Aalborg University

Identification of Randers Water Distribution Network for Optimal Control

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Student report

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Preface

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Nomenclature

Acronyms

EPA	Environmental Protection Agency
FCV	Flow Control Valve
GT	Graph Theory
MPC	Model Predictive Control
PRV	Pressure Regulating Valve
OD	Opening Degree
WSS	Water Supply System

Symbols

Symbol	**Description**	**Unit**
a	Description	$[\cdot]$
b	Description	$[\cdot]$

Graph theory

Symbol	**Description**	**Unit**
a	Description	$[\cdot]$
b	Description	$[\cdot]$

Glossary of mathematical notation

Description of the mathematical notation and terminology used in the report.

Contents

Nomenclature **v**

1 Introduction **1**

I System Analysis **3**

2 Description of Water Supply Systems **5**
2.1 Hydraulic system overview 5
2.1.1 Pipe networks 6
2.1.2 Elevated reservoirs 7
2.1.3 Pumps 7
2.1.4 Valves 8
2.1.5 Hydraulic head 8
2.2 The Randers water supply network (in progress) 8

3 System Modelling **11**
3.1 Hydraulic component modelling 11
3.1.1 Pipe model 11
3.1.2 Valve model 12
3.1.3 Pump model 12
3.1.4 Elevated reservoir model 12
3.2 Graph-based network modelling 12
3.2.1 Incidence matrix 13
3.2.2 Cycle matrix 13
3.2.3 Kirchhoff's and Ohm's law for hydraulic networks 14
3.2.4 Multi-inlet reduced network model 14
3.2.5 Inclusion of elevated reservoirs 17
3.3 EPANET modelling 17

4 Network simplification **19**
4.1 Purpose of the model reduction 19
4.2 State of the art model reduction analysis 19
4.2.1 Simplification of network model components 19
4.2.2 Black box simplification 19

II System Identification **21**

5 Unspecified **23**

III Conclusion and verification **25**

6 Accepttest **27**

7 Discussion **29**

8 Conclusion **31**

IV Appendices **33**

A Assumption List **35**

B System Description **37**
B.1 Components of the System . . . 37
B.2 System Topology . . . 38
B.3 Incidence Matrix . . . 38
B.4 Cycle Matrix . . . 38

C Measurements **39**

Bibliography **41**

1. Introduction

Due to the fast-paced technological development all over the world, the demand for industrial growth and energy resources has seen a rapid increase. Along with the industrial growth, the sudden rise in population has made the world realize that this shortage of energy sources is an actual and universally anticipated problem [1]. In order to cope with such shortage issues and to make the rapid development possible and less expensive, the world is moving towards more efficient use of resources and optimization of infrastructure. Therefore, technological development is also moving the focus on green energy, resulting in more and more renewable energy sources added to the grid [2].

Water Supply Systems(WSSs) are among the sectors which make the industrial growth possible. On top of this, WSSs are one of the most vital infrastructures of modern societies in the world. In Denmark typically, such networks are operating by making pumps transport water from reservoirs through the pipe network, to the end-users. In most cases, elevated reservoirs are exploited in these WSSs, such that they can even out the demand differences for the consumers. Although elevated reservoirs are usually an integrated part of these systems, providing drinking water is a highly energy-intensive activity. For instance, in the United States alone, the drinking water and waste water systems are typically the largest energy consumers, accounting for 25 to 40 percent of a municipality's total public expenditure. [3].

Since fresh water is limited, and due to the presence of global changes such as climate change and urbanization, new trends are emerging in the water supply sector. In the past few decades, several research and case study showed that WSSs and other energy distribution networks need to be improved due to the leakages in the system, high cost of maintenance and due to high energy consumption. Companies also realized that by using proper pressure management in their networks, the effect of leakages can be reduced, thereby huge amount of fresh water can be saved [4].

In Denmark recently, the larger water suppliers have been focusing on making the water supply sector more effective through introducing a benchmarking system focusing on the environment, the security of supply and the efficiency based on user demands. Since 1980, these efficiency activities has been an important issue [5]. It has been proved that by utilizing advanced, energy- or cost-optimizing control schemes and utilizing renewable energy sources, such as elevated reservoirs, the life of the existing infrastructure can be extended and money or energy can be saved [1]. Therefore there is a growing demand in industry for developing methods, leading towards more efficient WSSs.

The presented project is executed in collaboration with the company, Verdo A/S. It is in the interest of Verdo A/S to utilize an advanced model-based optimal control scheme on the WSS with several storages in Randers, Denmark. For a large municipality such as Randers, the water distribution network is complex and consists of thousands of elements. Since the control algorithm itself is complex and model-based, the computational effort is also high. Furthermore, the offline optimisation of a large-scale WSS means that any changes to the network may require significant changes in the optimisation method, which leads to high costs of the system maintenance [6]. Therefore typically a model reduction is required in such networks to make the online execution of the control algorithm possible.

The long-term goal of this project is to find a solution for implementing Model Predictive Control(MPC) on the Randers WSS. However, before the implementation of any control scheme would be possible, a proper and identified model is required. Therefore, as the first part of the project, the following problem statement can be formulated:

How can the WSS in Randers be simplified and identified, with storages included in the system, such that the reduced model preserves the original nonlinear behaviour and remains suitable for a plug-and-play commissionable Model Predictive Control scheme.

