

Identification notes

System equations

The output \bar{p}_K is given by *Equation: (1)*, which is a static equation

$$\bar{p}_K = K^T \bar{H}_T^{-T} f_T(A_2 q_C + A_3 K \bar{d}_K - A_3 D v_D \sigma) - \underbrace{K^T \bar{H}_T^{-T} \hat{H}_T^T (\hat{p} + \hat{h}) - K^T \bar{h}}_{\text{Linear term}}. \quad (1)$$

We consider it as an input-output model where the inputs:

$K^T \bar{h} = \bar{h}_K$	is the elevation of inlets ,
\hat{p}	is the pressure in the WTs,
\hat{h}	is the elevation of the WTs,
σ	is the total demand,
\bar{d}_K	is the inlet flows,
q_C	is the flows in \mathcal{C} .

One way to identify the input-output relation is to introduce a new output, where

$$\tilde{y} = \bar{p}_K + K^T \bar{H}_T^{-T} \hat{H}_T^T (\hat{p} + \hat{h}) + K^T \bar{h}. \quad (2)$$

In *Equation: (2)*, the known linear terms are summed with the pressure outputs \bar{p}_K . The right hand-side of the output equation in *Equation: (1)* is considered as a function, taking into account the input q_C implicitly

$$\tilde{f}_1(\bar{d}_K, \sigma) = K^T \bar{H}_T^{-T} f_T(A_2 q_C + A_3 K \bar{d}_K - A_3 D v_D \sigma). \quad (3)$$

The constraint for the system is on q_C , for which analytical solution cannot be given

$$f_C(q_C) - A_1(\hat{p} + \hat{h}) + A_2^T f_T(A_2 q_C + A_3 K \bar{d}_K - A_3 D v_D \sigma) = 0. \quad (4)$$

However, *Equation: (4)* is in the form of

$$q_C = q_C(\bar{d}_K, \sigma). \quad (5)$$

For this reason, the non-linear dependencies on q_C , \bar{d}_K and σ can be considered as

$$\tilde{y} = \tilde{f}_1(\bar{d}_K, \sigma). \quad (6)$$

Another way to approximate the input-output relation is to keep the linear terms on the right side of *Equation: (1)*. In this case we introduce linear parameters in the regression model. For this case, the model is in the form of

$$\bar{p}_K = \tilde{f}_2(\bar{d}_K, \sigma, (\hat{p} + \hat{h}), \bar{h}_K) \quad (7)$$

If this is chosen, we do not need to use the system matrices to subtract the linear terms. When real data is used, the structure might be different from the network in EPANET. Furthermore, the parameter v_D is considered to be constant. In this case, the consumption data can be calculated back from the mass-balance equation written up for the network, assuming that flow is measured in the WTs.

$$\sigma = 1^T \hat{d} + 1^T \bar{d}_K. \quad (8)$$

The equation which describes the dynamics is the following

$$\Lambda \dot{\hat{p}} = -(\hat{H}_C - \hat{H}_T \bar{H}_T^{-1} \bar{H}_C) q_C - \hat{H}_T \bar{H}_T^{-1} K \bar{d}_K + \hat{H}_T \bar{H}_T^{-1} D v_D \sigma. \quad (9)$$

For *Equation: (9)* the same considerations apply as for the output equation. Although it is a linear equation, by taking into q_C implicitly, the dynamics are non-linear. It can be written in the form

$$\hat{p}_{k+1} = \tilde{f}_3(\bar{d}_K, \sigma). \quad (10)$$

Example on a small model

The network is shown in *Figure 1*

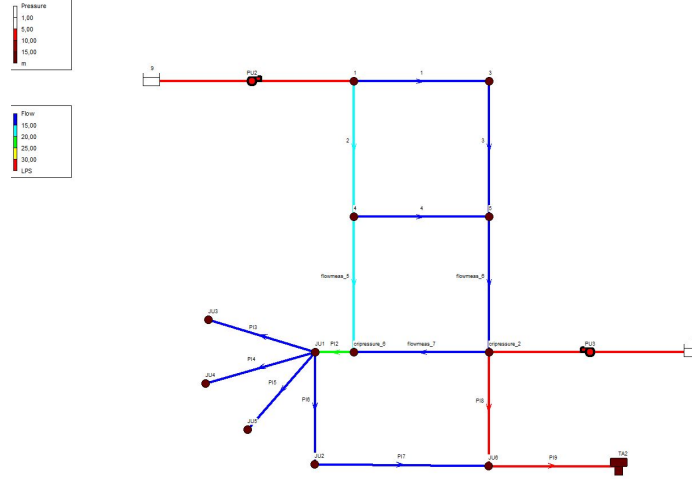


Figure 1: Example network.

