Dynamic Private Task Assignment under Differential Privacy

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***Abstract*—Data collection is indispensable for spatial crowdsourcing services, such as resource allocation, policymaking, and scientific explorations. However, privacy issues make it challenging for users to share their information unless receiving sufficient compensation. Differential Privacy (DP) is a promising mechanism to release helpful information while protecting individuals' privacy. However, most DP mechanisms only consider a fixed compensation for each user's privacy loss. In this paper, we design a task assignment scheme that allows workers to dynamically improve their utility with dynamic distance privacy leakage. Specifically, we propose two solutions to improve the total utility of task assignment results, namely Private Utility Conflict-Elimination (PUCE) approach and Private Game Theory (PGT) approach, respectively. We prove that PUCE achieves higher utility than the state-of-the-art works. We demonstrate the efficiency and effectiveness of our PUCE and PGT approaches on both real and synthetic data sets compared with the recent distance-based approach, Private Distance Conflict-Elimination (PDCE). PUCE is always better than PDCE slightly. PGT is 50% to 63% faster than PDCE and can improve 16% utility on average when worker range is large enough.**

***Index Terms*—Spatial Crowdsourcing, Differential Privacy**

I. INTRODUCTION

With the popularity of mobile computing, spatial crowdsourcing has emerged as a new paradigm for spatial task solutions involving human participation. Workers are encouraged to share their data with servers in exchange for benefits. However, sometimes workers are reluctant to share due to vital privacy leakage (e.g., location) which can lead to extensive attacks such as identity theft, physical surveillance and stalking and leakage of other sensitive information (e.g., individual health status, racial types, and religion views). For example, in ride-sharing, if a taxi driver submits his locations to the platform for task requests over a period of time (i.e., a month), a malicious platform attacker is able to guess the driver's range of activity and surveil him or her .

Differential Privacy (DP) [1] is often used to protect individual data. It trades off utility and privacy by well designing the privacy budget (ϵ). However, different people have different demands for both utility and privacy. For example, some confidential agencies pay great attention to privacy. They would rather gain high-level privacy protection by sacrificing some utility. In ride-sharing, some taxi drivers would like to sacrifice some personal location privacy for higher incomes by serving more passengers. Users need to adjust their utility by altering the privacy protection level themselves.

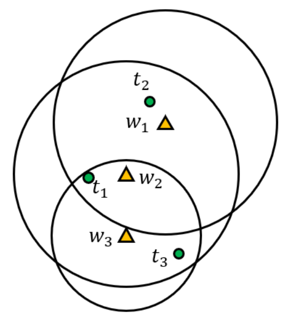


Fig. 1: Workers' locations with service areas and tasks' locations.

In this paper, we propose a dynamic private task assignment scheme such that workers can trade their location privacy for higher utilities. Consider the motivation example as follows:

Example 1. *As shown in Figure 1, there are three workers: w_1, w_2 and w_3, and three tasks: t_1, t_2 and t_3. Each worker w_j competes for tasks with smaller distances. However, in order to protect their locations, all workers employ a differential privacy mechanism to obfuscate their distances to tasks, and send the obfuscated distances to the server. Assume the server will assign workers to tasks based on their reported obfuscated distances to minimize the total distance as: $\{\langle t_1, w_3\rangle$, $\langle t_2, w_1\rangle$, $\langle t_3, w_2\rangle\}$. We also assume that the server is untrusted in this example, which means the obfuscated distances on the server can be accessed by workers if they want. Then, worker w_3 can sacrifice some of his location privacy to report a closer obfuscated distance with task t_3. The updated assignment will be $\{\langle t_1, w_2\rangle$, $\langle t_2, w_1\rangle$, $\langle t_3, w_3\rangle\}$ with a smaller total distance.*

In this paper, we study the privacy-aware task assignment (PA-TA) problem in spatial crowdsourcing, where workers can dynamically adjust their privacy protection levels for higher utilities. Specifically, we assume a privacy setting where spatial crowdsourcing workers are curious and want to protect their location privacy thus only report obfuscated distances to the server during the task assignment phase without relying on a trusted server. Similar to the existing location protection studies in spatial crowdsourcing [2], [3], [4], we assume the server is untrusted, and thus cannot guarantee the security of received obfuscated distances, which means other entities (e.g., curious workers) have access to the obfuscated distances from the server. In this paper, we handle the task assignment in a multi-proposal enabled batch-based style, where in a given time window each worker can propose to an available task for multiple times with different obfuscated distances to improve their utilities until the end of the time window. We first formally define the PA-TA problem. To improve the accuracy of comparing obfuscated distances, we propose a new comparison method, *Partial Probability Comparison Function* (PPCF), which can resolve the comparison between a real distance and an obfuscated distance. We prove our PPCF is better than the existing method, *Probability Compare Function* (PCF) [3], both theoretically and practically . To solve PA-TA, we propose two solutions, namely Private Utility Conflict-Elimination (PUCE) and Private Game Theoretic Approach (PGT). PUCE is a greedy-based algorithm. PGT is on a game-theoretic approach and can achieve higher accuracy than PUCE when the worker range is larger than 1.4 on synthetic data sets. The contributions of this paper are as follows.

(1) We formally define the privacy-aware task assignment problem to support dynamic privacy budget adjustment for workers in spatial crowdsourcing in Section III.

(2) We propose a greedy-based algorithm, namely Private Utility Conflict-Elimination (PUCE), in Section V, and a game-theoretic approach, namely Private Game Theoretic Approach (PGT), in Section VI.

(3) We test our approaches on both synthetic and real data sets to show their efficiency and effectiveness in Section VII.

II. Related Work

Task Assignment in Spatial Crowdsourcing. Most task assignments in spatial crowdsourcing focus on maximizing total utility. Deng et al. [5] define the total utility as the total number of performed tasks. Zhang et al. [6] maximize the total acceptance ratio of workers. Zhao et al. [7] propose algorithms to maximize the total rewards of the assigned tasks. Tong et al. [8] and Wang et al. [9] maximize the total expected rewards of the assigned tasks. The conventional methods to achieving optimized total utility are exact methods [10], [11] and greedy methods [12], [13], [14]. In order to achieve higher utilities, game-theoretic methods for task assignment are proposed recently. Ni et al. [15] declare that the tasks may have some dependencies among them and give the definition of dependency-aware spatial crowdsourcing (DA-SC). They propose a game-theoretic approach to solve DA-SC, and the experiment demonstrates that the game-theoretic approach is superior to the greedy algorithms . Zhao et al. [16] focus on the problem of Fairness-aware Task Assignment, which is to minimize the payoff difference among workers and to maximize the average worker payoff. They model the problem as a multiplayer game and propose two game-theoretic methods.

Privacy Protection in Spatial Crowdsourcing. Differential Privacy [1] is a golden tool for privacy protection and private data release. According to the existing of the trusted entity, it can be classified into two categories: 1) Central Differential Privacy (CDP) [17]; 2) Geo-Indistinguishability (Geo-I) [18] and Local Differential Privacy (LDP) [19].

To et al. [20] adopt *Private Spatial Decomposition* (PSD) [21] to create obfuscated data releases of workers and devise a geocast mechanism for task request dissemination to protect the privacy of workers' locations. However, it needs a trusted entity to help sanitize workers' location data. Wang et al. [22] study *Bayesian attack* [18], [23] on sparse mobile crowdsourcing and propose a privacy-preserving framework to reduce the data quality loss caused by differential location obfuscation. They provide the method to get the optimal location obfuscation matrix satisfying ϵ-differential privacy. It can be used to protect workers' location without relying on the trust entity. To et al. [2] propose a privacy-aware framework that protects the privacy of both tasks and workers in spatial crowdsourcing without any trusted entity. It employs Geo-I to transform both tasks' and workers' locations into obfuscated locations. The platform can identify a set of candidate workers for the task requester through these obfuscated locations without knowing the real locations of both workers and the task. Wang et al. [3] also assume that no trusted entity exist, but they propose a method that achieves local differential privacy. These works get rid of reliance on trusted third parties. However, they only protect individual privacy without inspiring tasks or workers to participate in the platform.

Private Data Compensation. In order to motivate requesters and workers to join spatial crowdsourcing platforms while protecting their location privacy, we need a connection between their utility and privacy cost. Jin and Zhang [24] provide a framework for spectrum-sensing participants selection, which achieves differential location privacy, approximate social cost minimization, and truthfulness simultaneously. Ghosh et al. [25] model the utility of competing agents considering privacy cost. They hold the privacy cost related to some unknown quantities v and suppose the privacy cost is changing linearly with privacy budget ϵ (ϵv). Nissim et al. [26] argue that ϵv should be the upper bound rather than the total privacy cost. They propose a privacy-aware mechanism with v below a certain threshold. Xiao [27] proposes two models for quantifying an agent's privacy cost using mutual information and max divergence, respectively. However, it requires the privacy variable $\delta > 0$. Wang et al. [3] propose a personalized privacy-preserving task allocation method. They define *Probability Compare Function* (PCF) to compare two noise values with the acknowledgment of their privacy budget. Besides, they propose *Probabilistic Winner Selection Mechanism* to minimize the total travel distance and *Vickrey Payment Determination Mechanism* to determine the appropriate payment to each

TABLE I: Notations.

Variable	Description
t_i	the i-th task
w_j	the j-th worker
$d_{i,j}$	the real distance from t_i to w_j
$\hat{d}_{i,j}$	the obfuscated distance from t_i to w_j
$\tilde{d}_{i,j}$	the effective obfuscated distance from t_i to w_j
$\boldsymbol{\epsilon_{i,j}}$	the privacy budget vector owned by w_j to propose to t_i
$\epsilon_{i,j}^{(u)}$	the u-th element in $\boldsymbol{\epsilon_{i,j}}$
$\tilde{\epsilon}_{i,j}$	the effective privacy budget
$\boldsymbol{b_{i,j}}$	the state vector corresponding to $\boldsymbol{\epsilon_{i,j}}$
$b_{i,j}^{(u)}$	the u-th element in $\boldsymbol{b_{i,j}}$ recording whether $\epsilon_{i,j}^{(u)}$ has been used
$s_{i,j}$	the state recording whether t_i matches w_j

winner of workers satisfying truthfulness, profitability, and probabilistic individual rationality. However, all workers can only have a fixed budget for each task and cannot dynamically compete for tasks with higher utilities.

III. Problem definition

Definition 1 (Spatial Tasks)**.** Let t_i denote a task. Its location and value are denoted as l_i and v_i, respectively.

Here, v_i is an inherent property of t_i, and a worker will gain v_i revenue if he serves t_i.

Definition 2 (Spatial Workers)**.** Let w_j denote a worker located at l_j. His service area is denoted as A_j with a service radius r_j.

A_j is a circle area centered at l_j with radius r_j (also called *worker range* in the experiment). Let set R_j denote all tasks in A_j. w_j only proposes to those tasks in R_j.

To make the distance and the privacy budget comparable with the task value, we define the Distance Value Function (f_d) in Definition 3 and Privacy Budget Function (f_p) in Definition 4 to unify the measurement.

Definition 3 (Distance Value Function, f_d)**.** Given a distance $d \in R^*$, a function $f_d : R^* \rightarrow R^*$ is called *distance value function*, which takes d as the input and outputs a value v. It satisfies that $f_d(0) = 0$, $f_d^{'}(\cdot) \geq 0$.

Definition 4 (Privacy Budget Value Function, f_p)**.** Given a privacy budget $\epsilon \in R^*$, $f_p : R^* \rightarrow R^*$ is a *privacy budget value function*, which takes ϵ as input and outputs a value v. It satisfies that $f_p(0) = 0$, $f_p^{'}(\cdot) \geq 0$ and $\forall \epsilon_1, \epsilon_2 \in R, f_p(\epsilon_1) + f_p(\epsilon_2) = f_p(\epsilon_1 + \epsilon_2)$.

f_d transforms a distance value into a task value. f_p transforms a privacy budget value into a task value. f_d and f_p are defined as monotone increasing functions and $f_d(0) = f_p(0) = 0$. Besides, f_p is a linear function in this paper and we will consider other types of functions in the future work.

Definition 5 (Privacy-aware Task Assignment Problem)**.** Given a set of tasks $\mathcal{T}$, a set of workers $\mathcal{W}$, and a set of obfuscated worker-and-task distances $\{\hat{d}_{i,j} | i \in [m], j \in [n]\}$, where each $\hat{d}_{i,j}$ is added with a noise $\eta_{i,j}$ subjecting to distribution $DF(\epsilon_{i,j})$, a PA-TA problem is to find a match M between workers and tasks subject to the working area constraint of workers, such that

$$
\begin{aligned}
\max \quad & \sum_{t_i \in \mathcal{T}} \sum_{w_j \in \mathcal{W}} (s_{i,j} \cdot (v_i - f_d(d_{i,j})) - f_p(\boldsymbol{b_{i,j}} \cdot \boldsymbol{\epsilon_{i,j}})) \\
s.t. \quad & \sum_{t_i \in \mathcal{T}} s_{i,j} \leq 1, \quad \forall i = 1, 2, ..., m \\
& \sum_{w_j \in \mathcal{W}} s_{i,j} \leq 1, \quad \forall j = 1, 2, ..., n \\
& \sum_{z \in Z} b_{i,j}^{(z)} \leq Z, \quad \forall z = 1, 2, ..., Z \\
& s_{i,j}, b_{i,j} \in \{0, 1\}, \ \forall i = 1, 2, ..., m; \forall j = 1, 2, ..., n
\end{aligned}
$$

where $s_{i,j}$ is the matching state representing whether task t_i is allocated to worker w_j. $s_{i,j} = 1$, if t_i is allocated to w_j; otherwise, $s_{i,j} = 0$. v_i is the value of task t_i. f_d is a Distance Value Function transforming distance to value cost. f_p is a Privacy Budget Value Function transforming privacy cost to value cost. $\boldsymbol{\epsilon_{i,j}} = \langle \epsilon_{i,j}^{(1)}, ..., \epsilon_{i,j}^{(Z)} \rangle$ is the privacy budget vector between task t_i and worker w_j, where $\epsilon_{i,j}^{(u)} (u \in Z)$ stands for the u-th proposal of worker w_j to task t_i. $\boldsymbol{b_{i,j}} = \langle b_{i,j}^{(1)}, ..., b_{i,j}^{(Z)} \rangle$ is the state vector corresponding to $\boldsymbol{\epsilon_{i,j}}$. Take $\boldsymbol{b_{1,2}} = \langle 1, 1, 0, 0, 0 \rangle$ as an example. It means in the total competition, w_2 can propose to t_1 five times and has already proposed twice with the privacy leakage $\epsilon_{1,2}^{(1)}$ and $\epsilon_{1,2}^{(2)}$.

