Task Allocation in Dependency-aware Spatial Crowdsourcing

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ABSTRACT

Ubiquitous smart devices and high-quality wireless networks enable people to participate in spatial crowdsourcing tasks easily, which require workers to physically move to specific locations to conduct their assigned tasks. Spatial crowdsourcing has attracted much attention from both academia and industry. In this paper, we consider a spatial crowdsourcing scenario, where the tasks may have some dependency on each other. Specifically, one task can only be conducted when its dependent tasks have already been finished. In fact, task dependency is quite common in many real life applications, such as house repairing and party preparation. We formally define the dependency-aware spatial crowdsourcing (DA-SC), which focuses on finding an optimal worker-and-task assignment under the constraints of dependencies, skills of workers, moving distances and deadlines with a goal of maximizing the successfully finished tasks. We prove that the DA-SC problem is NP-hard and thus intractable. Therefore, we propose two approximation algorithms, including greedy approach and game-theoretic approach, which can guarantee the approximate bounds of the results in each batch process. Through extensive experiments on both real and synthetic data sets, we demonstrate the efficiency and effectiveness of our DA-SC approaches.

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1. INTRODUCTION

Recently, with the popularity of smart devices and high speed wireless networks, a new kind of crowdsourcing systems, namely spatial crowdsourcing [1], become ubiquitous (e.g., Uber [2], TaskRabbit [3], and Waze [4]), and attract attention from both academia and industry. Specifically, in spatial crowdsourcing systems, workers are requested to physically move to particular locations to conduct their assigned tasks. The spatial tasks can be simple tasks that every normal worker can conduct, such as taking photos of a landmark (e.g., street view of Google Maps [5]), delivering foods (e.g., Uber Eats [6]), and queuing up for a popular restaurant (e.g., TaskRabbit [3]). However, some complex tasks require specific skills to finish, such as assembling furniture, cleaning home and repairing a house.

Previous studies [7, 8, 9] in multi-skill/complex tasks oriented spatial crowdsourcing focus on assigning a set of multi-skilled workers to a given complex task such that the required skills of the task can be fully covered by the union of the skill sets of the assigned workers. However, in practice, each complex task can be a combination of multiple subtasks and there are dependencies between the subtasks. For instance, if a requester wants to repair a house, she/he would propose some tasks like painting walls, installing pipe systems, and cleaning. Generally, the pipe systems should be installed before painting walls since some pipes are embedded in the walls. If installing pipe systems after painting, workers have to paint the wall again. Similarly, if the tasks of painting walls and installing pipe systems are not accomplished, the cleaning task could insignificant. In addition, each worker can only do one task at the same time. In spatial crowdsourcing platforms, the workers are free to come and leave and the tasks dynamically appear. If a requester creates the subtasks one-by-one satisfying their dependencies, he/she needs to keep on monitoring the progresses of the subtasks and submits proper subtasks to the platform once their dependent subtasks are all accomplished, which is not efficient with respect to time and requires a lot of effort from the requesters in the highly dynamic spatial crowdsourcing scenarios. Thus, a dependency-aware spatial crowdsourcing platform is needed to efficiently assign multi-skill workers to dependency-aware tasks such that they can be optimally conducted. In this paper, we consider an important problem in the spatial crowdsourcing system, namely *dependency-aware spatial crowdsourcing* (DA-SC), which assigns workers to those dependency-aware tasks, with the constraints of skills, dependencies, moving distance, and deadlines of spatial tasks.

In the sequel, we will illustrate the DA-SC problem through a motivation example.

Example 1. *In the example, a spatial crowdsourcing platform has three workers* $(r_1 - r_3)$ *and five tasks* $(t_1 - t_5)$ *as shown in Figure 1(a), where the dash arrow lines pointing from a task to its dependent tasks. The details about each worker and task are shown in Table 1 and Table 2. For simplicity, in this example we set all workers and tasks appear on the platform at the same time. In addition, the maximum moving distance of each worker is large enough and the moving speed of each worker is fast enough to reach the assigned tasks before their deadlines. The goal of the platform is to maximize the total number of valid assigned worker-task pairs such that*

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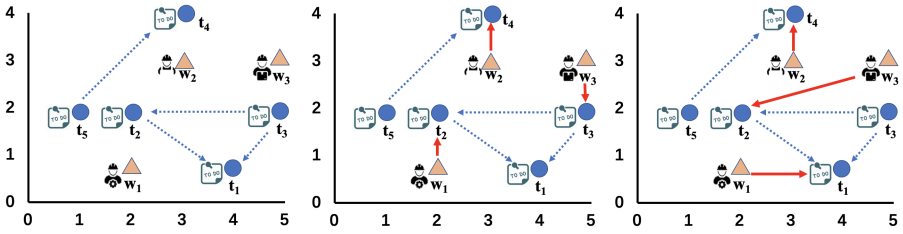


(a) Initial locations (b) Allocation by Greedy (c) Optimal Allocation

Figure 1: Motivation Example.

Table 1: Tasks' Detail.

Task	Location	Required Skill	Dependency
t_1	(4, 1)	ψ_1	$\emptyset$
t_2	(2, 2)	ψ_2	$\{t_1\}$
t_3	(5, 2)	ψ_3	$\{t_1, t_2\}$
t_4	(3, 4)	ψ_4	$\emptyset$
t_5	(1, 2)	ψ_3	$\{t_4\}$

Table 2: Workers' Detail.

Worker	Location	Skill Set
w_1	(2, 1)	$\{\psi_1, \psi_2\}$
w_2	(3, 3)	$\{\psi_4\}$
w_3	(5, 3)	$\{\psi_1, \psi_2, \psi_3\}$

the assigned worker has the required skill and the dependencies of the task have been satisfied.

If the platform greedily assigns the nearest skill-satisfied workers to the tasks but does not consider the dependencies, the allocation is shown by the red arrows in Figure 1(b). Note that, since t_1, which is t_2's dependency, has not been finished, the assignment (w_1, t_2) is invalid. Similarly, the assignment (w_3, t_3) is also invalid. Then, the total number of finished tasks is only 1.

However, if the platform takes dependencies into account and conducts dependent tasks separately, the allocation is shown by the red arrows in Figure 1(c). Each worker is assigned to one task and the dependencies of each assigned task are satisfied. Thus, the total number of finished tasks is 3.

Motivated by the example above, in this paper, we formalize the DA-SC problem, which focuses on efficiently assigning proper workers to dependency-aware spatial tasks, under the constraints of dependencies, skills, moving distances and deadlines, and the total number of the assigned worker-and-task pairs is maximized.

However, to achieve an optimal dependency-aware assignment result is not easy in DA-SC, where the tasks and workers are highly dynamic and their constraints are required to be all satisfied. Moreover, the dependencies between tasks make the problem more complex, since one task can only be conducted when its dependent tasks are all accomplished and finishing one critical task may lead to its subsequent tasks become ready to be conducted. Previous works in multi-skill oriented spatial crowdsourcing [7, 8, 9] do not consider the dependency of the tasks and assign a set of workers to fully support a task requiring a certain skill set, which is not efficient as some workers need to wait until the dependencies of their tasks are satisfied. Thus, no existing methods can be used to solve dependency-aware spatial crowdsourcing problem efficiently.

In this paper, we first prove that DA-SC problem is NP-hard by reducing it from the knapsack problem with conflict graphs (KCG) [10]. Thus, the DA-SC problem is intractable and hard to achieve the optimal results. We tackle the DA-SC problem with two approximation algorithms, namely *game-theoretic approach* and *greedy approach*, which can efficiently acquire near optimal results with proven approximation ratios for each batch process under the constraints of skills, dependencies, distance and deadlines.

