Space Distribution Method for Autonomous Vehicles at a Signalized Multi-Lane Intersection

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***Abstract*—Under the connected vehicle environment, autonomous vehicles (AVs) could bring numerous advantages including: improving the traffic flow, enhancing safety and alleviating air pollution. However, optimally operating AVs at signalized multi-lane intersections is a challenging problem due to the complex interaction of vehicles between lanes. It is thus a desire to manage and control the dynamics of AVs at signalized multi-lane intersections. To this end, this paper puts forward a bi-level control framework to optimize the intersection throughput. In our proposed method, the upper level (i.e. the intersection controller) is used to optimize the lane usages of each approach and the AVs' positions. In contrast, the lower level (i.e. the vehicle controllers) receives information from the upper level to control the AVs to get the maximum speed. More specifically, in the upper level, we apply a novel Space Distribution Method (SDM) for the AVs to maximize the throughput (i.e. a number of AVs) of the (multi-lane) intersection where signal timings are predefined. The SDM is divided into three steps: i) platoon formulation; ii) lane-mode optimization; and iii) AVs' position distribution. To maximize the throughput, the intersection controller receives information about the states of the AVs (e.g. the trajectories), then optimizes the lane usages for each approach, the desired speed, and the gap of the AVs as well as the AV's position along the approach. After that, each AV which is allowed to cross the intersection will determine its own trajectory and travel with the scheduled time without crash. Numerical simulations are set up to show that the throughput increases significantly, even more than twice of the throughput obtained from other methods in some circumstances.**

***Index Terms*—Autonomous Vehicles (AVs), intersection controller (IC), vehicle controllers (VCs), platoon, lane-mode, Space distribution method (SDM), throughput.**

I. INTRODUCTION

Traffic congestion is one of the major transportation challenges which the world is facing in the coming years. For example, it has been reported that between 2013 and 2030, traffic congestion will cost the UK economy a staggering £307bn, with the annual cost of congestion set to rise by 63% to £21.4bn over the same period[1]. This research will investigate a potential traffic control solution to address such problems by introducing emerging technologies in autonomous vehicles (AVs). In general, an AV can obtain neighbouring information via vehicle-to-vehicle (V2V) communication and/or vehicle-to-infrastructure (V2I) communication (hereafter, V2X communication for short), and then adopt a suitable control law to achieve a certain objective, such as maintaining constant inter-vehicle spacing within vehicles or smooth driving patterns. Recent studies have shown that the use of AVs in a traffic stream can significantly improve road safety, traffic efficiency, and the environmental sustainability.

Due to the V2X capability, the global environmental perception of drivers is broadened beyond the line of sight. Besides, the high-fidelity traffic data exchange sets up connection and cooperation between drivers and intersection controllers, forming cooperative intelligent transport systems (C-ITS). Furthermore, IEEE 1609 standards [1] using IEEE 802.11 based Dedicate Short Range Communication (DSRC) is established to assist the development of C-ITS, so many emerging signal control strategies were developed, such as in [2] and [3]. Particularly, the operations of AVs are getting significant attention and many methods have been then developed to coordinate the dynamics of the AVs and the central intersection management to reduce the total travel time, energy consumption and the pollution. Dresner [4] and Perronnet [5] proposed a reservation scheme to guide vehicles traveling on two directions of two approaches. In more detail, an intersection is divided into a grid of tiles and vehicles need to send the requested tiles on their planned route to the IC before crossing the intersection. Once the requirement of the vehicle is accepted if there is no conflict with other vehicles, the IC will allocate the tiles and the time slot for it. Otherwise, its request is denied so it might decelerate and wait until a new reservation is accepted.

Jin *et al.* [6] studied a platoon formation for the intersection control by coordinating the adjacent AVs. In this method, the leader and the followers exchange the trajectory information (e.g. speed and position) with the IC to calculate the time to cross the intersection. After that, the platoon leader and the followers shall optimize their trajectories to obey the assigned schedule under safety constraints. Generally, the platoon formation provides better traffic operations and leads to more efficient control strategies as reported in [7], [8], [9]. A bi-level controller for a crossroad was described by in [10], in which the lower level collects and evaluates the information of traffic flow to specify a control policy that stabilizes vehicle flows. Whereas, the upper level firstly utilizes a consensus algorithm to calculate the desired traffic density, then determines the speed of each vehicle. The adjacent intersections then share states of their traffic density among them to boost the system's throughput. Along this line, Zhao *et al.* [11] integrate an intersection control problem with a vehicle trajectory planning method as a bi-level programming problem, in which the upper level is designed to minimize the total travel time by a mixed integer linear programming model. In contrast, the lower level is a linear programming model with an objective function to maximize the total speed entering the intersection. The two levels are coupled by the arrival time and the terminal speed. Schmidt *et al.* [12] designed a similar bi-level control method, in which the upper layer assumes that the speed of each car

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[1]http://highwaysmagazine.co.uk/traffic-congestion-could-cost-uk-300bn

is equal, and estimates its time and order to merge in the control section. By using heuristic rules in the second layer, the acceleration for each car is optimized to avoid crash when merging.

To prevent a car accident, Alonso *et al*. [13] authorized AVs themselves to decide a suitable time to cross the intersection. The AVs must share information about their state such as: position, velocity, driving direction and order. After that, they use the priority tables to determine whether the vehicle should keep traveling or stop until the junction is clear. In the next phase, the look-up table is renewed when each vehicle updates its own priority. The decentralized method was proposed in [14] which determines the best vehicle sequence to go through the intersection. It is done by predicting the arrival time among the cars on the queue, then allowing the one having the shortest arrival time to cross as well as prohibiting the others to cross by sending messages to them. Avoiding the collision of vehicles simultaneously crossing a multi-lane intersection could be solved by adding an addition logic.

Lin and Liu [15] proposed a coordination method in which the road is divided into three sections and let the vehicles pass through the intersection with maximum speed at exactly the calculated time. Moreover, these vehicles are following the planned trajectories so that the fuel consumption was minimized. Markarem *et al*. [16] proposed an alternative control method in which AVs coordinate at the intersection in order to reduce the energy consumption and the pollution. Kamal *et al*. [17] developed a vehicle-intersection coordination scheme without using any traffic lights so that all AVs can pass through the intersection safely and smoothly. Yan [18], Lee [19], and J. Wu [20] managed to formulate an optimization problem in order to minimize the total travel time under safety constraints. They applied a dynamic programming method (DP) to find the heuristic solutions of the problems. Because the multi-lane intersection is very difficult to solve with the DP, the authors utilized Petri nets to model the system, then optimize the sum of the two queues length. Tlig *et al*. [21] used a centralized controller to cooperate the vehicles crossing the intersection alternately. Each vehicle calculates its own trajectory that coincides with three segments: a deceleration segment, a constant speed segment, and an acceleration segment. The deceleration and acceleration of the vehicle is determined, thus they only optimize the speed for the constant speed segment to minimize the total travel time and the energy consumption. To widen the issue for interconnected intersections, the authors studied a bi-level control framework in which intersections share information and adjust their phases to improve the traffic flow efficiency [22]. In the platoon based approach of [9], a receding horizon model predictive control method was proposed to minimize the fuel consumption for platoons and drive the platoons to pass the intersection during a green phase. The method is then extended to dynamic platoon splitting and merging rules for cooperation among AVs and conventional vehicles in response to the high variation of urban traffic flow. In [23] and [24], a parsimonious shooting heuristic algorithm as proposed to optimize the trajectories of a stream of AVs and considered multiple objective functions such as the fuel consumption and the travel time.