The objective of PA-TA is to find a one-to-one match that maximizes the total profit on the platform. In the objective function, there are three important parts to construct the matching profit between t_i and w_j: task value v_i, distance value cost $f_d(d_{i,j})$ and privacy cost $f_p(\boldsymbol{b_{i,j}} \cdot \boldsymbol{\epsilon_{i,j}})$. We model the matching profit as the linear combination of the three parts. Note that, the privacy cost is concerned for the process of "w_j proposing to t_i" but not for the final matching state. Thus, $f_p(\boldsymbol{b_{i,j}} \cdot \boldsymbol{\epsilon_{i,j}})$ is not affected by $s_{i,j}$.

We give some of the frequently used variables in Table I.

IV. Review of Conflict Elimination Algorithm

Conflict Elimination Algorithm (CEA) [3] is a related work that can resolve the winner conflict problem and can be used as a subroutine in our proposed algorithm, thus we first quickly review CEA. Here, workers are regarded as competitors. When there are more than one worker competing for one task, there will be a conflict, called winner conflict. The problem of resolving all these conflicts is called winner conflict problem.

Given all distances from each task-worker pair, CEA constructs the distance rank matrix $A_{m \times n} = (a_{i,k})_{m \times n}$ where $a_{i,k}$ stands for the index of the worker who is the k-th nearest from t_i. For example, $a_{i,k} = j$ means w_j is the k-th nearest worker of t_i.

For any conflict worker w_c selected by φ tasks, CEA allocates only one task to w_c and finds another candidate other than w_c for each of the rest $\varphi - 1$ conflict tasks. Thus, for each conflict worker w_c, there will be φ candidate distance choices as shown in equation 1:

$$
\begin{cases}
\mathcal{C}_1 : D_{c_1} = D(a_{c_1,1}) + D(a_{c_2,2}) + ... + D(a_{c_\varphi,2}) \\
\mathcal{C}_2 : D_{c_2} = D(a_{c_1,2}) + D(a_{c_2,1}) + ... + D(a_{c_\varphi,2}) \\
... \\
\mathcal{C}_\varphi : D_{c_\varphi} = D(a_{c_1,2}) + D(a_{c_2,2}) + ... + D(a_{c_\varphi,1})
\end{cases}
\tag{1}
$$

where $\mathcal{C}_u (1 \leq u \leq \varphi)$ stands for the u-th solution: allocating w_c to t_{c_u} and other tasks are allocated to the successive workers.

TABLE II: Distance rank matrix.

Task/Rank	1	2	3
t_1	w_1 (9.06)	w_2 (9.85)	w_3 (12.04)
t_2	w_3 (2.09)	w_1 (10.44)	w_2 (12.59)
t_3	w_3 (2.00)	w_2 (11.28)	w_1 (18.87)

To choose the best solution from φ choices in equation 1, we need to compare four distance values. For example, to compare $\mathcal{C}_u$ and $\mathcal{C}_v$ $(1 \leq u, v \leq \varphi)$, we need to compare $D(a_{c_u,1})+D(a_{c_v,2})$ with $D(a_{c_v,1})+D(a_{c_u,2})$. If the distances are obfuscated distances, we have to compare four Laplace random variables. In CEA, it supposes that the difference between the travel distances for different tasks is relatively small for the same worker (i.e., $D(a_{c_u,1}) \simeq D(a_{c_v,1})$). Then, CEA only needs to compare two Laplace random variables, which can be calculated by *Probability Compare Function* [3].

Definition 6 (Probability Compare Function [3])**.** Given two values d_a and d_b with their obfuscated values $\hat{d}_a = d_a + Lap(x, 1/\epsilon_a)$ and $\hat{d}_b = d_b + Lap(x, 1/\epsilon_b)$ ($Lap(x, y)$ is a random variable drawn from Laplace distribution with parameters x, y), a function $f : R^4 \to [0, 1]$ is called a probability compare function (PCF) if $PCF(\hat{d}_a, \hat{d}_b, \epsilon_a, \epsilon_b) = \Pr[d_a < d_b]$.

For Example, suppose there are 3 tasks and 3 workers, their distance rank matrix is shown in Table II. Each element in the table stands for the worker and his relative distance to the corresponding task. For w_3, both t_2 and t_3 will choose him at the first rank, thus w_3 is a conflict worker. We have $\mathcal{C}_1 : D_2 = D(a_{2,1}) + D(a_{3,2})$ and $\mathcal{C}_2 : D_3 = D(a_{2,2}) + D(a_{3,1})$. To make a choice between $\mathcal{C}_1$ and $\mathcal{C}_2$ (choose the minimal one), it supposes $D(a_{2,1}) \simeq D(a_{3,1})$, and thus only needs to compare $D(a_{3,2})$ with $D(a_{2,2})$. Since $D(a_{2,2}) < D(a_{3,2})$, $\mathcal{C}_2$ is selected.

V. Private Utility Conflict-Elimination (PUCE)

A direct method to solve our matching problem is collecting all workers' proposals to tasks with privacy budgets and obfuscated distances and using the Hungarian algorithm to get the optimal matching. Here, the Hungarian matching algorithm [28], also called the Kuhn-Munkres algorithm, is one classical method to exactly solve maximum bipartite matching problem with the time complexity of $O(n^3)$, where n is the number of vertices in either part of the bipartite graph. However, to use the Hungarian algorithm, we have to compare the path length calculated by summing many obfuscated distances, which needs complex comparisons and has low accuracy. In this section, we propose a private utility conflict-elimination (PUCE) algorithm to solve PA-TA problem. Due to each worker can propose to multiple tasks in each round, PUCE greedily chooses the worker-and-task pair that maximizes the subjective function of PA-TA.

A. Comparison and Estimation of Obfuscated Distances

Before introducing PUCE algorithm, we first explain three necessary techniques for solving: 1) how to calculate a suitable obfuscated distance when there is a series of obfuscated distances for a given task and a given worker; 2) how to compare a real distance with an obfuscated distance; 3) how to compare two utilities when knowing the obfuscated distances.

In this paper, according to the objective function of PA-TA, we define the utility of worker w_j conducting task t_i as:

$$U_j(i) = v_i - f_d(d_{i,j}) - \sum_{t_i \in \mathcal{T}} f_p(\boldsymbol{b_{i,j}} \cdot \boldsymbol{\epsilon_{i,j}}) \tag{2}$$

Effective Obfuscated Distance and Effective Privacy Budget. In the process of our task assignment, w_j may propose to t_i many times, which means w_j will submit more than one obfuscated distance $\hat{d}_{i,j}$ to the server. For the server, it needs to determine an obfuscated distance (we call it *effective obfuscated distance*) for $d_{i,j}$ to make comparison. For other workers, they also need the *effective obfuscated distance* to compare with the distances of themselves. Thus, we need a method to calculate the *effective obfuscated distance* in a series of obfuscated distances and ensure the *effective obfuscated distance* supports comparison (i.e., supporting PCF).

We first adopt *maximum likelihood estimation* (MLE) [29] to get a distance interval $\check{d}$ from a worker w's release set $\boldsymbol{DE} = \{(\hat{d}_1, \epsilon_1), (\hat{d}_2, \epsilon_2), ..., (\hat{d}_u, \epsilon_u)\}$ for a task t. Let $\boldsymbol{DE}.\boldsymbol{\hat{d}}$ denote the set $\{\hat{d}_1, \hat{d}_2..., \hat{d}_u\}$ in $\boldsymbol{DE}$. Let $L(X) = L(\hat{d}_1, \hat{d}_2, ..., \hat{d}_u; X) = \prod_{k=1}^{u} \Pr[\hat{d}_k; X]$, where $\Pr[\hat{d}_k; X]$ is the probability function of $Lap(\epsilon_k)$. When the server gets $\boldsymbol{DE}$, it calculates the estimation of d as follows.

$$\begin{aligned}\check{d} &= \arg\max_d \prod_{k=1}^{u} \frac{\epsilon_k}{2} \exp(-|\hat{d}_k - d| \cdot \epsilon_k) \\ &= \arg\min_d \sum_{k=1}^{u} \epsilon_k \cdot |\hat{d}_k - d|.\end{aligned}$$

The value of $\check{d}$ is all points on a line segment. We limit the domain of d in $\boldsymbol{DE}.\boldsymbol{\hat{d}}$ to get the only estimation of d (supporting comparison). This estimation of d is the *effective obfuscated distance*, and we denote it as $\tilde{d}$. We call the corresponding privacy budget (denoted by $\tilde{\epsilon}$) of $\tilde{d}$ in the pair as *effective privacy budget* and call the pair $(\tilde{d}, \tilde{\epsilon})$ as *effective distance-budget pair*.

For example, suppose w_1 releases 3 pairs of obfuscated distance and privacy budget to t_1: $\boldsymbol{DE} = \{(0.1, 0.2), (0.2, 0.9), (0.3, 0.1)\}$. Then we can calculate the effective distance-budget pair as $(\tilde{d} = 0.2, \tilde{\epsilon} = 0.9)$.

Partial Probability Compare Function (PPCF). If w_{j_1} want to compare his distance from himself to t_i with the effective obfuscated distance $\hat{d}_{i,j_2}$ of w_{j_2} to t_1, w_{j_1} can utilize the real distance d_{i,j_1} instead of $\hat{d}_{i,j_1}$ or $\tilde{d}_{i,j_1}$ to achieve a more accurate comparison result. Thus, we need a method for the comparison between a real distance and an obfuscated distance. Suppose there are two values d_i and d_j. The obfuscated value of d_j is $\hat{d}_j$, which is calculated by adding noise η_j drawn from a type of distribution $DF(\epsilon_j)$. Then, we have

$$\begin{aligned}\hat{d}_j &= d_j + \eta_j, \quad \eta_j \sim DF(\epsilon_j), \\ \Pr[d_i < d_j] &= \Pr[d_i < \hat{d}_j - \eta_j] \\ &= \Pr[\eta_j < \hat{d}_j - d_i].\end{aligned}$$

Let $f(x)$ be the probability density function of η_j, then

$$\Pr[d_i < d_j] = \int_{-\infty}^{\hat{d}_j - d_i} f(\eta_j) d\eta_j.$$

Algorithm 1: WorkerProposal

Input: Not winning worker set NWW
Output: Candidate list CL
1 Initialize candidate list CL as m empty sets;
2 **for** *each worker* w_j *in* NWW **do**
3   Initialize applicable list BL as empty list;
4   **for** *each task* t_i *in* R_j **do**
5   **if** w_j*'s privacy budget has been exhausted* **then**
6   continue;
7         $U_j(i) = v_i - f_d(d_{i,j}) - \sum_{t_i \in \mathcal{T}} f_p(\boldsymbol{b_{i,j}} \cdot \boldsymbol{\epsilon_{i,j}})$;
8   **if** $U_j(i) \leq 0$ **then**
9   continue;
10   Get $(\tilde{d}_{i,win(i)}, \tilde{\epsilon}_{i,win(i)})$ of $w_{win(i)}$ from the server;
11   Calculate new $(\tilde{d}_{i,j}, \tilde{\epsilon}_{i,j})$;
12   Calculate $\tilde{d}'_{i,win(i),j}$ by Equation 4;
13   **if** *PPCF*$(d_{i,j}, \tilde{d}'_{i,win(i),j}, \epsilon_{i,win(i)}) \leq 0.5$ **then**
14   continue;
15   **if** *PCF*$(\tilde{d}_{i,j}, \tilde{d}'_{i,win(i),j}, \tilde{\epsilon}_{i,win(i)}, \tilde{\epsilon}_{i,j}) \leq 0.5$ **then**
16   continue;
17   Add $\tilde{d}_{i,j}$ to $CL[i]$
18 **return** CL;

Similar to PCF, we define PPCF$(d_i, \hat{d}_j, \epsilon_j) = \Pr[d_i < d_j]$. If the distribution of $DF(\epsilon_j)$ is symmetric about the y-axis (e.g., Laplace distribution), then

$$\text{PPCF}(d_i, \hat{d}_j, \epsilon_j) > \frac{1}{2} \Leftrightarrow d_i < \hat{d}_j. \tag{3}$$

Our PPCF is better than PCF as shown in Theorem V.1. Please refer to the details of the proof in Appendix IX-A in our technical report [30].