To summarize, our main contributions are listed as follows:

- We formally define the dependency-aware spatial crowdsourcing (DA-SC) problem and prove it is NP-hard in Section 2.
- We propose two batch-based approximation algorithms, greedy and game-theoretic approaches, in Section 3 and Section 4 respectively. Both of the approaches have guaranteed approximate bounds for the final assignment results.
- We conduct extensive experiments on real and synthetic data sets, and show the efficiency and effectiveness of our proposed approaches in Section 5.

For the rest of the paper, we discuss the related work in Section 6 and conclude in Section 7.

2. PROBLEM DEFINITION

In this section, we present the formal definition of the dependency-aware spatial crowdsourcing (DA-SC) problem.

2.1 Heterogeneous Workers

We first define the heterogeneous workers in spatial crowdsourcing applications. Assume that $\Psi = \{\psi_1, \psi_2, \cdots, \psi_r\}$ is a universe of r abilities/skills.

Definition 1. (Heterogeneous Workers) A worker, denoted by $w =< l_w, s_w, w_w, v_w, d_w, WS_w >$, appears on the platform with an initial location i_w at timestamp s_w and waits at most w_w time for assigning a task. In addition, the worker w moves with the velocity v_w and has a maximum moving distance d_w. Moreover, each worker w has a set of skills $WS_w \subseteq \Psi$.

In Definition 1, the heterogeneous worker w locates at a spatial place l_w at timestamp s_w and will wait w_w time for assignment. In other words, the worker w no longer provides services on the platform after the timestamp $s_w + w_w$. Moreover, each worker w is associated with a set of skills WS_w. In addition, each worker can move dynamically in any direction with the velocity v_w. For simplicity, in this paper, we use Euclidean distance as our distance function, which is denoted as $dist(x, y)$ for the distance between locations x and y. Note that, our proposed approaches can also be used with other distance functions (e.g., road-network distance) to solve the DA-SC task assignment problem.

2.2 Dependency-aware Spatial Tasks

Next, we define dependency-aware tasks in the spatial crowdsourcing system, which are constrained by task locations, deadlines, skill requirements and dependencies.

Definition 2. (Dependency-aware Spatial Tasks) Let $t =< l_t, s_t, w_t, rs_t, D_t >$ denote a task. It appears on the platform with a spatial location l_t at timestamp s_t and needs to be served within w_t time. In addition, the task can be accomplished only by a worker who has the skill rs_t. Moreover, the task is dependent on a set of tasks D_t.

Table 3: Symbols and Descriptions.

Symbol	Description
Ψ	The universe of r abilities/skills Ψ_r
W	The set of n workers w
WS_w	The set of worker w's skills
v_w	The worker w's velocity
d_w	The worker w's maximum moving distance
T	The set of m tasks t
ts_t	The task t's required skill
D_t	The set of task t's dependent tasks
$l_w(l_t)$	The location of a worker(task)
$s_w(s_t)$	The start timestamp of a worker(task)
$w_w(w_t)$	The waiting time of a worker(task)
$ct_w(x, y)$	The time cost of the worker w's moving from x to y

A task requester creates a dependency-aware spatial task t, which requires a worker with skill rs_t physically moving to a specific location l_t and starting the service before the expiration time $s_t + w_t$. When the requesters propose the tasks, meanwhile, they can designate the dependency relationships between the tasks. For any task t, it can be conducted only after its immediate precedent tasks D_t are completed. Without loss of generality, the graph of the dependency relationships of tasks is acyclic, which means no tasks are dependent on its subsequent tasks.

2.3 The Dependency-aware Spatial Crowdsourcing Problem

We formally define the *dependency-aware spatial crowdsourcing* (DA-SC) problem as follows.

Definition 3. (Dependency-aware Spatial Crowdsourcing Problem) Given a set of workers W and a set of tasks T, the DA-SC problem is to obtain an assignment M among W and T to maximize the number of assigned worker-and-task pairs

$$Sum(M) = |M| = \sum_{w \in W, t \in T} I(w, t) \quad (1)$$

where $I(w, t) = 1$ if the pair (w, t) is matched in the assignment M, and otherwise $I(w, t) = 0$, such that the following constraints are satisfied:

1. **Skill constraint**. In order to accomplish the task t, the worker w must have the required skill ts_t (i.e., $ts_t \in WS_w$).
2. **Deadline constraint**. For any worker-task pair (w, t), it should satisfy the following two deadline conditions. (1) The task should appear before the worker leaves the platform, (i.e., $s_t \leq s_w + w_w$). (2) The worker should be able to arrive at the location of the assigned task before the deadline of the task (i.e., $w_t - \max\{s_w - s_t, 0\} - ct_w(l_w, l_t) \geq 0$, here $ct_w(l_w, l_t)$ is the travel cost from l_w to l_t).
3. **Exclusive constraint**. One task can be assigned to at most one worker (i.e., $\sum_{w \in W} I(w, t) \leq 1$) and any worker can be assigned to at most one task at each time.
4. **Dependency constraint**. The worker w can conduct the task t if and only if the task t's dependent tasks had been assigned. (i.e., $\prod_{t' \in D_t} a_{t'} = 1$, where $a_{t'} = \sum_{w \in W} I(w, t')$).

In Definition 3, our DA-SC problem aims to assign a worker w to a task t such that: (1) the worker w is able to reach the task t's location before its expired time and has the required skill; (2) the task t's dependent tasks are all assigned; (3) the worker/task has not been assigned with another task/worker; and (4) the number of assigned worker-and-task pairs should be maximized.

2.4 Hardness of DA-SC Problem

With m dependency-aware tasks and n workers, in the worst case, there is an exponential number of possible assignment strategies. In this subsection, we prove that our DA-SC problem is NP-hard, by reducing a well-known NP-hard problem, the *knapsack problem with conflict graphs* (KCG)[10], to our DA-SC problem.

Theorem 2.1. *(Hardness of the DA-SC problem) The Dependency-aware Spatial Crowdsourcing problem is NP-hard.*

Proof. We prove the theorem by a reduction from the *knapsack problem with conflict graphs (KCG)*[10], which can be described as follows:

Let n be the number of items, each of them with a profit p_j and weight w_j, $j = 1, \cdots, n$, and c the capacity of the knapsack. In addition, the conflict graph $G = (V, E)$ where every vertex corresponds uniquely to one item and an edge $(i, j) \in E$ indicates that items i and j can not be packed together. Moreover, each item can be selected no more than once. The problem is to maximize the sum of the profit of the items in the knapsack so that the sum of the weights is less than or equal to the knapsack's capacity.

We consider the special case of the offline version of the DA-SC problem, where each task and worker appears on the platforms at the same time, their waiting time is large enough and each worker has all skills. In addition, the structure of each task's dependency is a line. In other words, if task t_i and t_j are contained in the task t_l's dependency set, then t_i is contained in the task t_j's dependency set or t_j is contained in the task t_i's dependency set. Next, we construct a new set TC by combining each task with its dependency set, i.e., $TC = \{tc_t | tc_t = \{t_t\} \cup D_t, \forall t \in T\}$. Then, we construct a conflict graph $G' = (V', E')$ as follows:

$$\begin{cases} V' = TC \\ E' = \{(i, j) | tc_i \cap tc_j \neq \emptyset\} \end{cases}$$

We define each task combination's profit and weight are both its cardinality and the capacity of the knapsack is the number of workers. Hence, the objective of our DA-SC problem is transferred to find the task combination allocation which maximizes the sum of selected task combinations' profit so that the weights is less than or equal to the knapsack's capacity.