To the best of our knowledge, in the vast literature of intersection control methods for AVs (i.e. joint control of signal timings and trajectory planning), there are few attempts dedicated to the lane-based control of the AVs at signalized multi-lane intersections. It is worth mentioning that the lane-based control problem has been proposed in [25], [26] and [27] for human-driven vehicles (HVs) at an isolated signal-controlled intersection. However, with these methods, we cannot control the speeds and the positions of the HVs so that their lane-based optimization method is a passive control. Whereas, the focus of our paper is on the multi-lane intersection control problems for AVs in which we can influence the trajectories of the AVs (i.e. a lane-based control with trajectory planning method). Moreover, Matthew's method in [25] could only be applied for one directional movement in the roads (e.g. either straightforward or left/right turning), so it is impossible to be applied when there are more than two directional movements (e.g. both straightforward and left turning). Although some recent studies in [28], [29], and [30] concern the capacity for mixed HVs and AVs and the optimal lane allocation for connected vehicles, these are not ready for intersection control problems of the AVs. This is because they do not include the information of the queue, stopping at a stop bar, conflict area (AVs from different approaches), blocking by the preceding cars having other direction, etc. For example, in many situations, there are more AVs in one direction than in other directions, or a vehicle near the stop bar could not pass the intersection because of being blocked by the left turning vehicles. As a result, the throughput will be reduced, and this can lead to congestion in the worse cases (e.g. spill-back problems).

To overcome such issues, this paper proposes a bi-level control framework to optimize the throughput at a signalized multi-lane intersection where the signal timings are predefined. In our proposed method, the upper level (i.e. the intersection controller) is used to optimize the lane usages of each approach and the AVs' positions. In contrast, the lower level (i.e. the vehicle controllers) receives information from the upper level to control the AVs to get the maximum speed. The main challenging problem is in the upper level where the intersection controller estimates how many AVs could pass the intersection in their right of way for each phase. Because the number of lanes are a small positive integer, we could solve these problems easily by considering all combinations and choosing the case to get the maximum throughput. In order to do that, we apply a novel space distribution method (SDM) for AVs at a signalized multi-lane intersection with three steps: i) platoon formulation; ii) lane-based optimization; and iii) position distribution.

Formulating platoons is the first step when the AVs enter the network and move with the same speed as the preceding ones (see [8]). This platoon based operations make it easier and more efficient for traffic control strategies [9], [11]. Meanwhile, the vehicles send information about their states (e.g. position and speed) to the IC to maximize the throughput in the second step. Specifically, based on the received information, the IC with predefined signal timings will optimize *the lane usages variables and the number of AVs having permission to cross the intersection (called GAVs)*. Finally, each GAV will calculate its own trajectory (e.g. speed and acceleration) and position distribution (e.g. lane changes) in order to pass through the intersection with maximum speed and smallest gap.

The reminder of this paper is organized as follows. A kinematic vehicle model with safety constraints and the optimization problem are described in Section II. Next, Section III introduces the SDM to be used in our bi-level control method in order to utilize the benefits of the AVs. The numerical results are presented in Section IV to show the advantages of the proposed method. Finally, we conclude the paper in Section V.

II. Kinematic Vehicle Model and Optimization Problem

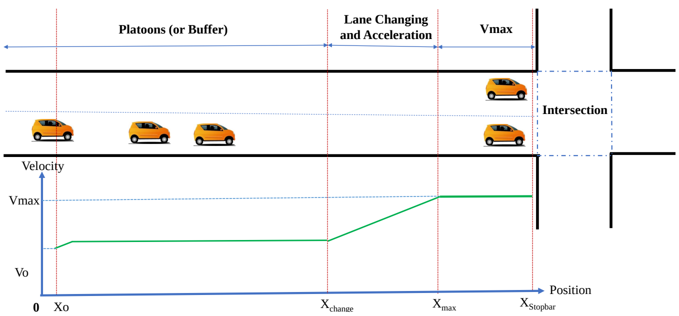


Fig. 1: The optimal AV's trajectory

In this paper, we will divide each approach into three segments as follows: i) platoon segment; ii) lane changing and acceleration segment; and iii) maximum speed segment (see Fig. 1). The first segment is dedicated to a constant speed pattern. In a platoon, each AV communicates with the adjacent ones to obtain their trajectories, then adjusts its own speed to satisfy the safety constraints. The gap between two adjacent vehicles in same lane and moving along the same direction is at least equal to a distance that they travel in 1 second. The second segment is used for changing lanes and accelerating to reach a maximum speed without a collision. Finally, in the last segment, the AVs maintain the maximum speed and smallest gap before reaching the stop bar, then pass through the intersection.

Next, we will describe the kinematic vehicle model and formulate the optimization problem.

A. Kinematic Vehicle Model

Fig. 2 describes the variables of AVs in the global reference frame $X_G Y_G$. We have the following equations of kinematic vehicle model [31]:

$$\dot{x}(t) = v(t)\cos(\theta(t)), \tag{1}$$
$$\dot{y}(t) = v(t)\sin(\theta(t)), \tag{2}$$
$$\dot{\theta}(t) = v(t)\sin(\alpha(t))/l_b, \tag{3}$$

where $(x(t)$, $y(t)$, $\theta(t))$ are the vehicle positions in the global reference frame; $v(t)$ and $\alpha(t)$ are the linear velocity and orientation of the vehicle front wheel, respectively; and l_b is the wheelbase of the vehicle. These equations are used to avoid collisions when the AVs change lanes, merge or diverge from the platoon.

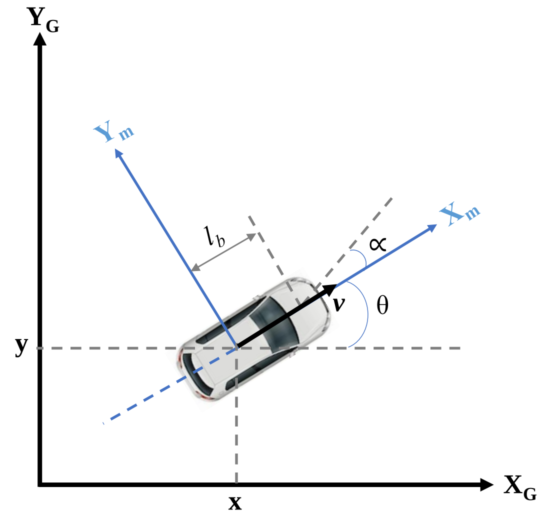


Fig. 2: The variables in the global reference frame

Let $\mathcal{L} = \{1, 2, ..., M\}$, $\mathcal{J} = \{$'west','east','north','south'$\}$, $\mathcal{P} = \{1, 2, 3, 4\}$, denotes the set of lanes, approaches, and phases, respectively. It is noted that all approaches has the same maximum number of lanes, which is equal to M. Let l, j and p be the elements in $\mathcal{L}$, $\mathcal{J}$, and $\mathcal{P}$, respectively. Moreover, the set of AVs in lanes l, approaches j is named as $\mathcal{I}_{lj}$ whose element is indicated as i.

If the AVs travel in the same lane of the same approach, we can simplify these equations as follows:

$$\dot{x}_{ilj}(t) = v_{ilj}(t), \tag{4}$$
$$\dot{v}_{ilj}(t) = a_{ilj}(t), \tag{5}$$
$$x_{ilj}(t) - x_{(i-1)lj}(t) = g_{ilj}(t) \geq v_{ilj}, \tag{6}$$
$$V_{\max} \geq v_{ilj}(t) \geq V_{\min}, \tag{7}$$
$$a_{\max} \geq a_{ilj}(t) \geq a_{\min}, \tag{8}$$

where $x_{ilj}(t)$ and $x_{(i-1)lj}(t)$ are the position of vehicle i and its follower $(i-1)$ in lane l, approach j, respectively; $v_{ilj}(t)$ and $a_{ilj}(t)$ are the speed and the acceleration of vehicle i on approach j, respectively; $g_{ilj}(t)$ denotes the distance between vehicle i and $(i-1)$ in lane l, approach j; $V_{\max}$ and $V_{\min}$ are the upper and lower bound of the vehicles' speed; $a_{\max}$ and $a_{\min}$ are the maximum and minimum acceleration, respectively.

It is worth noting that inequality (6) is a lower limit of the gap between adjacent AVs (i.e. the distance travelled in 1s). Inequalities (7) and (8) are the constraints for the feasible range of speed and acceleration of the AVs, respectively.

