Aalborg University

Identification of Randers Water Distribution Network for Optimal Control

Electronic & IT:
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Group:
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Preface

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Aalborg University, th of December 2017

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Nomenclature

Acronyms

D-W	Darcy-Weisbach
EPA	Environmental Protection Agency
FCV	Flow Control Valve
GT	Graph Theory
MPC	Model Predictive Control
PRV	Pressure Regulating Valve
OD	Opening Degree
WSS	Water Supply System

Acronyms - Randers Network

BKV	Bunkedal Vandværk
LZ	Low Zone
HBP	Hobrovej Pumpestation
HNP	Hornbæk Pumpestation
HSP	Hadsundvej Pumpestation
HZ	High Zone
OST	Oust Mølle Vandværk
TBP	Toldbodgade Pumpestation
VSV	Vilstrup Vandværk
ØSV	Østrup Skov Vandværk

Symbols

Symbol	Description	Unit
a	Description	$[\cdot]$
b	Description	$[\cdot]$

Graph theory

Symbol	Description
a	Description
b	Description

Constants

Symbol	Description
a	Description
b	Description

Glossary of mathematical notation

Description of the mathematical notation and terminology used in the report.

Contents

1. Introduction

Due to the fast-paced technological development all over the world, the demand for industrial growth and energy resources has seen a rapid increase. Along with the industrial growth, the sudden rise in population has made the world realize that this shortage of energy sources is an actual and universally anticipated problem [1]. In order to cope with such shortage issues and to make the rapid development possible and less expensive, the world is moving towards more efficient use of resources and optimization of infrastructure. Therefore, technological development is also moving the focus on green energy, resulting in more and more renewable energy sources added to the grid [2].

Water Supply Systems(WSSs) are among the sectors which make the industrial growth possible. On top of this, WSSs are one of the most vital infrastructures of modern societies in the world. In Denmark typically, such networks are operating by making pumps transport water from reservoirs through the pipe network, to the end-users. In most cases, elevated reservoirs are exploited in these WSSs, such that they can even out the demand differences for the consumers. Although elevated reservoirs are usually an integrated part of these systems, providing drinking water is a highly energy-intensive activity. For instance, in the United States alone, the drinking water and waste water systems are typically the largest energy consumers, accounting for 25 to 40 percent of a municipality's total public expenditure. [3].

Since fresh water is limited, and due to the presence of global changes such as climate change and urbanization, new trends are emerging in the water supply sector. In the past few decades, several research and case study showed that WSSs and other energy distribution networks need to be improved due to the leakages in the system, high cost of maintenance and due to high energy consumption. Companies also realized that by using proper pressure management in their networks, the effect of leakages can be reduced, thereby huge amount of fresh water can be saved [4].

In Denmark recently, the larger water suppliers have been focusing on making the water supply sector more effective through introducing a benchmarking system focusing on the environment, the security of supply and the efficiency based on user demands. Since 1980, these efficiency activities has been an important issue [5]. It has been proved that by utilizing advanced, energy- or cost-optimizing control schemes and utilizing renewable energy sources, such as elevated reservoirs, the life of the existing infrastructure can be extended and money or energy can be saved [1]. Therefore there is a growing demand in industry for developing methods, leading towards more efficient WSSs.

The presented project is executed in collaboration with the company, Verdo A/S. It is in the interest of Verdo A/S to utilize an advanced model-based optimal control scheme on the WSS with several storages in Randers, Denmark. For a large municipality such as Randers, the water distribution network is complex and consists of thousands of elements. Since the control algorithm itself is complex and model-based, the computational effort is also high. Furthermore, the offline optimisation of a large-scale WSS means that any changes to the network may require significant changes in the optimisation method, which leads to high costs of the system maintenance [6]. Therefore typically a model reduction is required in such networks to make the online execution of the control algorithm possible.

The long-term goal of this project is to find a solution for implementing Model Predictive Control(MPC) on the Randers WSS. However, before the implementation of any control scheme would be possible, a proper and identified model is required. Therefore, as the first part of the project, the following problem statement can be formulated:

How can the WSS in Randers be simplified and identified, with storages included in the system, such that the reduced model preserves the original nonlinear behaviour and remains suitable for a plug-and-play commissionable Model Predictive Control scheme.

Part I

System Analysis

2. Description of Water Supply Systems

This chapter gives a general overview of hydraulic systems and an introduction to the WSS in Randers. The basic topology and structures of water supply networks are explained. Furthermore, the basic components of hydraulic systems are discussed and the unit called head, as an alternative measure of pressure, is introduced.

2.1 Hydraulic system overview

WSSs are designed to deliver water to consumers in terms of sufficient pressure and appropriate chemical composition. Distribution systems as such are typically transport water from one geographical place to another. In practice, there are different methods exist to achieve this water transport. One example is the use of natural advantages such as the water stored in mountains, and thereby use the potential energy of the water to provide pressure in the network. Examples for this are countries like Norway where the advantages of the landscape are being exploited [7]. However, in this project the source of the water is considered as groundwater, considering that in Denmark all reservoirs in the network are tapping water from the ground. It worth noting that the quality of groundwater in Denmark is sufficiently good to use it for drinking water supply purposes. After tapping the water, it goes through an aeration process at the waterworks and afterwards the pure water is pumped into the network [8]. In WSSs, pumps and valves are the elements that enable the control and thereby the proper delivery of water to the consumers or to elevated reservoirs, storing water for later use. Such a network is illustrated in the figure below:

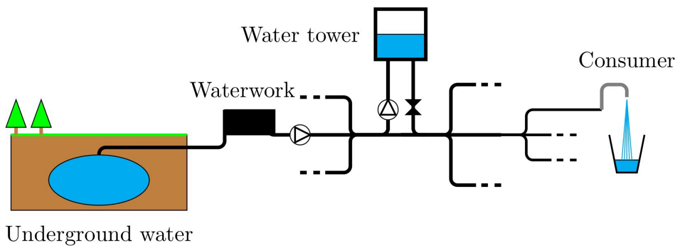


Figure 2.1: Illustration of a WSS [9].

The delivered water needs to fulfil a certain pressure criteria in order to reach consumers at higher levels. For example, in some cases the pressure has to be high enough to make it to the fourth floor of a building and still provide appropriate pressure in the water taps. Generally, in such cases booster pumps are placed in the basement of buildings, helping to supply the pressure. Too large pressure values, however increase water losses due to pipe waste [10].

Another criteria is that the flow through particular pipes need to stay within acceptable limits. A low flow rate can lead to water quality problems due to the undesirable microorganisms in the water and due to the metal and salt accumulation on the wall of the pipes [10].

As can be seen in *Figure 2.1*, typically WSSs consist of pipe, valve, reservoir, elevated reservoir(tank) and pump components. The common property of them is that they are all two-terminal components, therefore they can be characterized by the dynamic relationship between the pressure drop across their two corresponding endpoints and the flow through them [11].

2.1.1 Pipe networks

Pipes have a major role in WSSs since they are used for carrying pressurized water. They serve as a connection between components. Normally, the pipe network can be split into different sub-parts, taking into account the physical characteristics and the attributes of the pipes. Therefore, water supply networks can consist of transmission mains, arterial mains, distribution mains and service lines as shown in the example below:

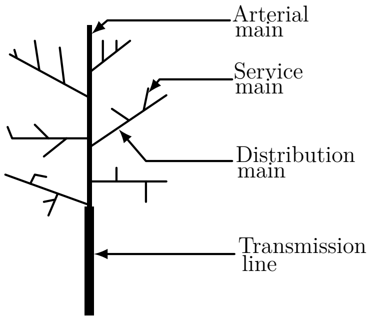


Figure 2.2: Illustration of pipe mains. Tree configuration.

Transmission mains deliver large amounts of water over long distances. Arterial and distribution mains provide intermediate steps towards delivering water to the end-users. Service lines transmit the water from the distribution mains straight to the end-users [12].

The transmission and distribution network can have a topology that is called a loop or a tree structure. *Figure 2.2* shows an example for a tree configuration. This type of configuration is most frequently used in rural areas [13]. Typically the network has only one path for the water to reach the end-users. A more frequent problem compared to looped configurations is, that on the outer parts of the system lower pressures can be experienced due to the pressure losses from long flow paths. The flow dynamics within this kind of systems therefore consist of large flows closer to the source that turn into smaller flows on the outer parts of the system. Main disadvantage of a purely tree structure system is that due to maintenance or momentary breakdowns, the system suffers disruption of service [13].

Loop networks have a configuration as shown in *Figure 2.3*.

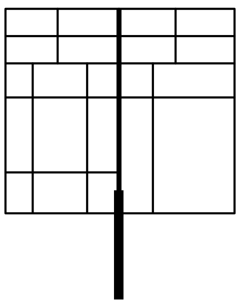


Figure 2.3: Loop configuration.

Loop networks are usually composed of smaller loops which are composed of smaller distribution mains, and larger loops that are connected to arterial or transmission mains. Elevated reservoirs are typically placed in the centre of the system due to pressure losses resulting from flows through the loop network [14]. This is reasonable because within a certain grid, the same pressure is provided by the tank, instead of providing the pressure through long pipelines to different distances. Furthermore, in the presence of a ring structure, the large loop around the area may be used to feed an internal distribution grid or a distribution grid attached to the outer part of the loop. Loop configurations are generally associated with larger suburban and city distribution systems such as larger cities[14]. The Randers WSS falls into this category.

2.1.2 Elevated reservoirs

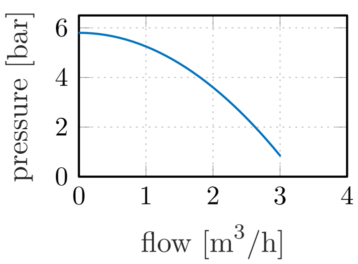
Elevated reservoirs, or tanks, are typically placed in the system to use them as buffers and level out the pressure and flow demand differences. When the demand is high, the waterworks might not be able to provide the sufficient amount of water in the network. In these cases, the elevated reservoir supplies the remaining demand. When the user consumption decreases, the system can be controlled such that the tank is being refilled to provide the required demand for the next peak time of consumption. Having such an elevated reservoir in the network, the system becomes more independent of the pump stations, as the refilled tank can itself maintain the desired pressure and flow for a limited time.