Note that, since the structure of each task's dependency is a line, if $tc_j \cap tc_i \neq \emptyset$, then it must hold that $tc_j \subset tc_i$ or $tc_i \subset tc_j$. Therefore, the task combination allocation we get satisfying the conflict constraint guarantees that each task $t \in T$ is selected no more than once.

Since the KCG problem is equivalent to that of the aforementioned special case, we can reduce the KCG problem to the special case of the offline version of the DA-SC problem. Therefore, the offline version of the DA-SC problem is NP-hard. □

Note that, even when we do not consider the dependency between tasks, the DA-SC problem is still NP-hard due to other constraints as shown in the existing work [11].

Thus, due to the NP-hardness of the DA-SC problem, in the subsequent sections, we propose two approximate algorithms, namely *greedy* and *game-theoretic* approaches, to efficiently solve DA-SC with proven approximation ratios for each batch process. Specifically, the spatial crowdsourcing platforms assign workers to tasks batch-by-batch for every constant time interval (e.g., 5 seconds). In each batch process, the server applies our approximate algorithms to assign workers to tasks under the constraints of dependencies, skills, moving distances and deadlines such that the number of finished tasks are maximized in the current batch process. Table 3 summarizes the commonly used symbols.

3. GREEDY APPROACH

In this section, we proposed a batch-based algorithm, namely *DASC_Greedy* approach, to quickly get a bounded result based on the submodular function [12]. Instead of taking all dependent tasks as a task group, in *DASC_Greedy*, we combine each task and its dependency set as an associative task set. Then, the algorithm iteratively selects the "current" biggest associative task set that can be finished by active workers.

3.1 Associative Task Set

We denote task t_i and its dependent tasks as one associative task set tc_i (i.e., $tc_i = \{t_i\} \cup D_i$). In *DASC_Greedy*, every time we will assign a set of workers to an associative task set tc_i such that the tasks in tc_i can be fully finished by the assigned workers. Note that after assigning one associative task set tc_i, we will update other related associative task sets which contain any tasks in tc_i. In particular, we will remove these tasks in tc_i from these related associative task sets. For instance, there is a task t_4 and its dependency is $\{t_1, t_2, t_3\}$. In addition, t_2 is t_3's dependency, but t_1 and t_2 are independent, Thus, there are four associative task sets, $\{\{t_1\}, \{t_2\}, \{t_2, t_3\}, \{t_1, t_2, t_3, t_4\}\}$. If we select the associative task set $\{t_2\}$ to work firstly, the rest associative task sets will be updated to $\{\{t_1\}, \{t_3\}, \{t_1, t_3, t_4\}\}$.

3.2 DASC_Greedy Algorithm

Algorithm 1 shows the pseudo code of our *DASC_Greedy* Algorithm, where we greedily select the associative task set with the largest size each time.

Algorithm 1: DASC_Greedy Algorithm

Input: A set W_b of n_b workers and a set T_b of m_b tasks in the batch b

Output: An assignment M_b in the batch b

1 generate the associative task sets TC_b;
2 **repeat**
3 $C = \emptyset$;
4 **foreach** *associative task set* $tc \in TC_b$ **do**
5 run the Hungarian Algorithm[13] to find a set of workers tw who can accomplish the associative task set tc;
6 **if** $tw \neq \emptyset$ **then**
7 add the assignment $\langle tw, tc\rangle$ into C;
8 move the the assignment $\langle tw, tc\rangle$ whose associative task set tc has the largest cardinality from C to M_b;
9 update TC_b and W_b;
10 **until** *no more associative task set can be assigned*
11 **return** M_b

Initially, we generate associative task sets TC_b (line 1). Then, in each round, we select one associative task set which can be accomplished by active workers and has the largest cardinality (line 2-10). Specifically, for each associative task set tc in TC_b, we try to find a set of workers tw who can accomplish tc by the Hungarian Algorithm[13] (line 5). Then, if the worker set tw can be found, then add $\langle tw, tc\rangle$ into the candidate set C (line6-7). After that, we move the assignment whose associative task set has the largest cardinality from the candidate set C to M_b (line 8). Next, we update the associative task sets in TC_b and the worker set W_b (line 9).

3.3 Theoretic Analyses

We first discuss the running time of DASC_Greedy.

Lemma 3.1. *The time complexity of* DASC_Greedy *is* $O(\min\{n_p, m_p\} \cdot m_p \cdot |D|_{max} \cdot n_p \cdot |WS|_{max})$*, where* $|D|_{max}$ *is the maximum cardinality of one task's dependency set and* $|WS|_{max}$ *is the maximum cardinality of one worker's skill set.*

Proof. The loop will be ended until there is no associative task set can be assigned. There are three cases: (1) the tasks are all assigned; (2) the workers are all assigned; (3) there are workers or tasks left, but no worker-and-task pairs can satisfy all constraints. Thus, the number of round can be bounded by $O(\min\{n_p, m_p\})$. In each round, there are $O(m_p)$ associative task sets should be scanned (line 4). Since the number of each associative task set's candidate workers and the number of tasks in each associative task set are both negligible compared with m_p and n_p, the time complexity of DASC_Greedy is $O(\min\{n_p, m_p\} \cdot m_p)$. □

This lemma shows that we can get the results within polynomial time. Next we discuss how good the results are. Firstly, we prove that the objective function of the DA-SC problem in Equation 1 is monotone and submodular.

Theorem 3.1. $Sum(M)$ *is monotone and submodular.*

Proof. Let $|tc|$ be the cardinality of the associative task set tc. Then, the objective function (as shown in Equation 1) can be rewritten as $Sum(M) = \sum_{w \in W, t \in T} I(w,t) = \sum_{tc \in M} |tc|$. Since $|tc|$ is always positive, $Sum(M)$ is monotone.

To proving its submodularity, we have:

$$\forall \langle tw_i, tc_i\rangle \notin M, \forall \langle tw_j, tc_j\rangle \notin M,$$
$$Sum(\widehat{M}) - Sum(\widetilde{M}) \leq Sum(\overline{M}) - Sum(M) \qquad (2)$$

where $\widehat{M} = M \cup \{\langle tw_i, tc_i\rangle, \langle tw_j, tc_j\rangle\}$, $\widetilde{M} = M \cup \{\langle tw_i, tc_i\rangle\}$ and $\overline{M} = M \cup \{\langle tw_j, tc_j\rangle\}$. Here tw is the set of workers to conduct the associative task set tc.

There are two cases:

- $\langle tw_i, tc_i\rangle$ is assigned in $\widehat{M}$ and $\overline{M}$, but the cardinality of tc_j in $\widehat{M}$ is larger than that in $\overline{M}$. Since when we update the associative task sets TC after assigning the associative task set tc_i, we will remove the tasks in tc_i from the related associative task sets of tc_i. The cardinality of the associative task set tc_j will not increase after assigning another associative task set.
- The worker set $\widehat{W_p}$ is a subset of the worker set $\overline{W_p}$, where $\widehat{W_p}$ ($\overline{W_p}$) is the active worker set after (before) finishing the associative task set tc_i (i.e., $\widehat{W_p} \subseteq \overline{W_p}$). If tc_j can be finished by the workers in $\overline{W_p}$ but cannot be finished by the workers in $\widehat{W_p}$, tc_j will not be assigned in $\widehat{M}$. Then, $Sum(\widehat{M}) - Sum(\widetilde{M}) = 0$, but $Sum(\overline{M}) - Sum(M) = |tc_j|$.

Thus, Equation 2 has been proven. In conclusion, $Sum(M)$ is monotone and submodular. □

Let $\rho_j(M) = Sum(M \cup \langle tw_j, tc_j\rangle) - Sum(M)$ and E is the universe of assignments. Inspired by the existing work [14], according to the monotonicity and submodularity of $Sum(M)$, we have the following lemma.