B. Optimization Problem

The goal of an optimization problem in this paper is to achieve the maximum throughput at a signalized multi-lane intersection, which is defined as follows:

$$\begin{aligned} \max \int_0^T \sum_{j\in\mathcal{J}} Q_j(t)dt &= \max \sum_{j\in\mathcal{J}} \int_0^T Q_j(t)dt \\ &= \max \sum_{j\in\mathcal{J}} \int_0^T \sum_{l=1}^{M} Q_{lj}(t)dt, \end{aligned} \tag{9}$$

where $Q_j(t)$ is the flow rate per second (i.e. the number of AVs crossing the intersection in a second) from approach j; $Q_{lj}(t)$ is the flow rate per second from lane l of approach j; T is the time horizon considered in the optimization process.

Without loss of generality, we only consider the optimization problem in one cycle of pre-determined signal timings. Let us define:

$$NA_j = \int_0^{T_c} Q_j(t)dt = \sum_{p\in\mathcal{P}} \int_0^{t_p} Q_j(t)dt = \sum_{p\in\mathcal{P}} NA_{jp}, \quad (10)$$

where NA_j is the total number of AVs crossing the intersection from approach j during a cycle; NA_{jp} is the total number of AVs crossing the intersection from road j during phase p; T_c is the cycle length; t_p is the predefined green time of phase p. Besides, for a multi-lane intersection, NA_{jp} can be computed as

$$NA_{jp} = \int_0^{t_p} \sum_{l=1}^{\sigma_{jp}} Q_{lj}(t)dt = \sum_{l=1}^{\sigma_{jp}} \int_0^{t_p} Q_{lj}(t)dt, \quad (11)$$

where σ_{jp} is the number of assigned lanes on approach j during phase p.

In each phase p, there are only two approaches which have vehicles crossing the intersection (during the green time) so $Q_{lj}(t) \geq 0$, while the others are held back (during the red time) so $Q_{lj}(t) = 0$. If $Q_{lj}(t)$ is a constant for all lane l during phase p then:

$$NA_{jp} = t_p \sigma_{jp} Q_{lj}. \quad (12)$$

From equations (10) and (11), the optimization problem (9) for one cycle can be rewritten as

$$\begin{aligned} \max \int_0^{T_c} \sum_{j\in\mathcal{J}} Q_j(t)dt &= \max \sum_{j\in\mathcal{J}} NA_j \\ &= \max \sum_{j\in\mathcal{J}} \sum_{p\in\mathcal{P}} NA_{jp}. \end{aligned} \quad (13)$$

To the best of our knowledge, the relationship between the AVs flow rate and the total travel time at a signalized multi-lane intersection has not been shown. Therefore, we have the following *Proposition*.

Proposition 1. *If we could increase the flow rate $Q(t)$ as quickly as possible then the total travel time of vehicles in the network will be reduced.*

Proof. First, we will deduce the total travel time as a function of the flow rate $Q(t)$, where $Q(t) = \sum_{j\in\mathcal{J}} Q_j(t)$. Indeed, this total travel time can be computed as:

$$\begin{aligned} TT &= \sum_{j\in\mathcal{J}} \sum_{l\in\mathcal{L}} \sum_{i\in\mathcal{I}_{lj}} (t_{ilj}^{out} - t_{ilj}^{in}) \\ &= \sum_{j\in\mathcal{J}} \sum_{l\in\mathcal{L}} \sum_{i\in\mathcal{I}_{lj}} t_{ilj}^{out} - \underbrace{\sum_{j\in\mathcal{J}} \sum_{l\in\mathcal{L}} \sum_{i\in\mathcal{I}_{lj}} t_{ilj}^{in}}_{C} \\ &= \int_0^{T_e} Q(t)tdt - C, \end{aligned} \quad (14)$$

where t_{ilj}^{out}, t_{ilj}^{in} are time when vehicle i in lane l, approach j enters the network and exits the intersection, respectively; C is a constant number which indicates the total time when vehicles enter the network; T_e is the time when the last vehicles exit the intersection.

Note that $\int_0^{T_e} Q(t)dt$ defines the total number of vehicles so $\int_0^{T_e} (Q(t)dt)t$ defines the total travel time. This is also in line with the definition of the total travel time of vehicles to exit the intersection in [32].

Therefore, the proposition can be stated as the following *Lemma*:

Lemma 1. *If there exist two flow rate sequences $Q(t)$ and $Q^*(t)$ which have the same number of vehicles entering (or exiting) the network, but have different flow rates at the same time, or satisfy the following conditions:*

$$\int_0^{T_e} Q(t)dt = \int_0^{T_e} Q^*(t)dt, \quad (15a)$$

$$Q(t) \geq Q^*(t), \quad \text{if} \quad 0 \leq t \leq t_1, \quad (15b)$$

$$Q(t) \leq Q^*(t), \quad \text{if} \quad t_1 < t \leq T_e. \quad (15c)$$

then, we have:

$$\int_0^{T_e} Q(t)tdt \leq \int_0^{T_e} Q^*(t)tdt \quad (16)$$

Proof. It is straightforward to show that:

$$\begin{aligned} &\int_0^{T_e} Q(t)tdt - \int_0^{T_e} Q^*(t)tdt \\ &= \int_0^{t_1} \big(Q(t) - Q^*(t)\big)tdt - \int_{t_1}^{T_e} \big(Q^*(t) - Q(t)\big)tdt \\ &\overset{(a)}{\leq} t_1 \int_0^{t_1} \big(Q(t) - Q^*(t)\big)dt - t_1 \int_{t_1}^{T_e} \big(Q^*(t) - Q(t)\big)dt \\ &\overset{(b)}{=} t_1 \int_0^{t_1} \big(Q(t) - Q^*(t)\big)dt - t_1 \int_0^{t_1} \big(Q(t) - Q^*(t)\big)dt \\ &= 0. \end{aligned} \quad (17)$$

In equation (17), (a) is derived from the inequalities (15b) and (15c); whereas (b) is a result of the constraint (15a). Hence, equation (16) holds. ■

Now, let us consider the flow sequence $Q^m(t)$ such that it has the maximum flow rate in the first time period as follows:

$$\int_0^{T_e} Q^m(t)dt = \int_0^{T_e} Q^k(t)dt, \quad (18)$$

$$Q^m(t) \geq Q^k(t), \quad \text{if} \quad 0 \leq t \leq t_k, \quad (19)$$

$$Q^m(t) \leq Q^k(t), \quad \text{if} \quad t_k < t \leq T_e, \quad (20)$$

where $Q^k(t)$ is an arbitrary flow sequence.

From (15a, 15b, 15c) and (16), the flow sequence $Q^m(t)$ will result in the smallest total travel time. In the other words, *Proposition 1* could be stated as: *maximizing the throughput will minimize the total travel time.* ■

It is worth mentioning here that the capacity does not correspond to the lower speed as in the case of the steady state of the well-known (link-based) fundamental diagrams. Because in the case of signalized intersections the traffic state is highly unlikely to be stable or steady because of stopping in the red phase (i.e. signal effects) and queuing effects. Therefore, the maximum throughput does occur at the maximum speed.

Let us define the AVs which have same direction (e.g. either turning left or going straight) in each approach as PAVs. Now, we will find the way to increase the throughput during a phase by using *Proposition 2*.

Proposition 2. *Assuming that the signal timings are determined so that the total number of PAVs crossing the intersection from approach j during phase p, denoted as NA_{jp}, is maximized if they travel in a platoon with the maximum speed, the smallest gap and the highest number of assigned lanes.*

Proof.

$$Q_{lj}(t) = v^A_{lj}(t)\rho_{lj}(t) = v^A_{lj}(t)\frac{n_{lj}}{D}, \tag{21}$$

where v^A_{lj}, ρ_{lj}, n_{lj} are the average speed, density, and the number of vehicles crossing the intersection into a constant distant D (e.g., $D = 100$ m) from lane l, approach j, respectively.
Then, the sum of n_{lj} vehicle's length and the gaps between them is smaller than the interval D, which is expressed as follows

$$n_{lj}L + \sum_{i\in\mathcal{I}_{lj}} g_{ilj} \le D, \tag{22}$$

where L is the (average) vehicle length.