Due to the elevation of the tank, when it is filled up, the pumping stations need to provide a pressure higher than the pressure in the water tank. Therefore when the tank is being emptied, the pumping stations can reduce the amount of pressure they provide to the system, since the pressure from the elevated reservoir becomes dominant. This is due to the fact that the dynamics of systems with large storages come primarily from the pressure of the tank [15]. However, it should be noted that normally the level in the tank is varying less than a meter. This means that the effect on the pump operation is limited. Due to these considerations, the dynamics of these elevated reservoirs has to be taken into account while modelling the system.

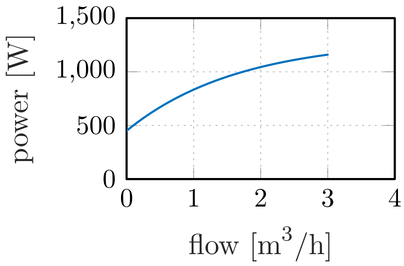
2.1.3 Pumps

Water pumps are used to increase pressure in hydraulic systems, thus making the water flow. Pumps are typically the main actuators of a WSS and they can be either flow or pressure controlled. Therefore, pumps can have controllers to produce a desired flow or pressure. This is done by changing the rotational speed of the pump. In this way, when the pump has a reference pressure or flow, simple control makes it possible to produce the desired flow or pressure respectively [16].The pressure required to make the water reach some height is the sum of the pressure required to overcome the elevation and the friction losses in the pipe network.

The most common pumps in WSSs are centrifugal pumps. Normally, the characteristics of such pumps are described by two pump curves. The two curves depict the volume flow versus the pressure and the power of the pump respectively. Normally the curves describe the characteristics for one particular speed, which is denoted the nominal speed [16]. An example of these pump curves is shown in *Figure 2.4*.



(a) Flow versus pressure difference



(b) Flow versus power consumption

Figure 2.4: Pump curves describing the performance of a centrifugal pump at nominal speed.

As can be seen, at a given flow, the pump can deliver a pressure with a maximum limit. This pressure decreases when the flow is increasing. At a certain flow and pressure value, the pump has an optimal point where the operation is the most energy efficient. Pumps are normally designed such that the optimal point lies in the operational area for the pumping application [9].

As in almost all WSSs, the flow is varying in the system, according to the flow demand from the end-consumers. Therefore, when dealing with varying flow in the system, pumps are often placed parallel at the pump stations such that they can keep their optimum points. As the flow increases, more pumps get activated to keep the pressure constant [9].

2.1.4 Valves

Valves in the WSS can be also seen as actuators along with pump elements. Unlike the pumps, valves are passive actuators in the sense that they do not consume energy. In principle, there are many types of valves existing. They can be categorized as non-return valves, control valves, shut-off valves and the combination of the two former one. Non-return valves allow waterflow only in one direction, while control valves can either adjust the flow or the pressure on their two endpoints. The former category is typically called a Flow Control Valve(FCV), while the latter is called a Pressure Reducer Valve or Pressure Regulating Valve(PRV). Shut-off valves are important components of the network since they can change the structure of the system, when for example doing maintenance or just redirecting the flow. This project deals with all three types of valves.

Valves can be controlled such that no flow passes through. In these cases the valve is closed and thereby certain parts of the system can be isolated as mentioned above. Other possibility is that the valve is fully open. In such case the pressure drop between the two endpoints is experienced because of the friction loss of the valve.

2.2 The Randers water supply network

The Randers drinking WSS is managed by Verdo A/S, which is the main supplier of drinking water and heating to the city of Randers. Verdo supplies water to approximately 46.000 customers in Randers Municipality [17]. The WSS is a complex, looped configuration with many different distribution areas. The coverage of the distribution areas are shown in *Figure 2.5* below.

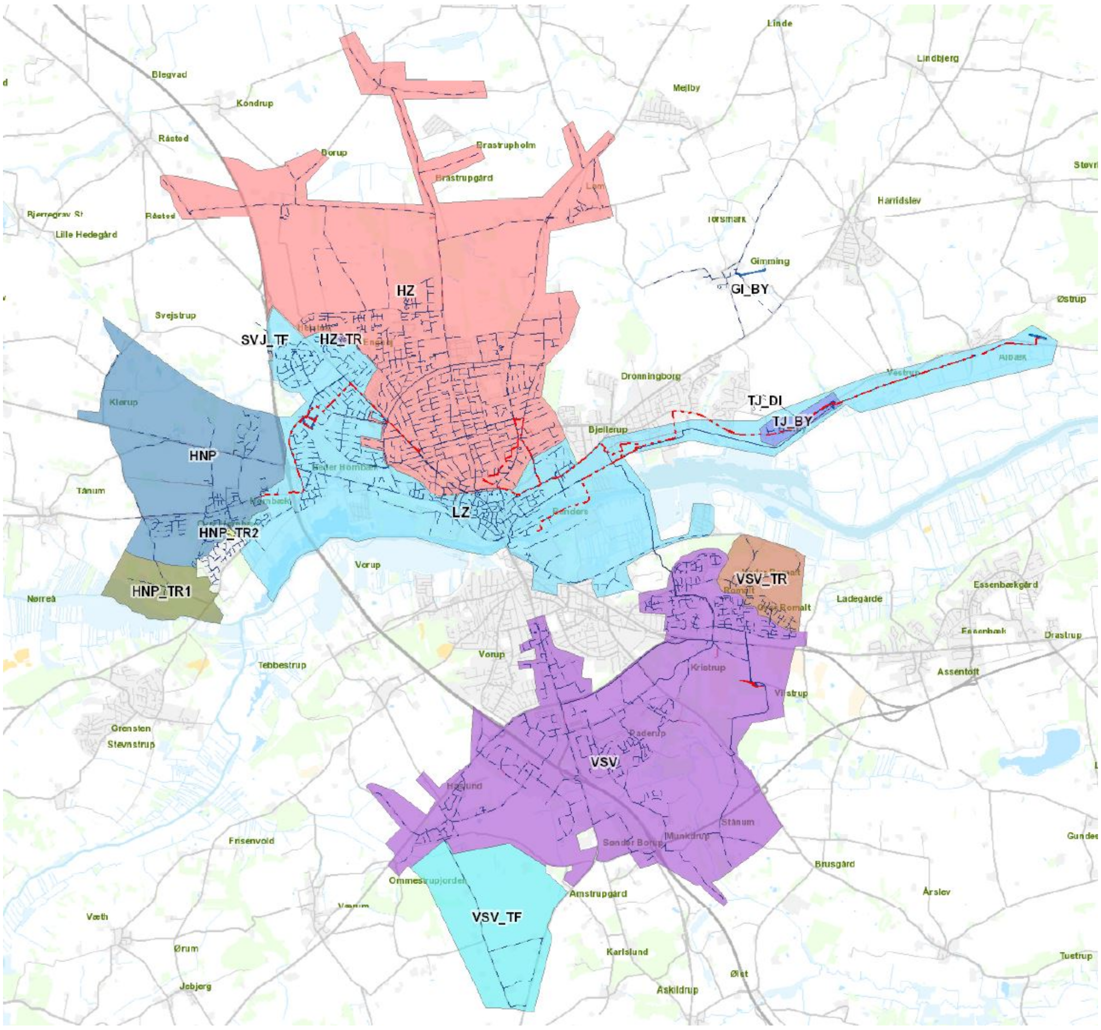


Figure 2.5: Distribution zones in the Randers WSS.

As can be seen in *Figure 2.5*, the network consists of many different distribution zones, however, the whole distribution map can be split into four main zones, according to the different geographical properties of the city. Furthermore, Randers is split into two regions, Randers North and Randers South. This is due to the fact that Randers fjord divides the city into two parts[17] .

The distribution network which is located in the southern part of the city is called Vilstrup zone. This zone has its own waterwork and pumping station which allows to supply the whole southern area by itself. This zone is shown in *Figure 2.6* below.

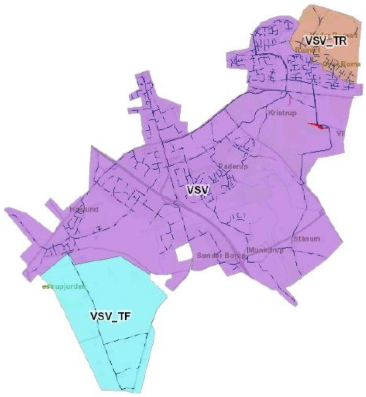


Figure 2.6: Vilstrup zone in Randers.

The only connection between Vilstrup and the northern region is through an emergency line, which is indeed used in emergency cases when the waterwork and pumping station in Vilstrup malfunctions or if there is a pollution in the water tanks. In these cases waterworks from the northern parts can provide water to the Vilstrup zone. Besides the emergency cases, Vilstrup does not rely on the waterworks and pumping stations in Randers North.

Randers North consists of three different areas, each having its own particular geographical property. The water distribution in these regions are normally relies on each other, meaning that during a certain time period, the schedulings of the pumps are interconnected.

The area shown in *Figure 2.7* below is called the High Zone(HZ) due to the high elevation level of the region.

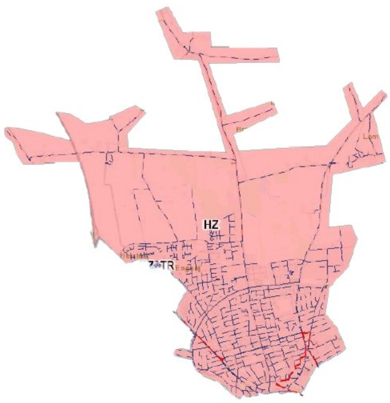


Figure 2.7: High Zone in Randers.

This part of the city lies approximately 55 meters above sea level, which means that big pumping effort is required to deliver the sufficient level of pressure to this area.