Theorem V.1. *For any given distance $d_x, d_y, \epsilon_x, \epsilon_y$ satisfying $d_x < d_y$. Let $\eta_x \sim Lap(0, 1/\epsilon_x), \eta_y \sim Lap(0, 1/\epsilon_y)$. Let $\hat{d}_x = d_x + \eta_x, \hat{d}_y = d_y + \eta_y$. Then $Pr[PCF(\hat{d}_x, \hat{d}_y, \epsilon_x, \epsilon_y) > \frac{1}{2}] \leq Pr[PPCF(d_x, \hat{d}_y, \epsilon_y) > \frac{1}{2}]$.*

Comparison Transformation from Utility to Distance. After receiving proposals of workers, the server needs to eliminate conflict among workers for each task. We can easily use CEA directly to choose only one worker for each task. However, in CEA, the comparison is based on obfuscated distances rather than utility functions, which does not satisfy our optimized goal. If we use the utility directly as the comparison object, the server will know the utility value in each round, which leaks the real distance between tasks and workers.

In order to handle the problem above, we convert the utility comparison into the distance comparison and then use CEA to choose the high-utility one under the distance form. For any two workers w_a and w_b, they hold tasks t_x and t_y, respectively. Their utilities are $U_a(x)$ and $U_b(y)$, respectively. Let $V_a(x) = U_a(x) + f_d(d_{x,a})$ and $V_b(y) = U_b(y) + f_d(d_{y,b})$. Then we have

$$\begin{aligned}\Pr(U_a(x) > U_b(y)) &= \Pr(V_a(x) - f_d(d_{x,a}) > V_b(y) - f_d(d_{y,b})) \\ &= \Pr(f_d^{-1}(V_a(x)) - d_{x,a} > f_d^{-1}(V_b(y)) - d_{y,b}) \\ &= \Pr(d_{x,a} < d_{y,b} + f_d^{-1}(V_a(x)) - f_d^{-1}(V_b(y))).\end{aligned}$$

Let
$$\hat{d}'_{y,b,a} = \hat{d}_{y,b} + f_d^{-1}(V_a(x)) - f_d^{-1}(V_b(y)), \tag{4}$$

thus,
$$\begin{aligned}\Pr(U_a(x) > U_b(y)) &= \Pr(\eta_{x,a} - \eta_{y,b} > \hat{d}_{x,a} - \hat{d}'_{y,b,a}) \\ &= \text{PCF}(\hat{d}_{x,a}, \hat{d}'_{y,b,a}, \epsilon_{x,a}, \epsilon_{y,b}).\end{aligned}$$

Therefore, we can calculate $\hat{d}'_{y,b,a}$ for each pair of w_a and w_b with the same task t_y and use PCF function to compare the

Algorithm 2: WinnerChosen

Input: Candidate list CL, last term allocation list AL'
Output: Allocation list AL, updating state upd
1 **if** *All set in* CL *are empty* **then**
2   **return** $(AL', false)$
3 Initialize each update task set list TSL as empty list;
4 Initialize AL as m null values;
5 Initialize competing table CT as empty table;
6 **for** *Each candidate set* CS_i *in* CL **do**
7   **if** CS_i *is empty* **then**
8   Set $AL[i] = AL'[i]$;
9   **else**
10   Add $CT[i] = CS_i \cup \{\tilde{d}_{i,win(i)}\}$ to CT;
11   Calculate $\hat{d}'_{i,a,b}$ for each pair in $CT[i]$;
12   Sort $CT[i]$ in descending order by PCF$(\hat{d}'_{i,a,b}, \hat{d}_{i,b}, \epsilon_{i,a}, \epsilon_{i,b})$;
13 Get updated matching M set by using CEA for CT;
14 Add M to AL;
15 **return** $(AL, true)$;

utility. Similarly, we can compare $U_a(x)$ and $U_b(y)$ through PPCF:

$$\begin{aligned}\Pr(U_a(x) > U_b(y)) &= \Pr(\eta_{y,b} < \hat{d}'_{y,b,a} - d_{x,a}) \\ &= \text{PPCF}(d_{x,a}, \hat{d}'_{y,b,a}, \epsilon_{y,b}).\end{aligned}$$

B. The PUCE Algorithm

We suppose that w_j will propose to all tasks R_j within area A_j. In order to further decline unnecessary privacy costs, we add an extra judgment for workers through the PPCF function.

The worker proposal process and winner-chosen algorithm are respectively shown in Algorithm 1 and Algorithm 2.

In Algorithm 1, each worker w_j checks all the tasks in his service area and judges whether it is worth to complete for the tasks (check whether $U_j(i) > 0$ for $t_i \in R_j$). Besides, he also judges whether he has advantages over the before-winner worker for these tasks by utility comparison. The utility comparison is shown from line 10 to line 16. If the two conditions are satisfied, w_j will propose to this task with a new privacy budget and obfuscated distance.

Algorithm 2 takes candidate list CL (constructed by Algorithm 1) and last term allocation list AL' as the input. It outputs the updating allocation list with the updating state upd. The $false$ value of upd means there is no change for AL. The candidate list will be partitioned into two parts. Ones with no workers' proposal are the same as the last term ones, which is shown from line 7 to line 8. The others containing workers' proposals will be added to a new competing table with the winners of the last term. Each set of workers for applied tasks in competing table will be sorted by the utility value (compared by PCF$(\hat{d}'_{x,a,b}, \hat{d}_{x,b}, \epsilon_{x,a}, \epsilon_{x,b})$) in descending order. The process is shown from line 10 to line 12.

By executing Algorithm 1 and Algorithm 2, we can construct our PUCE algorithm as shown in Algorithm 3. The total task set and the total worker set can be divided into several time window slices. We execute PUCE on each time window in a batch-based style. In the beginning, the not winning worker set NWW is initialized as the whole worker set W, and the allocation list AL is initialized as an empty set. We execute Algorithm 1 to get candidate allocation list CL. Then we execute Algorithm 2 to pick a new allocation list AL and get a updating state upd. When there are still some workers

Algorithm 3: PUCE

Input: Tasks $\mathcal{T}$ and workers $\mathcal{W}$ in the current time window
Output: The task-worker matching pairs TWM

Initialize not winning worker set NWW as $\mathcal{W}$;
Initialize halt state hs as $false$;
Initialize allocation list AL as m empty set list;
while hs *is not* $true$ **do**
  Get CL by executing Algorithm 1;
  Get AL and upd by executing Algorithm 2;
  Update NWW by removing new winners and adding new losers;
  Set $hs = upd$;
Set TWM as AL;
return TWM;

TABLE III: Task-worker distances.

Worker/Task	t_1	t_2	t_3
w_1	12.2	3.61	17.12
w_2	5	10.44	12.21
w_3	9.43	18.25	7.28

proposing to tasks (CL is not empty), upd will be set as $true$. We also update NWW by removing the new winner workers and adding the new loser workers. When no workers propose to any task, upd will be set as $false$. Thus, we get the final task-worker matching pairs TWM as AL.

Example 2 (Running Example of PUCE)**.** *We give a running example of the whole process of PUCE following the motivation example. As shown in Figure 1, three workers w_1, w_2 and w_3 have service areas 15, 15 and 10, respectively. Three tasks t_1, t_2 and t_3 have task values 12.4, 11 and 13, respectively. The distance between each task and worker is shown in Table III.*

Suppose there are three privacy budgets for each task-worker pair. The corresponding effective distance, the privacy budget and utility are shown in Table IV.

At the beginning, NWW is set as $\{w_1, w_2, w_3\}$. CL is set to NULL. w_1 firstly judges whether the tasks within his service area will be added to the CL. He calculates the utility for t_1 as $U_1(1) = 0.1 > 0$ and adds $\tilde{d}_{1,1}$ to $CL[1]$. Besides, he adds $\tilde{d}_{2,1}$ to $CL[2]$. And w_2, w_3 also add their selected tasks (by the judgement in Algorithm 1). And we can get the data in CL as shown in Table V (the utility values are shown in square brackets). Then we get CT by sorting CL, which is shown in Table VI.

After that, we find t_1 is allocated to w_3. Besides, t_2 and t_3 fall into conflict for w_2. After the comparison of CEA, t_3 is allocated to w_2. In the next round, there is only t_2 unallocated. And w_1 has not matched any task yet. w_1 can only propose for t_2. However the utility of $U_1(2)$ in this round is $-3.1 \le 0$. Thus there is no worker proposing to any tasks in this round. And the process is end.

Privacy Analysis. We define the query data set of worker w_j as X_j, which consists of all tasks in the service area of w_j (i.e., R_j). The neighboring data set of X_j is noted as X'_j. It satisfies that $\|X_j - X'_j\| = 1$, which means there is only one different task item between X_j and X'_j. We focus on the query f as '*Get each distance from w_j to his service tasks R_j*'. That means $f(X_j) = [d_{i_1,j}, ..., d_{i_{|R_j|},j}]$.

TABLE IV: Effective obfuscated distance, privacy budget and utility.

Matchable pair	$(\tilde{d}, \epsilon^{(1)})$, utility		$(\tilde{d}, \epsilon^{(2)})$, utility		$(\tilde{d}, \epsilon^{(3)})$, utility	
(t_1, w_1)	(12.7,0.1)	0.1	(12.4,0.3)		(12.3,0.4)	
(t_1, w_2)	(5.5,4.6)	2.8	(5.3,4.65)		(5.1,4.8)	
(t_1, w_3)	(9.93,0.1)	2.87	(9.63,0.4)		(9.53,0.4)	
(t_2, w_1)	(4.11,6.99)	0.4	(4.01,7.1)	-3.1	(3.81,7.2)	
(t_2, w_2)	(10.94,0.1)	0.46	(10.64,0.2)		(10.54,0.5)	
(t_3, w_2)	(12.71,0.1)	0.69	(12.51,0.3)		(12.31,0.4)	
(t_3, w_3)	(7.78,5.4)	0.32	(7.58,5.5)		(7.38,5.6)	

TABLE V: Candidate list CL.

CL			
1	$\tilde{d}_{1,1}$=12.7, [0.1]	$\tilde{d}_{1,2}$=5.5, [2.8]	$\tilde{d}_{1,3}$=9.93 [2.87]
2	$\tilde{d}_{2,1}$=4.11, [0.4]	$\tilde{d}_{2,2}$=10.94, [0.46]	
3	$\tilde{d}_{3,2}$=12.71, [0.69]	$\tilde{d}_{3,3}$=7.78, [0.32]	

TABLE VI: Competing table CT.

CT	1	2	3
1	$\tilde{d}_{1,3}$=9.93 [2.87]	$\tilde{d}_{1,2}$=5.5, [2.8]	$\tilde{d}_{1,1}$=12.7, [0.1]
2	$\tilde{d}_{2,2}$=10.94, [0.46]	$\tilde{d}_{2,1}$=4.11, [0.4]	
3	$\tilde{d}_{3,2}$=12.71, [0.69]	$\tilde{d}_{3,3}$=7.78, [0.32]	

Theorem V.2. *PUCE satisfies $(\sum_{t_i \in R_j} \boldsymbol{b_{i,j}\epsilon_{i,j}} r_j)$-local differential privacy for each worker w_j.*

Proof. Let $\mathcal{A}_j$ be the mechanism PUCE applying to w_j with query f defined above. Let X_j be the location of w_j. For query $f(X_j) = [d_{i_1,j}, ..., d_{i_{|R_j|},j}]$, we extend it to an equivalent query $\hat{f}(X_j) = f(X_j) \cdot \mathcal{J}$, where $\mathcal{J}$ is a block diagonal matrix:

$$\mathcal{J} = \begin{bmatrix} CP(\boldsymbol{b_{i_1,j}}) & & & \\ & CP(\boldsymbol{b_{i_2,j}}) & & \\ & & \ddots & \\ & & & CP(\boldsymbol{b_{i_{|R_j|},j}}) \end{bmatrix}$$

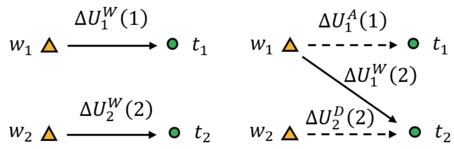
Here, $CP(\boldsymbol{b})$ is the compression of $\boldsymbol{b}$, which means removing all zero element of $\boldsymbol{b}$. For example, if $\boldsymbol{b} = [1, 1, 0, 0, 0]$, then $CP(\boldsymbol{b}) = [1, 1]$. $\hat{f}(X_j)$ means query $d_{i_u,j}$ for $sum(\boldsymbol{b_{i_u,j}})$ times for $u \in [|R_j|]$, where $sum(\boldsymbol{b_{i_u,j}})$ means the sum of all elements in $\boldsymbol{b_{i_u,j}}$. We denote the size of $\hat{f}(X_j)$ as $|\hat{f}|$ and the a-th element of $\hat{f}(X_j)$ as $\hat{f}(X_j)_a$.