Part I

System Analysis

2. Description of Water Supply Systems

This chapter gives a general overview of hydraulic systems and an introduction to the WSS in Randers. The basic topology and structures of water supply networks are explained. Furthermore, the basic components of hydraulic systems are discussed and the unit called head, as an alternative measure of pressure, is introduced.

2.1 Hydraulic system overview

WSSs are designed to deliver water to consumers in terms of sufficient pressure and appropriate chemical composition. Distribution systems as such are typically transport water from one geographical place to another. In practice, there are different methods exist to achieve this water transport. One example is the use of natural advantages such as the water stored in mountains, and thereby use the potential energy of the water to provide pressure in the network. Examples for this are countries like Norway where the advantages of the landscape are being exploited [7]. However, in this project the source of the water is considered as groundwater, considering that in Denmark all reservoirs in the network are tapping water from the ground. It worth noting that the quality of groundwater in Denmark is sufficiently good to use it for drinking water supply purposes. After tapping the water, it goes through an aeration process at the waterworks and afterwards the pure water is pumped into the network [8]. In WSSs, pumps and valves are the elements that enable the control and thereby the proper delivery of water to the consumers or to elevated reservoirs, storing water for later use. Such a network is illustrated in the figure below:

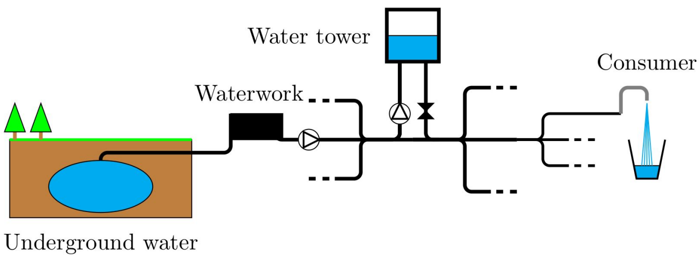


Figure 2.1: Illustration of a WSS [9].

The delivered water needs to fulfil a certain pressure criteria in order to reach consumers at higher levels. For example, in some cases the pressure has to be high enough to make it to the fourth floor of a building and still provide appropriate pressure in the water taps. Generally, in such cases booster pumps are placed in the basement of buildings, helping to supply the pressure. Too large pressure values, however increase water losses due to pipe waste [10].

Another criteria is that the flow through particular pipes need to stay within acceptable limits. A low flow rate can lead to water quality problems due to the undesirable microorganisms in the water and due to the metal and salt accumulation on the wall of the pipes [10].

As can be seen in *Figure 2.1*, typically WSSs consist of pipe, valve, reservoir, elevated reservoir(tank) and pump components. The common property of them is that they are all two-terminal components, therefore they can be characterized by the dynamic relationship between the pressure drop across their two corresponding endpoints and the flow through them [11].

2.1.1 Pipe networks

Pipes have a major role in WSSs since they are used for carrying pressurized water. They serve as a connection between components. Normally, the pipe network can be split into different sub-parts, taking into account the physical characteristics and the attributes of the pipes. Therefore, water supply networks can consist of transmission mains, arterial mains, distribution mains and service lines as shown in the example below:

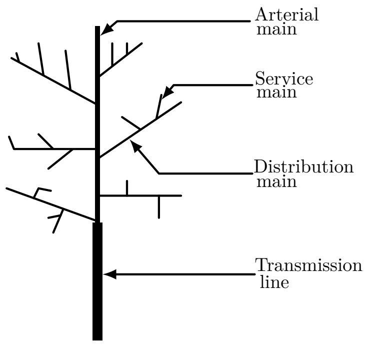


Figure 2.2: Illustration of pipe mains. Tree configuration.

Transmission mains deliver large amounts of water over long distances. Arterial and distribution mains provide intermediate steps towards delivering water to the end-users. Service lines transmit the water from the distribution mains straight to the end-users [12].

The transmission and distribution network can have a topology that is called a loop or a tree structure. *Figure 2.2* shows an example for a tree configuration. This type of configuration is most frequently used in rural areas [13]. Typically the network has only one path for the water to reach the end-users. A more frequent problem compared to looped configurations is, that on the outer parts of the system lower pressures can be experienced due to the pressure losses from long flow paths. The flow dynamics within this kind of systems therefore consist of large flows closer to the source that turn into smaller flows on the outer parts of the system. Main disadvantage of a purely tree structure system is that due to maintenance or momentary breakdowns, the system suffers disruption of service [13].

Loop networks have a configuration as shown in *Figure 2.3*.

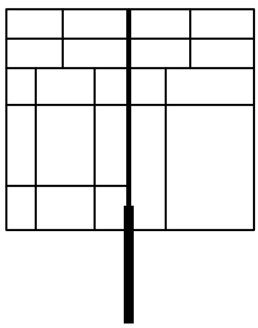


Figure 2.3: Loop configuration.

Loop networks are usually composed of smaller loops which are composed of smaller distribution mains, and larger loops that are connected to arterial or transmission mains. Elevated reservoirs are typically placed in the centre of the system due to pressure losses resulting from flows through the loop network [14]. This is reasonable because within a certain grid, the same pressure is provided by the tank, instead of providing the pressure through long pipelines to different distances. Furthermore, in the presence of a ring structure, the large loop around the area may be used to feed an internal distribution grid or a distribution grid attached to the outer part of the loop. Loop configurations are generally associated with larger suburban and city distribution systems such as larger cities[14]. The Randers WSS falls into this category.