The area underneath the HZ is called the Low Zone(LZ). This zone in Randers lies approximately on sea level. Therefore, the HZ and LZ have a significant elevation difference which requires special pumping solutions in this area. In order to get a visual overview of the geographical properties of the HZ and LZ, the elevation profile is shown between these two areas in *Appendix: A*, in *Figure A.1* and *Figure A.2*. The area itself is shown in *Figure 2.8*.

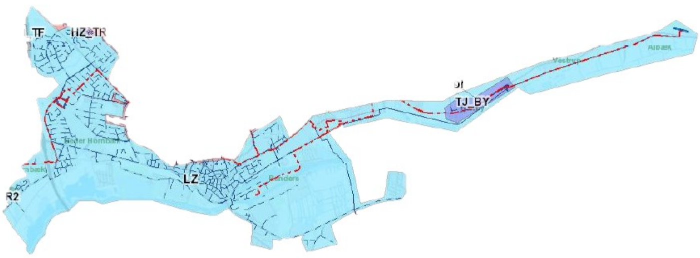


Figure 2.8: Low Zone in Randers.

The fourth main area in the Randers WSS is an area which according to its elevation neither belongs to the HZ, nor to the LZ. This area is called Hornbæk and shown in *Figure 2.9*.

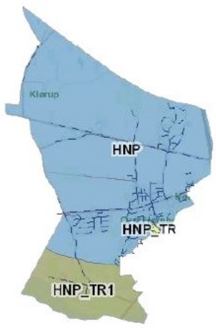


Figure 2.9: Hornbæk region in Randers.

The elevation in this area is around 30 meters above sea level and covers the western part of the city.

2.2.1 Waterworks and pumping stations

Verdo provides drinking water by pumping water from groundwater bases and treating the water in four different water works. Due to the high quality of ground water, this water treatment is only aeration in some cases. The WSS in Randers has four waterworks and four pumping stations, in different locations. In order to draw a better picture of the pumping and waterwork stations in the system, first they are listed and named and secondly their geographical locations and properties are described. The waterworks and pumping stations are the following

BKV	Bunkedal Waterwork
ØSV	Østrup Skov Waterwork
VSV	Vilstrup Waterwork
OST	Oust Mølle Waterwork

Table 2.1: Waterworks in the network.

HBP	Hobrovej Pumping Station
HSP	Hadsundvej Pumping Station
TBP	Toldbodgade Pumping Station
HNP	Hornbæk Pumping Station

Table 2.2: Pumping stations in the network.

In order to show the geographical location of the waterworks and pumping stations in the network, an illustration of the network model is shown where each pumping station and waterwork are labelled with its name. The network is shown in *Figure 2.10.*

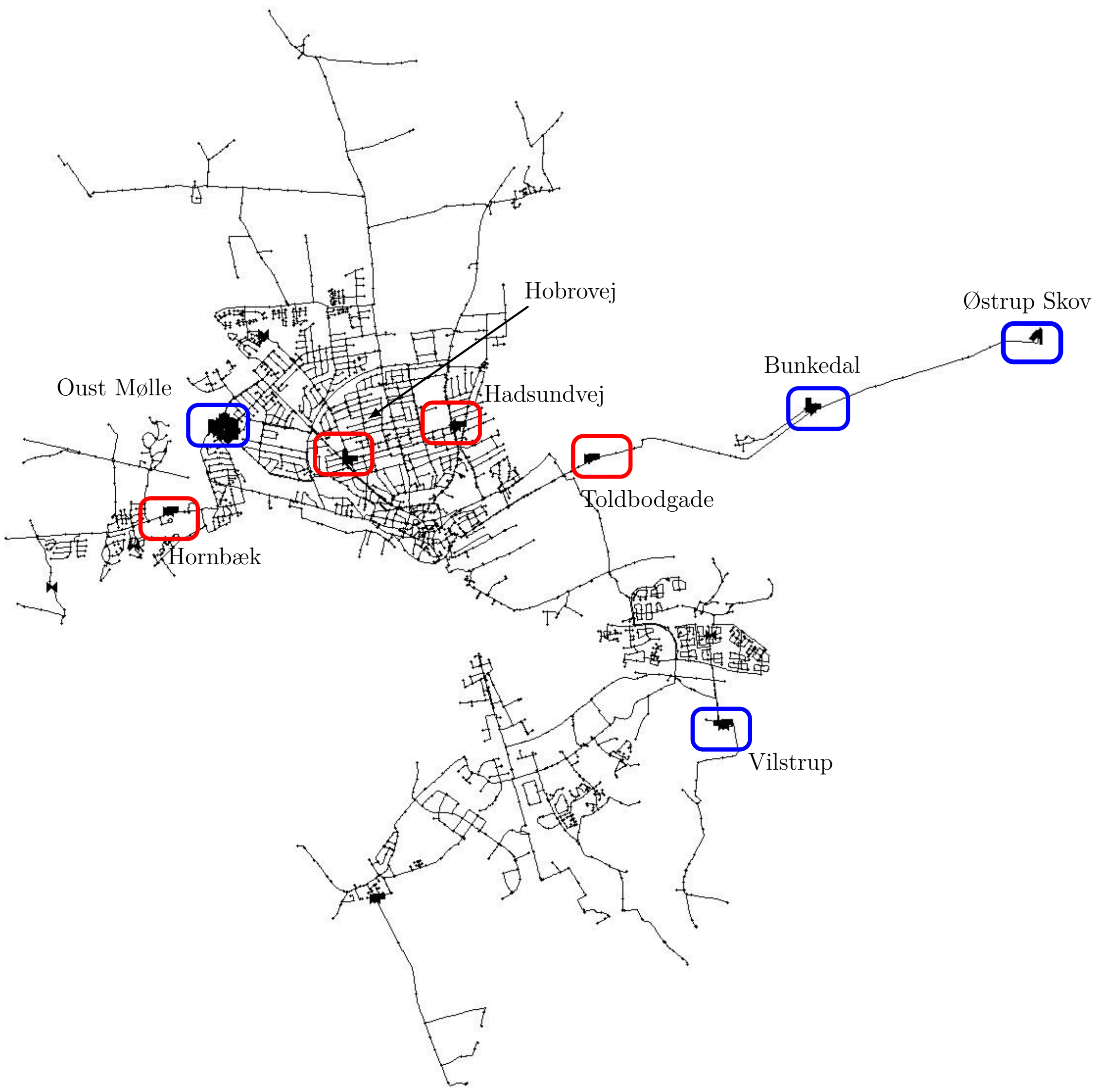


Figure 2.10: Waterworks(encircled in blue) and Pumping stations(encircled in red) in the Randers network.

The two main waterworks in the northern region are Bunkedal and Østrup Skov Waterworks. It is important to mention that in these areas, the water quality is sufficiently good, therefore clean water is pumped up to the surface. This is due to the fact that this groundwater lies under the ground such that it is protected by certain layers of the ground which makes it possible to provide this water without any kind of cleaning process, except aeration. These protection layers has been created by Randers fjord over the centuries due to glacial erosion (cite).

One of the drawbacks, however, is that the fresh water is located in the LZ, therefore the water has to be pumped from BKV and ØSV to locations with higher elevation. Since BKV and ØSV are the main sources to the HZ and LZ areas, in the worst case, water has to be pumped up approximately 55 meters above sea level. For this reason, the pumping station called Toldbodgade provides the sufficient amount of pressure to the HZ areas. In the HZ areas, at the border of LZ and HZ, there are two pumping stations, Hadsundvej and Hobrovej, which divides the water distribution in the HZ and LZ.

As can be seen in *Figure 2.10*, the network in the HZ is a grid structure. The two main pump stations, HSP and HBP provide the sufficient pressure and flow to the grid and to the LZ areas, such that they keep a balance in pressure and flow. Furthermore, there is a water tank placed both at HSP and HBP. Since the HZ has an elevation of approximately 55 meters, the static pressure in each water tank at the two pumping stations is sufficient to provide pressure in the LZ areas, without any pumping effort. Therefore HSP and HBP provide pressure to the LZ areas such that the geodesic properties are exploited. An illustration of the two pumping stations and the HZ grid is shown in *Figure 2.11*.

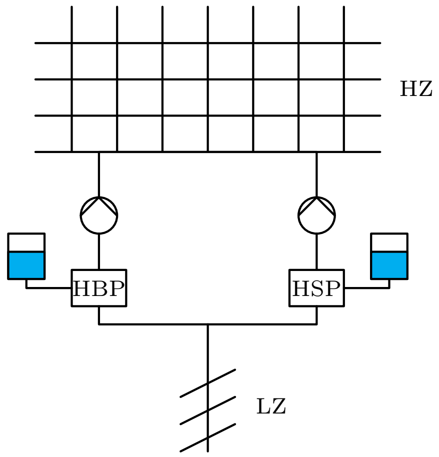


Figure 2.11: Hadsundvej and Hobrovej pumping stations with the HZ grid and LZ.

In order to avoid too high or too low pressure in the system, the pressure needs to be controlled alongside the flow. HBP is responsible for flow control, while HSP is responsible for the pressure control. Thereby it is avoided to provide the desired flow to the end-users with possibly with too high or too low pressure.

Furthermore, in Hornbæk zone, the elevation is above sea level, and the static pressure from the two main pumping stations can not provide the desired pressure to this region. Therefore, boosting is needed which is supplied by the Oust Mølle Waterwork and the Hornbæk Pumping Station. OST and HNP thereby are responsible for the Hornbæk zone.

As it is described in Section *2.2: The Randers water supply network*, the Vilstrup zone is an individual distribution network, if normal operation is assumed. Therefore, the Vilstrup Waterwork is able to provide all flow demands in Randers South, without the help of the other waterworks in Randers North. Due to this consideration, the WSS in Randers South can be discarded when the control of Randers North is analysed.

In this report, the control of the two main pumping stations, Hadsundvej and Hobrovej is taken into account. This means that the Vilstrup region is indeed ignored, and the end-users are considered in the HZ, LZ and in Hornbæk region. With these considerations, the network simplifies to the following shown in *Figure 2.12*

Figure 2.12: The simplified network map of the Randers WSS.

The modelling of this system is going to be used for control purposes, which means that the model needs to be simple, but at the same time needs to have the same characteristics as of the original network. In the WSS in Randdders, the control purpose is to find an optimal control scheme which is able to actuate the pumps at the two main pumping stations, such that the dynamic effect of the water tanks at each station are taken into account.