The input data for identification consists of both the linear and non-linear inputs. Furthermore, feed-forward backpropagation model has been chosen for training. *Figure 2* shows the non-constant training and target data.

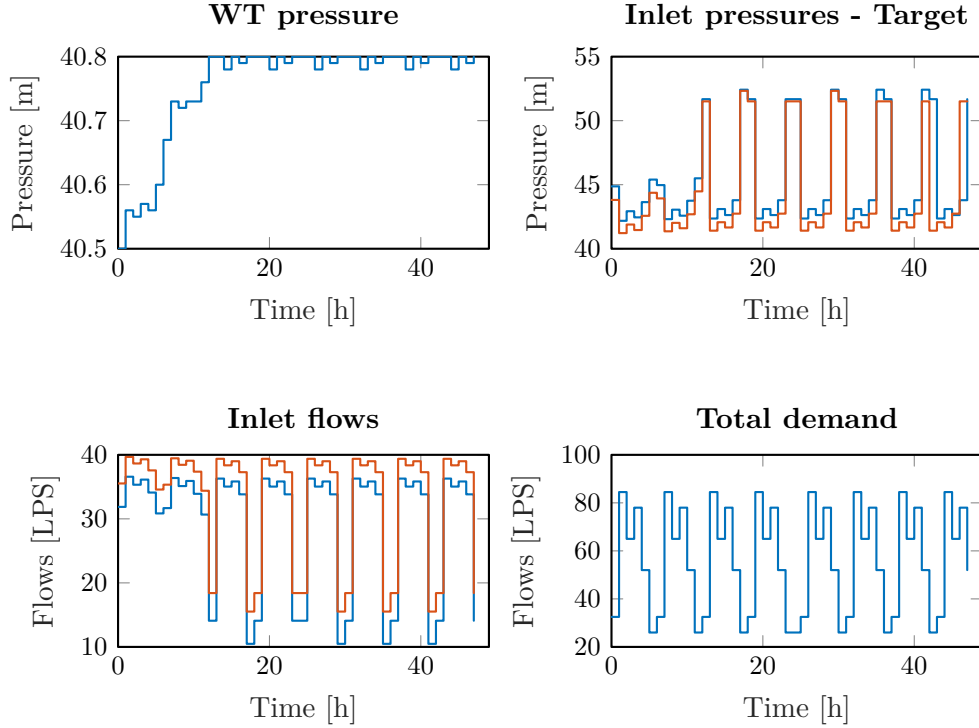


Figure 2: Input data for training.

The performance of the validation is shown in *Figure 3*

Best Validation Performance is 0.10993 at epoch 45

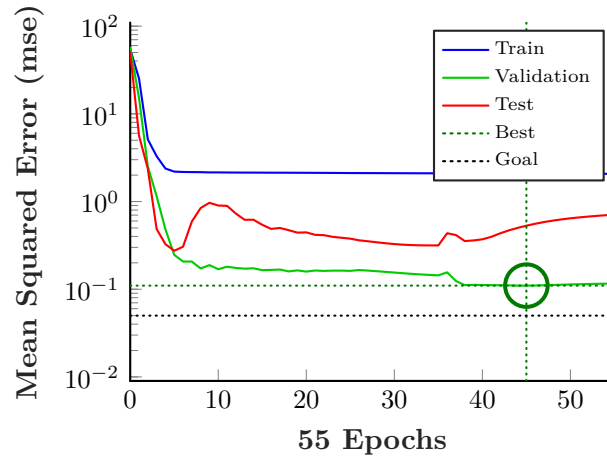
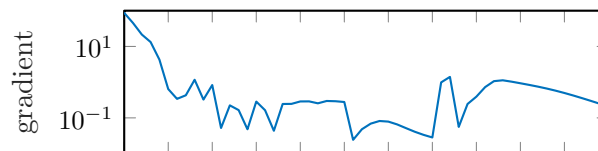


Figure 3: Performance - MSE.