Assuming that the AVs travel in a platoon, so they have the same speed. From equation (6), we have

$$g^{\max}_{lj} \ge g_{ilj} \ge g^{\min}_{lj} \ge v_{ilj} = v^A_{lj}(t), \tag{23}$$

where $g^{\min}_{lj}$ and $g^{\max}_{lj}$ are a minimum and maximum gap between adjacent vehicles in lane l, approach j, respectively. Then constraint (22) can be rewritten as

$$n_{lj}L + (n_{lj}-1)g^{\min}_{lj} \le n_{lj}L + \sum_{i\in\mathcal{I}_{lj}} g_{ilj} \le D. \tag{24}$$

Therefore, the density in the interval D is limited as

$$\rho_{lj}(t) = \frac{n_{lj}}{D} \le \frac{D + g^{\min}_{lj}}{(L+g^{\min}_{lj})D} \le \frac{D + v^A_{lj}(t)}{(L+v^A_{lj}(t))D}. \tag{25}$$

Then, the flow rate $Q_{jl}(t)$ given in equation (21) has the upper bound as:

$$\begin{aligned} Q_{jl}(t) &\le v^A_{lj}(t)\frac{D+v^A_{lj}(t)}{(L+v^A_{lj}(t))D} \\ &\le V_{\max}\frac{D+V_{\max}}{(L+V_{\max})D} = Q_{\max}. \end{aligned} \tag{26}$$

Consequently, the total number of vehicles NA_{jp} is represented as

$$\begin{aligned} NA_{jp} &= \sum_{l=1}^{\sigma_{jp}}\int_0^{t_p} Q_{lj}(t)dt \le \sum_{l=1}^{\sigma_{jp}}\int_0^{t_p} Q_{\max}dt \\ &\le \sigma^{\max}_{jp}Q_{\max}t_p = NA^{\max}_{jp}. \end{aligned} \tag{27}$$

The equality in (27) happens when

$$v^A_{lj}(t) = V_{\max}, \tag{28a}$$
$$g^{\max}_{lj} = g^{\min}_{lj} = v^A_{lj}(t), \tag{28b}$$
$$\sigma_{jp} = \sigma^{\max}_{jp}. \tag{28c}$$

As a result, *Proposition 2* holds. ■

Thanks to *Proposition 2*, the optimum throughput in one cycle, given in (13), can be calculated as follows:

$$\begin{aligned} &\max_{v^A_{lj},\ g_{ilj},\ \sigma_{jp}} \sum_{j\in\mathcal{J}} NA_j \\ &= \max_{v^A_{lj},\ g_{ilj},\ \sigma_{jp}} \sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}} NA_{jp} \\ &= \max_{v^A_{lj},\ g_{ilj},\ \sigma_{jp}} \sum_{p\in\mathcal{P}}\sum_{j\in\mathcal{J}} NA_{jp} \\ &= \max_{v^A_{lj},\ g_{ilj},\ \sigma_{jp}} \sum_{p\in\mathcal{P}}\sum_{j\in\mathcal{J}}\sum_{l=1}^{\sigma_{jp}}\int_0^{t_p} Q_{lj}(t)dt. \end{aligned} \tag{29}$$

III. Bi-level optimization and space distribution method

As we cannot control the speed and the position of HVs in the traffic network so that the lane-based optimization method in [26] is a passive control. One of the advantages of AVs is that we are able to cleverly distribute their positions and control their speed in order to maximize the throughput via V2X capability (i.e. trajectory planning method). To this end, we propose here a bi-level optimization framework using a space distribution method (SDM) for traffic control under the connected environment to get the benefits of AVs. The main idea of the proposed framework is that the positions nearest to the stop bar in all possible lanes are allocated for all PAVs having the ability to cross the intersection during their phase with the maximum speed and smallest gap. These AVs are called AAVs.

In more detail, *Proposiotion 2* states that the throughput from each approach j during phase p $\left(NA_{jp}\right)$ depends on how the positions (i.e. gap and lane) are allocated and the speed of the AAVs is controlled. This SDM allows to increase a necessary number of assigned lanes for each approach, maximize the speed and minimize the gap of the AAVs.

A. Architecture of the proposed method

Firstly, let us explain how to optimize the number of assigned lanes of each approach. Consider a 4-lane-4-approach intersection with 4 phases: left turning in phase 1 and 3, straight in phase 2 and 4 as Fig. 3. For the sake of simplicity, we ignore the right turning phase in this example. In case of HVs, during the straight phase, only the HVs in lane 1 (or 4) could pass through the intersection because vehicles in lane 2 (or 3) are prevented by the preceding ones. Consequently, the throughput might be reduced. In case of AVs, however, with the AVs' capability, we can allocate all PAVs which could cross the intersection during the green time of this phase to go nearest to the stop bar as Fig. 4b. Especially, if the number of AAVs in approach A (upstream) is much higher than that in approach B (downstream) we could assign three lanes for approach A and only one lane for approach B. As a result, the throughput during the straight phase could increase significantly (i.e. nearly three times in this example). Note that the number of lanes assigned during the left turning phase for HVs' case is only one lane or two lanes [27]. However, by applying the proposed SDM and a suitable phase sequence for AVs' case, we can allow the AAVs to cross the intersection in three lanes as Fig. 4a.

Secondly, as the information (i.e. the speed, position, etc.) of the AVs is known and the traffic signals are predefined, a PAV is considered an AAV if it satisfies the following condition:

$$t_{rp} \geq \frac{d^s_{ilj}}{V_{\max}}, \tag{30}$$

where t_{rp} is the remaining time to a red signal of phase p, d^s_{ijl} is the distance of vehicle i on lane l of approach j to the stop bar. Thus, we can estimate the maximum total number of AAVs and consider them a virtual queue waiting for crossing in the next phase.

Fig. 5 displays the layout of the proposed bi-level optimization framework and describes how to maximize the speed and minimize the gap of the AAVs. It is composed of two levels: the intersection controller (i.e. the upper level) and the vehicle controllers (i.e. the lower level). When the intersection controller (IC) receives information that there are AVs entering the intersection on approach j, it commands these AVs to join a platoon. Next, the IC checks the AVs' possibility to reach the stop bar during the next phase. If they could, the number of AAVs on approach j will be increased. Then, the numbers of AAVs for two approaches in this phase will be used in lane optimization stage to maximize the throughput. After the optimization problem is solved, the IC will send the position and lane information to each AAV waiting for a permission to get through the intersection.

In the lower level, the vehicle controllers (VCs) of these GAVs will calculate their trajectories by themselves in order to reach an advisory speed. Besides, they communicate and cooperate with each other in order to have the smallest gap. These trajectories will satisfy safety constraints (4-8). Finally, the GAVs execute this stage by the following steps: change lanes, accelerate and maintain the maximum speed and the

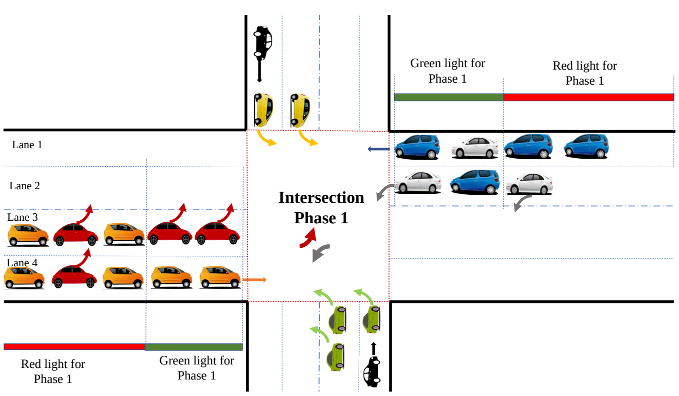


Fig. 3: The 4-lane-4-approach intersection and 4 phases with predefined signal times for human-driven vehicles. The red and white cars and will turn left, orange and blue cars will go straight, etc.

minimum gap at the stop bar with safety constraints. On the other hand, if the AAVs do not receive the permission, they must keep traveling in the platoon and waiting for the next cycle.