Lemma 3.2. *For any assignment* M *and* M'*, it holds,*

$$Sum(M') - Sum(M)$$
$$\leq \sum_{\langle tw_i, tc_i\rangle \in M'/M} \rho_i(M) - \sum_{\langle tw_i, tc_i\rangle \in M/M'} \rho_i(M \cup M' - \langle tw_i, tc_i\rangle)$$

Proof. Based $Sum(M)$'s submodularity, we can have

$$\rho_{tc}(M) \geq \rho_{tc}(M'), \forall M \subseteq M', tc \in E - M'.$$

Then, for arbitrary M and M' with $M' - M = \{j_1, \cdots, j_r\}$ and $M - M' = \{k_1, \cdots, k_q\}$ we have:

$$\begin{aligned}
&Sum(M \cup M') - Sum(M) \\
=&\sum_{t=1}^{r}[Sum(M \cup \{j_1, \cdots, j_t\}) - Sum(S \cup \{j_1, \cdots, j_{t-1}\})] \\
=&\sum_{t=1}^{r}\rho_{j_t}(M \cup \{j_1, \cdots, j_{t-1}\}) \leq \sum_{t=1}^{r}\rho_{j_t}(M) = \sum_{j \in M'-M}\rho_j(M)
\end{aligned}$$

Similarly, we also have:

$$\begin{aligned}
&Sum(M \cup M') - Sum(M') \\
=&\sum_{t=1}^{q}[Sum(M' \cup \{k_1, \cdots, k_t\}) - Sum(M' \cup \{k_1, \cdots, k_{t-1}\})] \\
=&\sum_{t=1}^{q}\rho_{k_t}(M' \cup \{k_1, \cdots, k_{t-1}\}) \\
\geq&\sum_{t=1}^{q}\rho_{k_t}(M' \cup M - \{k_t\}) = \sum_{j \in M-M'}\rho_j(M \cup M' - \{j\})
\end{aligned}$$

After subtracting above two inequality, we have:

$$\begin{aligned}
&Sum(M') \\
\leq&Sum(M' \cup M) - \sum_{k \in M-M'}\rho_k(M \cup M' - \{k\}) \\
\leq&Sum(M) + \sum_{j \in M'-M}\rho_j(M) - \sum_{k \in M-M'}\rho_k(M \cup M' - \{k\})
\end{aligned}$$

□

In addition, based on the observation that $0 \leq \rho_{tc}(M) \leq \psi$, $\forall M \subseteq E$, $tc \in E - M$, where ψ is the upper bound of $\rho_{tc}(M)$, we have:

$$Sum(M') \leq Sum(M) + \sum_{tc \in M'-M}\rho_{tc}(M), \forall M, M' \subseteq E.$$

Note that $|M| \leq K$, where $K = \min\{n_b, m_b\}$. Let $Sum(M_{opt})$ be the value of an optimal solution and M^t be the assignment set which is generated after t rounds, i.e., $Sum(M^t) = \sum_{i=0}^{t-1}\rho_i$. Suppose the *DASC_Greedy* Algorithm stops after K^* rounds, then we have the following lemma.

Lemma 3.3. *Suppose tc_t is added by* DASC_Greedy *Algorithm in the t-th round, let $\rho_{t-1} = \rho_{tc_t}(M^{t-1})$. The corresponding $\{\rho_i\}_{i=0}^{K^*-1}$ satisfy: $Sum(M_{opt}) \leq \sum_{i=0}^{t-1}\rho_i + K \cdot \rho_t, t = 0, \cdots, K^* - 1$.*

Proof. Since $\rho_j(M) \leq \rho_t, \forall j \in T - S$, according to Lemma 3.2, we have:

$$\begin{aligned}
Sum(M_{opt}) &\leq Sum(M^t) + \sum_{tc \in M_{opt}-M^t}\rho_{tc}(M^t) \\
&\leq \sum_{i=0}^{t-1}\rho_i + \sum_{tc \in M_{opt}-M^t}\rho_t
\end{aligned}$$

In addition, $|M_{opt} - M^t| \leq K$. Thus,

$$Sum(M_{opt}) \leq \sum_{i=0}^{t-1}\rho_i + \sum_{tc \in M_{opt}-M^t}\rho_t \leq \sum_{i=0}^{t-1}\rho_i + K \cdot \rho_t$$

□

Let M_{greedy} be the assignment gotten by *DASC_Greedy* Algorithm. Then, we have the following theorem to show that our greedy algorithm can provide solution with a theoretic guaranteed bound with respect to the optimal result.

Theorem 3.2. *The matching size returned by* DASC_Greedy *Algorithm is at least $(1 - \frac{1}{e}) \cdot |M_{opt}|$.*

Proof. Based on the Lemma 3.3, we have

$$Sum(M_{opt}) \leq K \cdot \rho_0 = K \cdot (Sum(M^1))$$

$$Sum(M_{opt}) - Sum(M^1) \leq K \cdot \rho_1 = K \cdot (Sum(M^2) - Sum(M^1))$$

Similarly, we can get the inequalities when $t = 2, \cdots, K$. Thus, we have:

$$\frac{Sum(M_{opt} - Sum(M_{greedy}))}{Sum(M_{opt})} \leq \left(\frac{K-1}{K}\right)^K$$

Thus, the matching size returned by *DASC_Greedy* Algorithm is at least $(1 - \frac{1}{e}) \cdot |M_{opt}|$. □

4. GAME THEORETIC APPROACH

In this section, we develop a game theoretic based framework, namely DASC_Game, to further improve the results achieved by DASC_Greedy. Specifically, we model the DA-SC problem as a strategic game, where each worker corresponds to a player: his goal is to find the task that maximize his own utility. We first introduce the general knowledge of game theory, and show that the strategic game can reach an equilibrium (i.e., a local optimal result where no worker has incentive to deviate from his/her assigned task). Then, we propose a game theoretic approach with theory analyses on its converge speed and quality of results.

4.1 Game Theory

Algorithm 2 illustrates a common framework for studying the dynamic process of decision-making in a strategic game [15]. In *strategic games*, players compete with each other over the same resources in order to optimize their individual objective functions. Under this framework, each player always tries to choose a *strategy* that maximizes his own utility without taking the effect of his choice on the other players' objectives into consideration. The input of the framework is a strategic game, which can be formally represented by a tuple $\langle W, \{S_w\}_{w \in W}, \{U_w : \times_{w \in W} S_w\}_{w \in W} \rightarrow \mathbb{R}\rangle$ where S_q represents all the possible tasks that worker w can take during the game to optimize his function U_w. The optimization of U_w depends on w's own strategy, as well as the strategies of the other workers. In [16], Nash points that a strategic game has a pure Nash equilibrium, if there exist a specific choice of strategies $s_w \in S_w$ such that the following condition is true for all $w \in W$:

$$U_w(s_1, \cdots, s_w, \cdots, s_{|W|}) \leq U_w(s_1, \cdots, s'_w, \cdots, s_{|W|}), \forall s'_w \in S_w$$

In other words, no player has incentive to deviate from his current strategy.