2.1.2 Elevated reservoirs

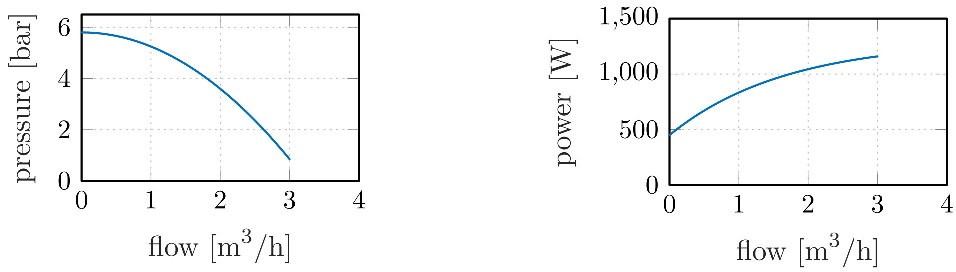
Elevated reservoirs, or tanks, are typically placed in the system to use them as buffers and level out the pressure and flow demand differences. When the demand is high, the waterworks might not be able to provide the sufficient amount of water in the network. In these cases, the elevated reservoir supplies the remaining demand. When the user consumption decreases, the system can be controlled such that the tank is being refilled to provide the required demand for the next peak time of consumption. Having such an elevated reservoir in the network, the system becomes more independent of the pump stations, as the refilled tank can itself maintain the desired pressure and flow for a limited time.

Due to the elevation of the tank, when it is filled up, the pumping stations need to provide a pressure higher than the pressure in the water tank. Therefore when the tank is being emptied, the pumping stations can reduce the amount of pressure they provide to the system, since the pressure from the elevated reservoir becomes dominant. This is due to the fact that the dynamics of systems with large storages come primarily from the pressure of the tank [15]. However, it should be noted that normally the level in the tank is varying less than a meter. This means that the effect on the pump operation is limited. Due to these considerations, the dynamics of these elevated reservoirs has to be taken into account while modelling the system.

2.1.3 Pumps

Water pumps are used to increase pressure in hydraulic systems, thus making the water flow. Pumps are typically the main actuators of a WSS and they can be either flow or pressure controlled. Therefore, pumps can have controllers to produce a desired flow or pressure. This is done by changing the rotational speed of the pump. In this way, when the pump has a reference pressure or flow, simple control makes it possible to produce the desired flow or pressure respectively [16].The pressure required to make the water reach some height is the sum of the pressure required to overcome the elevation and the friction losses in the pipe network.

The most common pumps in WSSs are centrifugal pumps. Normally, the characteristics of such pumps are described by two pump curves. The two curves depict the volume flow versus the pressure and the power of the pump respectively. Normally the curves describe the characteristics for one particular speed, which is denoted the nominal speed [16]. An example of these pump curves is shown in *Figure 2.4*.



(a) Flow versus pressure difference

(b) Flow versus power consumption

Figure 2.4: Pump curves describing the performance of a centrifugal pump at nominal speed.

As can be seen, at a given flow, the pump can deliver a pressure with a maximum limit. This pressure decreases when the flow is increasing. At a certain flow and pressure value, the pump has an optimal point where the operation is the most energy efficient. Pumps are normally designed such that the optimal point lies in the operational area for the pumping application [9].

As in almost all WSSs, the flow is varying in the system, according to the flow demand from the end-consumers. Therefore, when dealing with varying flow in the system, pumps are often placed parallel at the pump stations such that they can keep their optimum points. As the flow increases, more pumps get activated to keep the pressure constant [9].

2.1.4 Valves

Valves in the WSS can be also seen as actuators along with pump elements. Unlike the pumps, valves are passive actuators in the sense that they do not consume energy. In principle, there are many types of valves existing. They can be categorized as non-return valves, control valves, shut-off valves and the combination of the two former one. Non-return valves allow waterflow only in one direction, while control valves can either adjust the flow or the pressure on their two endpoints. The former category is typically called a Flow Control Valve(FCV), while the latter is called a Pressure Reducer Valve or Pressure Regulating Valve(PRV). Shut-off valves are important components of the network since they can change the structure of the system, when for example doing maintenance or just redirecting the flow. This project deals with all three types of valves.

Valves can be controlled such that no flow passes through. In these cases the valve is closed and thereby certain parts of the system can be isolated as mentioned above. Other possibility is that the valve is fully open. In such case the pressure drop between the two endpoints is experienced because of the friction loss of the valve.

2.1.5 Hydraulic head

Since EPANET uses head as the measure of pressure, probably it is more convenient to use head in the report

2.2 The Randers water supply network (in progress)

The Randers drinking WSS is managed by Verdo A/S and is the main supplier of drinking water and heating to the city of Randers. Verdo supplies water to approximately 46.000 customers in Randers Municipality [17]. The simulation model of the network is illustrated in *Figure 2.5*. As can be seen, the WSS in Randers is a complex, looped configuration with many different distribution areas. In overall, the system consists of around 4500 pipe elements, six tanks and more than 20 valves. The water in the network is controlled by eight pumping stations, located at different places throughout the network, thereby making it possible to deliver the desired pressure and flow to the end-users. At these pumping stations, pumps are often placed in parallel connections, due to the reason mentioned in Section *2.1.3: Pumps*.