Tom's comments:
-some additional info on the purpose of the control
-what challenges should we face while controlling such system, with WTs included

3. System Modelling

This chapter gives a mathematical description of the component modelling. Thus the different physical and mathematical measures of hydraulic systems are introduced. The similarities to electronic network modelling are shown by explaining the relevant properties of graph theory. A reduced model for multi-inlet networks is first introduced, then the inclusion of tanks is discussed. In the end, the EPANET-based modelling approach is introduced and compared to the real-world system.

3.1 Hydraulic component modelling

In this section the mathematical relation between pressure and flow is given for each component in a WSS system, in order to show their non-linear behaviour. The purpose here is not to derive the different models, rather to introduce the mathematical formalism which describes them.

Equation: (3.1) shows the dual variables which describe all two-terminal components in the network

$$\begin{bmatrix} \Delta p \\ q \end{bmatrix} = \begin{bmatrix} p_{in} - p_{out} \\ q \end{bmatrix}, \tag{3.1}$$

where

Δp	is the differential pressure across the element,	[m]
q	is the flow through the element,	[l/s]
p_{in}, p_{out}	are the absolute pressures.	[m]

3.1.1 Hydraulic head

As can be seen in *Equation: (3.1)*, the measure of the pressure is in meters, and the measure of volumetric flow is in liters per second. It is important to note here, before the network modelling, that the units used for calculations are in SI. However, at some points in the report, non-SI units are considered due to the fact that EPANET uses head and liters per seconds as the measure of pressure and flow. Therefore the conception of hydraulic head is shortly introduced. The derivation and connection between pressure and head is explained in more detail in (appref). In the further report, the measure of pressure is always in meters.

In general, the hydraulic head, or total head, is a measure of the potential of fluid at a specific measurement point. It relates the energy of an incompressible fluid to the height of an equivalent static column of that fluid. The different forms of energies concerning fluids can be measured in distance, and therefore these terms are sometimes referred to as heads. The total hydraulic head of a fluid is composed of the pressure head and elevation head. [1]

The total head is given

$$h = \underbrace{\frac{p}{\rho g}}_{h_p} + z, \tag{3.2}$$

[1] There is a third term, called the kinetic head which is discarded, since the flow is assumed to be turbulent and uniformly distributed along the cross sectional area of all pipes [20].

where

h	is the total head,	[m]
h_p	is the pressure head,	[m]
p	is the absolute pressure in pressure units,	[Pa]
z	is the elevation,	[m]
g	is the gravitational constant,	[m/s^2]
ρ	is the density of the fluid.	[kg/m^3]

Therefore, pressure head is a measurement of length, which is dependant on the density of the fluid but can be converted to the units of pressure. Using meters for describing pressure in the system is convenient for the reason, that pressure can be treated the same way as the elevation. During the modelling, this property is therefore exploited.

3.1.2 Pipe model

Pipes in the network are governed by the dynamic equation

$$\Delta p_i = J_i \dot{q}_i + f_i(q_i) - \Delta h_i, \tag{3.3}$$

where

J_i	is the mass inertia of the water in the pipes,
$f_i(q_i)$	is the pressure drop due to friction,
Δh_i	is the pressure drop due to geodesic level difference across the two terminals of pipe elements.

The dynamics of the pipes are discarded in the project, as it is shown in other works that the small time constant of the pipe dynamics are not dominant in the system, especially if there are elevated reservoirs included [9, 15]. Therefore the pressure drop across pipes can be written as

$$\Delta p_i = f_i(q_i) - \Delta h_i, \tag{3.4}$$

The pressure drop due to friction across the i^{th} edge is a diagonal map where $f : \mathbb{R}^m \to \mathbb{R}^m$ is strictly increasing.[2] As it is shown in *Equation: (3.5)*, f_i describes a flow dependant pressure drop due to the hydraulic resistance

$$f_i(q_i) = \rho_i |q_i| q_i, \tag{3.5}$$

where

$\rho_i > 0$	is the resistance coefficient, the parameter of the pipes.	[$\cdot$]

Equation: (3.5) is motivated by turbulent flow in the pipes, which is typical in water supply applications. The resistance coefficient is calculated according to the Darcy-Weisbach formula, which provides the theoretically most precise result [8, 19]. ρ is given as shown in *Equation: (3.6)* below[3].

$$\rho = \frac{c f_D(\epsilon, D, R) l}{D^5}, \tag{3.6}$$

where

f_D	is the Darcy friction factor,	[$\cdot$]
ϵ	is the roughness of the pipe,	[m]
D	is the diameter of the pipe,	[m]
R	is the Reynolds-number, which defines the type of the flow,	[$\cdot$]
c	is a constant in SI units,	[s^2/m]
l	is the length of the pipe.	[m]

The derivation of *Equation: (3.6)* is explained in more detail in appref [20].

[2] A map $f : \mathbb{R}^m \to \mathbb{R}^m$ is strictly increasing if $\langle x - y, f(x) - f(y) \rangle > 0$ for every $x, y \in \mathbb{R}^m$ such that $x \neq y$ [18].

[3] EPANET uses the same calculation for the resistance terms.

Furthermore, in the following sections it is assumed that each f_i has a structure shown in *Equation: (3.5)*.

It is important to note here that $f_i(\cdot)$ is a homogeneous map which means that if the argument is multiplied by a scalar, then its value is multiplied by some power of this scalar [4]. For $f_i(q_i)$, it can be shown that

$$\rho_i|(\alpha q_i)|(\alpha q_i) = f_i(\alpha q_i) = \alpha^2 f_i(q_i). \tag{3.7}$$

More precisely, with the given structure of $f(\cdot)$ the scaling would be $|\alpha|\alpha$, however $\alpha \geq 0$ is already assumed in *Equation: (3.7)*. This property is noted here and used later in the system description, in Section *3.2.4: Multi-inlet reduced network model*, where the scaling is indeed such that $\alpha \in \mathbb{R}_+$.

3.1.3 Valve model

Valves in the network are governed by the following algebraic expression

$$\Delta p_i = \mu_i(q_i, k_v) = \frac{1}{k_v(OD)^2}|q_i|q_i, \tag{3.8}$$

where

k_v	is the valve conductivity function, taking in its argument the Opening Degree(OD) of the valve [15].

$\mu_i(q_i, k_v)$ is a continuously differentiable and proper function which for $q_i = 0$ is zero and monotonically increasing.

3.1.4 Pump model

Centrifugal pumps are governed by the following expression [16]

$$\Delta p = -a_{h2}{q_i}^2 + a_{h1}\omega_r q_i + a_{h0}{\omega_r}^2 \tag{3.9}$$

where

	Δp	is the differential pressure produced by the pump,	[m]
	a_{h2},a_{h1},a_{h0}	are constants describing the pump,	[$\cdot$]
and	ω_r	is the impeller rotational speed.	$\left[\frac{\text{rad}}{\text{s}}\right]$

3.1.5 Elevated reservoir model

In elevated reservoirs, the rate of change in the volume of the fluid inside the tank is equal to the volumetric flow at which water enters or leaves the tank. Since the pressure on the bottom is due to the cross sectional area of the tank and the amount of water in it, proportional relation can be set between the pressure and the flow in and out of the tank. The dynamics of such a system can be described by a first order differential equation of the form

$$\dot{p}_i = -\tau_i\left(\frac{p_i}{h_i}\right)q_i \tag{3.10}$$

where

p_i	is the pressure at the node connected to the tank,
h_i	is the water level in the tank,
τ_i	is a parameter dependant on the cross sectional area and the pressure - water level ratio in the tank,
q_i	is the flow in the tank if $q_i > 0$ and flow out of the tank if $q_i < 0$.

[4] $g(\alpha v) = \alpha^k g(v)$

As can be seen in *Equation: (3.11)*, in general, the parameter of the tank depends on the pressure and water level ratio, if the cross sectional are is not constant along the height of the tank. However, it is assumed that tanks have the same cross sectional areas in the entire height. Then *Equation: (3.11)* simplifies to

$$\dot{p}_i = -\tau_i q_i. \tag{3.11}$$

3.2 Graph-based network modelling

Graph-based network modelling has the advantage of making use of tools from circuit theory. Most of these tools are developed based on Graph Theory(GT). These methods can be used to model WSSs as directed graphs, where components of the systems, such as valves, pipes, tanks and pumps can correspond to edges and each terminal of the network correspond to nodes, or equivalently, to vertices.

In case of WSSs, in order to track the pressure and flow in the desired part of the network, the equation system of the network has to be solved for the desired edges and vertices. The whole network can be described by writing up the equations for all edges in the network, based on the mathematical modelling of the different components in the system, as shown in Section *3.1: Hydraulic component modelling*. However, in case of complex systems as water networks for large cities, these systems of equations are difficult to handle individually and typically cannot be solved explicitly if there are loops in the system. Therefore the properties of GT are not only useful for setting up relations between flow and pressure, but to make handling of a algebraic constraints easier by exploiting the properties of the matrix algebra. Thereby making it convenient for implementing it in computer algorithms for iterative solving methods.

WSSs can be described by a directed and connected graph, such that [21]:

$$\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}, \tag{3.12}$$

where

$\mathcal{G}$	is a directed and connected graph,
$\mathcal{V}$	is the set of vertices, where $\mathcal{V} = \{v_1, ..., v_n\}$,
$\mathcal{E}$	is the set of edges, where $\mathcal{E} = \{e_1, ..., e_m\}$.