Let Y_j denote the set of all published obfuscated distances of the worker w_j to tasks in R_j. Then we have $Y_j = \hat{f}(X_j) + [\eta_1, \eta_2, ..., \eta_{|\hat{f}|}]$, where $\eta_a (1 \le a \le |\hat{f}|)$ is an i.i.d random variable drawn from $Lap(1/\epsilon_a)$. Hence we have

$$\begin{aligned}
\frac{\Pr[\mathcal{A}_j(X_j) = Y_j]}{\Pr[\mathcal{A}_j(X'_j) = Y_j]} &= \prod_{a \in [|\hat{f}|]} \left(\frac{\exp(-\epsilon_a |Y_{j,a} - \hat{f}(X_j)_a|)}{\exp(-\epsilon_a |Y_{j,a} - \hat{f}(X'_j)_a|)}\right) \\
&= \prod_{t_i \in R_j} \prod_{u \in [sum(\boldsymbol{b_{i,j}})]} \left(\frac{\exp(-\epsilon_{i,j}^{(u)} |\tilde{d}_{i,j}^{(u)} - d_{i,j}|)}{\exp(-\epsilon_{i,j}^{(u)} |\tilde{d}_{i,j}^{(u)} - d'_{i,j}|)}\right) \\
&\le \prod_{t_i \in R_j} \prod_{u \in [sum(\boldsymbol{b_{i,j}})]} (\exp(\epsilon_{i,j}^{(u)} (|d_{i,j} - d'_{i,j}|))) \\
&= \prod_{t_i \in R_j} \exp(\boldsymbol{b_{i,j}\epsilon_{i,j}} (|d_{i,j} - d'_{i,j}|)) \\
&\le \exp(\sum_{t_i \in R_j} \boldsymbol{b_{i,j}\epsilon_{i,j}} r_j).
\end{aligned}$$

Because X_j contains only one element, then we have PUCE satisfies $(\sum_{t_i \in R_j} \boldsymbol{b_{i,j}\epsilon_{i,j}} r_j)$-local differential privacy for each worker w_j. □

Time Cost Analysis. There are m tasks and n workers in $\mathrm{Cond}_{m,n}$. Each of the n workers has Z privacy budget for each task. Therefore, the worst time cost for PUCE is $O(m \cdot n \cdot Z)$.



(a) State 1 (b) State 2

Fig. 2: Utility change.

Algorithm 4: PGT

Input: Tasks $\mathcal{T}$ and workers $\mathcal{W}$ in the current time window
Output: The allocation list AL

```
Initialize AL as a list with m null value;
Initialize halt state hs as false;
while hs is false do
    Set hs as true;
    for each worker w_j ∈ W do
        Get the maximal UT_j for each task t_i ∈ R_j \ {AL[b]};
        if UT_j(st) is null or UT_j(st) ≤ 0 then
            continue;
        Set hs as false;
        Set t_c as w_j's already mateched task;
        Set t_b as the task with maximal UT_j;
        Set w_f as the worker matched t_b before;
        Update effective distance-budget pair between t_b and w_j;
        Set AL[c] = null;
        Set AL[b] = w_j;
return AL;
```

VI. Private Game Theoretic Approach (PGT)

In this section, we declare that each worker can compete for each task within their service area, whether they have already won a task. We modeled our problem as an exact potential game with at least one Nash equilibrium in pure strategy. To compare the utility values to make a choice, we approximate our utility function by replacing real distance with effective obfuscated distance.

A. Cases of Utility Change in Competition

There are three cases of utility change in each time of competition for each task-worker pair. They are *Winning Change*, *Abandoned Change* and *Defeated Change*. We denote them as $\Delta U_j^W(i)$, $\Delta U_j^A(i)$ and $\Delta U_j^D(i)$ respectively, which are expressed as follows:

$$\Delta U_j^W(i) = v_i - f_d(\tilde{d}_{i,j}) - f_p(\epsilon_{i,j}^{(z)}),$$
$$\Delta U_j^A(i) = -v_i + f_d(\tilde{d}_{i,j}),$$
$$\Delta U_j^D(i) = -v_i + f_d(\tilde{d}_{i,j}).$$

$\Delta U_j^W(i)$ means the utility change of winning task t_i for worker w_j. $\Delta U_j^A(i)$ means the utility change of abandoning task t_i (because each worker can only match at most one task) for worker w_j. $\Delta U_j^D(i)$ means the utility change of being defeated by some other competitor in competing for task t_i for worker w_j. It is the same with $\Delta U_j^A(i)$. We use $\Delta U_j^{W(k)}(i)$, $\Delta U_j^{A(k)}(i)$ and $\Delta U_j^{D(k)}(i)$ to denote the above three utility change in k-th competition.

We give examples of these three utility changes. Suppose there are two workers w_1, w_2 and two tasks t_1, t_2. At the first stage, w_1 competes for t_1 and w_2 competes for t_2. Then the corresponding $\Delta U_1^W(1)$ and $\Delta U_2^W(2)$ are shown in Figure 2(a). At the second stage, w_1 competes for t_2 and gets it successfully. As is shown in Figure 2(b). The utility change between w_1 and t_2 is $\Delta U_1^W(2)$. The utility change between w_1 and t_1 is $\Delta U_1^A(1)$. The utility change between w_2 and t_2 is $\Delta U_2^D(2)$.

B. Game Modeling and Nash Equilibrium

We approximate our PA-TA as *Privacy-aware Approximate Task Assignment* (PAA-TA) problem by replacing the real distance as effective distance. We formulate PAA-TA as an n-player strategic game, $\mathcal{G} =< \mathcal{W}, \boldsymbol{S}, \boldsymbol{UT} >$. $\mathcal{G}$ consists of players $\mathcal{W}$, strategy spaces $\boldsymbol{S}$, and utility functions $\boldsymbol{UT}$. We specify these three components as follows:

(1) $\mathcal{W} = \{w_1, ..., w_n\}$ denotes the finite set of n workers with $n \geq 2$. We will use worker and player interchangeably in the rest of the paper.

(2) $\boldsymbol{S} = \{S_j\}_{j=1}^n$ is the strategy spaces (i.e., the overall strategy set of all players). S_j is the finite set of strategies available to worker w_j. Here, one strategy of worker w_j indicates an action that he proposes to some task t_i with a privacy budget $\epsilon_{i,j}^{(u)}$ for the u-th proposal.

(3) $\boldsymbol{UT} = \{UT_j^{(k)}\}_{j=1}^n$ is the utility functions of all players w_j where k is the total competition number. For each chosen strategy $\boldsymbol{st} \in S$, $UT_j^{(k)}(\boldsymbol{st}) \in \mathbb{R}$ is the utility of player w_j. We calculate $UT_j^{(k)}(\boldsymbol{st})$ as follows:

$$\begin{aligned} UT_j^{(k)}(\boldsymbol{st}) &= \Delta U_j^{W(k)}(i_2) + \Delta U_{win(i_2)}^{D(k-1)}(i_2) + \Delta U_j^{A(k-1)}(i_1) \\ &= v_{i_2} - f_d(\tilde{d}_{i_2,j}^{(k)}) - f_p(\epsilon_{i_2,j}^{(z_k)}) - v_{i_2} + f_d(\tilde{d}_{i_2,win(i_2)}^{(k-1)}) \\ &\quad - v_{i_1} + f_d(\tilde{d}_{i_1,j}^{(k-1)}) \\ &= -f_d(\tilde{d}_{i_2,j}^{(k)}) - f_p(\epsilon_{i_2,j}^{(z_k)}) + f_d(\tilde{d}_{i_2,win(i_2)}^{(k-1)}) - v_{i_1} + f_d(\tilde{d}_{i_1,j}^{(k-1)}). \end{aligned} \tag{5}$$

In equation 5, w_j wins t_{i_1} and $w_{win(i_2)}$ wins t_{i_2} in $(k-1)$-th competition. w_j will compete for t_{i_2} in k-th competition.

In the following part, we define *exact potential game* (EPG) and prove that PAA-TA is an EPG.

Definition 7 (Exact Potential Game)**.** A strategic game, $\mathcal{G} =< \mathcal{W}, \boldsymbol{S}, \boldsymbol{UT} >$, is an Exact Potential Game (EPG) if there exists a function, $\Phi : \boldsymbol{S} \rightarrow \mathbb{R}$, such that for all $\boldsymbol{st}_j \in \boldsymbol{S}$, it holds that, $\forall w_j \in \mathcal{W}$, $\forall k \in N^+$,

$$\begin{aligned} &UT_j^{(k)}(st_j', \boldsymbol{st}_{-j}) - UT_j^{(k)}(st_j, \boldsymbol{st}_{-j}) \\ =&\Phi^{(k)}(st_j', \boldsymbol{st}_{-j}) - \Phi^{(k)}(st_j, \boldsymbol{st}_{-j}). \end{aligned}$$

Theorem VI.1. *PAA-TA is an Exact Potential Game (EPG).*

Proof: We define a potential function as

$$\Phi^{(k)}(\boldsymbol{st}) = \sum_{t_i \in \mathcal{T}} \sum_{w_j \in \mathcal{W}} (s_{i,j}^{(k)} \cdot (v_i - f_d(\tilde{d}_{i,j})) - f_p(\boldsymbol{b_{i,j}^{(k)}} \cdot \boldsymbol{\epsilon_{i,j}}))$$

which represents the total utility value of the matching result in k-th competition that all worker gain. Let $\tilde{U}_j^{(k)}(i) = v_i - f_d(\tilde{d}_{i,j}^{(k)}) - \sum_{t_i \in \mathcal{W}} f_p(\boldsymbol{b_{i,j}^{(k)}} \cdot \boldsymbol{\epsilon_{i,j}})$ be the approximate value of $U_j(i)$ by replacing the real distance $d_{i,j}$ with the effective obfuscated distance $\tilde{d}_{i,j}$. Then we get the recurrence relation of $\tilde{U}_j^{(k)}(i)$ for k as

$$\tilde{U}_j^{(k)}(i) = \begin{cases} \tilde{U}_j^{(k-1)}(i) + v_i - f_d(\tilde{d}_{i,j}^{(k)}) - f_p(\epsilon_{i,j}^{(z_k)}) & \sharp 1 \\ \tilde{U}_j^{(k-1)}(i) - v_i + f_d(\tilde{d}_{i,j}^{(k-1)}) & \sharp 2 \\ \tilde{U}_j^{(k-1)}(i) & \sharp 3 \end{cases}$$

where condition $\sharp 1$ means w_j wins t_i in k-th competition, condition $\sharp 2$ means w_j gives up his original task or is defeated in k-th competition and condition $\sharp 3$ means others. Suppose that w_j, w_{j_x}, w_{j_y} wins $t_{i_1}, t_{i_2}, t_{i_3}$ in $(k-1)$-th competition respectively and w_j will compete for t_{i_2} (st_j) or t_{i_3} (st'_j) in k-th competition, then we obtain

$$
\begin{aligned}
&\Phi^{(k)}(st'_j, \boldsymbol{st}_{-j}) - \Phi^{(k)}(st_j, \boldsymbol{st}_{-j}) \\
=&\tilde{U}_j^{(k)}(i_1) + \tilde{U}_j^{(k-1)}(i_2) + \tilde{U}_j^{(k)}(i_3) + \tilde{U}_{jy}^{(k)}(i_3) + \tilde{U}_{jx}^{(k-1)}(i_2) \\
&-(\tilde{U}_j^{(k)}(i_1) + \tilde{U}_j^{(k)}(i_2) + \tilde{U}_j^{(k-1)}(i_3) + \tilde{U}_{jy}^{(k-1)}(i_3) + \tilde{U}_{jx}^{(k)}(i_2)) \\
=&\tilde{U}_j^{(k)}(i_3) - \tilde{U}_j^{(k-1)}(i_3) - (\tilde{U}_j^{(k)}(i_2) - \tilde{U}_j^{(k-1)}(i_2)) \\
&+\tilde{U}_{jy}^{(k)}(i_3) - \tilde{U}_{jy}^{(k-1)}(i_3) - (\tilde{U}_{jx}^{(k)}(i_2) - \tilde{U}_{jx}^{(k-1)}(i_2)) \\
=&v_{i_3} - f_d(\tilde{d}_{i_3,j}^{(k)}) - f_p(\epsilon_{i_3,j}^{(z_k)}) - (v_{i_2} - f_d(\tilde{d}_{i_2,j}^{(k)}) - f_p(\epsilon_{i_2,j}^{(z_k)})) \\
&-v_{i_3} + f_d(\tilde{d}_{i_3,jy}^{(k-1)}) - (-v_{i_2} + f_d(\tilde{d}_{i_2,jx}^{(k-1)})) \\
=&-f_d(\tilde{d}_{i_3,j}^{(k)}) - f_p(\epsilon_{i_3,j}^{(z_k)}) + f_d(\tilde{d}_{i_3,jy}^{(k-1)}) \\
&+f_d(\tilde{d}_{i_2,j}^{(k)}) + f_p(\epsilon_{i_2,j}^{(z_k)}) - f_d(\tilde{d}_{i_2,jx}^{(k-1)}) \\
=&v_{i_3} - f_d(\tilde{d}_{i_3,j}^{(k)}) - f_p(\epsilon_{i_3,j}^{(z_k)}) - v_{i_3} + f_d(\tilde{d}_{i_3,jy}^{(k-1)}) - v_{i_1} + f_d(\tilde{d}_{i_1,j}^{(k-1)}) \\
&-(v_{i_2} - f_d(\tilde{d}_{i_2,j}^{(k)}) - f_p(\epsilon_{i_2,j}^{(z_k)}) - v_{i_2} + f_d(\tilde{d}_{i_2,jx}^{(k-1)}) - v_{i_1} + f_d(\tilde{d}_{i_1,j}^{(k-1)})) \\
=&\Delta U_j^{W(k)}(i_3) + \Delta U_{jy}^{D(k-1)}(i_3) + \Delta U_j^{A(k-1)}(i_1) \\
&-(\Delta U_j^{W(k)}(i_2) + \Delta U_{jx}^{D(k-1)}(i_2) + \Delta U_j^{A(k-1)}(i_1)) \\
=&UT_j^{(k)}(st'_j, \boldsymbol{st}_{-j}) - UT_j^{(k)}(st_j, \boldsymbol{st}_{-j})
\end{aligned}
$$

According to Definition 7, the strategic game of the PAA-TA is an exact potential game. Therefore, PAA-TA has pure Nash equilibrium according to Theorem 2.3 in Ref [31].