In order to express the objective functions of all the workers, [17] proposed a single function $\Phi : \times_{w \in W} S_w \rightarrow \mathbb{R}$, called the *potential function* in *potential games*, which constitute a special class of strategic games. Let $\overline{s_w}$ denote the set of strategies followed by all workers except w (i.e., $\overline{s_w} = \{s_1, \cdots, s_{w-1}, s_{w+1}, \cdots, s_{|W|}\}$). A potential game is *exact*, if there exists a potential function Φ, such that for all s_w and all possible combinations of $\overline{s_i} \in \times_{j \in N \setminus \{i\}} S_j$, the following condition holds:

$$U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w}) = \Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w})$$

[17] proves that for potential games, the best-response framework as shown in Algorithm 2 always converges to a pure Nash equilibrium. Therefore, we can use the best-response framework of Algorithm 2 to obtain the solution for DA-SC problem. Specifically,

Algorithm 2: Best Response Framework

Input: Strategic game $\langle V, \{S_v\}_{v\in V}, \{C_v : \times_{u\in V} S_u\}_{v\in V} \to \mathbb{R}\rangle$
Output: Nash equilibrium

Assign a random strategy to each player
repeat
  foreach *player* $v \in V$ **do**
    compute v's best strategy wrt the other players' strategies
    let v follow his best strategy
until *Nash equilibrium//no player has changed his strategy*
return *the strategy of each player*

we propose a game-theoretic algorithm, namely DASC_Game, and analyze its converge speed and result quality.

4.2 Game-Theoretic Algorithm

We model the DA-SC problem as a game, which is represented by a tuple $\langle W_b, S_{w\,w\in W_b}, \{U_w : \times_{w\in W_b} S_w \to \mathbb{R}\}_{w\in W_b}\rangle$, where W_b is the worker set in the batch b. W_b contains the workers who appear in the previous batches but still wait for assignments, and the new workers who just come in this batch. Similarly, T_b denotes the task set in the batch b. Specifically, each worker $w \in W_b$ has a set of strategies $S_w \in T_b$. Let $s_w \in S_w$ be a specific strategy of worker w, which represents the task in this batch that the worker w can do, and nw_{s_w} is the number of workers who also try to do the task s_w. Given s_w and the strategies $\overline{s_w}$ of the other players, the total utility $U_w(s_w, \overline{s_w})$ of w is the summation of (i) the shared utility of task s_w, and (ii) the contribution to the tasks which are dependent on s_w. The goal of each worker $w \in W$ is to find the task s_w that maximizes his own total utility as expressed by Equation 3:

$$U_w(s_w, \overline{s_w}) = \begin{cases} \frac{(\alpha-1)\cdot\prod_{f\in D_{s_w}} a_f}{\alpha\cdot nw_{s_w}} + \sum_{s_w\in D_t} \frac{\prod_{f\in D_t\cup\{t\}} a_f}{\alpha\cdot|D_t|\cdot nw_{s_w}}, & D_{s_w} \neq \emptyset \\ \frac{1}{nw_{s_w}} + \sum_{s_w\in D_t} \frac{\prod_{f\in D_t\cup\{t\}} a_f}{\alpha\cdot|D_t|\cdot nw_{s_w}}, & D_{s_w} = \emptyset \end{cases} \tag{3}$$

where α is the normalization parameter. Because of the dependency constraint, each task's utility can be obtained iff its dependent tasks are assigned, i.e., $\prod_{f\in D_t} a_f = 1$. Since each task's utility is effected by the assignment of itself and the assignments of its dependent tasks, we divide the utility of each task t into two parts. First part, namely *Utility_Self*, is the utility of the assignment of itself, whose value is $\frac{\alpha-1}{\alpha}$, and the second part,*Utility_Dependency*, is the utility of the assignments of its dependent tasks, whose value is $\frac{\alpha-1}{\alpha}$. Note that, when the task's dependency set is empty, the task does not share any *Utility_Dependency* to other tasks and its *Utility_Self* is 1. In addition, the task t has $|D_t|$ dependent tasks, thus, we divide the *Utility_Dependency* into $|D_t|$ shares and add each share to each dependent task. In other words, each task's utility is formed by two parts, the first part is the left utility of itself and the extra utilities from the tasks whose dependency sets contain this task. In addition, since each task may be assigned to multiple workers in the *Best Response* procedure, we divide the utility of the task t into nw_t shares where nw_t is the number of workers who try to do the same task t, and each worker who try to do this task has one share of the utility. Hence, in Equation 3, the first term is the shared *Utility_Self* and the second term is the sum of the shared *Utility_Dependency* from the tasks whose dependency sets contain this task s_w.

Significantly, we have the observation that the objective function of DA-SC problem in Equation 1 is equal to the summation of all individual worker's utility (i.e., $Sum(M) = \sum_{w\in W} U_w(s_w, \overline{s_w})$). This decomposition of the DA-SC objective into the summation of individual utility functions provides a natural motivation for modeling DA-SC as a game, since workers' own goals are considered for reaching a global target. In addition, we define the potential function $\Phi(S)$ as follows:

$$\begin{aligned}\Phi(S) &= -\sum_{w\in W_b}\sum_{\substack{t\in\cup S_w\\ \wedge t\neq s_w}} \frac{\prod_{f\in D_t\cup\{t\}} a_f}{(nw_t+1)\cdot(n_b-nw_t)} \\ &= -\sum_{w\in W_b}\sum_{\substack{t\in\cup S_w\\ \wedge t\neq s_w}} \frac{(\alpha-1)\cdot\prod_{f\in D_t\cup\{t\}} a_f}{\alpha\cdot(nw_t+1)\cdot(n_b-nw_t)} \\ &\quad -\sum_{w\in W_b}\sum_{\substack{t\in\cup S_w\\ \wedge t\neq s_w}}\sum_{t\in D_l} \frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(nw_t+1)\cdot|D_l|\cdot(n_b-nw_t)} \\ &\quad -\sum_{w\in W_b}\sum_{\substack{t\in\cup S_w\wedge\\ t\neq s_w\wedge D_t=\emptyset}} \frac{\prod_{f\in D_t\cup\{t\}} a_f}{\alpha\cdot(nw_t+1)\cdot(n_b-nw_t)}\end{aligned}$$

We divide the utility of task t by $(nw_t+1)\cdot(n_b-nw_t)$ shares and the utility of task t can be obtained iff t is assigned and its dependent tasks are assigned, i.e., $\prod_{f\in D_t\cup\{t\}} a_f = 1$. In addition, for each worker $w \in W_b$, we calculate the sum of one utility share of each task t in the strategy universe $\cup S_w$ except the task that the worker w selects to conduct, i.e., s_w. Then, we define the potential function $\Phi(S)$ as the negative of the sum of these value. Recall that, each task's value can be separated into two parts, thus, we can represent the potential function $\Phi(S)$ by three term. The first term is the sum of the shared *Utility_Self*, the second term is the sum of the shared *Utility_Dependency* and the third term is the sum of the left *Utility_Self* for the tasks whose dependency sets are empty.

Algorithm 3: DASC_Game Algorithm

Input: A set W_b of n_b workers and a set T_b of m_b tasks in the batch b
Output: An assignment M_b in the batch b

1 **foreach** *each worker* $w \in W$ **do**
2 assign w with a random task $t \in S_w$;
3 **repeat**
4 **foreach** *worker* $w \in W$ **do**
5 $maxUtility = -\infty$;
6 **foreach** *task* $t \in S_w$ **do**
7 compute worker w's individual utility function $U_w(s_w, \overline{s_w})$;
8 **if** $U_w(s_w, \overline{s_w}) > maxUtility$ **then**
9 $maxUtility = U_w(s_w, \overline{s_w})$;
10 $s_w = t$;
11 **until** *Nash equilibrium*
12 According to the Nash equilibrium, get the assignment set I;
13 **return** M_b

Algorithm 3 shows the pseudo code of our *DASC_Game* Algorithm. Initially, *DASC_Game* assigns each worker with a random task t from worker's possible task set S_w (line 1 – 2). Then, it starts the best-response procedure (lines 3 – 11). Specifically, in each iteration for each worker w, it computes the utility of assigning worker w with each possible tasks, then assigns the task associated with the maximum utility to the worker w. The iteration terminates when no worker changes his/her assigned task during a round. Then, according the selected task of each worker, we can get the assignment set M_b (line 12). Note that, we finally remove the assignments of that the tasks whose dependencies are not fully satisfied. And for the task which more than one worker want to do,

we randomly select one worker to the task. In addition, the performance of *DASC_Game* can be improved by some simple heuristics. Specifically, in Line 2, instead of a random initialization, we can run the *DASC_Greedy* for the initialization. We examine the effect of this heuristic in the experimental evaluation (Section 5).