3.2.1 Incidence matrix

The incidence matrix, H, of a connected graph, $\mathcal{G}$, is a matrix where the number of rows and columns correspond to the number of vertices and edges, respectively. Therefore $H \in \mathbb{R}^{n\times m}$. In case of hydraulic networks, edges are directed in order to keep track of the direction of the flow in the system.

$$H_{i,j} = \begin{cases} 1 & \text{if the } j^{th} \text{ edge is incident out of the } i^{th} \text{ vertex.} \\ -1 & \text{if the } j^{th} \text{ edge is incident into the } i^{th} \text{ vertex.} \\ 0 & \text{if the } j^{th} \text{ edge is not connected to the } i^{th} \text{ vertex.} \end{cases} \tag{3.13}$$

It is worth mentioning that the reduced incidence matrix can be obtained by removing any arbitrary row from H. Therefore H always have $(n-1)$ row rank. This statement can be explained by the mass conservation in the network, which is explained in the following section, Section *3.2.3: Kirchhoff's and Ohm's law for hydraulic networks.*

3.2.2 Cycle matrix

Purely tree structure of a WSS is not common when considering water distribution systems. However, trees can be arbitrarily chosen from the underlying graph of the system.[5] A tree, $\mathcal{T}$, of the graph is a connected sub-graph where any two vertices are connected by exactly one path [22]. Therefore a certain sub-graph which is a tree of the network can be represented as follows

$$\mathcal{T} = \{\mathcal{V}_{\mathcal{T}}, \mathcal{E}_{\mathcal{T}}\} \tag{3.14}$$

A special case of connected tree sub-graphs is the spanning tree of the network. A spanning tree contains all the vertices of $\mathcal{G}$ and has no cycles, since it is a tree. A spanning tree of the network therefore can be represented as

$$\mathcal{T} = \{\mathcal{V}, \mathcal{E}_{\mathcal{T}}\} \tag{3.15}$$

In order to obtain a spanning tree, an edge has to be removed from each cycle. The removed edges are $\mathcal{G} - \mathcal{T}$, and called the chords of $\mathcal{T}$ with respect to $\mathcal{G}$. By adding a chord to $\mathcal{T}$, a cycle is created which is called a fundamental cycle. A graph is conformed by as many fundamental cycles as the number of chords [22].

The set of fundamental cycles correspond to the fundamental cycle matrix, B, such that the number of rows and columns are defined by the number of chords and edges, respectively. The cycle matrix of the system is given by

$$B_{i,j} = \begin{cases} 1 & \text{if the } j^{th} \text{ edge belongs to the } i^{th} \text{ cycle and their directions agree} \\ -1 & \text{if the } j^{th} \text{ edge belongs to the } i^{th} \text{ cycle and their directions are opposite} \\ 0 & \text{if the } j^{th} \text{ edge does not belong to the } i^{th} \text{ cycle} \end{cases} \tag{3.16}$$

3.2.3 Kirchhoff's and Ohm's law for hydraulic networks

In this project, the hydraulic system is considered to be an open network with pipes, valves, pumps and the storage tanks, where water is able to enter and leave the network at a subset of the vertices. For such system, Kirchhoff's vertex law corresponds to conservation of mass in each vertex and described by

$$Hq = d, \tag{3.17}$$

where

$d \in \mathbb{R}^n$	is the vector of nodal demands, with $d_i > 0$ when demand flow is into vertex i and $d_i < 0$ when demand flow is out of vertex i.	$\left[\frac{1}{\text{s}}\right]$

Nodal demands can be seen as the end-user consumption, which means that water is taken out from the network. The mass conservation corresponds to the fact that what is consumed from the system must also be produced. Due to mass conservation, there can be only $(n-1)$ independent nodal demands in the network

$$d_n = -\sum_{i=1}^{n-1} d_i. \tag{3.18}$$

As a matter of fact, *Equation: (3.18)* is not an additional constraint since it follows from *Equation: (3.17)*. This can be sown by using the knowledge that 1_n is the left kernel[6] of H.

In the further report, a distinction is made between inlet and non-inlet nodes. It is assumed that the demand at non-inlet nodes fulfil the following constraint

[5] Recall that a tree with n vertices has $n-1$ edges [22].

[6] The kernel of matrix $A \in \mathbb{R}^{m\times n)}$ is $\{x \in \mathbb{R}^n | Ax = 0\}$.

$$d_i \geq 0. \tag{3.19}$$

It is worth noting however, that in closed hydraulic networks the vertex law becomes

$$Hq = 0. \tag{3.20}$$

Ohm's law for hydraulic networks can be expressed with the incidence matrix, when H^T is applied to the vector of absolute pressures, p. Important to point out that the description below in *Equation: (3.21)* is valid if edges of the underlying graph are considered as only pipe elements

$$\Delta p = H^T p = f(q) - H^T h. \tag{3.21}$$

In *Equation: (3.21)*, the differential pressure is described across each edge in the network, taking into account the pressure loss due to friction, $f(q)$, and the pressure drop due to geodesic level differences, where $h \in \mathbb{R}^n$ is the vector of geodesic levels at each vertex expressed in units of potential, i.e. pressure. It is noted that the pressure loss, $f(q)$ and the geodesic level h are both considered in the unit head. Therefore the measure is meter, and the units fit.

3.2.4 Multi-inlet reduced network model

The system is a water network supplied from more than one pumping stations and the distribution is to several end-users. In the underlying graph therefore the nodes are pipe connections, with possible water demand from the end-users, and the edges are only pipes.

The aim of the modelling here is to obtain a reduced order network model which is able to capture the dependence of the measured output pressures on the flows and pressures at the inlets. The inclusion of storage tanks is the next step of the model development, therefore it is described in a following section, in Section *3.2.5: Inclusion of elevated reservoirs.*

It is assumed that the inlet pressures and demands are measured. Furthermore, pressure measurement is available in certain parts of the remaining network, at the end-users. Considering generality, the model is described for c inlets, however it should be noted that regarding the Randers WSS, two inlet vertices are taken into account.

In order to put the system into a form which can handle the measured pressure dependencies on the control inputs, the underlying graph of the network is first partitioned. The n vertices of the graph are separated into two sets

$$\mathcal{V} = \{\bar{\mathcal{V}}, \hat{\mathcal{V}}\}, \tag{3.22}$$

where

	$\hat{\mathcal{V}} = \{\hat{v}_1, ..., \hat{v}_c\}$	represents the vertices corresponding to the inlet points,
and	$\bar{\mathcal{V}} = \{\bar{v}_1, ..., \bar{v}_{n-c}\}$	represents the remaining vertices in the graph.

The partitioning for the m edges of the graph is being chosen such that

$$\mathcal{E} = \{\mathcal{E}_{\mathcal{T}}, \mathcal{E}_{\mathcal{C}}\}, \tag{3.23}$$

where
$\mathcal{E}_\mathcal{T} = \{e_{\mathcal{T},1}, ..., e_{\mathcal{T},n-c}\}$
and
$\mathcal{E}_\mathcal{C} = \{e_{\mathcal{C},1}, ..., e_{\mathcal{C},m-n+c}\}$.

The subsets regarding edges and the partitioning is chosen such that the sub-matrix, which maps edges in $\mathcal{E}_\mathcal{T}$ to vertices in $\bar{\mathcal{V}}$, is invertible. It is worth mentioning that such partitioning is always possible for connected graphs.

Therefore the incidence matrix can be split into four sub-matrices, as shown in *Equation: (3.24)* below

$$H = \left[\begin{array}{c:c} \bar{H}_\mathcal{T} & \bar{H}_\mathcal{C} \\ \hdashline \hat{H}_\mathcal{T} & \hat{H}_\mathcal{C} \end{array}\right], \tag{3.24}$$

where
$\bar{H}_\mathcal{T} \in \mathbb{R}^{(n-c)\times(n-c)}$ is the sub-matrix, mapping edges in $\mathcal{E}_\mathcal{T}$ to vertices in $\bar{\mathcal{V}}$,
$\bar{H}_\mathcal{C} \in \mathbb{R}^{(n-c)\times(m-n+c)}$ is the sub-matrix, mapping edges in $\mathcal{E}_\mathcal{C}$ to vertices in $\bar{\mathcal{V}}$,
$\hat{H}_\mathcal{T} \in \mathbb{R}^{c\times(n-c)}$ is the sub-matrix, mapping edges in $\mathcal{E}_\mathcal{T}$ to vertices in $\hat{\mathcal{V}}$,
$\hat{H}_\mathcal{C} \in \mathbb{R}^{c\times(m-n+c)}$ is the sub-matrix, mapping edges in $\mathcal{E}_\mathcal{C}$ to vertices in $\hat{\mathcal{V}}$.

It is worth noting that the only requirement for the edge partitioning is $\bar{H}_\mathcal{T}$ being invertible[7]. Furthermore, the set $\mathcal{T} = \{\mathcal{V}, \mathcal{E}_\mathcal{T}\}$ is not necessarily a tree of the underlying graph, it can be any form of graph that fulfils the requirements. The set here, $\mathcal{T}$, is not connected due to the requirement of $\mathcal{E}_\mathcal{T} \geq (n-1)$. For the multi-inlet case, $c > 1$, therefore $\mathcal{E}_\mathcal{T} = (n-c)$. However, one special case is given when $c = 1$, meaning that the network has only one inlet. In this case, $\mathcal{T}$ is indeed a spanning tree.

With the chosen partition, Kirchhoff's vertex law in *Equation: (3.17)* can be rewritten as

$$\bar{d} = \bar{H}_\mathcal{T} q_\mathcal{T} + \bar{H}_\mathcal{C} q_\mathcal{C}, \tag{3.25}$$

$$\hat{d} = \hat{H}_\mathcal{T} q_\mathcal{T} + \hat{H}_\mathcal{C} q_\mathcal{C}, \tag{3.26}$$

and Ohm's law in *Equation: (3.21)*, separating the pressure drop due to hydraulic resistance

$$f_\mathcal{T}(q_\mathcal{T}) = \bar{H}_\mathcal{T}^T(\bar{p} + \bar{h}) + \hat{H}_\mathcal{T}^T(\hat{p} + \hat{h}), \tag{3.27}$$

$$f_\mathcal{C}(q_\mathcal{C}) = \bar{H}_\mathcal{C}^T(\bar{p} + \bar{h}) + \hat{H}_\mathcal{C}^T(\hat{p} + \hat{h}). \tag{3.28}$$

Writing up *Equation: (3.27)* and *Equation: (3.28)* in matrix form

$$\begin{bmatrix} f_\mathcal{T}(q_\mathcal{T}) \\ f_\mathcal{C}(q_\mathcal{C}) \end{bmatrix} = \underbrace{\begin{bmatrix} \bar{H}_\mathcal{T}^T & \hat{H}_\mathcal{T}^T \\ \bar{H}_\mathcal{C}^T & \hat{H}_\mathcal{C}^T \end{bmatrix}}_{\begin{bmatrix} \bar{H}^T & \hat{H}^T \end{bmatrix}} \begin{bmatrix} (\bar{p} + \bar{h}) \\ (\hat{p} + \hat{h}) \end{bmatrix} \tag{3.29}$$

As it is shown in *Equation: (3.29)*, the transposed incidence matrices can be written up as the two sub-matrices partitioned according to inlet and non-inlet nodes.

