Mathematical Derivations from the EISA-RIA Framework

Extracted from EISA-RIA Collaboration Papers

1 Algebraic Structure and Fundamental Definitions

1.1 The EISA Superalgebra

The framework is built upon the triple-graded superalgebra:

$$\mathcal{A}_{\text{EISA}} = \mathcal{A}_{\text{SM}} \otimes \mathcal{A}_{\text{Grav}} \otimes \mathcal{A}_{\text{Vac}}$$

where the tensor product is defined over the representation spaces. The full Hilbert space is:

$$\mathcal{H} = \mathcal{H}_{\text{SM}} \otimes \mathcal{H}_{\text{Grav}} \otimes \mathcal{H}_{\text{Vac}}$$

1.2 Vacuum Sector Generators

The vacuum sector is generated by 16 anticommuting operators:

$$\{\zeta^k, \zeta^l\} = 2\delta^{kl} I, \quad k, l = 1, \ldots, 16$$

From these, a composite scalar field emerges:

$$\phi \sim \operatorname{Tr}(\zeta^\dagger \zeta)$$

The vacuum density matrix is defined as:

$$\rho_{\text{vac}} = \frac{\exp\left(-\beta \sum_k \zeta^k \zeta^{k\dagger}\right)}{Z}, \quad Z = \operatorname{Tr}\left[\exp\left(-\beta \sum_k \zeta^k \zeta^{k\dagger}\right)\right]$$

1.3 Super-Jacobi Identity

Mathematical consistency is ensured by the graded Super-Jacobi identity:

$$(-1)^{|X||Z|}[X, [Y, Z]] + (-1)^{|Y||X|}[Y, [Z, X]] + (-1)^{|Z||Y|}[Z, [X, Y]] = 0$$

for all generators X, Y, Z with grades $|X|, |Y|, |Z|$.

2 Recursive Info-Algebra (RIA) and Entropy Minimization

2.1 Loss Function

The core of the RIA mechanism is the minimization of the composite loss function:

$$\mathcal{L} = S_{\text{vN}}(\rho) + (1 - F(\rho, \rho_{\text{target}})) + \frac{1}{2}(1 - \text{Tr}(\rho^2))$$

where:

- $S_{\text{vN}}(\rho) = -\text{Tr}(\rho \log \rho)$ is the von Neumann entropy
- $F(\rho, \sigma) = \left[\text{Tr}\sqrt{\sqrt{\rho}\,\sigma\,\sqrt{\rho}}\right]^2$ is the fidelity
- The purity term $\frac{1}{2}(1 - \text{Tr}(\rho^2))$ penalizes mixed states.

2.2 Variational Quantum Circuit (VQC) Ansatz

Optimization is performed using a classically simulated VQC with a layered structure:

$$U(\vec{\theta}, \vec{\phi}) = \prod_{l=1}^{N_{\text{layers}}} \left[\bigotimes_{q=1}^{d/2} U_{\text{RX}}^{(q)}(\theta_{l,q}) U_{\text{RY}}^{(q)}(\phi_{l,q}) \right] \cdot U_{\text{ENT}}$$

where $U_{\text{RX/RY}}$ are single-qubit rotations and U_{ENT} provides entanglement.

3 Modified Dirac Equation and Fermion Mass Generation

3.1 Origin of the Scalar Coupling

A Yukawa-like coupling emerges from integrating out high-energy modes via the Operator Product Expansion (OPE):

$$(\bar{\psi}\psi)(\zeta^\dagger\zeta) \to \kappa(\bar{\psi}\psi)|\phi|^2$$

Dimensional analysis confirms $[\kappa] = [\text{mass}]^{-1}$. The matching condition gives:

$$\kappa \approx \frac{g^2}{\Lambda^2}$$

3.2 Modified Dirac Equation

The equation for a fermion field ψ becomes:

$$(iD - m - \kappa|\phi|^2)\psi = 0$$

This increases the effective mass:

$$m_{\text{eff}} = m + \kappa\langle|\phi|^2\rangle$$

3.3 Sourcing Spacetime Curvature

The scalar field ϕ sources curvature through its energy-momentum tensor. Under the approximation of slow variation ($|\partial_\mu \phi| \ll m_\phi |\phi|$) and dominance of vacuum energy, the trace-reversed Einstein equations yield:

$$R \approx \kappa^2 |\phi|^2$$

A more complete derivation including a non-minimal coupling term $-\frac{1}{2}\xi R|\phi|^2$ in the action gives:

$$\xi = \frac{\kappa^2}{16\pi G}$$

4 Gravitational Wave Background from Vacuum Fluctuations

4.1 Evolution of the Scalar Field

The dynamics of the composite scalar field ϕ are governed by:

$$\frac{\partial \phi}{\partial t} = \mathcal{D}[\phi] + \alpha \left(\int |\phi|^2 d^3x \right) \cdot \big(1 + \beta \ln\big(|\phi|^2 + \epsilon\big)\big) + \kappa \nabla^2 \phi$$

where $\mathcal{D}[\phi]$ represents dissipative terms.

4.2 Gravitational Wave Spectrum

The resulting stochastic GW background spectrum is given by:

$$\frac{d\Omega_{\mathrm{GW}}(f)}{d \ln f} = \frac{1}{\rho_c} \left(\frac{f}{f_{\mathrm{ref}}} \right)^{n_t} \int d\tau \, a^4(\tau) \langle \delta T_{ij} \delta T^{ij} \rangle$$

where:

$$\begin{aligned} \rho_c &= \frac{3H_0^2}{8\pi G} \quad \text{(critical density)} \\ f_{\mathrm{ref}} &\sim 10^{-8}\,\mathrm{Hz} \quad \text{(reference frequency)} \\ n_t &\approx 0 \quad \text{(tilt)} \end{aligned}$$

The stress-energy tensor correlation $\langle \delta T_{ij} \delta T^{ij} \rangle$ is sourced by vacuum fluctuations.

4.3 Characteristic Strain

The characteristic strain for PTA observations is:

$$h_c(f) = A \left(\frac{f}{f_{\mathrm{ref}}} \right)^{\frac{3-n_t}{2}}$$

with an amplitude:

$$A \approx \sqrt{\frac{2}{3\pi^2} \frac{\Omega_{\mathrm{GW}} h^2}{H_0^2} f_{\mathrm{ref}}^2} \sim 10^{-15}$$

for $\Omega_{\mathrm{GW}} h^2 \approx 10^{-10}$.

5 Cosmological Evolution and Hubble Tension

5.1 Modified Friedmann Equation

The cosmic evolution includes a transient vacuum energy component $\Omega_v(\tau)$:

$$\left(\frac{da}{d\tau}\right)^2 = a^2 \left(\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda + \Omega_v(\tau) + \delta(\tau)\right)$$

A common parameterization is:

$$\Omega_v(\tau) = A_v \exp(-\tau/\tau_{\text{decay}})$$

5.2 Resolution of the Hubble Tension

The model adds a correction to the Hubble parameter:

$$\Delta H^2 \approx \frac{8\pi G}{3} \langle T_{\mu\nu} \rangle g^{\mu\nu} \sim \frac{\kappa^2 \langle |\phi|^2 \rangle}{\Lambda^2}$$

Numerically, this yields:

$$\Delta H \approx 3\,\text{km/s/Mpc}$$

leading to a revised Hubble constant:

$$H_0^{\text{EISA}} \approx \sqrt{(67.4)^2 + (3)^2} \approx 70 \pm 1\,\text{km/s/Mpc}$$

which alleviates the tension between Planck and SH0ES measurements.

6 CMB Power Spectrum Modifications

6.1 Angular Power Spectrum

The CMB temperature anisotropy power spectrum is:

$$D_\ell = \frac{\ell(\ell+1)}{2\pi} C_\ell, \quad C_\ell = \frac{2}{\pi} \int_0^\infty dk\, k^2 P(k) |\Theta_\ell(k)|^2$$

The transfer function is approximated by:

$$\Theta_\ell(k) \propto \int d\tau\, a(\tau)^2 \Omega_v(\tau) j_\ell(k\tau)$$

6.2 Vacuum Contribution to Primordial Power Spectrum

Vacuum fluctuations from $\mathcal{A}_{\text{Vac}}$ modify the primordial power spectrum:

$$P(k) \sim k^{n_s - 1} + \Delta P(k), \quad \Delta P(k) \sim \frac{\kappa^2 \langle |\phi|^2 \rangle}{k\Lambda^2}$$