PGT Algorithm. The server executes the competition process with the aid of workers. Each worker w_j needs to repeat choosing the best task t_b for the maximal utility value. If the maximal value is positive, w_j will update his effective distance-budget pair for t_b and ask the server to update the allocation list.

We give the process in Algorithm 4. The critical step is to calculate the best response information (maximal UT_j) shown in line 6. The state variable hs is a boolean variable that indicates whether there still exists a task that can improve a utility function UT_j for any $w_j \in \mathcal{W}$. If there is no such task, the process will halt.

Example 3 (Running Example of PGT). *Consider the example in Table III, and the effective obfuscated distance and privacy budgets are shown in Table IV. As shown in Table VII, suppose in the k-th competition, the winners of t_1, t_2 and t_3 are w_1, w_2 and w_3 respectively. And w_1, w_2 and w_3 have consumed their first privacy budgets ϵ_1 for all three tasks. Besides, they public the obfuscated distances relevant to ϵ_1 for all tasks (so that all the effective obfuscated distances related to ϵ_1 are able to calculated by the server and all workers). Suppose f_d and f_p are both identity functions (i.e., $f_d(x) = x$, $f_p(x) = x$).*

In the $(k+1)$-th competition, it is w_1's turn to compete. w_1 can only compete for t_2. He first uses his new privacy budget $\epsilon_{2,1}^{(z_k+1)} = \epsilon_{2,1}^{(2)} = 7.1$ and calculates the new effective obfuscated distance $\tilde{d}_{2,1}^{(k+1)} = 4.01$. After that, he calculates $UT_1^{(k+1)} = -f_d(\tilde{d}_{2,1}^{(k+1)}) - f_p(\epsilon_{2,1}^{(z_k+1)}) + f_d(\tilde{d}_{2,2}^{(k)}) - v_1 + f_d(\tilde{d}_{1,1}^{(k)}) = 0.13 > 0$. Then, he publishes his privacy budget $\epsilon_{2,1}^{(2)} = 7.1$ with the corresponding obfuscated distance $\hat{d}_{2,1}^{(2)}$ to the server. The server can also calculate the new effective obfuscated distance $\tilde{d}_{2,1}^{(k+1)}$ and $UT_1^{(k+1)}$. It finds that $UT_1^{(k+1)}$ is positive, which means w_1 wins t_2. The server then alters the allocation table AL by setting the winner of t_2 as w_1 and the winner of t_1 as NULL.

TABLE VII: Allocation list from the k-th competition.

Task	k-th	$(k+1)$-th	$(k+2)$-th – $(k+6)$-th
t_1	w_1	NULL	w_2
t_2	w_2	w_1	w_1
t_3	w_3	w_3	w_3

TABLE VIII: The timeline of effective distances and privacy budgets.

<table>
<tr><th>Pair/Times</th><th>k</th><th>$k+1$</th><th>$k+2$</th><th>$k+3$</th><th>$k+4$</th><th>$k+5$</th><th>$k+6$</th></tr>
<tr><td>(t_1, w_1)</td><td colspan="4">(12.7,0.1)</td><td>(12.7,0.1)
(12.4,0.3)</td><td colspan="2">(12.7,0.1)</td></tr>
<tr><td>(t_1, w_2)</td><td colspan="2">(5.5,4.6)</td><td>(5.5,4.6)
(5.3,4.65)</td><td colspan="4">(5.3,4.65)</td></tr>
<tr><td>(t_1, w_3)</td><td colspan="3">(9.93,0.1)</td><td>(9.93,0.1)
(9.63,0.4)</td><td colspan="2">(9.93,0.1)</td><td>(9.93,0.1)
(9.63,0.4)</td></tr>
<tr><td>(t_2, w_1)</td><td>(4.11,6.99)</td><td>(4.11,6.99)
(4.01,7.1)</td><td colspan="5">(4.01,7.1)</td></tr>
<tr><td>(t_2, w_2)</td><td colspan="5">(10.94,0.1)</td><td>(10.94,0.1)
(10.64,0.2)</td><td>(10.94,0.1)</td></tr>
<tr><td>(t_3, w_2)</td><td colspan="2">(12.71,0.1)</td><td>(12.71,0.1)
(12.51,0.3)</td><td colspan="2">(12.71,0.1)</td><td>(12.71,0.1)
(12.51,0.3)</td><td>(12.71,0.1)</td></tr>
<tr><td>(t_3, w_3)</td><td colspan="7">(7.78,5.4)</td></tr>
</table>

In the $(k+2)$-th competition, it is w_2's turn to compete. w_2 can compete for both t_1 and t_3. He calculates $UT_2^{(k+2)}[t_1] = v_1 - f_d(\tilde{d}_{1,2}^{(k+2)}) - f_p(\epsilon_{1,2}^{(z_k+2)}) = 2.45 > 0$ and $UT_2^{(k+2)}[t_3] = -f_d(\tilde{d}_{3,2}^{(k+2)}) - f_p(\epsilon_{3,2}^{(z_k+2)}) + f_d(\tilde{d}_{3,3}^{(k+1)}) = -5.03 < 0$. After that, w_2 sets $UT_2^{(k+2)}$ as $UT_2^{(k+2)}[t_1]$, which is the maximal positive value in set $\{UT_2^{(k+2)}[t_1], UT_2^{(k+2)}[t_3]\}$. Then, w_2 applies to the server for t_1 by proposing $(\hat{d}_{2,1}^{(2)}, \epsilon_{2,1}^{(2)})$. After similar calculations, the server alters AL by setting the winner of t_1 as w_2.

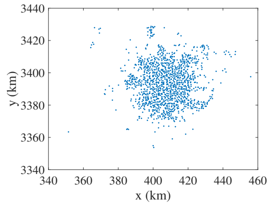
In the $(k+3)$-th competition, it is w_3's turn to compete. w_3 can only propose to t_1. However, the value $UT_3^{(k+3)} = -9.95 < 0$. Therefore, w_3 does not compete for any tasks.

These three steps are repeated until all workers do not propose to any tasks (i.e., until the 6-th competition). Table VIII records the changing of effective obfuscated distances and privacy budgets. The red one (with $UT > 0$) means there is a new winner who publishes a new privacy budget and updates the corresponding effective obfuscated distance. The green one (with $UT \leq 0$) means the competitor fails to compete for the task and will publish neither his new obfuscated distance nor his new privacy budget.

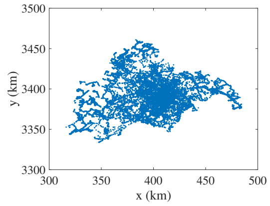
Convergence Analysis. In order to answer the convergence speed of PGT, we need to know how many rounds it takes to find a pure Nash equilibrium. For the corresponding potential game of a PAA-TA instance, $\mathcal{G} =< \mathcal{W}, \boldsymbol{S}, \boldsymbol{UT} >$, we assume there is an equivalent game with potential function $\Phi_{\mathbb{Z}}(\boldsymbol{st}) = d \cdot \Phi(\boldsymbol{st})$, where d is a positive multiplicative factor satisfying that $\Phi_{\mathbb{Z}}(\boldsymbol{st}) \in \mathbb{Z}$ for $\forall \boldsymbol{st} \in \boldsymbol{S}$. Let $\boldsymbol{st}^*$ be the best strategy the workers can choose in this PAA-TA game instance. Based on the above assumption, we prove that PGT executes at most $\Phi_{\mathbb{Z}}(\boldsymbol{st}^*)$ rounds.

Theorem VI.2. *PGT executes at most $\Phi_{\mathbb{Z}}(\boldsymbol{st}^*)$ rounds to achieve a pure Nash equilibrium, where $\Phi_{\mathbb{Z}}(\boldsymbol{st}^*) = d \cdot \Phi(\boldsymbol{st}^*)$ is a scaled potential function with integer value d and $\boldsymbol{st}^*$ is the optimal strategy the workers can choose in the potential PAA-TA game instance.*

Proof. We say PGT converges when no workers deviate from their current strategies. If PGT has not converged, then at least one worker w_j deviates from his current strategy in



(a) Order location distribution



(b) Taxi location distribution

Fig. 3: Orders and taxies of Chengdu Didi data set.

each round. Besides the new change strategy st_j' of w_j is better than his current strategy st_j. And the change will improve at least 1 (i.e., $\Phi_{\mathbb{Z}}(st_i', \boldsymbol{s}_{-i}) - \Phi_{\mathbb{Z}}(st_i, \boldsymbol{s}_{-i}) \geq 1$) for potential games. Because the maximum value of scaled potential function is $\Phi_{\mathbb{Z}}(\boldsymbol{st}^*)$, and the total utility is always positive, PGT needs at most $\Phi_{\mathbb{Z}}(\boldsymbol{st}^*)$ rounds to converge to a pure Nash equilibrium. $\square$

Quality Analysis. Since the distance in our game is rather real distance than effective obfuscated distance, here we give the upper bound of expectation of *price of stability* (EPoS) and the lower bound of expectation of *price of anarchy* (EPoA). Let

$$U_j^L(i) = v_i - f_d(d_{i,j}) - f_p(\sum_{t_k \in R_j} sum(\boldsymbol{\epsilon_{k,j}}))$$

$$U_j^H(i) = v_i - f_d(d_{i,j}) - f_p(min(\boldsymbol{\epsilon_{i,j}}))$$

$$U_{min}^+(i) = \begin{cases} \min\limits_{R_j \ni t_i, U_j^L(i)>0} U_j^L(i), & \text{if there exists } U_j^L(i) > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$U_{max}^+(i) = \begin{cases} \max\limits_{R_j \ni t_i} U_j^H(i), & \text{if there exists } U_j^H(i) > 0 \\ 0, & \text{otherwise} \end{cases}$$

Then we have Theorem VI.3 as follows.

Theorem VI.3. *In the strategic game of PGT, the lower bound of EPoA is* $\frac{\sum_{t_i \in \mathcal{T}} U_{min}^+(i)}{\sum_{t_i \in \mathcal{T}} U_{max}^+(i)}$ $(\sum_{t_i \in \mathcal{T}} U_{max}^+(i) \neq 0)$ *and the upper bound of EPoS is 1.*

Please refer to details of the proof of Theorem VI.3 in Appendix B in our technical report [30].

Theorem VI.4. *PGT satisfies* $(\sum_{t_i \in R_j} \boldsymbol{b_{i,j}} \boldsymbol{\epsilon_{i,j}} r_j)$*-local differential privacy for each worker* w_j.

The proof is similar to Theorem V.2, and please refer to the details in Appendix C in our technical report [30].

VII. Experiment

A. Data Sets

We test our mechanisms in real and synthetic data sets.

Real Data Set. We use Didi Chuxing[32] in Chengdu, China, as our real data set. We choose the day with the most requests for evaluation (November 18, 2016 in Chengdu) and perform the same preprocess in the existing work [33], which is denoted as *chengdu*.

Chengdu contains 259347 orders and 30000 taxis. Each order tuple is a taxi request consisting of a release time, a pickup location, a drop-off location, and some passengers. Each taxi tuple is a basic message consisting of the original location of the taxi and its capacity. The location distribution of taxis is shown in Figure 3(b).

Synthetic Data Set. We generate two data sets with 2-dimensional uniform distribution and normal distribution, respectively. For the uniform distribution data set, we randomly generate 300000 points for tasks and 900000 for workers in a plane with a range of 100×100. Each point follows a 2-dimensional uniform distribution with an average of 0.

For the normal distribution data set, we generate 300k and 900k points for tasks and workers, respectively. The expectation and variance for all points are 0 and 150, respectively.

TABLE IX: Methods.

	Private version	Non-Private version	Non-PPCF version
Distance Elimination	PDCE [3]	DCE	PDCE-nppcf
Utility Elimination	PUCE	UCE	PUCE-nppcf
Game Theory	PGT	GT	—
Greedy	—	GRD	—

TABLE X: Experimental settings.

Parameters	Values
worker-task ratio	$1, 1.5, \mathbf{2}, 2.5, 3$
task values	$1.5, 3, \mathbf{4.5}, 6, 7.5$
worker range	$0.8, 1.1, \mathbf{1.4}, 1.7, 2.0$
privacy budget	$[0.5, 0.75], [0.75, 1.00], [1.00, 1.25], [1.25, 1.50], [1.50, 1.75]$; **[0.5,1.75]**
privacy budget group size	**7**

B. Experimental Setup

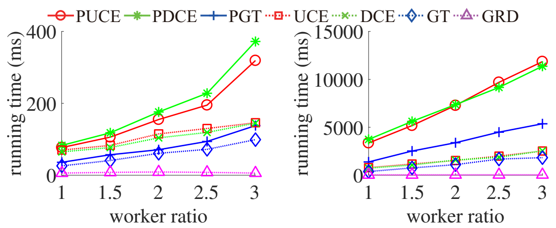
We split the orders into batches by timestamp. Each batch contains at most 1000 orders. Figure 3(a) is a batch example of the order distribution. We also split the taxis into ten groups for the real data set, each containing 3000 taxis. We use each worker group circularly for each batch. We set the start locations of orders as task locations and the start locations of taxis as worker locations.

Let S_T and S_W be two sets for tasks and workers. We define the value $p_{wt} = \frac{|S_W|}{|S_T|}$ as *worker-task ratio*, which stands for the ratio between worker number, and task number.

