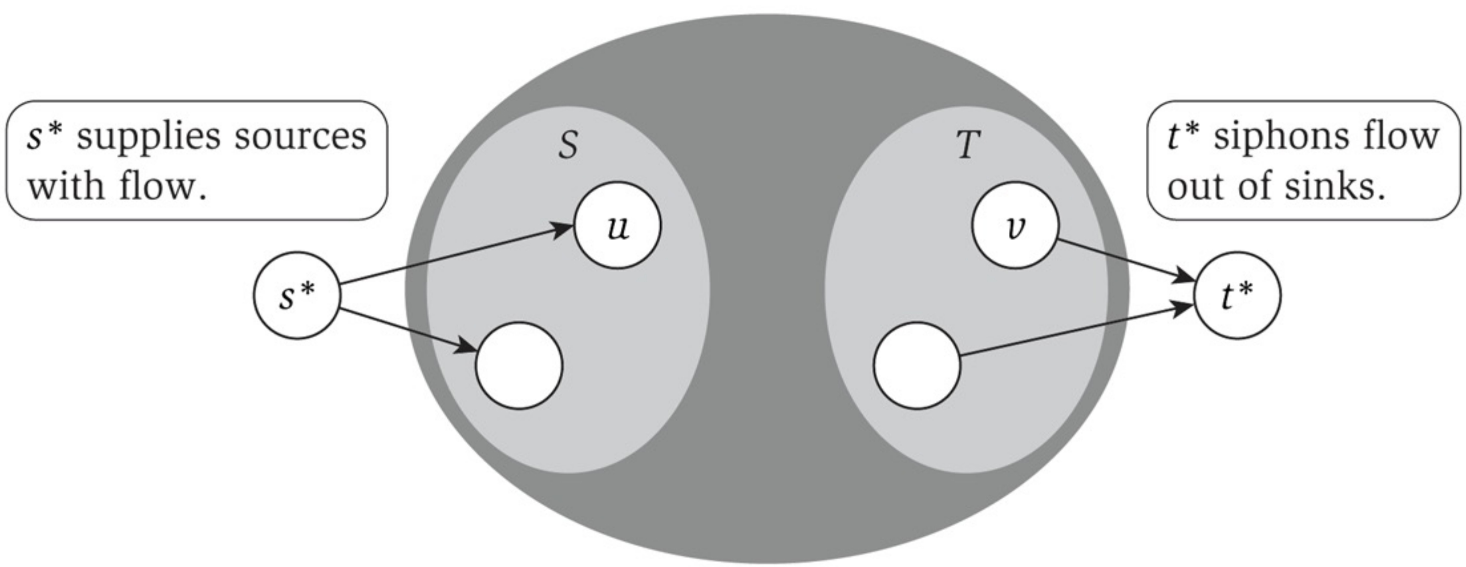
Max-flow variations

Finding a Circulation

- Real world applications don't have just one source and sink
  - Instead there are multiple ones: power production / consumption, etc.
- We designate a set S to be all the nodes that are sources
  - We can also view them has having negative demand
- Likewise, we designate a set T to be all the nodes that are sinks
  - They have positive demand
- Networks with multiple sources and sinks (modeled using demand) are called *circulation networks*