The relative deviation is:

$$\frac{\Delta P(k)}{P(k)} \sim 10^{-7} \quad \text{at} \quad k \sim 0.01\,\text{Mpc}^{-1}$$

This translates to a deviation in the angular power spectrum:

$$\frac{\Delta C_\ell}{C_\ell} \sim 10^{-7} \quad \text{at low multipoles}\,(\ell)$$

7 Renormalization Group Flow and Asymptotic Safety

7.1 Beta Functions

The RG flow of the Yukawa-like coupling g is given by:

$$\beta(g) = \mu \frac{dg}{d\mu} = -\frac{bg^3}{16\pi^2}$$

The coefficient $b = 7$ is derived from the group theory of $\mathcal{A}_{\text{EISA}}$:

$$b = \frac{11}{3} C_G - \frac{2}{3} \sum_f T(R_f) - \frac{1}{6} \sum_s T(R_s)$$

Contributions come from:

$$\begin{aligned} b_{\text{SM}} &\approx 8.35 \\ \Delta b_{\text{Grav}} &\sim -0.35 \\ \Delta b_{\text{Vac}} &\sim -1.0 \\ &\Rightarrow b = 8.35 - 0.35 - 1.0 = 7 \end{aligned}$$

7.2 Extended System and UV Fixed Point

The system is extended to include couplings $g_i = \{g, \kappa, \lambda, \xi\}$. Their one-loop beta functions are:

$$\begin{aligned} \beta(g) &= -\frac{7g^3}{16\pi^2} + \frac{32g\lambda}{16\pi^2} + \frac{g\xi\kappa^2}{32\pi^2} \\ \beta(G') &= 2G' + \frac{20G'^2}{16\pi^2} + \frac{G'g^2\Lambda^2}{16\pi^2}, \quad G' = \kappa^2\mu^2 \\ \beta(\lambda) &= \frac{10\lambda^2 + 2\lambda g^2 + 4g^4}{16\pi^2} + \frac{\xi^2\mu^2}{16\pi^2} \\ \beta(\xi) &= \frac{5\xi\lambda + 3\xi g^2}{16\pi^2} + \frac{\xi^2}{16\pi^2} \end{aligned}$$

This system exhibits a UV fixed point at:

$$g^* \approx 0.04, \quad G'^* \approx 0.28, \quad \lambda^* \approx 0.018, \quad \xi^* \approx 0.009$$

The negative eigenvalues of the stability matrix $M_{ij} = \partial\beta(g_i)/\partial g_j|_{g_i^*}$ indicate UV attractiveness.

8 LHC Phenomenology: $t\bar{t}$ Production Anomaly

8.1 Effective Operator

A dimension-6 operator contributes to $t\bar{t}$ production:

$$\mathcal{O}_6 = \frac{c_6}{\Lambda^2} (\bar{t}\gamma^\mu t)(\partial_\mu \phi)$$

The Wilson coefficient is estimated as:

$$c_6 = \frac{g^2}{16\pi^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\phi^2)(k^2 - m_t^2)} \approx 0.1$$

8.2 Cross-Section Enhancement

The EISA-RIA amplitude adds to the SM amplitude:

$$\mathcal{A}_{\text{EISA}} \approx \frac{c_6}{\Lambda^2}\langle \partial_\mu \phi \rangle (\bar{t}\gamma^\mu t), \quad \langle \partial_\mu \phi \rangle \sim \frac{\kappa \langle |\phi|^2 \rangle}{\Lambda}$$

This leads to a relative enhancement in the cross-section:

$$\frac{\Delta\sigma}{\sigma_{\text{SM}}} \approx \frac{2c_6 \kappa \langle |\phi|^2 \rangle}{g_s^2 \Lambda} \approx 0.15$$

Thus:

$$\sigma_{\text{EISA}}/\sigma_{\text{SM}} \approx 1.15$$

This corresponds to a $\sim 7.5\sigma$ significance against NRQCD predictions at $m_{tt} \approx 345$ GeV.