The advantage of using such bi-level control framework is that it alleviates the IC's computational demand for AVs' trajectories. In particular, if the number of AVs grows (e.g. more than hundreds) or there are some disturbances (e.g. obstacle or interrupted communication), then the computational demand of a centralized controller may rise exponentially to guarantee the safety constraints. Accordingly, the centralized IC no longer gets resources to do other works in the future. For instant, it could optimize the traffic signals or cooperate with the neighbor intersection controllers in a multi-intersection traffic network.

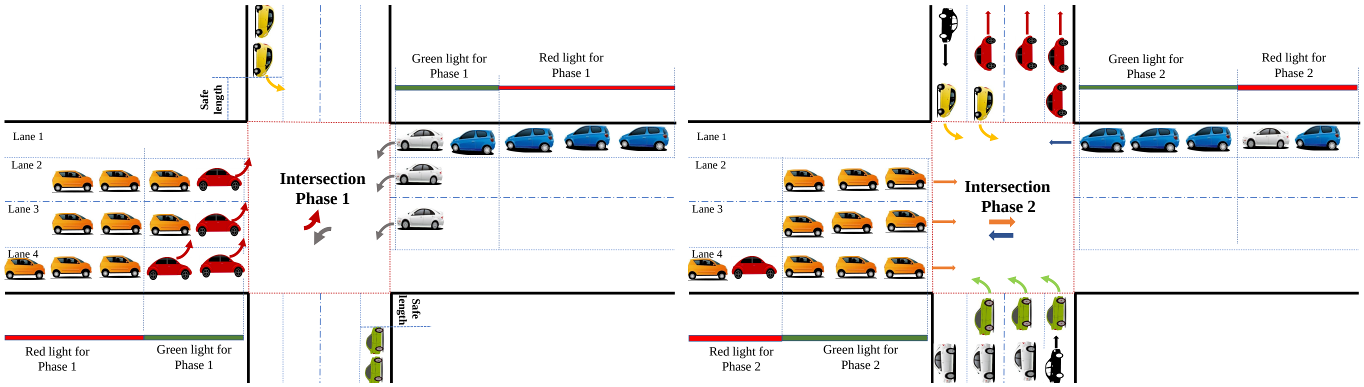
B. Distributing AVs' positions

In this section, we describe how to allocate the vehicle's positions on the upstream approach so that they could easily change lanes and accelerate without blocking the vehicles in the other phase. To this end, approach j is divided into 4 segments: Origin, Platoon (or buffer), Lane Changing, and Maximum Speed (see Fig. 6a).

The Origin is the first segment when vehicles enter network randomly. The next one is a platoon formulation segment where vehicles are arranged in a platoon and travel with the same speed under safety constraints. If traffic on approach j is free-flow or much less than that on the other approaches, the AVs should be allocated in one lane as Fig. 6a. The AAVs allowed to cross the intersection in the next phase will be nearest to the stop bar. This allocation will permit other congested approaches to get more lanes in order to boost their throughput. In contrast, if approach j is congested, then the AVs could be distributed over two lanes as in Fig. 6c. In both cases, the AVs (i.e. the red vehicles) are not blocked by the others (e.g. the yellow vehicles) and can move to the third segment easily. Having reached the last segment, the speed of the GAVs is nearly maximum, and they attempt to minimize their gaps. Finally, they reach the stop bar in time with the maximum speed and minimum gap.

Moreover, we present here ways to avoid a collision in the downstream approach j when the upstream approach is assigned with many lanes (e.g. 3 out of 4 lanes). If there are few AVs on approach j, only one lane will be used as shown in Fig. 6b. However, it should be careful when approach j is congested because it could use more than two lanes. Avoiding collision in Lane 3 can be done by merging the GAVs (coming from Lane 3) as soon as possible, as shown in Fig. 6d. The first method to merge GAVs is that the coming platoon in Lane 3 slows down, then joins in the tail of Lane 2. Another way is slowing down the platoon in Lane 2 and increasing the gap between the vehicles so that each GAV in the third platoon can leave it. Therefore, the length of the two segments (i.e. the Lane changing and maximum speed) must be long enough to prevent a crash but not too long, because it will alleviate the number of AAVs to approach the stop bar during the green time of the next phase.

It is noted that, the order of consecutive phases is very important because it helps to avoid conflicts at the stop bar



(a) If Phase 1 is active

(b) If Phase 2 is active

Fig. 4: Space Distribution Method for AVs

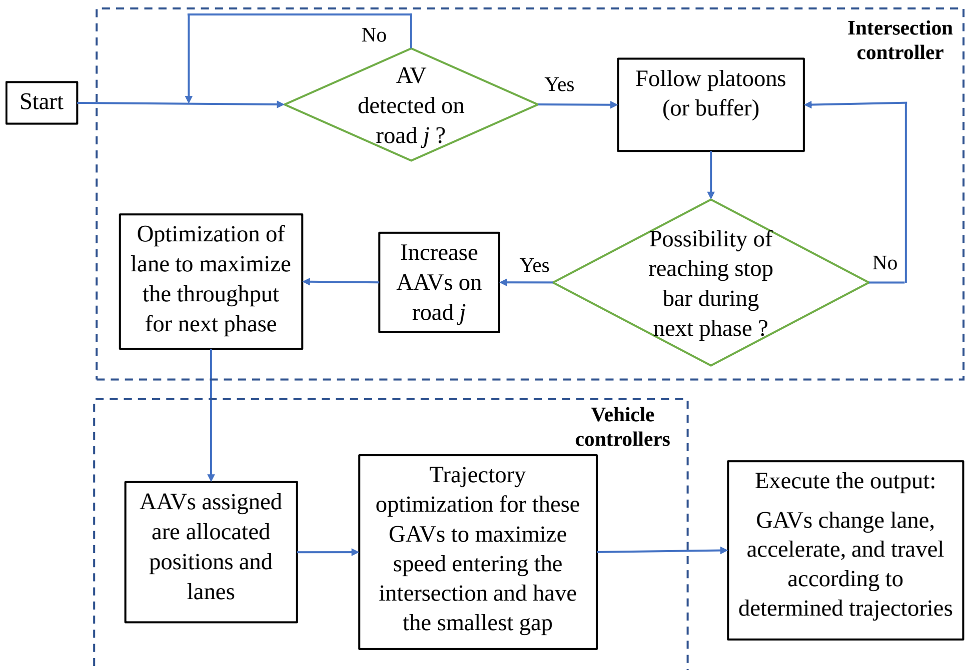


Fig. 5: The layout of the bi-level optimization framework

between the GAVs crossing the intersection to approach j and the AAVs approaching from approach j. Take the situation in Fig. 6d as an example, if the next green time is for the red vehicles, the red AAVs will reach the stop bar in three lanes and collide with the blue GAVs (crossing) at the stop bar. The red AAVs (in the westbound) are only permitted to travel to the stop bar when the prior green time is designated for other approaches (i.e. the northbound and southbound approaches). The phase sequence defined as Fig. 7 will guarantees this constraint.