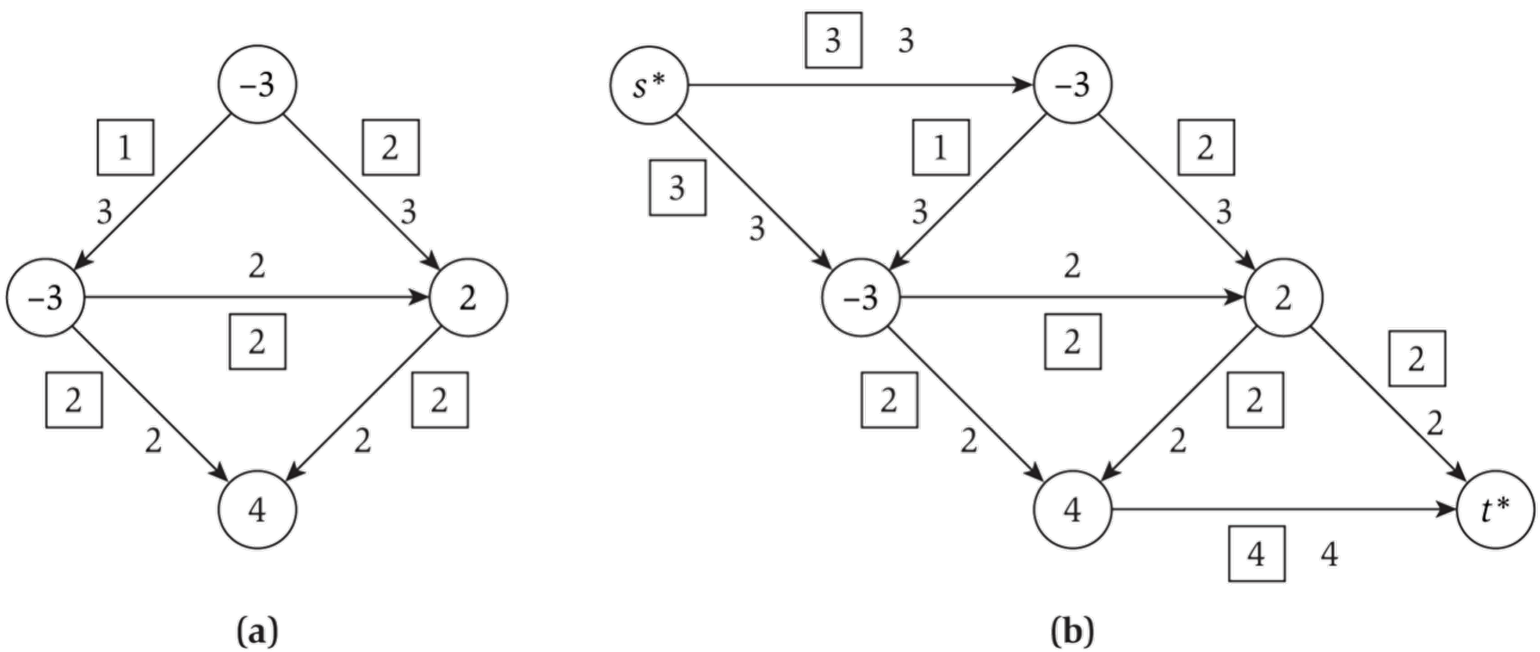
Reduction to max-flow

- With a few modifications, we can make this a max-flow problem:
  - Create a 'super source' s* with edges to each node in S
    - The capacity of that edge is the size of the source of the node in S
  - Likewise with the set T



Conversion example

- Converting a graph with multiple sources and sinks to a single-source-single-sink max-flow problem:



Circulation notes

- A circulation problem is aiming for *feasibility*, not max flow
  - But we use max flow to solve it
- We set each edge from the super-source to each individual source to be the absolute value as the individual source's demand
- Max-flow is then run
- If the total amount leaving the single-source is the SAME as the capacity of each outgoing edge, then the circulation is feasible

Edge lower bounds

- So far, we have considered only the capacity of an edge: the upper bound on the flow
- We also want to consider a lower bound on the flow on an edge
  - i.e. forcing a certain amount of flow through an edge
- We will reduce this to a circulation problem
  - Which can then be reduced to a max-flow problem

Handling lower bounds

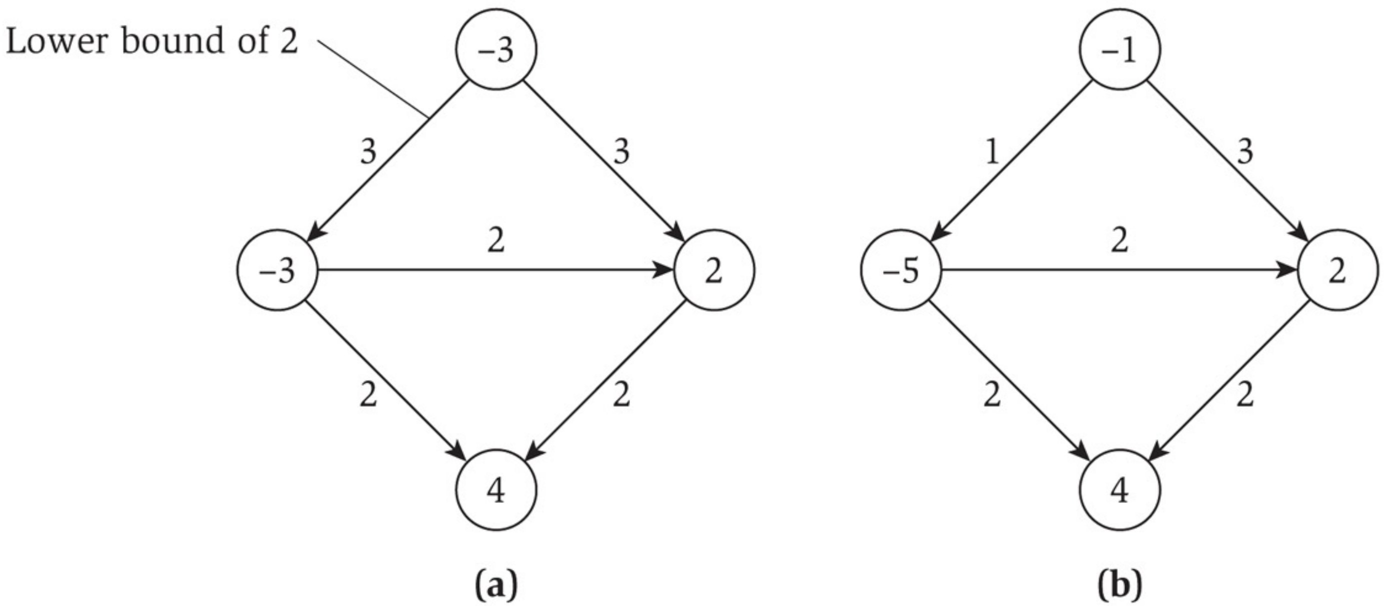
- A lower bound forces flow across an edge
  - Which increases demand at the start of the edge (to compensate for the flow across the edge)
  - And decreases demand at the terminus of the edge (as some flow is fulfilling the demand)

Solving a flow with lower bounds

- Given a circulation network G, construct a new graph G' such that for each edge e from u to v with a lower bound l_e:
  - We decrease the capacity on that edge by l_e
    - As that is the flow that is moving through the edge
  - We increase the demand at u by l_e
  - We decrease the demand at v by l_e
- Then solve G' as a circulation problem
  - i.e. add a super-sink and super-terminus, and solve as a max-flow problem

Eliminating a lower bound

- Diagrammatically…



Is this feasible? Can this work if we enforce the lower bound?

- No!
- How to find out? Convert to NW Flow, solve, check amount leaving the single source.

Final Words

- Circulation problems
  - Like max-flow but slightly different. Check if feasible (not max)
  - Variation: put a lower-bound on some connections
- Note all this is really just more reductions!
  - Solving variations of max-flow by converting the problem into an instance of "normal" max-flow.