

Church's lambda calculus reduces all of computation to what?

The definition and application of functions.

What is the bridge from lambda calculus to a language like ML or Haskell?

The addition of features definable in the lambda calculus itself.

Some are easily defined: *e.g.*, syntax for numbers and collections.

Others are more challenging, *e.g.*, mutable reference cells.

What are the three kinds of terms in lambda calculus?

Variable:  $v$

Abstraction:  $\lambda x. t$

Applications:  $t \ t$

What's the difference between an *internal* and *external* language?

An external language contains derived forms and can be translated to an internal language which uses only core features.



Which way does function application  
associate?

To the left.

The following two are equivalent:

$f \circ g \circ u$

$(f \circ g) \circ u$

The body of a lambda abstraction extends to  
where?

As far to the right as possible.

What's the difference between a *bound* and *free* variable.

What's a *closed term*?

A variable is free if it is not bound by an enclosing abstraction.

Closed terms (aka *combinators*) have no free variables.

Describe computation in the lambda calculus.

A *redex* term:

$(\lambda x. t_1) t_2$

Evaluation by *beta reduction*: a new term, namely  $t_1$  with each instance of  $x$  in it replaced by  $t_2$ .



What is an evaluation strategy?

Do not confuse with ...

A rule for determining which redex in a term can be evaluated when.

... *calling convention* which is unrelated, even though some of the evaluation strategies have "call" in their name.

Name some evaluation strategies.

Which is the most popular?

- Full beta-reduction
- Normal order
- Call-by-name
- Call-by-value

Call-by-value is the most common.

What is *full beta-reduction*?

An evaluation strategy which allows any redex in a term to be evaluated in the next step.

What is the *normal order* strategy?

An evaluation strategy in which only the outermost redex can be evaluated in the next step.



What is the *call-by-name* strategy?

An evaluation strategy that follows the normal order strategy but does not allow reductions inside the body of an abstraction.

What is a *value*?

A term that cannot be further reduced.

What is the *call-by-value* strategy?

An evaluation strategy that follows the normal order strategy but in which redexes are reduced by first reducing its argument to a value.

What's the difference between *strict* and *non-strict* evaluation strategies?

In strict evaluation strategies arguments to abstractions are evaluated regardless of whether they are used in the body of a function. In a non-strict (aka *lazy*) evaluation strategy an argument may not be evaluated if it's not used in the body.



What evaluation strategy does Haskell use?

*Call-by-need*, which is call-by-value with memoization. More specifically, after an argument is evaluated all other instances of that argument will be replaced with the value.

How does the lambda calculus represent multiple argument functions?

Using *currying*, a technique for representing multiple argument functions as higher-order unary functions.

In general:

$$f = \lambda (x, y) . s$$
$$\lambda \{x\} . \lambda \{y\} . s$$

What is *Church encoding*?

Techniques for embedding data and operators into the lambda calculus. It is equivalent to *Gödel numbering*. The latter is used for natural numbers, the former for lambda abstractions.

Define the two Church booleans.

They are binary functions. `true` simply returns its first argument, `false` its second.

```
true =  $\lambda t.\lambda f.t$  false =  
 $\lambda t.\lambda f.f$ 
```



Church encode `if/else` expressions.

$\text{ifelse} = \lambda l. \lambda m. \lambda n. \text{if } l \text{ then } m \text{ else } n$

**Now**  $\text{ifelse } b \ v \ w$  **will return**  $v$  **if**  $b$  **is** `true`, **and**  $w$  **if**  $b$  **is** `false`.