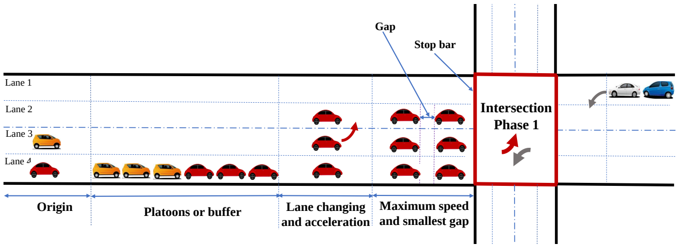
C. Optimization Sub-problems

So far, we have represented how the proposed method (i.e. the SDM in the upper level) could control the GAVs to obtain the maximum speed and smallest gap on each approach. Consequently, *Proposiotion 2* specifies that the maximum throughput of each approach NA_{jp} only depends on the number of lanes assigned. Thus, the optimization problem for a maximum throughput (29) transforms to how the lane usages for each approach during each phase are assigned when the maximum number of AAVs (or the length of the virtual queues) are known. Since the traffic signals and the phase sequences are predefined, the lane variables in each phase are independent on each other. As a result, the problem becomes simpler and could be divided into 4 (optimization) sub-problems for 4 phases as follows.

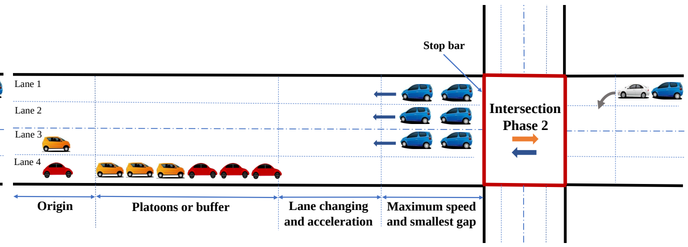
$$\max_{\sigma_{jp}} \sum_{j \in \mathcal{J}} NA_{jp}, \tag{31}$$

where σ_{jp} is the number of the assigned lanes for approach j on phase p.

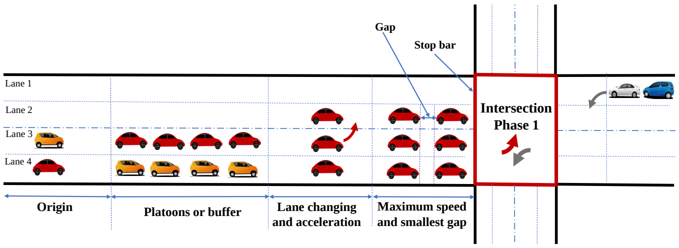
In each phase, there are only two approaches with switching green times, so let σ_{ap} and σ_{bp} denote their corresponding number of assigned lanes. In the straight phases (i.e. phase 2 or 4), to avoid a collision from two opposite directions (i.e. the



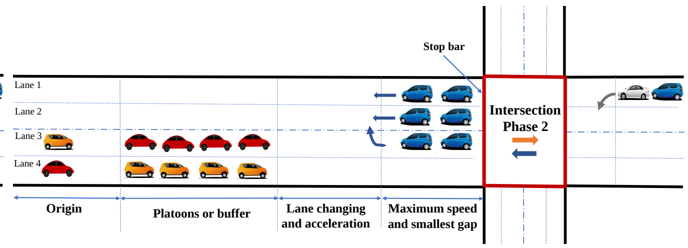
(a) If approach j (upstream) is free-flow



(b) If approach j (downstream) is free-flow



(c) If approach j (upstream) is congested



(d) If approach j (downstream) is congested

Fig. 6: Distributing AVs' Positions in the approach j

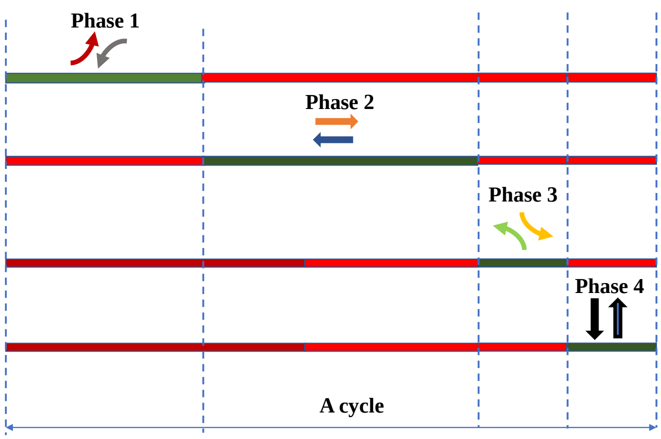


Fig. 7: A predetermined cycle for traffic signals

north-southbound or the west-eastbound) we need to impose the following constraint:

$$\sigma_{ap} + \sigma_{bp} = M \tag{32}$$

In additions, the constraints in the left turning phases (i.e. phase 1 or 3) are following:

$$\sigma_{ap} \leq M \tag{33a}$$
$$\sigma_{bp} \leq M \tag{33b}$$

It shall be noted that, σ_{ap} and σ_{bp} could be equal to M when there is no vehicle ($AAVs = 0$) on the downstream during phase p. Having crossed the intersection, they must merge/change lanes as soon as possible to avoid a collision.

Moreover, when solving the optimization problem for cases in which the number of AAVs on approach A is much higher than that on approach B during phase p, it could lead to $\sigma_{ap} = M$. It means that the AAVs on approach B may have to wait for a long time to cross the intersection. To balance this, we should have the following constraint in case the number of AAVs on approach B is non zero:

$$\sigma_{ap} < M \tag{34}$$

In summary, the optimization sub-problems for each phase become as follows:

$$\max_{\sigma_{ap},\sigma_{bp}} (NA_{ap} + NA_{bp}) \tag{35}$$

s.t

$$\begin{array}{ll} (33a), (33b) & \text{if} \quad p \in \{1, 3\}, \\ (32) & \text{if} \quad p \in \{2, 4\}, \\ (34) & \text{if there are some AAVs in the downstream} \end{array}$$

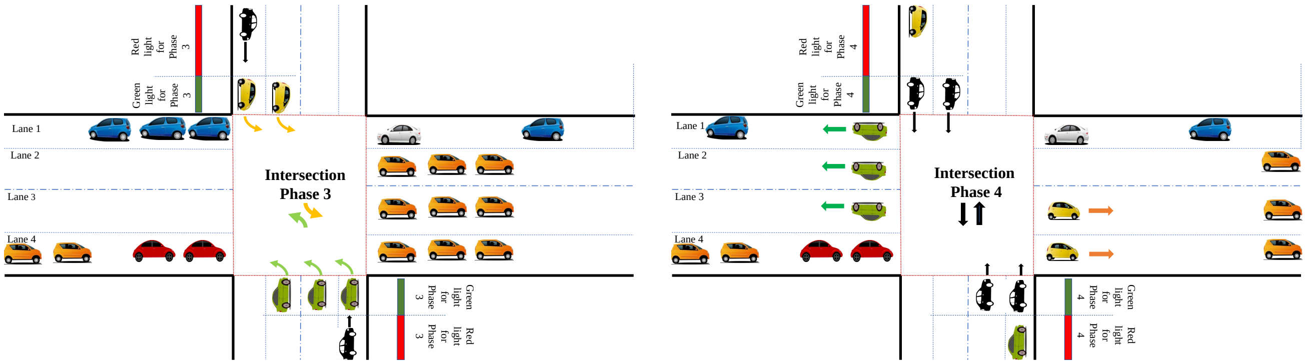
IV. Numerical Studies

A. Simulation setup

Let us consider again the 4-lane-4-approach intersection as in Fig. 3 and the traffic signals for 4 phases are predefined as in Fig. 7. The order of 4 phases sequence is left turning from the west to the north, straight from the west to the east, left turning from the north to the east, and straight from the north to the south. It is important to obey this order because it helps to avoid collisions and gives time for the AAVs in the next phase to move to the assigned positions.

Vehicles travel with maximum velocity which is $20m/s$, and minimum velocity which is $3m/s$. The maximum and minimum accelerations of vehicles are set at $4.0m/s^2$ and $-3.0m/s^2$, respectively. There are total 36 vehicles considered in this test including: 17 vehicles from the west, 8 vehicles from the east, 5 vehicles from the north and 6 vehicles from the south. The red and white AVs turn left in Phase 1, the orange and blue ones go straight in Phase 2, the yellow and green ones turn left in Phase 3 , and the black ones go straight in Phase 4.

