

Output equation

Let us write the output and constraint equations in discrete time. The output $\bar{p}_{\mathcal{K},k}$ is given by *Equation: (1)*.

$$\bar{p}_{\mathcal{K},k} = K^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(A_2 q_{\mathcal{C},k} + A_3 K \bar{d}_{\mathcal{K},k} - A_3 D v_{\mathcal{D}} \sigma_k) - K^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T (\hat{p}_k + \hat{h}) - \bar{h}_{\mathcal{K}}. \quad (1)$$

The constraint for the system is on $q_{\mathcal{C}}$

$$f_{\mathcal{C}}(q_{\mathcal{C},k}) - A_1(\hat{p}_k + \hat{h}) + A_2^T f_{\mathcal{T}}(A_2 q_{\mathcal{C},k} + A_3 K \bar{d}_{\mathcal{K},k} - A_3 D v_{\mathcal{D}} \sigma_k) = 0. \quad (2)$$

Reformulating *Equation: (2)* yields

$$q_{\mathcal{C},k} = q_{\mathcal{C},k}((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k). \quad (3)$$

Substituting *Equation: (3)* into *Equation: (1)*, we get the following

$$\begin{aligned} \bar{p}_{\mathcal{K},k} + \bar{h}_{\mathcal{K}} &= K^T \bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(A_2 q_{\mathcal{C},k}((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k) + A_3 K \bar{d}_{\mathcal{K},k} - A_3 D v_{\mathcal{D}} \sigma_k) \\ &\quad - K^T \bar{H}_{\mathcal{T}}^{-T} \hat{H}_{\mathcal{T}}^T (\hat{p}_k + \hat{h}). \end{aligned} \quad (4)$$

In *Equation: (4)*, we have the elevation of the pumping stations $(\bar{p}_{\mathcal{K},k} + \bar{h}_{\mathcal{K}})$ given by the expression on the right-hand side. Let us write *Equation: (4)* in a form where the expression is replaced with a non-linear function \tilde{f}_1 with an unknown structure but with the same variables in the argument.

$$\tilde{y}_k = \bar{p}_{\mathcal{K},k} + \bar{h}_{\mathcal{K}} = \tilde{f}_1((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k). \quad (5)$$

State equation

The state equation in continuous form is given by *Equation: (6)*.

$$\Lambda \dot{\hat{p}} = -(\hat{H}_{\mathcal{C}} - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}}) q_{\mathcal{C}} - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} K \bar{d}_{\mathcal{K}} + \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} D v_{\mathcal{D}} \sigma. \quad (6)$$

After substituting the constraint on $q_{\mathcal{C},k}$, the discrete form, using Euler-method, is given by *Equation: (7)*.

$$\begin{aligned} \Lambda \frac{1}{T_s} (\hat{p}_{k+1} - \hat{p}_k) &= -(\hat{H}_{\mathcal{C}} - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} \bar{H}_{\mathcal{C}}) q_{\mathcal{C}}((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k) \\ &\quad - \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} K \bar{d}_{\mathcal{K},k} + \hat{H}_{\mathcal{T}} \bar{H}_{\mathcal{T}}^{-1} D v_{\mathcal{D}} \sigma_k. \end{aligned} \quad (7)$$

Original idea:

The original idea was to choose a structure for *Equation: (7)* such that

$$\hat{p}_{k+1} = \tilde{f}_2((\hat{p}_k + \hat{h}), \bar{d}_{\mathcal{K},k}, \sigma_k) + a_2 \hat{p}_k, \quad (8)$$

where a is a linear parameter.

Other idea:

Equation: (7) can be written in the form

$$\begin{aligned}\hat{p}_{k+1} - \hat{p}_k = T_s \Lambda^{-1} [& - (\hat{H}_C - \hat{H}_T \bar{H}_T^{-1} \bar{H}_C) q_{C,k}((\hat{p}_k + \hat{h}), \bar{d}_{K,k}, \sigma_k) \\ & - \hat{H}_T \bar{H}_T^{-1} K \bar{d}_{K,k} + \hat{H}_T \bar{H}_T^{-1} D v_D \sigma_k].\end{aligned}\quad (9)$$

In this arrangement, let us keep the tank pressures \hat{p}_k on the left hand-side. Thus, \hat{p}_{k+1} can be reconstructed by adding the current values \hat{p}_k to both sides in the equation. Thereby, we do not need to introduce the linear parameter when we identify the state equation.

(Λ is invertible, as it is a diagonal matrix, with positive values in each diagonal entry.)

+1 on the state model:

Let us recall *Equation:* (8).

$$\hat{p}_{k+1} = \tilde{f}_2((\hat{p}_k + \hat{h}), \bar{d}_{K,k}, \sigma_k) + a_2 \hat{p}_k, \quad (10)$$

In this equation, the present values of the states are already represented inside the non-linear term \tilde{f}_2 as $(\hat{p}_k + \hat{h}_k)$. \hat{p} and $(\hat{p}_k + \hat{h}_k)$ are dependant variables, and we did not introduce linear terms for variables that are dependant e.g. in the output equation in *Equation:* (5).
