# Water distribution networks with water towers

## 1 Multiply water tower model

We propose a model framework that can handle networks with multiply water towers. Additionally, here we assumed that the control nodes are controlled by flow inputs.

As proposed in [3] we partition the network model such that the incidence matrix

$$H = \begin{pmatrix} \bar{H}_T & \bar{H}_C \\ \hat{H}_T & \hat{H}_C \end{pmatrix}$$

where $\bar{H}_T$ is invertible. This leads to the following patition of the model equations

$$f_T(q_T) = \bar{H}_T^T(\bar{p} + \bar{z}) + \hat{H}_T^T(\hat{p} + \hat{z}) \tag{1a}$$

$$f_C(q_C) = \bar{H}_C^T(\bar{p} + \bar{z}) + \hat{H}_C^T(\hat{p} + \hat{z}) \tag{1b}$$

$$\bar{H}_T q_T + \bar{H}_C q_C = \bar{d} \tag{1c}$$

$$\hat{H}_T q_T + \hat{H}_C q_C = \hat{d} \tag{1d}$$

Rewriting the model leads to the following three model equations

$$\bar{p} = \bar{H}_T^{-T} f_T(-\bar{H}_T^{-1}\bar{H}_C q_C + \bar{H}_T^{-1}\bar{d}) - \bar{H}_T^{-T}\hat{H}_T^T(\hat{p} + \hat{z}) - \bar{z} \tag{2}$$

$$\hat{d} = (\hat{H}_C - \hat{H}_T\bar{H}_T^{-1}\bar{H}_C)q_C + \hat{H}_T\bar{H}_T^{-1}\bar{d} \tag{3}$$

$$f_C(q_C) - \bar{H}_C^T\bar{H}_T^{-T} f_T(-\bar{H}_T^{-1}\bar{H}_C q_C + \bar{H}_T^{-1}\bar{d}) = (\hat{H}_C^T - \bar{H}_C^T\bar{H}_T^{-T}\hat{H}_T^T)(\hat{p} + \hat{z}) \tag{4}$$

We define $\hat{d}$ as the flows out of the water towers, and $\hat{p}$ the pressures at the nodes that connects the water towers. The matrix $A$ be diagonal with the the cross area of the individual water towers at diagonal, then the water tower dynamics can be described by

$$A\dot{\hat{p}} = -\hat{d} \tag{5}$$

Now we split the vectors related to the nodes marked with bars $\bar{p}$ and $\bar{d}$ such that

$$\bar{p} = K\bar{p}_{\mathcal{K}} + D\bar{p}_{\mathcal{D}} \qquad , \qquad \bar{d} = K\bar{d}_{\mathcal{K}} + D\bar{d}_{\mathcal{D}} \tag{6}$$

where $\bar{d}_{\mathcal{K}}$ are the controlled inlet flows, and $\bar{d}_{\mathcal{D}} = \bar{v}_{\mathcal{D}}\sigma$ are the end user demand flows. The matrices $K$ and $D$ are "pick out" matrices meaning that $\bar{p}_{\mathcal{K}} = K^T\bar{p}$ and $\bar{p}_{\mathcal{D}} = D^T\bar{p}$. Introducing the expressions (6) and (5) in (2), (3), and (4) leads to

$$A\dot{\hat{p}} = -(\hat{H}_C - \hat{H}_T\bar{H}_T^{-1}\bar{H}_C)q_C - \hat{H}_T\bar{H}_T^{-1}K\bar{d}_{\mathcal{K}} - \hat{H}_T\bar{H}_T^{-1}D\bar{v}_{\mathcal{D}}\sigma \tag{7}$$

$$\bar{p}_{\mathcal{K}} = K^T\bar{H}_T^{-T} f_T(-\bar{H}_T^{-1}\bar{H}_C q_C + \bar{H}_T^{-1}K\bar{d}_{\mathcal{K}} + \bar{H}_T^{-1}D\bar{v}_{\mathcal{D}}\sigma) - K^T\bar{H}_T^{-T}\hat{H}_T^T(\hat{p} + \hat{z}) - K^T\bar{z} \tag{8}$$

where $q_C$ is given by the solution to the following implicit expression

$$f_C(q_C) - \bar{H}_C^T\bar{H}_T^{-T} f_T(-\bar{H}_T^{-1}\bar{H}_C q_C + \bar{H}_T^{-1}K\bar{d}_{\mathcal{K}} + \bar{H}_T^{-1}D\bar{v}_{\mathcal{D}}\sigma) = (\hat{H}_C^T - \bar{H}_C^T\bar{H}_T^{-T}\hat{H}_T^T)(\hat{p} + \hat{z}) \tag{9}$$

## References

[1] C.S. Kallesøe, T.N. Jensen and R. Wisniewski. Adaptive Reference Control for Pressure Management in Water Networks. *ECC 2015, Linze, Austrians*. 2015.

[2] C.S. Kallesøe, T.N. Jensen and J.D. Bendtsen. Plug-and-Play Model Predictive Control for Water Supply Networks with Storage. *IFAC 2017, Toulouse, France*. 2017.

[3] T.N. Jensen, C.S. Kallesøe, J.D. Bendtsen and R. Wisniewski. Plug-and-play Commissionable Models for Water Networks with Multiple Inlets. *submitted to ECC2018*.