

Module

1. Formally prove that the *Bi-Partite Matching* algorithm we saw in class is optimal (i.e, it always find the optimal matching between nodes in the bi-partite graph). *HINT: Assume the algorithm is not optimal and show that you must be able to still find an augmenting path through the network, contradicting your assumption that max-flow terminated.*
2. (**Note:** Problem wording was updated on 4/28/2021 at 11:30am.)

Delta Airlines is trying to convince more customers to become *SUPER DUPER PLATINUM MEMBERS* (SDPMs), to raise their profits. However, after surveying customers, they discovered that the most attractive feature of the SDPM program is the ability to sit in the *member's lounge* at certain select airports.

To make the SDPM program more attractive, Delta wants to put a *member's lounge* in every airport they service but they simply cannot afford it yet. After a long discussion with the executive task force on SPDM member benefits, it is determined that for now, Delta would like to make sure that for all flights, every airport or an airport connected to that airport (or maybe both) is guaranteed to have a lounge.

Given a graph representing the airports Delta flies to and the flights between them, the SDPM problem is to decide whether k *member's lounges* can be placed in order to ensure that every airport or an airport connected to it has a lounge. In other words, if an airport does not have a lounge, one of its "neighbors" must have one.

Show that SDPM problem is NP-Complete.

Note: You are not being asked to explicitly solve the SDPM problem; you are only required to show that it is NP-Complete.