Now, defining a matrix Γ, in which the orientation of the edges are the same as for the incidence matrix, H. Γ is defined as follows

[7] $\exists\{\mathcal{V}, \mathcal{E}\} : \bar{H}_\mathcal{T}^{-1} \because rank(H) = (n-1)$ [22]

$$\Gamma = \begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \tag{3.30}$$

It should be noted that the expressions in matrix Γ are of the same structure as the structure of a partitioned cycle matrix. However, the set $\mathcal{T}$ does not define a spanning tree when $c > 1$, therefore matrix Γ is not a cycle matrix corresponding to any spanning tree. Multiplying H with Γ from the left-hand side

$$\Gamma H^T = \begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \begin{bmatrix} \bar{H}_{\mathcal{T}}^T & \hat{H}_{\mathcal{T}}^T \\ \bar{H}_{\mathcal{C}}^T & \hat{H}_{\mathcal{C}}^T \end{bmatrix} = \begin{bmatrix} 0 & -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T + \hat{H}_{\mathcal{C}}^T \end{bmatrix}. \tag{3.31}$$

Γ is defined such that $\Gamma \bar{H}^T = 0$ [22].

Multiplying with Γ from the left in *Equation: (3.29)*

$$\begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \begin{bmatrix} f_{\mathcal{T}}(q_{\mathcal{T}}) \\ f_{\mathcal{C}}(q_{\mathcal{C}}) \end{bmatrix} = \begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \begin{bmatrix} \bar{H}_{\mathcal{T}}^T & \hat{H}_{\mathcal{T}}^T \\ \bar{H}_{\mathcal{C}}^T & \hat{H}_{\mathcal{C}}^T \end{bmatrix} \begin{bmatrix} (\bar{p} + \bar{h}) \\ (\hat{p} + \hat{h}) \end{bmatrix} \tag{3.32}$$

induces the following expression

$$f_{\mathcal{C}}(q_{\mathcal{C}}) - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(q_{\mathcal{T}}) = (\hat{H}_{\mathcal{C}}^T - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T)(\hat{p} + \hat{h}). \tag{3.33}$$

From *Equation: (3.25)*, the vector $q_{\mathcal{T}}$, of flows in edges $\mathcal{E}_{\mathcal{T}}$ can be expressed

$$q_{\mathcal{T}} = -\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}} q_{\mathcal{C}} + \bar{H}_{\mathcal{T}}^{-1} \bar{d}. \tag{3.34}$$

Therefore using *Equation: (3.34)*, *Equation: (3.33)* can be rewritten

$$f_{\mathcal{C}}(q_{\mathcal{C}}) - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}} q_{\mathcal{C}} + \bar{H}_{\mathcal{T}}^{-1} \bar{d}) = (\hat{H}_{\mathcal{C}}^T - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T)(\hat{p} + \hat{h}). \tag{3.35}$$

Now expressing the vertex demands at non-inlet vertices, $\bar{d}$, such that

$$\bar{d} = -v\sigma \tag{3.36}$$

where

$\bar{d} \in \mathbb{R}^{n-c}$	is the vector of nodal demands in non-inlet vertices,
$\sigma \in \mathbb{R}_+$	is the total demand in the network, representing the total consumption of the end-users,
$v \in \mathbb{R}_{n-c}$	represents the distribution vector of nodal demands among the non-inlet vertices with the property $\sum_i v_i = 1$ and $v_i \in (0; 1)$.

Furthermore, introduce a vector, $a_{\mathcal{C}}$, such that

$$q_{\mathcal{C}} = a_{\mathcal{C}} \sigma. \tag{3.37}$$

Such an $a_{\mathcal{C}}$ can always be defined in this manner as long as $\sigma \neq 0$.

Having $\bar{d}$ and $q_{\mathcal{C}}$ introduced as the linear function of the total demand, σ, in the network, *Equation: (3.35)* can be expressed such that

$$\begin{aligned} &f_{\mathcal{C}}(q_{\mathcal{C}}) - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}} q_{\mathcal{C}} + \bar{H}_{\mathcal{T}}^{-1} \bar{d}) = \\ &f_{\mathcal{C}}(a_{\mathcal{C}}\sigma) - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}} a_{\mathcal{C}}\sigma - \bar{H}_{\mathcal{T}}^{-1} v\sigma) = \\ &f_{\mathcal{C}}(a_{\mathcal{C}})\sigma^2 - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}} a_{\mathcal{C}} - \bar{H}_{\mathcal{T}}^{-1} v)\sigma^2. \end{aligned} \tag{3.38}$$

where the latter equality is due to the homogeneity property of the pressure drops due to frictions, explained in Section *3.1.2: Pipe model.*

Defining a function $F_v : \mathbb{R}^{m-n+c} \to \mathbb{R}^{m-n+c}$, parametrized with v such that it takes $a_{\mathcal{C}}$ as input, the following expression can be formed

$$F_v(a_{\mathcal{C}}) = f_{\mathcal{C}}(a_{\mathcal{C}}) - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{C}}a_{\mathcal{C}} - \bar{H}_{\mathcal{T}}^{-1}v) \tag{3.39}$$

Furthermore, $F_v(\cdot)$ equals to the following, according to *Equation: (3.35)*

$$F_v(a_{\mathcal{C}}) = \frac{1}{\sigma^2}(\hat{H}_{\mathcal{C}}^T - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T}\hat{H}_{\mathcal{T}}^T)(\hat{p} + \hat{h}). \tag{3.40}$$

An algebraic expression for a_c can be found iff $\exists F_v^{-1}(\cdot)$. It can be shown, however that $\exists F_v^{-1}(\cdot)$ by showing that $F_v(\cdot)$ is a homeomorphism[8], which is done in [18].

As a result of using the inverse mapping of F_v, an expression can be obtained for $a_{\mathcal{C}}$

$$a_{\mathcal{C}} = F_v^{-1}A(\hat{p} + \hat{h}), \tag{3.41}$$

where
$A = \hat{H}_{\mathcal{C}}^T - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T}\hat{H}_{\mathcal{T}}^T \in \mathbb{R}^{(m-n+c \times c)}$.

A has a non-trivial kernel, and for every unique value of $1/\sigma^2 A(\hat{p}+\hat{h})$, there is a unique $a_{\mathcal{C}}$.

The main objective of writing up $a_{\mathcal{C}}$ is to show that it can be expressed in terms of $v, \sigma(t), \hat{p}(t)$ and $\hat{h}$, where $\hat{h}$ and $\sigma(t)$ are assumed to be known signals and parameters, $v(t)$ is an unknown parameter and $\hat{p}(t)$ is the control signal. The difficulty about the constraint on $a_{\mathcal{C}}$ in *Equation: (3.41)* is that its structure is unknown.

However, assuming that $(\hat{p} + \hat{h}) \neq 0 \in \; ker(A)$, then $a_{\mathcal{C}}$, *Equation: (3.40)* can be expressed such that

$$a_{\mathcal{C}} = F_v^{-1}(0). \tag{3.42}$$

Equation: (3.42) shows, that in the special case when the input vertices are chosen such that the product $A(\hat{p}+\hat{h}) = 0$, then $a_{\mathcal{C}}$ becomes only dependent on the parameter v.

Now, using the equations for Ohm's law in *Equation: (3.27)* and the vector $q_{\mathcal{T}}$, of flows in edges $\mathcal{E}_{\mathcal{T}}$ in *Equation: (3.34)*, the vector $\bar{p}$ of pressures at non-inlet vertices is expressed

$$\begin{aligned}\bar{p} =& \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{C}}q_{\mathcal{C}} + \bar{H}_{\mathcal{T}}^{-1}\bar{d}) - \bar{H}_{\mathcal{T}}^{-T}\hat{H}_{\mathcal{T}}^T(\hat{p} + \hat{h}) - \bar{h} \\ =& \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{C}}a_c + \bar{H}_{\mathcal{T}}^{-1}v)\sigma^2 - \bar{H}_{\mathcal{T}}^{-T}\hat{H}_{\mathcal{T}}^T(\hat{p} + \hat{h}) - \bar{h}\end{aligned} \tag{3.43}$$

As shown in *Equation: (3.43)*, the output vector which consists of the pressures in the non-inlet vertices can be written up in terms of $\sigma(t)$, $\hat{p}(t)$ time-varying signals, in terms of $\hat{h}$ and $\bar{h}$ constants and in terms of the parameter $a_{\mathcal{C}}$ and v. In the non-general case, as shown in *Equation: (3.42)*, a_c is a parameter which is governed by the behaviour of the total demand distribution among the non-inlet vertices. In case vector v is constant, thereby time-invariant, which means that the distribution of nodal demands are the same in all vertices in the network at all time, the output pressure in the i^{th} non-inlet vertices can be written as follows:

[8]Two functions are homeomorphic if they can be formed into each other by continuous, invertible mapping [23]. However, here invertibility is a sufficient condition.

$$\bar{p}_i(t) = \alpha_i \sigma^2(t) + \sum_j \beta_{ij} \hat{p}_j(t) + \gamma_i \tag{3.44}$$

where

$$\alpha_i = (\bar{H}_{\mathcal{T}}^{-T})_i f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}} a_{\mathcal{C}} + \bar{H}_{\mathcal{T}}^{-1} v)$$
$$\beta_{ij} = -(\bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^{T})_{ij}$$
$$\gamma_i = -(\bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^{T})_i \hat{h} - \bar{h}_i$$

However in WSSs, the above-mentioned consideration for v is unrealistic, meaning that the distribution of nodal demands in the non-inlet vertices should depend on time, as the end-user water consumption is not the same in every hour. This consumption behaviour of the end-users, however, is assumed to be periodic, which is a fair assumption, taking into account that the daily consumption shows approximately the same trends every day.

