\ s \Rightarrow S\ (ET\ m)\ s$ **where**
 $sSet\ s = lift\ (sSet\ s)$
 $sGet = lift\ sGet$

Exercise 1: Running Transf. Monads

Let

```
ex2 = eFail `eHandle` return 1
```

1. Suggest a possible type for $ex2$.
(Assume $1 :: Int$.)
2. Given your type, use the appropriate combination of “run functions” to run $ex2$.

Exercise 1: Solution

ex2 :: ET I Int

ex2 = eFail 'eHandle' return 1

ex2result :: Int

ex2result = runI (runET ex2)

The State Monad Transformer (1)

```
newtype ST s m a = ST (s → m (a, s))  
unST (ST m) = m
```

Any monad transformed by *ST* is a monad:

```
instance Monad m ⇒ Monad (ST s m) where  
  return a = ST (λs → return (a, s))  
  m >>= f = ST $ λs → do  
    (a, s') ← unST m s  
    unST (f a) s'
```

The State Monad Transformer (2)

We need the ability to run transformed monads:

$$runST :: Monad\ m \Rightarrow ST\ s\ m\ a \rightarrow s \rightarrow m\ a$$
$$runST\ stf\ s0 = \mathbf{do}$$
$$(a, _)\leftarrow unST\ stf\ s0$$
$$\mathbf{return}\ a$$

The State Monad Transformer (2)

We need the ability to run transformed monads:

$$runST :: Monad\ m \Rightarrow ST\ s\ m\ a \rightarrow s \rightarrow m\ a$$
$$runST\ stf\ s0 = \mathbf{do}$$
$$(a, _)\leftarrow unST\ stf\ s0$$
$$\mathbf{return}\ a$$

(We are again discarding information to keep things simple. Returning the final state along with result would be more general.)

The State Monad Transformer (3)

ST is a monad transformer:

```
instance Monad m =>
    MonadTransformer (ST s) m where
    lift m = ST (\s -> m >>= \a -> return (a, s))
```


The State Monad Transformer (3)

Any monad transformed by ST is an instance of S :

instance *Monad* $m \Rightarrow S (ST\ s\ m)\ s$ **where**
 $sSet\ s = ST\ (\backslash_ \rightarrow return\ ((), s))$
 $sGet = ST\ (\lambda s \rightarrow return\ (s, s))$

The State Monad Transformer (4)

An error monad transformed by ST is an error monad:

instance $E\ m \Rightarrow E\ (ST\ s\ m)$ **where**
 $eFail = lift\ eFail$
 $m1\ 'eHandle'\ m2 = ST\ \$\ \lambda s \rightarrow$
 $unST\ m1\ s\ 'eHandle'\ unST\ m2\ s$

Exercise 2: Effect Ordering

Consider the code fragment

$$\begin{aligned} ex3a &:: (ST\ Int\ (ET\ I))\ Int \\ ex3a &= (sSet\ 42 \gg eFail)\ 'eHandle'\ sGet \end{aligned}$$

Note that the exact same code fragment also can be typed as follows:

$$\begin{aligned} ex3b &:: (ET\ (ST\ Int\ I))\ Int \\ ex3b &= (sSet\ 42 \gg eFail)\ 'eHandle'\ sGet \end{aligned}$$

What is

$$\begin{aligned} &runI\ (runET\ (runST\ ex3a\ 0)) \\ &runI\ (runST\ (runET\ ex3b)\ 0) \end{aligned}$$

Exercise 2: Solution

$$\text{runI} (\text{runET} (\text{runST} \text{ ex3a} 0)) = 0$$

$$\text{runI} (\text{runST} (\text{runET} \text{ ex3b}) 0) = 42$$

Why? Because:

$$\begin{aligned} \text{ST } s (\text{ET } I) a &\approx s \rightarrow (\text{ET } I) (a, s) \\ &\approx s \rightarrow I (\text{Maybe } (a, s)) \\ &\approx s \rightarrow \text{Maybe } (a, s) \\ \text{ET } (\text{ST } s I) a &\approx (\text{ST } s I) (\text{Maybe } a) \\ &\approx s \rightarrow I (\text{Maybe } a, s) \\ &\approx s \rightarrow (\text{Maybe } a, s) \end{aligned}$$

MTL: Monad Transformer Library

Provides a number of standard monads, associated transformers, and all possible liftings in the style we have seen; e.g.:

- State (*Control.Monad.State*, lazy and strict)
- Exceptions (*Control.Monad.Except*)
- Lists (*Control.Monad.List*)
- Reader (*Control.Monad.Reader*)
- Writer (*Control.Monad.Writer*)
- Continuations (*Control.Monad.Cont*)

MTL: State

```
class Monad m => MonadState s m | m -> s where  
    get  :: m s  
    put  :: s -> m ()  
    state :: (s -> (a, s)) -> m a
```

Transformer: `newtype StateT s (m :: * -> *) a`

Run functions:

```
runState :: State s a -> s -> (a, s)  
evalState :: State s a -> s -> a  
execState :: State s a -> s -> s
```

MTL: Exception

```
class Monad m =>
    MonadError e m | m -> e where
    throwError :: e -> m a
    catchError :: m a -> (e -> m a) -> m a
```

Transformer: `newtype ExceptT e (m :: * -> *) a`

Run function:

```
runExcept :: Except e a -> Either e a
```

MTL: Reader

```
class Monad m =>
    MonadReader r m | m -> r where
    ask      :: m r
    local    :: (r -> r) -> m a -> m a
    reader   :: (r -> a) -> m a
```

Transformer: *ReaderT*

Run function:

```
runReader :: Reader r a -> r -> a
```


MTL: Writer

```
class (Monoid w, Monad m) =>
    MonadWriter w m | m -> w where
    writer :: (a, w) -> m a
    tell   :: w -> m ()
    listen :: m a -> m (a, w)
    pass   :: m (a, w -> w) -> m a
```

Transformer: `newtype WriterT w (m :: * -> *) a`

Run function:

```
runWriter :: Writer w a -> (a, w)
```

Problems with Monad Transformers

- With one transformer for each possible effect we get a quadratic number of combinations; each has to be instantiated explicitly.
- Jaskelioff (2008,2009) has proposed a possible, more extensible alternative:
 - Traditional approach: unsystematic lifting on case-by-case basis.
 - Jaskelioff: systematic lifting based on theoretical principles where each operation is paired with a type of its implementation allowing implementations to be transformed generically.

Reading (1)

- Nick Benton, John Hughes, Eugenio Moggi. Monads and Effects. In *International Summer School on Applied Semantics 2000*, Caminha, Portugal, 2000.
- Sheng Liang, Paul Hudak, Mark Jones. Monad Transformers and Modular Interpreters. In *Proceedings of the 22nd ACM Symposium on Principles of Programming Languages (POPL'95)*, January 1995, San Francisco, California

Reading (2)

- Mauro Jaskelioff. Monatron: An Extensible Monad Transformer Library. In *Implementation of Functional Languages (IFL'08)*, 2008.
- Mauro Jaskelioff. Modular Monad Transformers. In *European Symposium on Programming (ESOP,09)*, 2009.