Verdo provides the drinking water by drilling from 21 different groundwater bases and has four waterworks for water treatment [17]. The geographical position of the reservoirs at these waterworks is shown in *Figure 2.5*, encircled in red.

Missing parts:

- Which pump stations are to be controlled
- Are PRVs and FCVs really placed at the pumping stations in reality, or just for simulation purposes
- Which tanks
- Why tanks are installed at the place where they are

- which distribution areas are (mostly) dependant on which pumping stations - this would be useful to know in order to simplify the network and discard some areas in the network

Figure 2.5: Reservoirs in the Randers WSS (encircled in red).

As can be seen, reservoirs are placed in the outer parts of the city, typically next to the main pumping stations. The locations of the six tanks, or tank stations, in the system is are shown in (figref below)

Figure 2.6: Elevated reservoirs in the Randers WSS (encircled in blue).

3. System Modelling

This chapter gives a mathematical description of the component modelling. Thus the different physical and mathematical measures of hydraulic systems are introduced. The similarities to electronic network modelling are shown by explaining the relevant properties of graph theory. The reduced model for multi-inletsystems is introduced first, then the inclusion of tanks is discussed. In the end, the EPANET-based modelling approach is introduced which is used for simulation purposes within this project.

3.1 Hydraulic component modelling

In this section the mathematical relation between pressure and flow is given for each component in a WSS system, in order to show their non-linear behaviour. The purpose here is not to derive the different models, rather to introduce the mathematical formalism which describes them.

Equation: (3.1) shows the dual variables which describe all two-terminal components in the network

$$\begin{bmatrix} \Delta p \\ q \end{bmatrix} = \begin{bmatrix} p_{in} - p_{out} \\ q \end{bmatrix}, \tag{3.1}$$

where

Δp	is the differential pressure across the elements,	[m]
q	is the flow through the element,	[m]
p_{in}, p_{out}	are the absolute pressures. is the flow through the element.	$\left[\frac{\mathrm{l}}{\mathrm{s}}\right]$

3.1.1 Pipe model

Pipes in the network are governed by the dynamic equation

$$\Delta p_i = J_i \dot{q}_i + f_i(q_i) - \Delta h_i, \tag{3.2}$$

where

J_i	is the inertia of the pipes,
$f_i(q_i)$	is the pressure drop due to friction,
Δh_i	is the pressure drop due to geodesic level difference across the two terminals of pipe elements.

The dynamics of the pipes are discarded in the project, as it is shown in other works that the small time constant of the pipe dynamics are not dominant in the system, especially if there are elevated reservoirs included [9, 15]. Therefore the pressure across pipes can be written as

$$\Delta p_i = f_i(q_i) - \Delta h_i, \tag{3.3}$$

Q1

The pressure drop due to friction across the i^{th} edge is a diagonal map where $f : \mathbb{R}^m \to \mathbb{R}^m$ is strictly increasing.[1] As it is shown in *Equation: (3.4)*, f_i describes a flow dependant pressure drop due to the hydraulic resistance

[1] A map $f : \mathbb{R}^m \to \mathbb{R}^m$ is strictly increasing if $\langle x - y, f(x) - f(y) \rangle \geq 0$ for every $x, y \in \mathbb{R}^n$ such that $x \neq y$ [18].

$$f_i(q_i) = \rho_i |q_i| q_i, \tag{3.4}$$

where

$\rho_i > 0$ is the parameter of the pipes.

The form in *Equation: (3.4)* is motivated by turbulent flow in the pipes in the network, which is typical in water supply applications[8]. In the following sections it is assumed that each f_i has a structure shown in *Equation: (3.4)*.

It is important to note here that $f_i(\cdot)$ is a homogeneous map which means that if the argument is multiplied by a factor, then its value is multiplied by some power of this factor [2]. For $f_i(q_i)$, it can be shown that

$$\rho_i |(\alpha q_i)| (\alpha q_i) = f_i(\alpha q_i) = \alpha^2 f_i(q_i). \tag{3.5}$$

This property is noted here and used later in the system description, in Section *3.2.4: Multi-inlet reduced network model*.

3.1.2 Valve model

Valves in the network are governed by the following algebraic expression

$$\Delta p_i = \mu_i(q_i, k_v) = \frac{1}{k_v(OD)^2} |q_i| q_i, \tag{3.6}$$

where

k_v is the valve conductivity function, taking in its argument the Opening Degree(OD) of the valve [15].

Furthermore, $\mu_i(q_i, k_v)$ is a continuously differentiable and proper function which for $q_i = 0$ is zero and monotonically increasing.

3.1.3 Pump model

(in progress)

3.1.4 Elevated reservoir model

(in progress)

3.2 Graph-based network modelling

Graph-based network modelling has the advantage of making use of tools from circuit theory. Most of these tools are developed based on Graph Theory(GT). These methods can be used to model WSSs as directed graphs, where components of the systems, such as valves, pipes, tanks and pumps correspond to edges and each terminal of the network correspond to nodes, or equivalently, to vertices.