Therefore the demand in non-inlet vertices, described in *Equation: (3.36)* can be rewritten such that

$$\bar{d}(t) = -v(t)\sigma(t) \tag{3.45}$$

where
$v(t+T) = v(t)$,
$\sigma(t+T) = \sigma(t)$,
and T is the length of the period.

If the non-inlet demands are time-varying, but periodic behaviour is assumed and on top of this, the input vertices are arranged such that *Equation: (3.42)* is fulfilled, *Equation: (3.46)* can be rewritten as follows

$$\bar{p}_i(t) = \alpha_i(t) \sigma^2(t) + \sum_j \beta_{ij} \hat{p}_j(t) + \gamma_i, \tag{3.46}$$

where α_i is also a time-varying parameter of the model.

3.2.5 Inclusion of elevated reservoirs

As it is described in *Equation: (3.19)*, a distinction is made between non-inlet and inlet vertices, by assuming that non-inlet vertices have only positive or zero nodal demand. However, when the inclusion of a tank is considered, a special type of node has to be introduced. A node which can have a demand in both positive and negative directions, meaning that the demand is positive when the tank is being filled and negative when it is being emptied. For this reason, the input demands, pressures and elevations are separated such that

$$\begin{aligned} \hat{d} &= F\hat{d}_t + G\hat{d}_c \\ \hat{h} &= F\hat{h}_t + G\hat{h}_c \\ \hat{p} &= F\hat{p}_t + G\hat{p}_c \end{aligned} \tag{3.47}$$

where

- $\hat{d}_t \in \mathbb{R}^{(l\times 1)}$ is the vector including the nodal demands of the tanks,
- $\hat{d}_c \in \mathbb{R}^{(c-l\times 1)}$ is the the vector including the nodal demands of the pump inputs,
- $\hat{h}_t \in \mathbb{R}^{(l\times 1)}$ is the vector including the elevation of the tanks,
- $\hat{h}_c \in \mathbb{R}^{(c-l\times 1)}$ is the the vector including the elevation of the pump stations,
- $\hat{p}_t \in \mathbb{R}^{(l\times 1)}$ is the vector including the absolute pressures in the tanks,
- $\hat{p}_c \in \mathbb{R}^{(c-l\times 1)}$ is the the vector including the absolute pressures of the pump inputs,
- $F \in \mathbb{R}^{(c\times l)}$ is a mapping which selects the nodes belonging to tanks,
- $G \in \mathbb{R}^{(c\times c-l)}$ is a mapping which selects the nodes belonging to pump inputs.

The inclusion of tanks means that the static model description in *Equation: (3.46)* is not sufficient any more. The model without the tanks is a static description because it does not contain any dynamics, since the dynamics of the pipes were discarded in *Equation: (3.3)*. However, as the tanks are included in the system, besides the pumps, the tanks have pressure contribution as inputs. Therefore when the system is described with tanks, the system dynamics are according to the pressure and demand in the tank and are constrained by the algebraic equation in *Equation: (3.46)*. The block diagram of such system is shown in *Figure 3.1*.

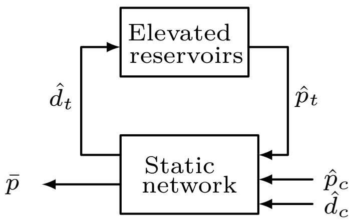


Figure 3.1: Block diagram of the system with WTs.

where the 'Elevated reservoirs' block represents a subsystem with dynamics, as the reservoirs act as integrators on the demand flow into or out of the tank.

In case of elevated reservoirs, usually the in- and outflow,$\hat{d}_t$, is controlled due to the reason that a pressure error can be significantly high because of the big time constant of such buffers in the network. The aim therefore is to set up a model which is capable of handling the dynamics of the tanks, constrained by the static network. Such dynamic system is expected to be derived in a form shown in eqref.

$$\bar{p} = f(\sigma, \hat{p}, \bar{p}) \tag{3.48}$$

$$\dot{\hat{p}} = g(\sigma, \hat{d}_t, \bar{p}) \tag{3.49}$$

$$\hat{d}_t = h(\bar{p}, \bar{d}, \hat{p}, \hat{d}) \tag{3.50}$$

3.3 Multi-inlet network example

In order to simulate and thereby verify the network model for the static system, it is first tested on a simple, two-source, two-loop pipe network. In this case the simulations can be easily compared to simulations in EPANET, as all non-inlet pressures and flows represent steady-state values in the network. Therefore the simulation results of the model is compared to the calculations made by EPANET.

The underlying graph of such network is shown in *Figure 3.2* below

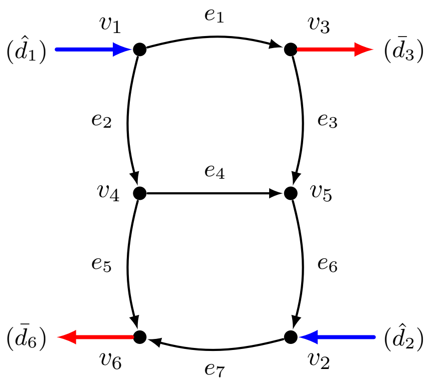


Figure 3.2: Graph of a simple multi-inlet network.

In *Figure 3.2*, the arrows are present only to illustrate the external in-and outflows due to the input flows by the pumping stations at v_1 and v_2, and the the user-consumption in v_3 and v_6.

In order to have a $\bar{H}_{\mathcal{T}}$ which is invertible, the orientation of the edges is chosen as shown in *Equation: (3.51)*.

$$\mathcal{E} = \{e_{\mathcal{T},1}, e_{\mathcal{T},5}, e_{\mathcal{T},3}, e_{\mathcal{T},4}, e_{\mathcal{C},2}, e_{\mathcal{C},6}, e_{\mathcal{C},7}\}. \tag{3.51}$$

The describing signals and parameters are the following

$$p(t) = \begin{bmatrix} \bar{p}_3(t) \\ \bar{p}_4(t) \\ \bar{p}_5(t) \\ \bar{p}_6(t) \\ \hat{p}_1(t) \\ \hat{p}_2(t) \end{bmatrix}, \quad d(t) = \begin{bmatrix} \bar{d}_3(t) \\ 0 \\ 0 \\ \bar{d}_6(t) \\ \hat{d}_1(t) \\ \hat{d}_2(t) \end{bmatrix}, \quad h = \begin{bmatrix} \bar{h}_3 \\ \bar{h}_4 \\ \bar{h}_5 \\ \bar{h}_6 \\ 0 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} v_3 \\ 0 \\ 0 \\ v_6 \end{bmatrix}. \tag{3.52}$$

There are two demand nodes in the network, therefore the total flow consumption can be written as

$$\sigma(t) = \bar{d}_3(t) + \bar{d}_6(t). \tag{3.53}$$

The specifications of this example network such as the elevation in different vertices, the distribution vector and the demands are listed in (appref).

In order to determine the output pressures, i.e. the pressures in the non-inlet vertices, first recall *Equation: (3.35)*,

$$g(q_{\mathcal{C}}) = f_{\mathcal{C}}(q_{\mathcal{C}}) - Bf_{\mathcal{T}}(Cq_{\mathcal{C}} + D\bar{d}) - A(\hat{p} + \hat{h}) = 0. \tag{3.54}$$

where

$$B = \bar{H}_{\mathcal{C}}^{T} \bar{H}_{\mathcal{T}}^{-T},$$
$$C = -\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}},$$
$$D = \bar{H}_{\mathcal{T}}^{-1}.$$

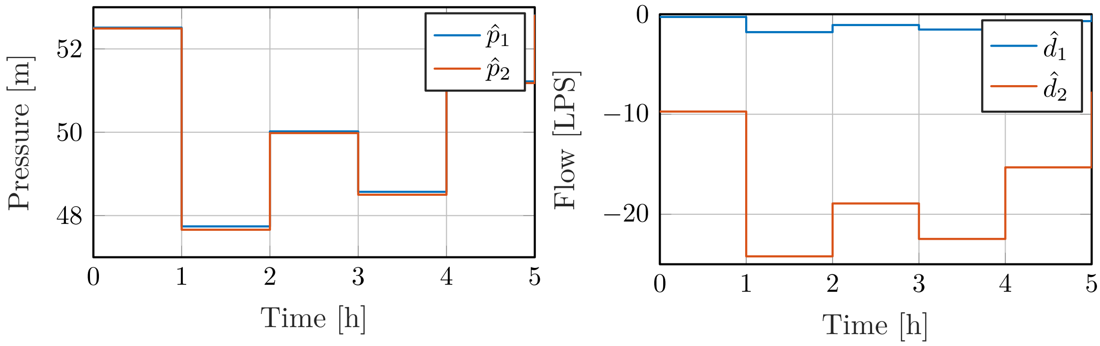
As there is not any analytical solution for $q_{\mathcal{C}}$ in *Equation: (3.54)*, it needs to be solved numerically. As $F(q_{\mathcal{C}})$ is differentiable with respect to $q_{\mathcal{C}}$, root finding algorithms such as Newton's method can be used. Furthermore, $g(q_{\mathcal{C}})$ is an increasing function, therefore the root of this function is the global minimum. By solving *Equation: (3.54)*, the unique value for $a_{\mathcal{C}}$ can be obtained. With this $a_{\mathcal{C}}$ vector, the non-inlet pressures and all flows in the network can be calculated in terms of the input pressures and the total demand in the network. *Equation: (3.54)* is solved in each time step in an extended-simulation in order to calculate the non-inlet pressures in different time periods.

In the first case, the distribution parameter among the non-inlet demands, v, is considered to be constant and the demand nodes are scaled up or down. The demands therefore has a base demand which is modified according to the scaling pattern in the simulation, similarly as in EPANET.