We alter the method in Ref [3] by constraining the workers' proposing range and replacing the PCF with PPCF in order to get reasonable comparison with our PUCE. We denote this altering method in Ref [3] as Private Distance Conflict-Elimination (PDCE). The difference between PUCE and PDCE is the optimization objective. In PDCE, the goal is to minimize all the travelling distance on the platform, which only consider the distance variable. However, in PUCE, the goal is to maximize the utility function of the platform, which considers the task value, travel distance and privacy budget.

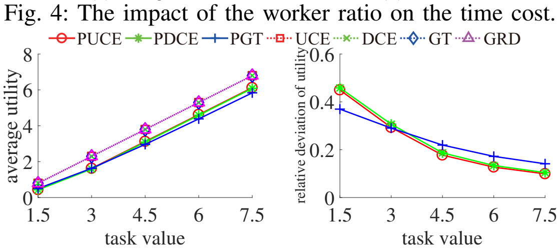
We compare our PUCE and PGT with PDCE. Besides, we construct the non-privacy solution of each privacy solution by eliminating the privacy budget cost in the utility function and replacing obfuscated distance with real distance. These non-privacy solutions are Utility Conflict-Elimination (UCE), Game Theory (GT), Distance Conflict-Elimination (DCE) and Greedy (GRD). Here, GRD always greedily chooses the current best worker-task pair (with the highest utility) for each worker. We also construct the non-PPCF solution of PUCE and PDCE by replacing the PPCF part with the PCF part. We denote these non-PPCF solutions as PUCE-nppcf and PDCE-nppcf. We compare all these methods above and summarize them in Table IX.

We show the parameter settings in table X, where the default values are marked in bold. As for distance value function f_d and privacy budget value function f_p, we model them as



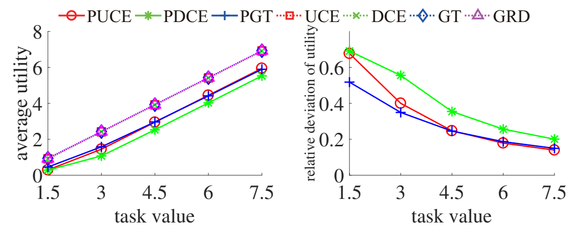
(a) *chengdu* (b) *normal*

Fig. 4: The impact of the worker ratio on the time cost.



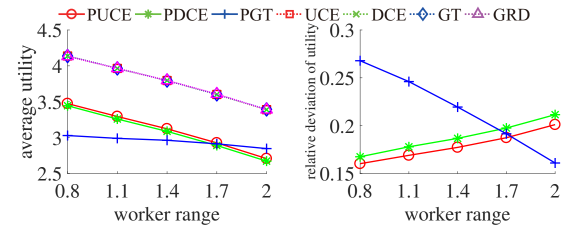
(a) Average Utility (b) Relative Deviation of Utility

Fig. 5: The impact of the task value on the utility (*chengdu*).



(a) Average Utility (b) Relative Deviation of Utility

Fig. 6: The impact of the task value on the utility (*normal*).



(a) Average Utility (b) Relative Deviation of Utility

Fig. 7: The impact of the worker range on the utility (*chengdu*).

linear functions and use $f_d(x) = \alpha x$ and $f_p(x) = \beta x$ in our experiment. We set $\alpha = 1$ and $\beta = 1$.

We run our experiment on an Intel(R) Xeon(R) Silver 4210R CPU @ 2.4GHz with 128 GB RAM in Java.

C. Measures

We design a utility-based empirical measure of the efficiency of our proposed mechanisms.

Average Utility: We define the average utility U_{AVG} as $\frac{\sum_{(i,j)\in M} U_j(i)}{|M|}$, which means the average utility value of a successful task-worker pair.

Relative Deviation of Utility: Let the utility of non-privacy solutions be U_{NP} and privacy ones be U_{P}. We define the relative deviation of utility U_{RD} as $\frac{U_{\text{NP}}-U_{\text{P}}}{U_{\text{NP}}}$.

Average Travel Distance: We define the average travel distance D_{AVG} as $\frac{\sum_{(i,j)\in M} d_{i,j}}{|M|}$, which means the average travel distance of a successful task-worker pair.

Relative Deviation of Distance: Let the distance of non-privacy solutions be D_{NP} and privacy ones be D_{P}. We define the relative deviation of distance D_{RD} as $\frac{D_{\text{P}}-D_{\text{NP}}}{D_{\text{NP}}}$.

D. Experimental Result

1) Time Cost: Figure 4 shows the time cost on different worker ratio from 1 to 3 while the other parameters are in the default values in Table X. We can see that the time cost increases linearly with the worker ratio. That is because when we fix the task quantity, as the worker ratio becomes larger, the competition between workers will become more fierce, and it will cost more time to finish the whole competition.

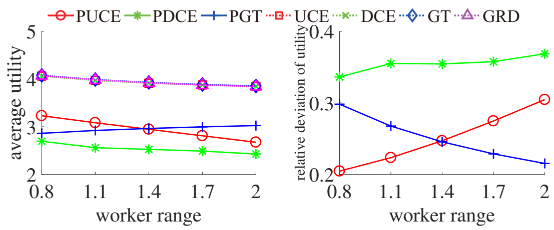
Besides, we can find that PUCE costs nearly the same time over the change of worker ratio. PGT costs much less time than PUCE and PDCE. Compared with PDCE, PGT costs about 52%–63% less time in *chengdu* and 50%–63% in *normal*.

2) Average Utility: Figure 5 and 6 show the relation between the utility and the task value on *chengdu* and *normal* respectively. We change the task value from 1.5 to 7.5 and set other parameters as the default values.

In Figure 5(a) and 6(a), the utility increases approximately linear with the task value. We can see that PGT performs worse than PDCE slightly in *chengdu*, but better in *normal*. PGT even performs better than PUCE in *normal*. The reason is that PGT takes advantage over the other two when the workers' service area contains many tasks. The data in *chengdu* is of road network data which is sparser than that in *normal*. Thus fixed the service area, a worker in *chengdu* can propose to fewer tasks than that in *normal* on average, which leads to poor utility for PGT. (The following experiment result in Figure 7(a) proves this inference.) PUCE performs better than PDCE in both of the two data sets. The relative deviation of utility impacted by the task value is shown in Figure 5(b) and 6(b). We can see that the relative deviation of utility decreases with the task value increase from 1.5 to 7.5, which means the absolute deviation between the private and non-private solutions keeps nearly stable. And when the task value becomes larger and larger, the utility of private solutions equals that of non-private solutions asymptotically.

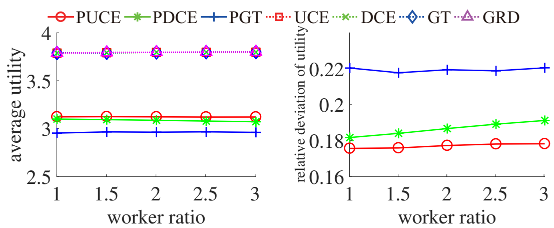
Figure 7 and 8 show the relation between the utility and the worker range on *chengdu* and *normal* respectively. The worker service area (denoted as worker range) increases from 0.8 to 2, and the other parameters are set as default values. The average utility depends on the total utility and the matching quantity. Specifically, in Figure 7(a), the average utility of all solutions decreases when the worker range increases from 0.8 to 2. It is because when worker service areas become larger, more workers (who have no task to propose to in some small range conditions, denoting them as $\mathcal{W}_L$) will be able to propose to some tasks. With the ratio of $\mathcal{W}_L$ becoming larger, the average distance to all matching tasks becomes larger, making the average utility smaller.

Besides, we can see that the utility of PGT decreases slower than both PUCE and PDCE. The utility of PGT is no less than 88% when the worker range is no more than 1.6. And as the worker range increases, the utility of PGT will exceed the other two. The reason why PGT keeps lower decrease is that PGT can avoid ineffective competition. When the service area becomes larger, the competition becomes more intense, and the advantage of PGT becomes more apparent.



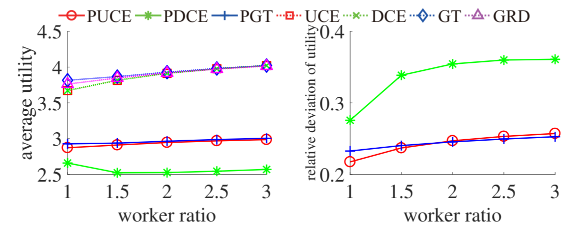
(a) Average Utility (b) Relative Deviation of Utility

Fig. 8: The impact of the worker range on the utility (*normal*).



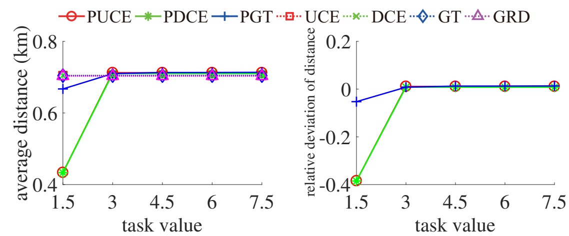
(a) Average Utility (b) Relative Deviation of Utility

Fig. 9: The impact of the worker ratio on the utility (*chengdu*).



(a) Average Utility (b) Relative Deviation of Utility

Fig. 10: The impact of the worker ratio on the utility (*normal*).



(a) Average Distance (b) Relative Deviation of Distance

Fig. 11: The impact of the task value on the distance (*chengdu*).

Figure 7(b) shows the relative deviation of utility affected by the worker range. We can see that the utility of PGT will tend to that of its non-privacy solution when the worker range becomes larger and larger. However PUCE and PDCE deviate more as the worker range becomes larger. That is because when the worker's service area becomes larger, it has a greater possibility of disturbing a large real distance to a small obfuscated distance or a small real distance to a large obfuscated distance. Without the guarantee of total utility function ST, the total proposing workers' utilities in PUCE and PDCE decrease dramatically when the worker range increases.

From Figure 8(a), we can get the similar conclusion to that in Figure 7(a). Besides, we can find when the worker range becomes large enough, the decline rate of average utility for PUCE and PDCE tend to be small. That is because being too far away will make the utility value non-positive, and the server will not choose. The average utility of PGT increases slightly, which is 16% larger than PDCE on average. That is because PGT can increase the total utility more rapidly than the matching quantity.

Figure 9 and 10 show the relation between the utility and the worker ratio. From figure 9(a) and 10(a), we can see that the worker ratio does not affect the average utility very much. That is because the increase of workers does not significantly increase proposing workers. Besides, we can see that PUCE always keeps a higher average utility than PDCE. And PGT performs worse than PDCE in *chengdu* but better in *normal*.

3) Average Travel Distance: Figure 11 to Figure 16 show the influence of the task value, worker range and worker ratio on the distance. PDCE is better than PUCE and PGT in most cases. That is because the goal of PDCE is only to minimize the total travel distance on the platform without considering task value and privacy budget cost. Besides, we can see that different data sets lead to different comparison results for PUCE, PGT and PDCE. The average travel distance of PDCE on *normal* outperforms the other two on *chengdu*.

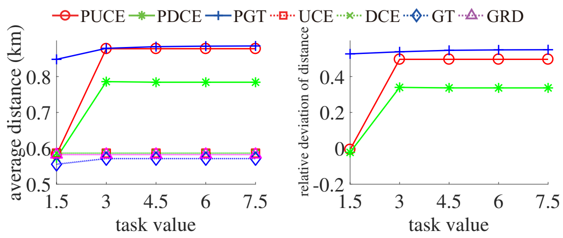
Figure 11 and 12 show the relation between the average distance and the task values. We can see that task values do not affect the average distance when the task value is larger than 3. That is because when the task value is large enough, it will not affect the difference between the two utility values. Workers will not choose many tasks in their range when the task value is minimal, leading to a small average distance. Besides, PUCE is better than PGT slightly but worse than PDCE. However, the difference of the distances between PUCE and PDCE keeps stable as task value increases.

Figure 13 and 14 show the relation between the average distance and the worker service area. We can see that the average distance increases when worker range increases. That is because a larger range will lead to more proposing workers with far distance, making the average distance larger. The average distances of PUCE and PGT are nearly equal. They are worse than PDCE with nearly fixed difference distance value when the worker range is larger than 1.4.

Figure 15 and 16 show the relation between the average distance and the worker ratio. Especially in figure 15(a), the average distance in non-privacy solutions decreases when the worker ratio increases. That is because, with the increase in workers, the competition has become rigorous. The number of tasks limits the increase of workers' proposals, and a task will be allocated to the worker at a small distance. Therefore, the average becomes smaller with the worker ratio becoming larger. As for privacy solutions, competition will also cost more privacy budget on utility value, which will relieve the reduction in privacy solutions. Similar to the comparison result before, PDCE is better than the other two schemes when the worker ratio is larger than 1.5.

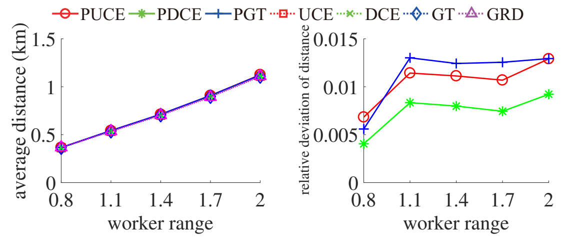
4) PPCF and Non-PPCF: We compare our PUCE and the PDCE with non-PPCF ones (PUCE-nppcf and PDCE-nppcf). We fix the task value as 4.5, the worker range distance as 1.4, and the worker ratio as 2. We divide the privacy budget range into 5 groups shown in Table X.