In case of WSSs, in order to track the pressure and flow in the desired part of the network, the equation system of the network has to be solved for the desired edges and vertices. The whole network can be described by writing up the equations for all edges in the network, based on the mathematical modelling of the different components in the system, as shown in Section *3.1: Hydraulic component modelling*. However, in case of complex systems as water networks for large cities, these systems of equations are hard to handle individually

[2] $g(\alpha v) = \alpha^k g(v)$

and typically cannot be solved explicitly if there are loops in the system. Therefore the properties of GT are not only useful for setting up relations between flow and pressure, but to make handling of a algebraic constraints easier by exploiting the properties of the matrix representation. Thereby making it convenient for implementing it in computer algorithms for iterative solving methods.

WSSs can be described by a directed and connected graph, such that [19]:

$$\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}, \tag{3.7}$$

where

$\mathcal{G}$	is a directed and connected graph,
$\mathcal{V}$	is the set of vertices, where $\mathcal{V} = \{v_1, ..., v_n\}$,
$\mathcal{E}$	is the set of edges, where $\mathcal{E} = \{e_1, ..., e_m\}$.

3.2.1 Incidence matrix

The incidence matrix, H, of a connected graph, $\mathcal{G}$, is a matrix where the number of rows and columns correspond to the number of vertices and edges, respectively. Therefore $H \in \mathbb{R}^{n \times m}$. In case of hydraulic networks, edges are directed in order to keep track of the direction of the flow in the system.

$$H_{i,j} = \begin{cases} 1 & \text{if the } j^{th} \text{ edge is incident out of the } i^{th} \text{ vertex.} \\ -1 & \text{if the } j^{th} \text{ edge is incident into the } i^{th} \text{ vertex.} \\ 0 & \text{if the } j^{th} \text{ edge is not connected to the } i^{th} \text{ vertex.} \end{cases} \tag{3.8}$$

It is worth mentioning that the reduced incidence matrix can be obtained by removing any arbitrary row from H. Therefore H always have $(n-1)$ row rank. This statement can be explained by the mass conservation in the network, which is explained in the following section, Section *3.2.3: Kirchhoff's and Ohm's law for hydraulic networks.*

3.2.2 Cycle matrix

Purely tree structure of a WSS is not common when considering water distribution systems. However, trees can be arbitrarily chosen from the underlying graph of the system.[3] A tree, $\mathcal{T}$, of the graph is a connected sub-graph where any two vertices are connected by exactly one path [20]. Therefore a certain sub-graph which is a tree of the network can be represented as follows

$$\mathcal{T} = \{\mathcal{V}_\mathcal{T}, \mathcal{E}_\mathcal{T}\} \tag{3.9}$$

A special case of connected tree sub-graphs is the spanning tree of the network. A spanning tree contains all the vertices of $\mathcal{G}$ and has no cycles, since it is a tree. A spanning tree of the network therefore can be represented as

$$\mathcal{T} = \{\mathcal{V}, \mathcal{E}_\mathcal{T}\} \tag{3.10}$$

In order to obtain a spanning tree, an edge has to be removed from each cycle. The removed edges are $\mathcal{G} - \mathcal{T}$, and called the chords of $\mathcal{T}$ with respect to $\mathcal{G}$. By adding a chord to $\mathcal{T}$, a cycle is created which is called a fundamental cycle. A graph is conformed by as many fundamental cycles as the number of chords [20].

The set of fundamental cycles correspond to the fundamental cycle matrix, B, such that the number of rows and columns are defined by the number of chords and edges, respectively. The cycle matrix of the system is given by

[3]Recall that a tree with n vertices has $n-1$ edges [20].

$$B_{i,j} = \begin{cases} 1 & \text{if the } j^{th} \text{ edge belongs to the } i^{th} \text{ cycle and their directions agree} \\ -1 & \text{if the } j^{th} \text{ edge belongs to the } i^{th} \text{ cycle and their directions are opposite} \\ 0 & \text{if the } j^{th} \text{ edge does not belong to the } i^{th} \text{ cycle} \end{cases} \tag{3.11}$$

3.2.3 Kirchhoff's and Ohm's law for hydraulic networks

In this project the hydraulic system is considered to be an open network with pipes, valves, pumps and the storage tanks, where water is able to enter and leave the network at a subset of the vertices. For such a system Kirchhoff's vertex law corresponds to conservation of mass in each vertex and described by

$$Hq = d, \tag{3.12}$$

where

$d \in \mathbb{R}^n$ is the vector of nodal demands, with $d_i > 0$ when demand flow is into vertex i and $d_i < 0$ when demand flow is out of vertex i. $\left[\frac{\mathrm{L}}{\mathrm{s}}\right]$

Nodal demands can be seen as the end-user consumption, which means that water is taken out from the network. The mass conservation corresponds to the fact that what is consumed from the system must also be produced. Due to mass conservation, there can be only $(n-1)$ independent nodal demands in the network

$$d_n = -\sum_{i=1}^{n-1} d_i. \tag{3.13}$$

In the further project, a distinction is made between inlet and non-inlet nodes. It is assumed that the demand at non-inlet nodes fulfil the following constraint

$$d_i \geq 0. \tag{3.14}$$

Q2

It is worth noting however, that in closed hydraulic networks the vertex law is

$$Hq = 0. \tag{3.15}$$

Ohm's law for hydraulic networks therefore can be expressed with the incidence matrix, when H^T is applied to the vector of absolute pressures, p. Important to point out that the edges of the underlying graph are considered as only pipe elements

$$\Delta p = H^T p = f(q) - H^T h. \tag{3.16}$$

In *Equation: (3.16)* the differential pressure is described across each edge in the network, taking into account the pressure loss due to friction, $f(q)$ and the pressure drop due to geodesic level differences, where $h \in \mathbb{R}^n$ is the vector of geodesic levels at each vertex expressed in units of potential, i.e. pressure.