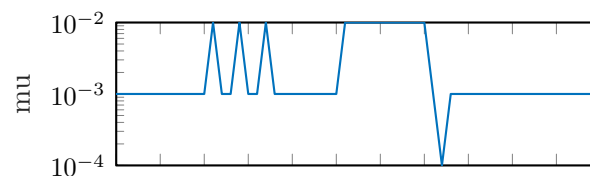
The validation goal is 0.05 in Mean Squared Error (MSE). The training is stopped if either the goal is reached or the validation failure is more than 10 epochs.

The training states are shown in *Figure 4*

Gradient = 0.20823, at epoch 55



Mu = 0.001, at epoch 55



Validation Checks = 10, at epoch 55

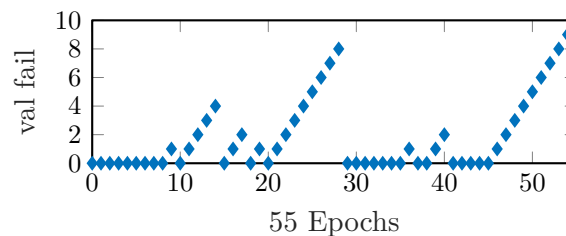


Figure 4: Training states.

For the training, Levenberg-Marquardt optimization was chosen to update weight and bias values. The validation of the output on the same data is shown in *Figure 5*

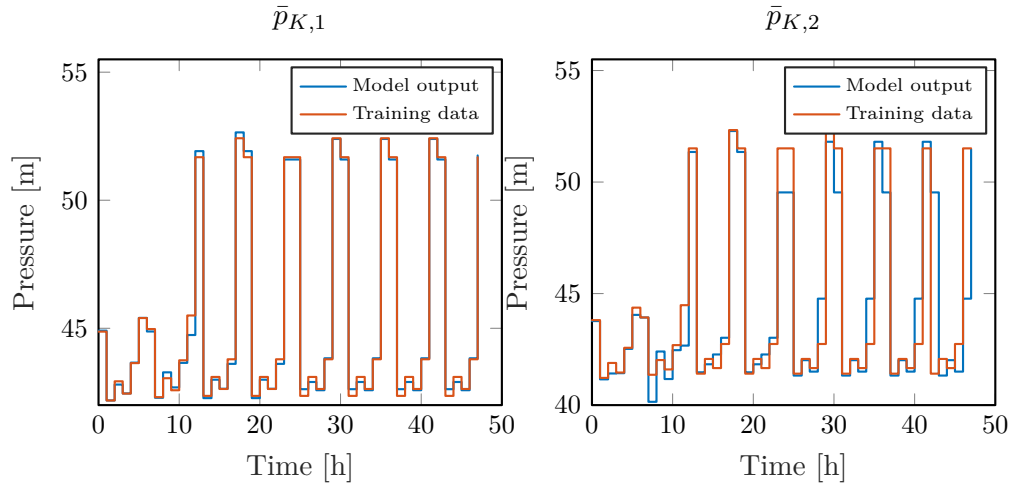


Figure 5: Outputs.

The neurons used in this identification use Tan-Sigmoid transfer functions. Therefore the regression model with the corresponding weights and biases can be given

$$y = B_2 + LW \cdot \text{transig}(B_1 + IW \cdot u). \quad (11)$$

The documentation of theory and introduction of neural networks and system identification is in progress. First, I used this simple example to study and understand this approach. Some concerns and questions came up however, which we could discuss at the meeting.

Question reminders

Q1: Creating the input data

Q2: Number of neurons

Q3: RBF vs. MPL, training.

Q4: Validation on different data, check overfitting.

Q5: How to define linear/non-linear mixed model