Figure 17 shows the relation between the average utility and the privacy budget. We mark the median of each interval as the value of the x-axis.



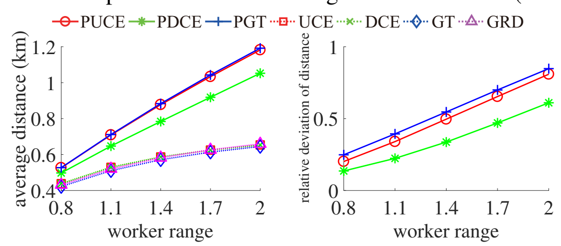
(a) Average Distance (b) Relative Deviation of Distance

Fig. 12: The impact of the task value on the distance (*normal*).



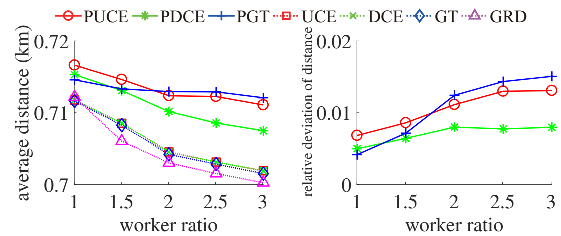
(a) Average Distance (b) Relative Deviation of Distance

Fig. 13: The impact of the worker range on the distance (*chengdu*).



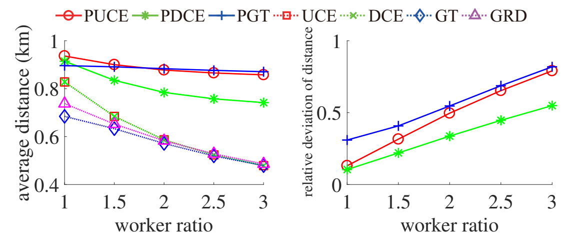
(a) Average Distance (b) Relative Deviation of Distance

Fig. 14: The impact of the worker range on the distance (*normal*).



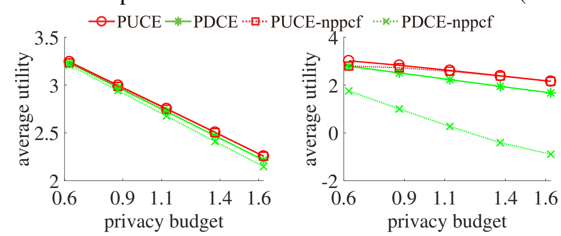
(a) Average Distance (b) Relative Deviation of Distance

Fig. 15: The impact of the worker ratio on the distance (*chengdu*).



(a) Average Distance (b) Relative Deviation of Distance

Fig. 16: The impact of the worker ratio on the distance (*normal*).



(a) *chengdu* (b) *normal*

Fig. 17: The impact of privacy on the utility.

The solutions with PPCF are better than that without PPCF when the privacy budget is small. It means PPCF is suitable for high-privacy situations and is continuously more effective than that without PPCF. As the privacy budget increases, the average utility decreases. That is because large privacy budgets give large average privacy budget cost for workers. Although high privacy budgets are able to lead to high utility match, it also leads to high privacy budget cost. Besides, as the privacy budget increases, the difference between PPCF and non-PPCF is eliminated. That is because the larger the privacy budget, the more accurate the obfuscated distance, and the smaller difference between PPCF and PCF.

VIII. Conclusion

In this paper, we formalize the Privacy-aware Task Assignment (PA-TA) Problem, which assigns a task to a worker to get a high utility value. In order to make use of obfuscated distance published by workers, we propose new notations called *effective obfuscated distance* and *effective privacy budget*. To get a higher utility value, we offer a new comparison function called PPCF and prove that it can achieve better effectiveness than PCF in both theory and practice. Besides, we propose another game theoretic approach to solve the problem. Extensive experiments have been conducted to show the efficiency and effectiveness of our methods on both real and synthetic data sets.

Our PUCE and PGT only consider the distance privacy of one worker in his service area. If the service area of a worker is small enough and the quantity of tasks in this area is large enough, attackers can locate the worker's position through trilateration by viewing the entire area as a position. That is because too much effective obfuscated distance from a worker to many tasks will outline the worker's service area. Our subsequent work will focus on this problem and consider how to hide correlation privacy caused by the relation between different worker service areas. Besides, in our goal function, we suppose the task value is only related to the task itself. Our subsequent work will extract the payment from the task value and research on the assumption that: the task value is related to task itself, travel distance and privacy cost.

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X. Appendix

A. Proof for Theorem V.1

Before the proof of Theorem V.1, we declare and prove Lemma X.1 and Lemma X.2 as follows.

Lemma X.1. *For any* $d_x, d_y, \epsilon_x, \epsilon_y$, $\hat{d}_x = d_x + Lap(0, 1/\epsilon_x), \hat{d}_y = d_y + Lap(0, 1/\epsilon_y)$, *we have* $PCF(\hat{d}_x, \hat{d}_y, \epsilon_x, \epsilon_y) > \frac{1}{2} \Leftrightarrow \hat{d}_x < \hat{d}_y$.

Proof. Let $\eta_x \sim \epsilon_x, \eta_y \sim \epsilon_y$. Then we have

$$PCF(\hat{d}_x, \hat{d}_y, \epsilon_x, \epsilon_y) = \iint_{D_R} f(\eta_x, \eta_y)$$

where $f(\eta_x, \eta_y) = \frac{\epsilon_x \epsilon_y}{4} e^{-\epsilon_x |\eta_x| - \epsilon_y |\eta_y|}$ and D_R is the plane set satisfying $D_R = \{(\eta_x, \eta_y) : \eta_y - \eta_x < \hat{d}_y - \hat{d}_x\}$. Note that $f(\eta_x, \eta_y)$ is symmetry about both x-axis and y-axis and D_R is part of plane split by line $l_\eta : \eta_y = \eta_x + \hat{d}_y - \hat{d}_x$. Thus, we know that only when l_η crosses the origin ($\hat{d}_y = \hat{d}_x$), $PCF(\hat{d}_x, \hat{d}_y, \epsilon_x, \epsilon_y)$ equals $\frac{1}{2}$. When $\hat{d}_y - \hat{d}_x < 0$, $PCF(\hat{d}_x, \hat{d}_y, \epsilon_x, \epsilon_y) < \frac{1}{2}$, and $\hat{d}_y - \hat{d}_x > 0$, $PCF(\hat{d}_x, \hat{d}_y, \epsilon_x, \epsilon_y) > \frac{1}{2}$. Therefore, $PCF(\hat{d}_x, \hat{d}_y, \epsilon_x, \epsilon_y) > \frac{1}{2} \Leftrightarrow \hat{d}_x < \hat{d}_y$. □

Lemma X.2. *For any two continue and differentiable non-negative functions f, g defined in $\mathbb{R}$, if there exists an interval $[a, +\infty)$ satisfying that $\int_a^{+\infty} f(x)dx = \int_a^{+\infty} g(x)dx$ and there exists a point $x_0 \in (a, +\infty)$ satisfying $f(x) \geq g(x)$ for $x \in (a, x_0]$ and $f(x) \leq g(x)$ for $x \in (x_0, +\infty)$, then $\int_a^x f(x)dx \geq \int_a^x g(x)dx$ for all $x \in [a, +\infty)$.*

Proof. For any $x \in [a, +\infty)$, we can divide it into two cases: (1) $x \in [a, x_0]$; (2) $x \in (x_0, +\infty]$. If (1) holds, according to $f(x) \geq g(x)$ *for* $x \in (a, x_0]$, we have directly get

$$\int_a^x f(x)dx \geq \int_a^x g(x)dx \quad \text{for } x \in [a, x_0]. \tag{6}$$

If (2) holds, then we have $\int_{x_0}^{+\infty} f(x)dx \leq \int_{x_0}^{+\infty} g(x)dx$. And we can get

$$\begin{aligned}
&\int_a^x f(x)dx - \int_a^x g(x)dx \\
=&\int_a^{+\infty} f(x)dx - \int_{x_0}^{+\infty} f(x)dx - (\int_a^{+\infty} g(x)dx - \int_{x_0}^{+\infty} g(x)dx) \\
=&\int_{x_0}^{+\infty} g(x)dx - \int_{x_0}^{+\infty} f(x)dx \geq 0.
\end{aligned}$$

Therefore, we have

$$\int_a^x f(x)dx \geq \int_a^x g(x)dx \quad \text{for } x \in (x_0, +\infty). \tag{7}$$

From Equation 6 and 7, we can have $\int_a^x f(x)dx \geq \int_a^x g(x)dx$ for $x \in [a, +\infty)$. □

Based on Lemma X.1 and Lemma X.2, we give the proof of Theorem V.1 as follows.

Proof. From Lemma X.1, we have $PCF(\hat{d}_x, \hat{d}_y, \epsilon_x, \epsilon_y) > \frac{1}{2} \Leftrightarrow \hat{d}_x < \hat{d}_y$. From Equation 3, we have $\Pr[d_x < d_y] > \frac{1}{2} \Leftrightarrow d_x < \hat{d}_y$. Therefore, we only need to prove $Pr[\hat{d}_x < \hat{d}_y] \leq Pr[d_x < \hat{d}_y]$ for any d_x, d_y satisfying $d_x < d_y$.

According to the definition, we have

$$\begin{aligned}
\Pr[\hat{d}_x < \hat{d}_y] &= \Pr[d_x + \eta_x < d_y + \eta_y] = \Pr[\eta_y > \eta_x + d_x - d_y] \\
&= \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{\eta_y - d_x + d_y} \frac{\epsilon_x \epsilon_y}{4} e^{-(\epsilon_x |\eta_x| + \epsilon_y |\eta_y|)} d\eta_x \right) d\eta_y
\end{aligned}$$

and

$$\begin{aligned}
\Pr[d_x < \hat{d}_y] &= \Pr[d_x < d_y + \eta_y] = \Pr[\eta_y > d_x - d_y] \\
&= \int_{d_x - d_y}^{+\infty} \frac{\epsilon_y}{2} e^{-\epsilon_y |\eta_y|} d\eta_y.
\end{aligned}$$

Let $s = d_y - d_x$. Let $F : s \to \Pr[\hat{d}_x < \hat{d}_y]$ and $G : s \to \Pr[d_x < \hat{d}_y]$. From the definition, we know $s > 0$, $\lim_{s \to 0} F(s) = \lim_{s \to 0} G(s) = \frac{1}{2}$ and $\lim_{s \to +\infty} F(s) = \lim_{s \to +\infty} G(s) = 1$. And we have

$$\begin{aligned}
\frac{\partial F(s)}{\partial s} &= \frac{\epsilon_x \epsilon_y}{4} \left(\frac{e^{-s\epsilon_x} + e^{-s\epsilon_y}}{\epsilon_x + \epsilon_y} - \frac{e^{-s\epsilon_x} - e^{-s\epsilon_y}}{\epsilon_x - \epsilon_y} \right) \\
&= \frac{\epsilon_x \epsilon_y}{2} \cdot \frac{e^{-s\epsilon_y} \epsilon_x - e^{-s\epsilon_x} \epsilon_y}{(\epsilon_x + \epsilon_y)(\epsilon_x - \epsilon_y)} > 0, \\
\frac{\partial G(s)}{\partial s} &= \frac{\epsilon_y}{2} e^{-s\epsilon_y} > 0, \\
\frac{\partial F(s)}{\partial s} / \frac{\partial G(s)}{\partial s} &= \frac{\epsilon_x (\epsilon_x - e^{s(\epsilon_y - \epsilon_x)} \epsilon_y)}{(\epsilon_x + \epsilon_y)(\epsilon_x - \epsilon_y)}.
\end{aligned}$$

Let $\frac{\partial F(s)}{\partial s} / \frac{\partial G(s)}{\partial s} \leq 1$. Then we have $s \leq \frac{1}{\epsilon_x - \epsilon_y} \ln \frac{\epsilon_x}{\epsilon_y}$. Let $\frac{\partial F(s)}{\partial s} / \frac{\partial G(s)}{\partial s} \geq 1$. Then we have $s \geq \frac{1}{\epsilon_x - \epsilon_y} \ln \frac{\epsilon_x}{\epsilon_y}$. That is to say $\frac{\partial G(s)}{\partial s} \geq \frac{\partial F(s)}{\partial s}$ for $s \in (0, \frac{1}{\epsilon_x - \epsilon_y} \ln \frac{\epsilon_x}{\epsilon_y})$ and $\frac{\partial G(s)}{\partial s} \leq \frac{\partial F(s)}{\partial s}$ for $s \in (\frac{1}{\epsilon_x - \epsilon_y} \ln \frac{\epsilon_x}{\epsilon_y}, +\infty)$. According to Lemma X.2, we have $F(s) \leq G(s)$ for $s \in (0, +\infty)$. □

B. Proof for Theorem VI.3

Proof. Let $\hat{U}(\boldsymbol{st})$ be the overall utility of the strategy $\boldsymbol{st}$ with (i.e., $\hat{U}(\boldsymbol{st}) = \Phi(\boldsymbol{st})$). Besides, we note the global optimal strategy as $\hat{\boldsymbol{st}}$, the strategy of achieving best competing utility value as $\boldsymbol{st}^*$ and the worst competing utility value as $\boldsymbol{st}^\sharp$. Then we have $\hat{U}(\hat{\boldsymbol{st}}) = \Phi(\hat{\boldsymbol{st}})$, $\hat{U}(\boldsymbol{st}^*) = \Phi(\boldsymbol{st}^*)$ and $\hat{U}(\boldsymbol{st}^\sharp) = \Phi(\boldsymbol{st}^\sharp)$. Thus,

$$EPoS = \frac{E(\hat{U}(\boldsymbol{st}^*))}{E(OPT)} = \frac{E(\hat{U}(\boldsymbol{st}^*))}{E(\hat{U}(\hat{\boldsymbol{st}}))} \leq 1.$$

