Identification notes

Output equation

Let us write the output and constraint equations in discrete time. The output $\bar{p}_{\mathcal{K},k}$ is given by *Equation: (1)*.

$$\bar{p}_{\mathcal{K},k} = K^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(A_2 q_{\mathcal{C},k} + A_3 K \bar{d}_{\mathcal{K},k} - A_3 D v_{\mathcal{D}} \sigma_k) - K^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T (\hat{p}_k + \hat{h}) - \bar{h}_{\mathcal{K}}. \tag{1}$$

The constraint for the system is on $q_{\mathcal{C}}$

$$f_{\mathcal{C}}(q_{\mathcal{C},k}) - A_1(\hat{p}_k + \hat{h}) + A_2^T f_{\mathcal{T}}(A_2 q_{\mathcal{C},k} + A_3 K \bar{d}_{\mathcal{K},k} - A_3 D v_{\mathcal{D}} \sigma_k) = 0. \tag{2}$$

Reformulating *Equation: (2)* yields

$$q_{\mathcal{C},k} = q_{\mathcal{C},k}((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k). \tag{3}$$

Substituting *Equation: (3)* into *Equation: (1)*, we get the following

$$\begin{aligned} \bar{p}_{\mathcal{K},k} + \bar{h}_{\mathcal{K}} = {} & K^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(A_2 q_{\mathcal{C},k}(\boxed{(\hat{p}_k + \hat{h})}, \bar{d}_{\mathcal{K},k}, \sigma_k) + A_3 K \bar{d}_{\mathcal{K},k} - A_3 D v_{\mathcal{D}} \sigma_k) \\ & - K^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T \boxed{(\hat{p}_k + \hat{h})}. \end{aligned} \tag{4}$$

In *Equation: (4)*, we have the elevation of the pumping stations ($\bar{p}_{\mathcal{K},k} + \bar{h}_{\mathcal{K}}$) given by the expression on the right-hand side. Let us write *Equation: (4)* in a form where the expression is replaced with a non-linear function $\tilde{f}_1$ with an unknown structure but with the same variables in the argument.

$$\tilde{y}_k = \bar{p}_{\mathcal{K},k} + \bar{h}_{\mathcal{K}} = \tilde{f}_1((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k). \tag{5}$$

State equation

The state equation in continuous form is given by *Equation: (6)*.

$$\Lambda \dot{\hat{p}} = -(\hat{H}_{\mathcal{C}} - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}}) q_{\mathcal{C}} - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} K \bar{d}_{\mathcal{K}} + \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} D v_{\mathcal{D}} \sigma. \tag{6}$$

After substituting the constraint on $q_{\mathcal{C},k}$, the discrete form, using Euler-method, is given by *Equation: (7)*.

$$\begin{aligned} \Lambda \frac{1}{T_s}(\hat{p}_{k+1} - \hat{p}_k) = {} & -(\hat{H}_{\mathcal{C}} - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}}) q_{\mathcal{C}}((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k) \\ & - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} K \bar{d}_{\mathcal{K},k} + \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} D v_{\mathcal{D}} \sigma_k. \end{aligned} \tag{7}$$

Original idea:

The original idea was to choose a structure for *Equation: (7)* such that

$$\hat{p}_{k+1} = \tilde{f}_2((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k) + a_2 \hat{p}_k, \tag{8}$$

where a is a linear parameter.

Other idea:

Equation: (7) can be written in the form

$$\begin{aligned}\hat{p}_{k+1} - \hat{p}_k = T_s \Lambda^{-1} \big[&- (\hat{H}_{\mathcal{C}} - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}}) q_{\mathcal{C},k}\big((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k\big) \\ &- \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} K \bar{d}_{\mathcal{K},k} + \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} D v_{\mathcal{D}} \sigma_k \big].\end{aligned} \tag{9}$$

In this arrangement, let us keep the tank pressures $\hat{p}_k$ on the left hand-side. Thus, $\hat{p}_{k+1}$ can be reconstructed by adding the current values $\hat{p}_k$ to both sides in the equation. Thereby, we do not need to introduce the linear parameter when we identify the state equation.

(Λ is invertible, as it is a diagonal matrix, with positive values in each diagonal entry.)

+1 on the state model:

Let us recall *Equation: (8)*.

$$\hat{p}_{k+1} = \tilde{f}_2\big((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k\big) + a_2 \hat{p}_k, \tag{10}$$

In this equation, the present values of the states are already represented inside the non-linear term $\tilde{f}_2$ as $(\hat{p}_k + \hat{h}_k)$. $\hat{p}$ and $(\hat{p}_k + \hat{h}_k)$ are dependant variables, and we did not introduce linear terms for variables that are dependant e.g. in the output equation in *Equation: (5)*.