To compare with the state-of-art results, four methods are studied to optimize the throughput as follows: human-driven



(a) If Phase 3 is active

(b) If Phase 4 is active

Fig. 8: Space Distribution Method for AVs

	Phase 1		Phase 2		Phase 3		Phase 4		NA
	WL	EL	WT	ET	NL	SL	NT	ST	
HVM (L)	1	1	1	1	1	1	1	1	
NA_{jp}	1	1	2	1	1	1	0	1	8
NLM (L)	1	1	2	2	1	1	1	1	
NA_{jp}	2	1	4	4	1	1	1	1	15
LM (L)	2	2	3	1	2	2	2	1	
NA_{jp}	3	2	7	2	2	2	2	1	21
SDM (L)	3	3	3	1	2	3	2	2	
NA_{jp}	4	3	9	3	2	3	2	2	28

TABLE I: The number of lanes assigned and throughput of the intersection for each phase with 4 methods: HVM, NLM, LM and SDM

vehicles method (HVM), non lane base method (NLM), lane base method (LM), space distribution method (SDM). In more detail, the HVM is used for only HVs without communication between them, so they only attempt to reach the stop bar quickly. Whereas the NLM and the LM are adopted for AVs and attempt to avoid stop-and-go waves by calculating their trajectories, so that the AVs could reach the stop bar with maximum speed under safety constraints. However, the NLM does not optimize the number of lanes, but the LM does. The SDM proposed in this paper does not only calculate the maximum number of lanes, but also estimates and allocates the AAVs' positions as well as minimizes their gaps.

B. Simulation results

Let us look at Fig. 3 again in which we will optimize the throughput for each phase.

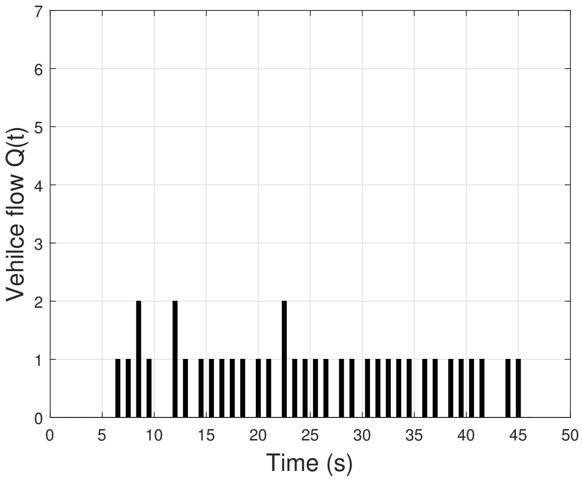
Phase 1: Note that most methods do not allow the vehicles to change lanes or overtake when they are near the stop bar. It can be seen that for Phase 1 there are only maximum 3 HVs on both approaches (i.e. the west and the east one) which could pass through the intersection. It is because some vehicles (i.e. the red and the white ones) allowed to go in this phase are blocked by the others (i.e. the orange and the blue ones) in the next phase. Due to the fact that the IC in case of HVM is not able to control the HVs' speed and gap, so only 1 or 2 HVs could cross the intersection.

Nonetheless, we can allocate the positions of the AAVs in more lanes and should have a safe length for the other approaches to avoid a collision when turning left as Fig. 4a. In this method, the IC gets information (i.e. the position, speed, etc.) from the AVs, then estimates that maximum 7 AAVs (i.e. 4 AAVs on the west and 3 AAVs on the east) could pass the intersection during the green time of Phase 1. It is thus straightforward to solve sub-problem (36) which meets both constraints (33a, 33b) and (34), so that the results are $\sigma_{w1} = \sigma_{e1} = 3$. In this phase, both the west and the east approaches could allow the vehicles on maximum three lanes to cross the intersection because of the fact that there are some AAVs in the north and the south (downstream) approach. Accordingly, 4 red cars in the west and 3 white cars in the east will speed up and change lanes in Lane changing segment under safety constraints. Consequently, these GAVs (also AAVs in this phase) will be distributed among three lanes of the upstream approaches (i.e. the west and the east), and the AVs in the downstream approaches (i.e. the north and the south) must be allocated in one lane. Having crossed the intersection, they should move into two lanes to avoid a collision. In Phase 1, hence, we have:

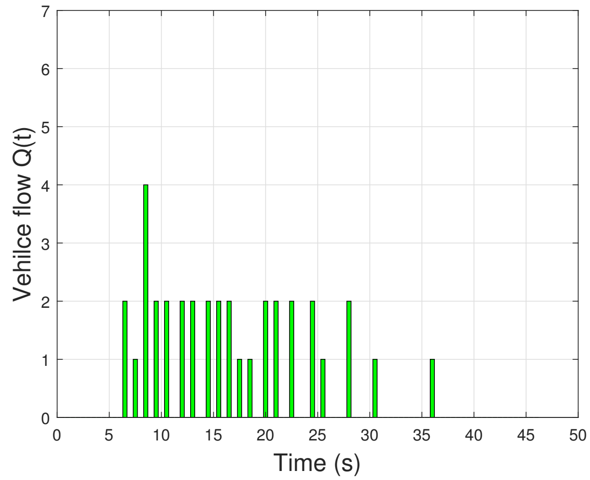
$$\sum_{j \in \mathcal{J}} NA_{j1} = 7$$

Phase 2: It is noted that each phase may affect the consequent ones in three methods HVM, NLM, and LM. For further information, as the red and orange vehicles share the same Lane 3 and if the red one cannot move in Phase 1, it will prevent the orange ones to move in Phase 2. Apparently, the flow rate in the next phases is negatively affected and the throughput of the intersection shall be alleviated. On the other hand, by using the optimal space distribution in the proposed framework, the red platoons and the orange platoons do not influence each other, hence, maximizing the throughput for each phase is independent.

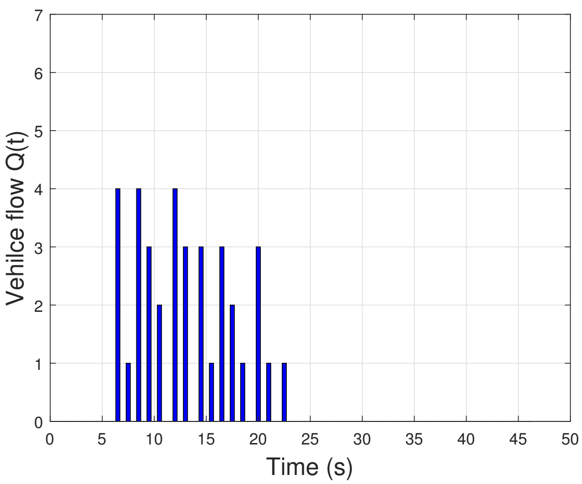
Similarly, the maximum number of vehicles able to get a permission to cross the intersection in the straight phase are 7 for three methods HVM, NLM, and LM. Nevertheless, by using the proposed method, we can estimate the maximum number of the AAVs on the west approach and the east approach to be 9 and 4, respectively. By solving the optimization sub-problem for lane variables with constraints (32) and (34), we obtain $\sigma_{w2} = 3, \sigma_{e2} = 1$. That is three lanes are assigned for the west approach and one lane for the east approach. As



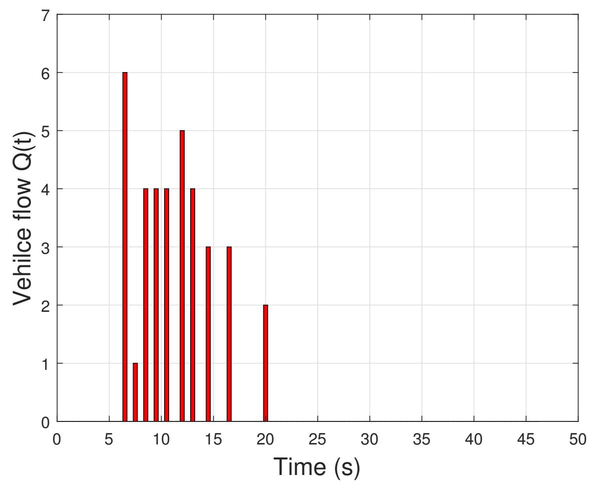
(a) Human-driven vehicles (HVM)