3.2.4 Multi-inlet reduced network model

The system is considered to be a water network supplied from more than one pumping stations and several end-users. In the underlying graph therefore the nodes are pipe connections, with possible water demand from the end-users, and the edges are pipes.

The inclusion of storage tanks is the next step of the model development, therefore it is described in a following section, in Section *3.2.5: Inclusion of elevated reservoirs.*

The aim of the modelling is to obtain a reduced order network model which is able to capture the dependence of the measured output pressures on the flows and pressures at the inlets. Therefore it is assumed that the inlet pressures and demands are measured. Furthermore, pressure measurement is available in the remaining network, at the end-users. Considering generality, the model is described for c inlets, however it should be noted that regarding the Randers WSS, two inlet points are considered.

In order to put the system into a form which can handle the measured pressure dependencies on the control inputs, the underlying graph of the network is first partitioned. The n vertices of the graph are separated into two sets

$$\mathcal{V} = \{\bar{\mathcal{V}}, \hat{\mathcal{V}}\}, \tag{3.17}$$

where

$\hat{\mathcal{V}} = \{\hat{v}_1, ..., \hat{v}_c\}$ represents the vertices corresponding to the inlet points,

and

$\bar{\mathcal{V}} = \{\bar{v}_1, ..., \bar{v}_{n-c}\}$ represents the remaining vertices in the graph.

The partitioning for the m edges of the graph is being chosen such that

$$\mathcal{E} = \{\mathcal{E}_{\mathcal{T}}, \mathcal{E}_{\mathcal{C}}\}, \tag{3.18}$$

where

$\mathcal{E}_{\mathcal{T}} = \{e_{\mathcal{T},1}, ..., e_{\mathcal{T},n-c}\}$

and

$\mathcal{E}_{\mathcal{C}} = \{e_{\mathcal{C},1}, ..., e_{\mathcal{C},m-n+c}\}$.

The subsets regarding edges and the partitioning is chosen such that the sub-matrix, which maps edges in $\mathcal{E}_{\mathcal{T}}$ to vertices in $\bar{\mathcal{V}}$, is invertible.

Therefore the incidence matrix can be split into four sub-matrices, as shown in *Equation: (3.19)* below

$$H = \left[\begin{array}{c:c} \bar{H}_{\mathcal{T}} & \bar{H}_{\mathcal{C}} \\ \hdashline \hat{H}_{\mathcal{T}} & \hat{H}_{\mathcal{C}} \end{array}\right], \tag{3.19}$$

where

$\bar{H}_{\mathcal{T}} \in \mathbb{R}^{(n-c)\times(n-c)}$ is the sub-matrix, mapping edges in $\mathcal{E}_{\mathcal{T}}$ to vertices in $\bar{\mathcal{V}}$,
$\bar{H}_{\mathcal{C}} \in \mathbb{R}^{(n-c)\times(m-n+c)}$ is the sub-matrix, mapping edges in $\mathcal{E}_{\mathcal{C}}$ to vertices in $\bar{\mathcal{V}}$,
$\hat{H}_{\mathcal{T}} \in \mathbb{R}^{c\times(n-c)}$ is the sub-matrix, mapping edges in $\mathcal{E}_{\mathcal{T}}$ to vertices in $\hat{\mathcal{V}}$,
$\hat{H}_{\mathcal{C}} \in \mathbb{R}^{c\times(m-n+c)}$ is the sub-matrix, mapping edges in $\mathcal{E}_{\mathcal{C}}$ to vertices in $\hat{\mathcal{V}}$.

It is worth noting that the only requirement for the edge partitioning is $\bar{H}_{\mathcal{T}}$ being invertible[4]. Furthermore, the set $\mathcal{T} = \{\mathcal{V}, \mathcal{E}_{\mathcal{T}}\}$ is not necessarily a tree of the underlying graph, it can be any form of a connected graph that fulfils the requirements. However, one special case is when $c = 1$, meaning that the network has only one inlet. In this case, $\mathcal{T}$ is indeed a spanning tree.