Normalization Issues. In some cases, the first term and the second term in Equation 3 may not be comparable. For instance, In some complex cases, one task may be in thousands tasks' dependency set. Then many workers may prefer to conduct the same task who are dependent by thousands of other tasks, since the total utility is dominated by the second term in Equation 3. To avoid this issue, we use the normalization parameter α to guarantee that the second term's range is $[0, 1]$, i.e., it should hold that $\sum_{s_w \in D_t} \frac{1}{\alpha \cdot |D_t|} \leq 1$. In other words, $\alpha \geq \frac{ND_{max}}{D_{max}}$, where ND_{max} is the maximum number of dependency sets that the same task belongs to and D_{max} is the maximum cardinality of one task's dependency set.

4.3 Theoretic Analyses

We first prove that DA-SC game is an exact potential game.

Theorem 4.1. *DA-SC problem constitutes an exact potential game.*

Proof. As shown in Section 4.1, it suffices to have that for every worker w, who changes his strategy from the current one s_w to his/her best-response s'_w, and for all possible combinations of the other players' strategies $\overline{s_w}$ it holds that:

$$U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w}) = \Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w})$$

Suppose nw_{s_w} and $nw_{s'_w}$ are the numbers of workers who are assigned to the tasks s_w and s'_w in the assignment $(s_w, \overline{s_w})$, respectively. Similarly, $\overline{nw_{s_w}}$ and $\overline{nw_{s'_w}}$ are the numbers of workers who are assigned to the tasks s_w and s'_w in the assignment $(s'_w, \overline{s_w})$, respectively. Note that, $nw_{s_w} = \overline{nw_{s_w}} + 1$ and $\overline{nw_{s'_w}} = nw_{s'_w} + 1$. Indeed, when $D_{s_w} \neq \emptyset$ and $D_{s'_w} \neq \emptyset$, we have:

$$\begin{aligned}
&\Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w}) \\
&= -\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s'_w}} a_f}{\alpha\cdot(nw_{s'_w}+1)} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(nw_{s'_w}+1)\cdot|D_l|}\right) \\
&\quad + \left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s_w}} a_f}{\alpha\cdot(\overline{nw_{s_w}}+1)} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(\overline{nw_{s_w}}+1)\cdot|D_{s_w}|}\right) \\
&= \left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s_w}} a_f}{\alpha\cdot nw_{s_w}} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot nw_{s_w}\cdot|D_{s_w}|}\right) \\
&\quad - \left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s'_w}} a_f}{\alpha\cdot\overline{nw_{s'_w}}} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot\overline{nw_{s'_w}}\cdot|D_{s'_w}|}\right) \\
&= U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w})
\end{aligned}$$

Similarly, when (1) $D_{s_w} = \emptyset$ and $D_{s'_w} \neq \emptyset$; (2) $D_{s_w} \neq \emptyset$ and $D_{s'_w} = \emptyset$; (3) $D_{s_w} = \emptyset$ and $D_{s'_w} = \emptyset$, we can have the same result: $\Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w}) = U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w})$. Due to the space limitation, we do not show the details here. For the full proof, please refer to Appendix A of our technical report [18].

Then, we proved DA-SC is an exact potential game. □

Since DA-SC problem is an exact potential game, and the set of strategic configurations S is finite, a Nash equilibrium can be reached after workers changing their strategies a finite number of times. For simplicity, we prove the upper bound for the number of rounds required to reach the convergence of *DASC_Game* Algorithm by a scaled version of the problem where the objective function takes integer values. Specifically, we assume an equivalent game with potential function $\Phi_{\mathbb{Z}}(S) = d \cdot \Phi(S)$, where d is a constant which depends on the nw_{s_w} such that $\Phi_{\mathbb{Z}}(S) \in \mathbb{Z}, \forall S$. Obviously, this does not scale with the size of the problem. Then, we can prove the following lemma.

Lemma 4.1. *The number of rounds required by* DASC_Game *Algorithm in each batch converges to an equilibrium is upper bounded by* $d \cdot \min\{n_b, m_b\}$.

Proof. The scaled version of *DASC_Game* Algorithm with the potential function as $\Phi_{\mathbb{Z}}(S) = d \cdot \Phi(S)$ will converge to a Nash equilibrium in the same number of rounds as *DASC_Game* Algorithm. Since $\Phi_{\mathbb{Z}} \in \mathbb{Z}$, the cost increase in $\Phi_{\mathbb{Z}}$ after each strategy change of a worker is at least 1. Therefore, the upper bound of the number of rounds is the maximum value, Φ_{max}, minus the minimum value, Φ_{min}. We can see that in *DASC_Game* Algorithm, $\Phi_{max} \leq 0$ and $\Phi_{min} \geq -d \cdot \min\{n_b, m_b\}$. Consequently, the number of rounds to reach an equilibrium is upper-bounded by $d \cdot \min\{n_b, m_b\}$, as stated by the lemma. □

Then, we can prove the time complexity of *DASC_Game* Algorithm as follows.

Lemma 4.2. *The time complexity of* DASC_Game *Algorithm in each batch is* $O(d \cdot n_b \cdot \min\{n_b, m_b\})$*, where* m_b *is the number of tasks and* n_b *is the number of workers in the batch b.*

Proof. As discussed above, the number of iteration is upper bounded by $O(d \cdot \min\{n_b, m_b\})$. In addition, we need run the best response for n_b workers. For each worker w, the best response requires computing the individual utility function for at most $|S|_{max}$ possible tasks (line 4-6). Furthermore, the time cost of computing individual utility function is $O(|D|_{max})$ (line 7). However, compared with n_b and m_b, $|S|_{max}$ and $|D|_{max}$ are negligible. Thus, the time complexity is $O(d \cdot n_b \cdot \min\{n_b, m_b\})$. □

After proving that DASC_Game can converges within polynomial time, we discuss how good the resulting solution is. Usually, researchers use *social optimum (OPT)*, *price of stability(PoS)*, and *price of anarchy(PoA)* to evaluate the quality of an equilibrium. Specifically, the OPT is the solution that yields the optimal values to all the objective functions, so that their total utility is maximum. The PoS of a game is the ratio between the best value among its equilibrium and the global optimum and the PoA of a game is the ratio between the worst value among its equilibriums and the global optimum.

Let $U(S)$ denotes the summation of all workers' utility, $U(S) \triangleq \sum_{w \in W_b} U_w(s_w, \overline{s_w})$, and recall that $U(S)$ is equal to the *DA-SC* object function. Then, we have the following lemma to bound the potential function $\Phi(S)$.