If we get the lower bound of $E(\hat{U}(\boldsymbol{st}^\sharp))$ and upper bound of $E(\hat{U}(\hat{\boldsymbol{st}}))$, then we can get the value of EPoA. As for $E(\hat{U}(\boldsymbol{st}^\sharp))$, we have

$$\begin{aligned}
E(\hat{U}(\boldsymbol{st}^\sharp)) &\geq \min_k \sum_{t_i \in \mathcal{T}} \sum_{w_j \in \mathcal{W}} (s_{i,j}^{(k)} \cdot (v_i - f_d(d_{i,j})) - f_p(\boldsymbol{b_{i,j}^{(k)}} \cdot \boldsymbol{\epsilon_{i,j}})) \\
&\geq \sum_{t_i \in \mathcal{T}} \min_{R_j \ni t_i, U_j^L(i) > 0} U_j^L(i) = \sum_{t_i \in \mathcal{T}} U_{min}^+(i)
\end{aligned}$$

As for $E(\hat{U}(\hat{\boldsymbol{st}}))$, we have

$$\begin{aligned}
E(\hat{U}(\hat{\boldsymbol{st}})) &\leq OPT(\sum_{t_i \in \mathcal{T}} \sum_{w_j \in \mathcal{W}} (s_{i,j} \cdot (v_i - f_d(\tilde{d}_{i,j})) - f_p(\boldsymbol{b_{i,j}} \cdot \boldsymbol{\epsilon_{i,j}}))) \\
&\leq \sum_{t_i \in \mathcal{T}} \max_{R_j \ni t_i} U_j^H(i) = \sum_{t_i \in \mathcal{T}} U_{max}^+(i)
\end{aligned}$$

Therefore, we have

$$EPoA = \frac{E(\hat{U}(\boldsymbol{st}^*))}{E(OPT)} \geq \frac{\sum_{t_i \in \mathcal{T}} U_{min}^+(i)}{\sum_{t_i \in \mathcal{T}} U_{max}^+(i)}$$

□

C. Proof for Theorem VI.4

Proof. Let $\mathcal{A}_j$ be the mechanism PGT applying to w_j with query f defined above. Let X_j be the location of w_j. For query $f(X_j) = [d_{i_1,j}, ..., d_{i_{|R_j|},j}]$, we extend it to an equivalent query $\hat{f}(X_j) = f(X_j) \cdot \mathcal{J}$, where

$$\mathcal{J} = \begin{bmatrix} CP(\boldsymbol{b_{i_1,j}}) & & & \\ & CP(\boldsymbol{b_{i_2,j}}) & & \\ & & \ddots & \\ & & & CP(\boldsymbol{b_{i_{|R_j|},j}}) \end{bmatrix}$$

is a block diagonal matrix. Actually, $\hat{f}(X_j)$ means query $d_{i_u,j}$ for $sum(\boldsymbol{b_{i_u,j}})$ times for $u \in [|R_j|]$. We denote the size of $\hat{f}(X_j)$ as $|\hat{f}|$ and the a-th element of $\hat{f}(X_j)$ as $\hat{f}(X_j)_a$.

Let Y_j denote the set of all published obfuscated distances of the worker w_j to tasks in R_j. Then we have $Y_j = \hat{f}(X_j) + [\eta_1, \eta_2, ..., \eta_{|\hat{f}|}]$, where $\eta_a (1 \le a \le |\hat{f}|)$ is an i.i.d random variable drawn from $Lap(1/\epsilon_a)$. Hence we have

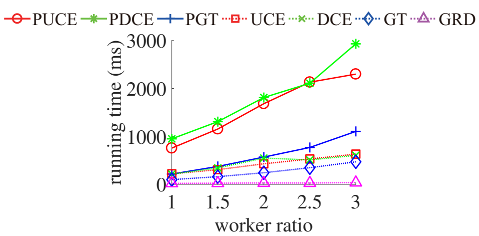
$$\begin{aligned}
\frac{\Pr[\mathcal{A}_j(X_j) = Y_j]}{\Pr[\mathcal{A}_j(X'_j) = Y_j]} &= \prod_{a\in[|\hat{f}|]} \left(\frac{\exp(-\epsilon_a |Y_{j,a} - \hat{f}(X_j)_a|)}{\exp(-\epsilon_a |Y_{j,a} - \hat{f}(X'_j)_a|)}\right) \\
&= \prod_{t_i \in R_j} \prod_{u \in [sum(\boldsymbol{b_{i,j}})]} \left(\frac{\exp(-\epsilon^{(u)}_{i,j} |\tilde{d}^{(u)}_{i,j} - d_{i,j}|)}{\exp(-\epsilon^{(u)}_{i,j} |\tilde{d}^{(u)}_{i,j} - d'_{i,j}|)}\right) \\
&\le \prod_{t_i \in R_j} \prod_{u \in [sum(\boldsymbol{b_{i,j}})]} (\exp(\epsilon^{(u)}_{i,j}(|d_{i,j} - d'_{i,j}|))) \\
&= \prod_{t_i \in R_j} \exp(\boldsymbol{b_{i,j}\epsilon_{i,j}}(|d_{i,j} - d'_{i,j}|)) \\
&\le \exp(\sum_{t_i \in R_j} \boldsymbol{b_{i,j}\epsilon_{i,j}} r_j).
\end{aligned}$$

Because X_j contains only one element, then we have PGT satisfies $(\sum_{t_i\in R_j} \boldsymbol{b_{i,j}\epsilon_{i,j}} r_j)$-local differential privacy for each worker w_j. □

D. Experiment Result for the Uniform Data set

The experiment result of the uniform data set is shown in this section.

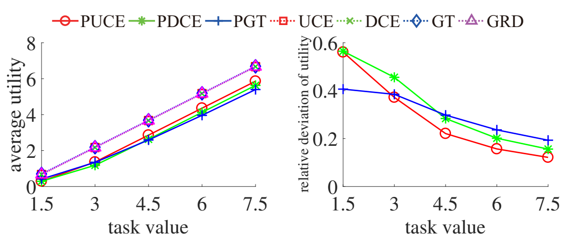
The time cost is shown in Figure 18.



(a) *uniform*

Fig. 18: The impact of the worker ratio on the time cost.

The impact of task value on utility is shown in Figure 19.



(a) Average Utility (b) Relative Deviation of Utility

Fig. 19: The impact of the task value on the utility for *uniform*.

The impact of worker range on utility is shown in Figure 20.

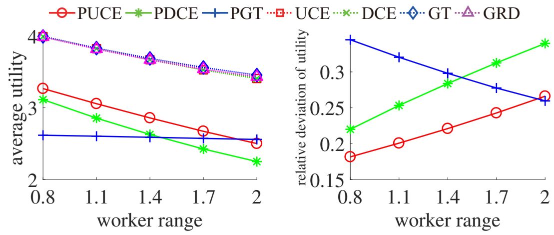
The impact of worker ratio on utility is shown in Figure 21.

The impact of task value on travel distance is shown in Figure 22.

The impact of worker range on travel distance is shown in Figure 23.

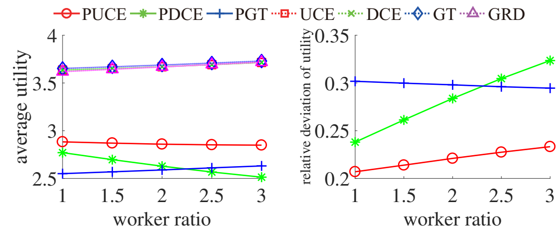
The impact of worker ratio on travel distance is shown in Figure 24.

The impact of worker ratio on PPCF and non-PPCF is shown in Figure 25.



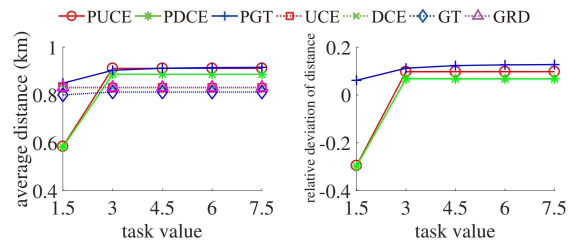
(a) Average Utility (b) Relative Deviation of Utility

Fig. 20: The impact of the worker range on the utility for *uniform*.



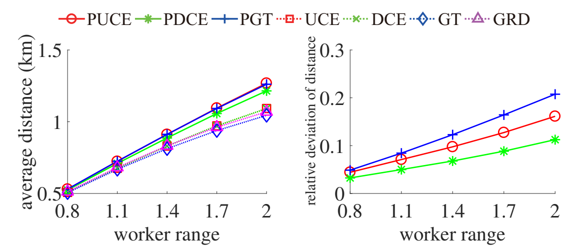
(a) Average Utility (b) Relative Deviation of Utility

Fig. 21: The impact of the worker ratio on the utility for *uniform*.



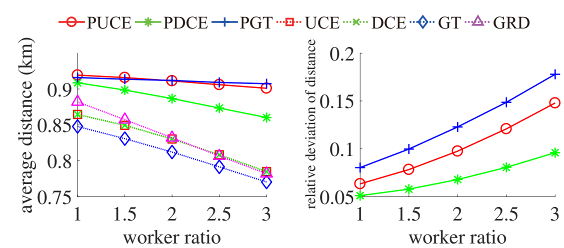
(a) Average Distance (b) Relative Deviation of Distance

Fig. 22: The impact of the task value on the distance for *uniform*.



(a) Average Distance (b) Relative Deviation of Distance

Fig. 23: The impact of the worker ratio on the distance for *uniform*.

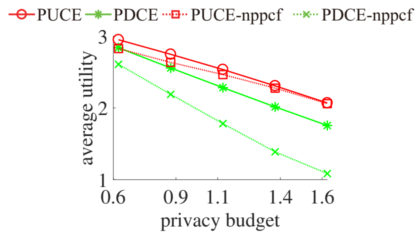


(a) Average Distance (b) Relative Deviation of Distance

Fig. 24: The impact of the worker ratio on the distance for *uniform*.

E. Additional Definitions

Definition 8 (One-to-one Match)**.** Let $G = (U, E, V)$ be a bipartite graph, and $M \subseteq E$ be a match in G. M is called a one-to-one match if for any two different edges $e_{u,v}, e_{u',v'} \in M$, $e_{u,v} \cap e_{u',v'} = \phi$.



(a) Average Distance

Fig. 25: The impact of privacy on the utility.

Let $S_G(M) = [s_{u,v}]_{u \in U, v \in V}$ denote a state matrix of M. Here, $s_{u,v} = 1$ when $e_{u,v} \in M$, $s_{u,v} = 1$; otherwise, $s_{u,v} = 0$.

We utilize Differential Privacy [17] to disturb the raw data and measure the privacy cost of workers' proposals for tasks.

Definition 9 (Differential Privacy [17], DP)**.** A randomized algorithm $\mathcal{A}$ with domain $\mathbb{N}^{|\mathcal{X}|}$ is (ϵ, δ)-differential private if for all $\mathcal{S} \subseteq$ Range$(\mathcal{A})$ and for all $x, y \in \mathbb{N}^{|\mathcal{X}|}$ such that $||x - y||_1 \leq 1$:

$$\Pr[\mathcal{A}(x) \in \mathcal{S}] \leq \exp(\epsilon)\Pr[\mathcal{A}(y) \in \mathcal{S}] + \delta,$$

where $|| \cdot ||_1$ is the ℓ_1 norm of an vector, Pr$[\cdot]$ denotes the probability of an event. Especially, when $\delta = 0$, $\mathcal{A}$ is ϵ-differential private.

When x and y consists of single element, $\mathcal{S}$ is also called a local randomizer, which provides *local differential privacy* (LDP) guarantees [34].

The Laplace mechanism [17] is the most well know perturbation methods for numeric values that satisfy the definition of differential privacy. Given a function f outputting a numeric value vector $f(\cdot)$, the laplace mechanism is able to transform f into a differentially private algorithm by adding random noise to each entry of $f(\cdot)$. The random noise is sampled from laplace distribution.

The scale of the random noise is relevant to the ℓ_1-sensitivity of f as well as a predetermined privacy budget ϵ. The ℓ_1-sensitivity of f is defined as the maximum possible difference between any two vector x and y with their ℓ_1 distance as 1:

Definition 10 (ℓ_1-sensitivity [17])**.** The ℓ_1-sensitivity of a function $f : \mathbb{N}^{|\mathcal{X}|} \to \mathbb{R}^k$ is:

$$\Delta f = \max_{x,y \in \mathbb{N}^{|\mathcal{X}|}, ||x-y||_1 = 1} ||f(x) - f(y)||_1.$$

Thus, the laplace mechanism is stated as follows:

Definition 11 (The Laplace Mechanism [17])**.** Given any function $f : \mathbb{N}^{|\mathcal{X}|} \to \mathbb{R}^k$, the Laplace mechanism is defined as:

$$\mathcal{A}_L(x, f(\cdot), \epsilon) = f(x) + (Y_1, ..., Y_k)$$

where Y_i ($i \in [k]$) is an i.i.d. random variable drawn from $Lap(\Delta f / \epsilon)$.

In order to compare two disturbed value conveniently, we introduce the Probability Compare Function [3] in Definition 6.