With the chosen partition, Kirchhoff's vertex law in *Equation: (3.12)* can be rewritten as

$$\bar{d} = \bar{H}_{\mathcal{T}} q_{\mathcal{T}} + \bar{H}_{\mathcal{C}} q_{\mathcal{C}}, \tag{3.20}$$

$$\hat{d} = \hat{H}_{\mathcal{T}} q_{\mathcal{T}} + \hat{H}_{\mathcal{C}} q_{\mathcal{C}}, \tag{3.21}$$

[4] $\exists\{\mathcal{V}, \mathcal{E}\} : \bar{H}_{\mathcal{T}}^{-1} \because rank(H) = (n-1)$ [20]

and Ohm's law in *Equation: (3.16)*, separating the pressure drop due to hydraulic resistance

$$f_{\mathcal{T}}(q_{\mathcal{T}}) = \bar{H}_{\mathcal{T}}^T(\bar{p} + \bar{h}) + \hat{H}_{\mathcal{T}}^T(\hat{p} + \hat{h}), \tag{3.22}$$

$$f_{\mathcal{C}}(q_{\mathcal{C}}) = \bar{H}_{\mathcal{C}}^T(\bar{p} + \bar{h}) + \hat{H}_{\mathcal{C}}^T(\hat{p} + \hat{h}). \tag{3.23}$$

Writing up *Equation: (3.22)* and *Equation: (3.23)* in matrix form

$$\begin{bmatrix} f_{\mathcal{T}}(q_{\mathcal{T}}) \\ f_{\mathcal{C}}(q_{\mathcal{C}}) \end{bmatrix} = \underbrace{\begin{bmatrix} \bar{H}_{\mathcal{T}}^T & \hat{H}_{\mathcal{T}}^T \\ \bar{H}_{\mathcal{C}}^T & \hat{H}_{\mathcal{C}}^T \end{bmatrix}}_{\begin{bmatrix} \bar{H}^T & \hat{H}^T \end{bmatrix}} \begin{bmatrix} (\bar{p} + \bar{h}) \\ (\hat{p} + \hat{h}) \end{bmatrix} \tag{3.24}$$

Q3

As it is shown in *Equation: (3.24)*, the transposed incidence matrices can be written up as the two sub-matrices partitioned according to inlet and non-inlet nodes.

Introducing matrix B, in which the arrangement of the edges are the same as for the incidence matrix, H, meaning that B is regarding to the graphs $H_{\mathcal{T}}$ and $H_{\mathcal{T}}$. Then B can be given

$$B = \begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \tag{3.25}$$

It should be noted that since the set $\mathcal{T}$ does not define a spanning tree when $c > 0$, matrix B is not a cycle matrix corresponding to any spanning tree. However, it is defined the same way as cycle matrices are given. For this reason, the following can be written

$$BH^T = \begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \begin{bmatrix} \bar{H}_{\mathcal{T}}^T & \hat{H}_{\mathcal{T}}^T \\ \bar{H}_{\mathcal{C}}^T & \hat{H}_{\mathcal{C}}^T \end{bmatrix} = \begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \hat{H}^T. \tag{3.26}$$

It should be noted, that $B\bar{H}^T = 0$ by the definition of matrix B [20].

Multiplying with B from the left in *Equation: (3.24)*

$$\begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \begin{bmatrix} f_{\mathcal{T}}(q_{\mathcal{T}}) \\ f_{\mathcal{C}}(q_{\mathcal{C}}) \end{bmatrix} = \begin{bmatrix} -\bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} & I \end{bmatrix} \begin{bmatrix} \bar{H}_{\mathcal{T}}^T & \hat{H}_{\mathcal{T}}^T \\ \bar{H}_{\mathcal{C}}^T & \hat{H}_{\mathcal{C}}^T \end{bmatrix} \begin{bmatrix} (\bar{p} + \bar{h}) \\ (\hat{p} + \hat{h}) \end{bmatrix} \tag{3.27}$$

induces the following expression

$$f_{\mathcal{C}}(q_{\mathcal{C}}) - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(q_{\mathcal{T}}) = (\hat{H}_{\mathcal{C}}^T - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T)(\hat{p} + \hat{h}). \tag{3.28}$$

From *Equation: (3.20)*, the vector $q_{\mathcal{T}}$, of flows in edges $\mathcal{E}_{\mathcal{T}}$ can be expressed

$$q_{\mathcal{T}} = -\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}} q_{\mathcal{C}} + \bar{H}_{\mathcal{T}}^{-1} \bar{d}. \tag{3.29}$$

Therefore using *Equation: (3.29)*, *Equation: (3.28)* can be rewritten

$$f_{\mathcal{C}}(q_{\mathcal{C}}) - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}} q_{\mathcal{C}} + \bar{H}_{\mathcal{T}}^{-1} \bar{d}) = (\hat{H}_{\mathcal{C}}^T - \bar{H}_{\mathcal{C}}^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T)(\hat{p} + \hat{h}). \tag{3.30}$$

Now expressing the vertex demands at non-inlet vertices, $\bar{d}$, such that

$$\bar{d} = -v\sigma \tag{3.31}$$

where

$\bar{d} \in \mathbb{R}^{n-c}$	is the vector of nodal demands in non-inlet vertices,
$\sigma \in \mathbb{R}_+$	is the total demand in the network, representing the total consumption of the end-users,
$v \in \mathbb{R}_{n-c}$	represents a vector with the property $\sum_i v_i = 1$ and $v_i \in (0;1)$.

Furthermore, introduce a vector, $a_\mathcal{C}$, such that

$$q_\mathcal{C} = a_\mathcal{C}\sigma \tag{3.32}$$

Q4

is a unique solution to *Equation: (3.28)*. The proof of this proposition is shown for the one-inlet case in [18].