Lemma 4.3.

$$\frac{1}{nw_{max}+1} \cdot U(S) \leq |\Phi(S)| \leq \frac{n_b}{\overline{nw} \cdot (n_b - \overline{nw})} \cdot U(S)$$

where $\overline{nw} = \min\{nw_{min}, n_b - nw_{max}\}$*,* nw_{max} *is the maximum number of workers who are assigned to the same task and* nw_{min} *is the minimum number of workers who are assigned to the same task.*

Proof. Since $nw_{max} \geq nw_t, \forall t \in \cup S_w$, we have

$$\sum_{w \in W_b} \sum_{\substack{t \in \cup S_w \\ \wedge t \neq s_w}} \frac{\prod_{f \in D_t \cup \{t\}} a_f}{(nw_t + 1) \cdot (n_b - nw_t)}$$

$$= \sum_{t \in \cup S_w} \frac{\prod_{f \in D_t \cup \{t\}} a_f}{nw_t + 1}$$

$$\geq \frac{1}{nw_{max} + 1} \cdot \sum_{t \in \cup S_w} \prod_{f \in D_t \cup \{t\}} a_f$$

Thus, $|\Phi(S)| \geq \frac{1}{nw_{max}+1} \cdot U(S)$. In addition, we have:

$$\sum_{w \in W_b} \sum_{\substack{t \in \cup S_w \\ \wedge t \neq s_w}} \frac{\prod_{f \in D_t \cup \{t\}} a_f}{(nw_t + 1) \cdot (n_b - nw_t)}$$

$$\leq \sum_{w \in W_b} \sum_{\substack{t \in \cup S_w \\ \wedge t \neq s_w}} \frac{\prod_{f \in D_t \cup \{t\}} a_f}{nw_t \cdot (n_b - nw_t)}$$

$$\leq \sum_{w \in W_b} \sum_{t \in \cup S_w} \frac{\prod_{f \in D_t \cup \{t\}} a_f}{nw_t \cdot (n_b - nw_t)} = \sum_{t \in \cup S_w} \frac{n_b \cdot \prod_{f \in D_t \cup \{t\}} a_f}{nw_t \cdot (n_b - nw_t)}$$

$$\leq \frac{n_b}{\overline{nw} \cdot (n_b - \overline{nw})} \cdot \sum_{t \in \cup S_w} \prod_{f \in D_t \cup \{t\}} a_f$$

Thus, we have $|\Phi(S)| \leq \frac{n_b}{\overline{nw} \cdot (n_b - \overline{nw})} \cdot U(S)$. □

Based on the potential function's bounds, we can prove the bounds of the equilibrium obtained by *DASC_Game*.

Theorem 4.2. *In* DASC_Game*, the PoS of each batch is bounded by $\frac{\overline{nw} \cdot (n_b - \overline{nw})}{n_b \cdot (nw_{max}+1)}$. In addition, the PoA of each batch is bounded by $\frac{\overline{nw} \cdot (n_b - \overline{nw})}{n_b \cdot \min\{n_b, m_b\}} \cdot \widehat{|\Phi(S)|}_{min}$, where $\widehat{|\Phi(S)|}_{min}$ is the minimum of $\Phi(S)$'s local minimums.*

Proof. Let S^* be the optimal set of strategies in this batch that maximize $U(S)$, and let $OPT = U(S^*)$. Further, let S^{**} be the set of strategies that yields the minimum of the potential function $\Phi(S)$, i.e., the best Nash equilibrium. From the Lemma 4.3 and since $U(S^*) \geq U(S), \forall S$ and $|\Phi(S^{**})| \geq |\Phi(S)|, \forall S$, we have:

$$U(S^{**}) \geq \frac{\overline{nw} \cdot (n_p - \overline{nw})}{n_p} \cdot |\Phi(S^{**})|$$

$$\geq \frac{\overline{nw} \cdot (n_b - \overline{nw})}{n_b} \cdot |\Phi(S^*)|$$

$$\geq \frac{\overline{nw} \cdot (n_b - \overline{nw})}{n_b \cdot (nw_{max} + 1)} \cdot U(S^*)$$

Since $OPT = U(S^*)$, $PoS = \frac{U(S^{**})}{OPT} \geq \frac{\overline{nw} \cdot (n_b - \overline{nw})}{n_b \cdot (nw_{max}+1)}$.

Next, let $S^{\#}$ be the strategic configuration of any Nash equilibrium obtained by *DASC_Game*. Thus, we have:

$$U_w(s_w^{\#}, \overline{s_w^{\#}}) \geq U_w(s'_w, \overline{s_w^{\#}}), \forall w \in W, \forall s'_w \in S_w$$

That is, $\forall s'_w \in S_w$, $\Phi(s_w^{\#}, \overline{s_w^{\#}}) - \Phi(s'_w, \overline{s_w^{\#}}) = U_w(s_w^{\#}, \overline{s_w^{\#}}) - U_w(s'_w, \overline{s_w^{\#}}) \geq 0$. Hence, $\Phi(s_w^{\#}, \overline{s_w^{\#}})$ is a local maximum and $|\Phi(s_w^{\#}, \overline{s_w^{\#}})|$ is a local minimum. Thus, we have:

$$U(S^{\#}) \geq \frac{\overline{nw} \cdot (n_b - \overline{nw})}{n_b} \cdot |\Phi(S^{\#})|$$

$$\geq \frac{\overline{nw} \cdot (n_b - \overline{nw})}{n_b} \cdot \widehat{|\Phi(S)|}_{min}$$

Since $U(S^*) \leq \min\{n_b, m_b\}$, we have:

$$PoA = \frac{U(S^{\#})}{U(S^*)} \geq \frac{\overline{nw} \cdot (n_b - \overline{nw})}{n_b \cdot \min\{n_b, m_b\}} \cdot \widehat{|\Phi(S)|}_{min}$$

□

Table 4: Experimental Settings on Real Data.

Parameters	Values
the start time range $[st^-, st^+]$	[0, 150], [0, 175], **[0, 200]**, [0, 225], [0, 250]
the waiting time range $[wt^-, wt^+]$	[1, 3], [2, 4], **[3, 5]**, [4, 6], [5, 7]
the velocity range $[v^-, v^+] * 0.01$	[0.1, 0.5], [0.5, 1], **[1, 1.5]**, [1.5, 2], [2, 2.5]
the distance range $[d^-, d^+] * 0.01$	[2, 2.5], [2.5, 3], **[3, 3.5]**, [3.5, 4], [4, 4.5]

5. EXPERIMENTAL STUDY

5.1 Data Sets

We use both real and synthetic data to test our proposed DA-SC approaches. Specifically, for real data, we use Meetup data set from [19], which was crawled from meetup.com between Oct. 2011 and Jan. 2012. There are 5,153,886 users, 5,183,840 events, and 97,587 groups in the Meetup data set, where each user is associated with a location and a set of tags, each group is associated with a set of tags, and each event is associated with a location and a group who creates the event. We use the locations and tags of users in the Meetup data set to initialize the locations and the practiced skills of workers in our DA-SC problem, where each tag is considered as a skill in our experiments. In addition, we utilize the locations and tags of groups which create the events to initialize the locations, the required skills and the dependencies of tasks in our experiments. Since workers are unlikely to move between two distant cities to conduct one spatial task, and the constraints of expired time and maximum moving distance also prevent workers from moving too far, we only consider those user-and-event pairs located in one city. Specifically, we select one famous and popular city, Hong Kong, and extract Meetup records from the area of Hong Kong (with latitude from $22.209°$ to $22.609°$ and longitude from $113.843°$ to $114.283°$), in which we obtain 1,282 tasks and 3,525 workers.

For real data, we generate tasks' dependency based on events' groups. Specifically, we consider one event as a task group and the tag set of the event as the required skill set of this task group. Then, we generate the dependent tasks in each task group. Each task requires one skill in the required skill set of the task group and its dependency set is the subset of this task group. Specifically, we randomly add the tasks which are in the same task group and have been generated before task t into t's dependency set. The reason of only adding the tasks generated before task t is to avoid the dependency cycle. Note that, when we add t_j into t_i's dependency set, we also add the t_j's dependency set D_j into t_i's dependency set. The reason is that, if t_a is dependent on t_b and t_b is dependent on t_c, then t_a must be dependent on t_c.