(b) Non Lane Method (NLM)



(c) Lane Method (LM)



(d) Space Distribution Method (SDM)

Fig. 9: Flow rate of 4 methods: HVM, NLM, LM, SDM

	Phase 1		Phase 2		Phase 3		Phase 4		NA
	WL	EL	WT	ET	NL	SL	NT	ST	
HVM (L)	1	1	1	1	1	1	1	1	
NA_{jp}	1	1	2	1	1	1	0	1	8
NLM (L)	1	1	2	2	1	1	1	1	
NA_{jp}	2	1	4	4	1	1	1	1	15
LM (L)	2	2	3	1	2	2	2	1	
NA_{jp}	3	2	7	2	2	2	2	1	21
SDM (L)	3	3	3	1	2	3	2	2	
NA_{jp}	4	3	9	3	2	3	2	2	28

TABLE II: The number of lanes assigned and the throughput of the intersection for each phase with 4 methods: HVM, NLM, LM and SDM

a result, the GAVs' numbers of these approaches are 9 and 3 in that order. This indicates that:

$$\sum_{j\in\mathcal{J}} NA_{j2} = 12$$

Phase 3 (Fig. 8a) and **Phase 4** (Fig. 8b) could be done in the same way as in **Phase 1** and **Phase 2**, ultimately.

Table I displays the best results of the four methods in each phase, where WL, ET, NL, etc., denote the upstream approach in each phase. That is in the WL (west-left): vehicles cross the intersection from the west in left turning phase (Phase 1); in the NT (north-straight): vehicles cross the intersection from the north in the go straight phase (Phase 3). There are only two upstream approaches which allow vehicles to cross the intersection in this phase such as: the west-left (WL) and the east-left (EL) in Phase 1. In each method, the first row describes the number of lanes assigned for the upstream approaches at each phase. For example, the SDM permits 3 lanes for WL and 3 lanes for EL to travel at Phase 1, but it assigns 3 lanes for WT and only 1 lane for ET at Phase 2. On the other hand, the second row exhibits the throughput from the upstream approach j during phase p (NA_{jp}) as well as the total throughput (i.e. in the last column). Table I shows that the SDM always obtains the greatest throughput ($NA = 28$) along with a number of lanes assigned in four methods. In some cases, the throughput from the SDM could be nearly twice or three times in comparison with that from the NLM,

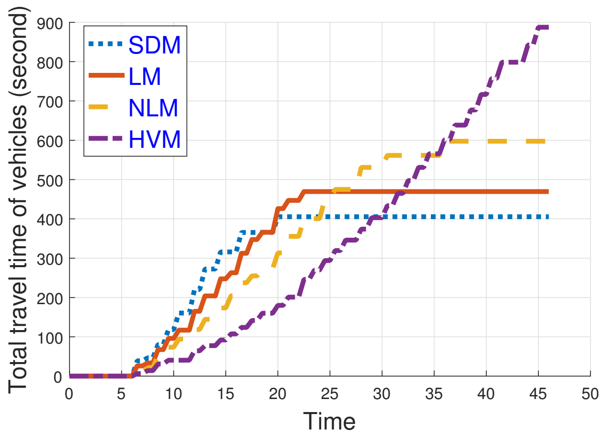


Fig. 10: Total travel time produced from 4 methods

	HVM	NLM	LM	SDM
Average Travel Time (s)	24.6528	16.5972	13.0417	11.2639
Average Delay Time (s)	18.5	10.444	6.8889	5.111
Reduction in delay (Compare with NLM)	-77.12%	0%	34.04%	51.06%

TABLE III: The average travel time and delay time of vehicles from 4 methods: HVM, NLB, LB, SDM

especially in the left turning phase. The LM ($NA = 21$) is better than the NLM ($NA = 15$) because it allows the AVs to travel in more lanes. The average speed, gap and lanes from the HVM cannot be controlled, thus it has the least throughput ($NA = 8$).

Furthermore, Fig. 9a- Fig. 9d illustrate the flow rate $Q(t)$ over time in four methods: HVM, NLM, LM, and SDM, respectively. In the first period time (from 6s to 16s), the SDM always has the largest flow rate and the shortest time ($T_e = 21$s) for the last vehicle to exit the intersection. The LM needs a little more time ($T_e = 23$s) than the SDM, whereas the NLM and the HVM require much higher time, i.e. $T_e = 36$s and $T_e = 45$s, respectively, to do this. The reason is that the NLM and the LM do not exploit the superiority of the AVs while the HVM is not able to control the vehicles' speed and position.

As a result, the total travel time or the average time delay from the SDM is always smallest as shown in Fig. 10 and Table II, which is consistent with *Proposition 1*. The delay from the SDM is reduced by more than a half (51.06%) compared with that from the NLM, and by one third of the delay from the HVM.

From the above results, the SDM shows that it can achieve the most efficient performance.

V. Conclusions

A. Remarks

This paper has presented a bi-level optimization framework using a space distribution method for the operations of the AVs at a signalized multi-lane intersection with predetermined signal timings. By exploiting the superiority of the AVs and knowing the predefined signal timings as well as the phase sequence, the intersection controller (i.e. the upper level optimization) will get information from the AVs, then optimize the lane variables and their positions to maximize the throughput. In the vehicle controllers (i.e. the lower level), each AV having permission to cross the intersection will evaluate its own trajectory to reach the stop bar in time as well as the exact position, and obtain the maximum speed and the minimum gap. This method enhances the throughput and reduces the total travel time remarkably. More specifically, our numerical results have shown that the proposed method outperformed the other methods in most cases. Particularly, the throughput from our proposed framework is increased nearly twice and the delay is reduced by more than 51% in comparison with those from the non-lane based method. Nevertheless, this improvement is only limited to simple cases at this proof of concept stage. More intensive experiments with realistic data should be carried out in the future to confirm our findings.

B. Recommendations

The limitation of this paper is that all vehicles are required fully connected and automated, and the IC and the AVs must have perfect communications without any delays or interruption. Because, if there are some vehicles (e.g. left turning) which do not follow the instructions from the intersection controller due to some reasons (e.g. human vehicles, interrupted communication), it could prevent the others (going straight) to change lanes to their assigned position or block them to cross the intersection at this time.

In addition, because SDM method needs three segments on the road, and the platoon segment is used to store the vehicles without permission to move, then the road needs to be long enough to reserve these vehicles. Finally, the more the speed of AVs increases, the more the collision risk between the AVs in two consecutive phases at the intersection. Hence, their time and position are required to measure exactly.

Due to these above reasons, the vehicles need to be perfectly connected and automated, then we assume that they can communicate with no delay (5G) and quickly respond (0.1 second). Therefore, the car-following model is released to simplify the problem (easier for calculation and simulation) in which the follower can simultaneously change the velocity with the leader. Future research should overcome these and consider those situations where some HVs (i.e. without communication capability) or connected vehicles (i.e. without speed control capability) are present (i.e. heterogeneous traffic flow).

The problem in this paper is solved when the signal timings are known. It is much more challenging if we do not know them in advance and thus should optimize these variables. Future research should consider determining the best traffic plans in the proposed framework to achieve better results. Besides, in each signalized intersection, the optimization problem depends on the original states (i.e. velocity and position) of vehicles, the number of lanes and the length on each road, the information (i.e. duration and sequence) of phases, and the interaction of these intersections, so the variables of network are large and mutually related. Therefore, it is difficult to search all combinations to find the optimum solutions in a network. To solve this problem, we plan to use the decentralized methods to find the solutions in the future works. For example, we can use Alternating Direction Method of Multiplier (ADMM) to divide the intersection network into many smaller problems in which the local optimum solutions are easier to find.

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