Having $\bar{d}$ and $q_\mathcal{C}$ introduced as the linear function of the total demand, σ, in the network, *Equation: (3.30)* can be expressed such that

$$\begin{aligned} &f_\mathcal{C}(q_\mathcal{C}) - \bar{H}_\mathcal{C}^T \bar{H}_\mathcal{T}^{-T} f_\mathcal{T}(-\bar{H}_\mathcal{T}^{-1}\bar{H}_\mathcal{C} q_\mathcal{C} + \bar{H}_\mathcal{T}^{-1}\bar{d}) = \\ &f_\mathcal{C}(a_\mathcal{C}\sigma) - \bar{H}_\mathcal{C}^T \bar{H}_\mathcal{T}^{-T} f_\mathcal{T}(-\bar{H}_\mathcal{T}^{-1}\bar{H}_\mathcal{C} a_\mathcal{C}\sigma - \bar{H}_\mathcal{T}^{-1} v\sigma) = \\ &f_\mathcal{C}(a_\mathcal{C})\sigma^2 - \bar{H}_\mathcal{C}^T \bar{H}_\mathcal{T}^{-T} f_\mathcal{T}(-\bar{H}_\mathcal{T}^{-1}\bar{H}_\mathcal{C} a_\mathcal{C} - \bar{H}_\mathcal{T}^{-1} v)\sigma^2, \end{aligned} \tag{3.33}$$

where the latter equality is due to the homogeneity property of the pressure drops due to frictions, explained in Section *3.1.1: Pipe model*.

Q5 : F_v

Q6 : *pdependenci*

3.2.5 Inclusion of elevated reservoirs

As it is described in *Equation: (3.14)*, a distinction is made between non-inlet and inlet vertices, by assuming that non-inlet vertices have only positive or zero nodal demand. However, when the inclusion of a tank is considered, a special type of node has to be introduced. A node which can have a demand in both positive and negative directions, meaning that the demand is positive when the tank is being filled and negative when it is being emptied.

3.3 EPANET modelling

EPANET is an open source software, created by the United States Environmental Protection Agency (EPA) for simulating hydraulic networks [21]. EPANET allows to track the flow of water in each pipe, the pressure at each node and the height of water in each tank. Furthermore, it uses a node-based model approach which means that the components in the network are either treated as nodes or links. Valves, pumps, reservoirs and tanks are considered as nodes due to their fixed geographical location and geodesic level. Pipes are considered as the links between the nodes in the network. Therefore, nodes are termination points for one or more pipes. The end-user consumption flow demand is considered as an attribute of certain nodes. Such nodes are called demand nodes and they have a certain water withdrawal. Attributes of nodes in the network are the geographical and geodesic coordinates, the flow demand, the total, and the available head. [21]

In EPANET, there is a function to carry out simulations within an extended period. Time patterns can be created that make demands at the nodes vary in a periodic way over the course of the time period. Nodal demands, reservoirs and pump schedules can all have time patterns associated with them, thus making the hydraulic simulation of the network more realistic. In order to create a schedule plan for changing reservoir levels or schedules for the pumping strategy, it is sufficient to have simulation data only at certain time steps. Therefore it is sufficient to solve the network using a set of hourly time steps (snapshots) over a period of 24 hours, and use the static, steady-state solutions for pump scheduling [21]. The main function of EPANET is this, and therefore is used within this project for extended period analysis. During the analysis, pressure and flow values, along with the demand pattern can be simulated for periodic time steps.

Since all nodes and links in the network have their unique IDs, during the project, the name of certain components will always have a reference to the original IDs in EPANET, for the better and clearer trackability.

4. Network simplification

4.1 Purpose of the model reduction

As it is described in Section *3.3: EPANET modelling*, planning pump schedules and tank level changes in EPANET can be carried out by solving iteratively for long time steps. However, when a complex network is being used for developing different control methods, typically the aim is to find an optimisation method which determines the optimal input to the system. In order to run such an optimizing algorithm on a complex system, the network needs to be solved numerically and several online execution runs are required.

In case of complex networks with a tree structure, there is only one way from a source node to any node in the system. Therefore the network can be described by solving the equation system of the model explicitly, therefore all pressures can be directly calculated in terms of the flows. In case of loop systems however, the procedure is more dificult, since there is more path to each nodes in the network. The system of equations have to be solved numerically which can dramatically increase the simulation time of an optimization algorithm. The objective of the system reduction and simplification is threfore to reduce the number of network components and especially the loops in the network, such that the accuracy of the simplified model is as close as possible to that of the original network model.

4.2 State of the art model reduction analysis

123

4.2.1 Simplification of network model components

Simplification directly, by keeping the basic components but combining and replacing the individual components. (For example, series, parallel pipes. Tree substructure in the system.)

4.2.2 Black box simplification

Replacing the system partially or fully with a system which provides the same function with less complexity.

- static simplification with linearization and simplification
- neural network approach

Part II

System Identification

5. Unspecified

Part III

Conclusion and verification

6. Accepttest

7. Discussion

8. Conclusion

Part IV

Appendices

A. Assumption List

Number	Assumptions	Section reference
1	Assumption 1.	Section ??: ??
2	Assumption 2.	Section ??: ??

Table A.1: List of assumptions

B. System Description

B.1 Components of the System

B.2 System Topology

123

B.3 Incidence Matrix

123

B.4 Cycle Matrix

C. Measurements

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Rettelser

Todo list

Q1 11
Q2 14
Q3 16
Q4 17
Q5 : F_v 17
Q6 : $\bar{p}dependencies$ 17