COMMENT1: *The following simulation results, i think, are qualitatively correct, but there are some matters about it. Which is probably a topic for the supervisor meeting.*

COMMENT2: *There isn't any constraint on the input flows.*

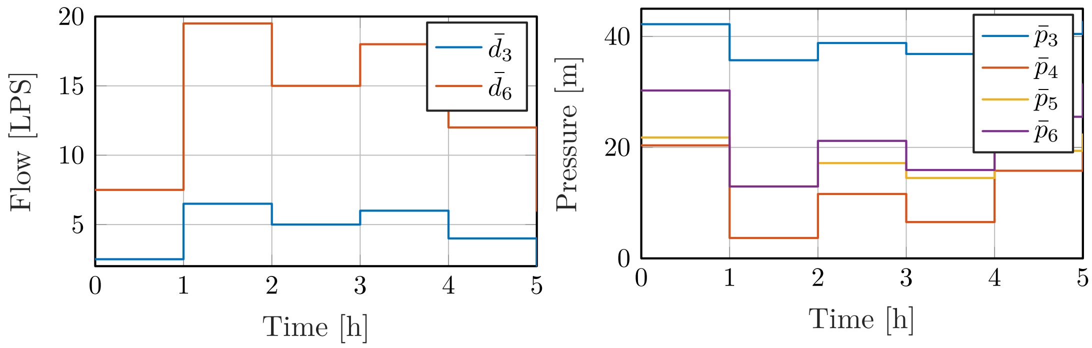
In this simulation, the pressure inputs for the two pumping stations are set for the same value.



(a) Inlet pressures, ($\hat{p}$)

(b) Inlet demands, ($\hat{d}$)

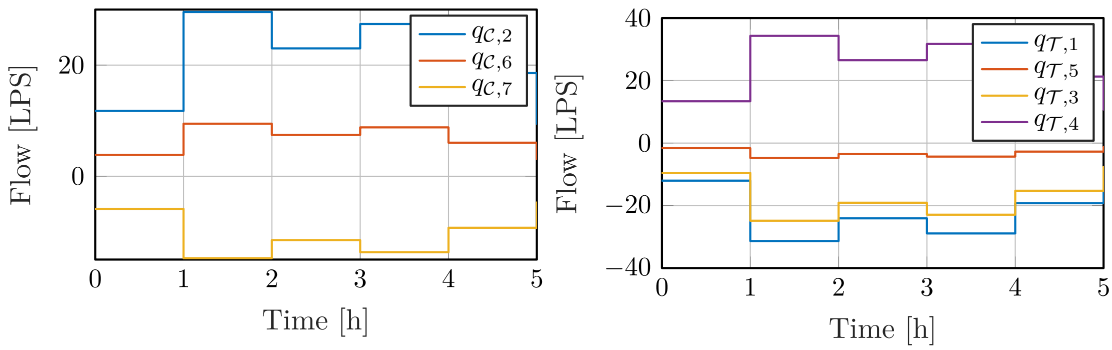
Figure 3.3: Signals describing the input pressures(left) and flows(right) of the pumping stations.



(a) Non-inlet demands, ($\bar{d}$)

(b) Non-inlet pressures, ($\bar{p}$)

Figure 3.4: Signals describing the demand flows by the end-users(left) and output pressures(right) in the network.



(a) Flows in $\mathcal{C}$, ($q_{\mathcal{C}}$)

(b) Flows in $\mathcal{T}$, ($q_{\mathcal{T}}$)

Figure 3.5: Signals describing the flows in all pipes in the network.

3.3.1 Simulation in EPANET

In EPANET, the simulation is built up in the same way as the network model. The simulation model is shown in the figure below.

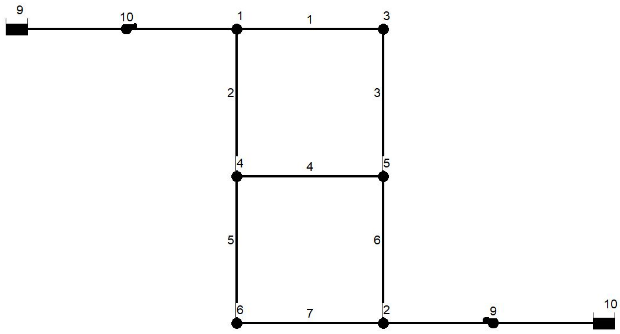


Figure 3.6: Non-inlet pressures in .

As can be seen in *Figure 3.6*, reservoirs and pumps are present as extra links and nodes in this simulation. These extra nodes and links are removed when data is extracted from the EPANET model due to the reason that the input pressures can be measured on v_1 and v_2. The input flows can be measured through the links, connecting the reservoirs to the nodes, v_1 and v_2. Furthermore, the elevations and the demands are attributes of the nodes. In this simulation only demands are changing according to hourly time steps.

3.4 Multi-inlet network example extended with a tank

4. Network simplification

4.1 Purpose of the model reduction

As it is described in Section **??**: **??**, planning pump schedules and tank level changes in EPANET can be carried out by solving iteratively for long time steps. However, when a complex network is being used for developing different control methods, typically the aim is to find an optimisation method which determines the optimal input to the system. In order to run such an optimizing algorithm on a complex system, the network needs to be solved numerically and several online execution runs are required.

In case of complex networks with a tree structure, there is only one way from a source node to any node in the system. Therefore the network can be described by solving the equation system of the model explicitly, therefore all pressures can be directly calculated in terms of the flows. In case of loop systems however, the procedure is more dificult, since there is more path to each nodes in the network. The system of equations have to be solved numerically which can dramatically increase the simulation time of an optimization algorithm. The objective of the system reduction and simplification is threfore to reduce the number of network components and especially the loops in the network, such that the accuracy of the simplified model is as close as possible to that of the original network model.

4.2 State of the art model reduction analysis

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4.2.1 Simplification of network model components

Simplification directly, by keeping the basic components but combining and replacing the individual components. (For example, series, parallel pipes. Tree substructure in the system.)

4.2.2 Black box simplification

Replacing the system partially or fully with a system which provides the same function with less complexity.

- static simplification with linearization and simplification
- neural network approach

Part II

System Identification

5. Multi-inlet reduced network model - example

Part III

Conclusion and verification

6. Accepttest

7. Discussion

8. Conclusion

Part IV
Appendices

A. Elevation Profile from HZ to LZ

Figure A.1: Elevation profile along the High and Low Zones 1.

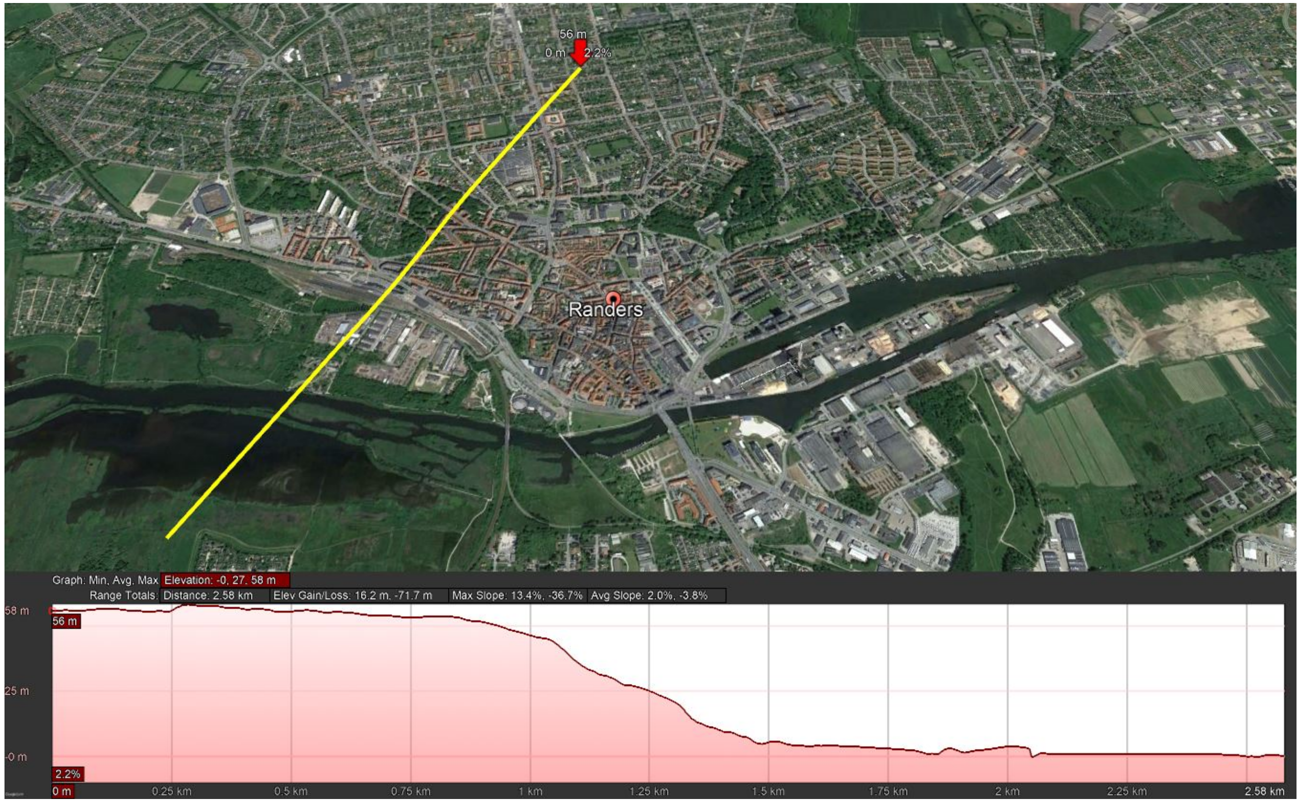

Figure A.2: Elevation profile along the High and Low Zones 2.

B. Assumption List

Number	**Assumptions**	**Section reference**
1	Assumption 1.	Section **??**: **??**
2	Assumption 2.	Section **??**: **??**

Table B.1: List of assumptions

C. Example Network

C.1 unspecified1

C.2 System Topology

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C.3 Incidence Matrix

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C.4 Cycle Matrix

D. Example Network with elevated reservoir

D.1 unspecified1

E. Measurements

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Rettelser

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