For synthetic data, we generate locations of workers and tasks in a 2D data space $[0, 0.5]^2$, following the uniform distribution. We vary the number of workers, tasks, and the cardinality of the skills' universe to mimic a wide scale of real-world application scenarios. In addition, we simulate the cardinality of each worker's skill set with the uniform distribution within the range $[sp^-, sp^+]$ from $[1, 5]$ to $[1, 25]$. Besides, we set the cardinality of each task's dependency with the uniform distribution within the range from $[0, 50]$ to $[0, 90]$. For each task t, we randomly add tasks generated before t and thier dependency set into t's dependency set until the cardinality of t's dependency set reaches the generated value.

For both real and synthetic data sets, we simulate the velocity of each worker with the uniform distribution within the range $[v^-, v^+]$ from $[0.01, 0.02]$ to $[0.05, 0.06]$. For the maximum moving distance of each worker, we generate it following the uniform distribution within the range $[d^-, d^+]$. For temporal constraints, we set each worker's and each task's start time and waiting time following the uniform distribution within the range $[st^-, st^+]$ and $[wt^-, wt^+]$.

Table 5: Experimental Settings on Synthetic Data.

Parameters	Values
the number, m, of workers	3K, 4K, **5K**, 6K, 7K
the number, n, of tasks	2K, 3.5K, **5K**, 6.5K, 8K
the start time range $[st^-, st^+]$	[0, 65], [0, 70], **[0, 75]**, [0, 80], [0, 85]
the waiting time range $[wt^-, wt^+]$	[8, 13], [9, 14], **[10, 15]**, [11, 16], [12, 17]
the velocity range $[v^-, v^+] * 0.01$	[1, 2], [2, 3], **[3, 4]**, [4, 5], [5, 6]
the distance range $[d^-, d^+] * 0.1$	[1, 2], [2, 3], **[3, 4]**, [4, 5], [5, 6]
the dependency size range	[0,50], [0,60], **[0, 70]**, [0, 80], [0, 90]
the number, r, of skill universe	1100, 1300, **1500**, 1700, 1900
the skill set range for each worker	[1, 5], [1, 10], **[1, 15]**, [1, 20], [1, 25]

5.2 Approaches and Measurements

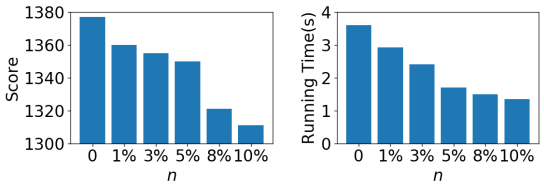
At beginning, we conduct an experiment on small scale data sets. The main purpose is to investigate the effectiveness of approximation methods regarding the optimal solution. We propose a depth-first search algorithm, namely *DFS Algorithm*, which exactly enumerates all possible assignments to find the optimal assignment. Each level in the search tree represents a worker in the worker set and the children of one node is the valid tasks that the worker represented by the next level can take. In addition, we propose two baseline methods. The first algorithm, namely *Closest*, greedily selects worker-and-task pairs with the lowest moving distance for each worker. The second algorithm, namely *Random*, randomly selects worker-and-task pair for each worker.

After that, we conduct experiments on large scale data sets to evaluate the effectiveness and efficiency of our two approaches, which includes DASC_Greedy and DASC_Game, in terms of the cardinality of the valid assignment and the running time. As proved in Section 2.4, the DA-SC problem is NP-hard, thus it is infeasible to calculate the optimal result as the ground truth in large scale datasets. Alternatively, we compare our approaches with two baseline methods. Note that, for *DASC_Game*, if we strictly set the termination condition as there is no worker changed his strategy in the last iteration, the converging speed is very slow. In practice, we usually set a threshold of the utility updating ratio instead. In other words, if the utility updating ratio in a round is lower than the threshold, we terminate the iteration in *DASC_Game*. Although with the increase of the threshold, the running time of *DASC_Game* would decreases, but the score of *DASC_Game* would decreases too. Thus, we have to select a proper threshold to trade off the score and the running time.

We run an experiment on the real datasets with different termination condition. Specifically, the parameters used in the experiment is the default values as shown in Table 4. We vary the value of the threshold from 0 (there is no worker changed his strategy in the last iteration) to 10%. As shown in Figure 2(a), when the threshold is larger than 5%, the score decrease sharply. And Figure 2(b) illustrates that, with the increase of the threshold, the running time decrease. Thus, to trade off the score and the running time, we set threshold as 5%.

In the following experiments, we examine the *DASC_Game* with strict termination condition, namely *Game*, and the *DASC_Game* with the threshold as 5%, namely *Game-5%*. In addition, we examine the heuristic that using *DASC_Greedy* for *DASC_Game*'s initialization, namely *G-G*. For simplicity, we represent the *DASC_Greedy* by *Greedy*.

Table 4 and Table 5 show our experimental settings on real data and synthetic data, where the default values of parameters are in bold font. In each set of experiments, we vary one parameter, while setting other parameters to their default values. For each experiment, we report the running time and the assignment score (the number of the valid assigned worker-and-task pairs) of our tested approaches. All our experiments were run on an Intel i7 CPU @2.2 GHz with 16GM RAM in Java.



(a) Score (b) Running Time

Figure 2: Effect of the Threshold

5.3 Results on Small-Scale Datasets

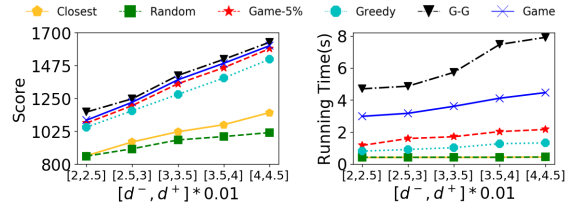
We run the experiment on a small scale synthetic data sets. Specifically, we set the number of workers is 20, the number of tasks is 40, and the cardinality of the skill universe is 10. In addition, the range of each worker's skill set's cardinality is [1,3] and the range of each task's dependency set's cardinality is [0, 8]. Other parameters are the default values of the experimental settings on synthetic data (Table 5). Table 6 shows the score and the running time of each algorithm. We can see that the scores of four proposed methods follow the bound which we proved in Section 3 and Section 4. Besides, the two baseline algorithms' score is much worse than the two proposed methods. In addition, we can see that the two proposed methods cost very little running time while the *DFS Algorithm* spends vast amounts of time. Thus, we can draw a conclusion that *DFS Algorithm* is only suitable for small scale data sets and our proposed algorithms have good acceptances compared with the optimal results.

Table 6: Experimental Results on Small-Scale Datasets

Algorithm	Score	Running Time (ms)
DFS	17	955514.9
Game-5%	17	1.9
Greedy	16	1.5
Closest	13	1.1
Random	12	1.4
G-G%	17	1.9
Game%	17	1.9

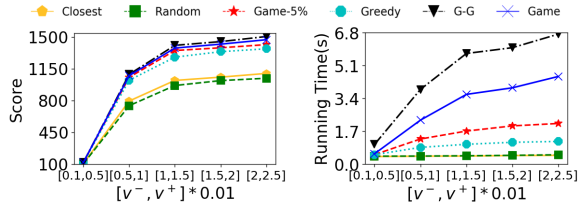
5.4 Results on Real Datasets

Effect of the Range of the Maximum Moving Distance $[d^-, d^+]$. Figure 3 illustrates the experimental results on different ranges, $[d^-, d^+]$, of each worker's maximum moving distance d from $[0.02, 0.025]$ to $[0.04, 0.045]$. In Figure 3(a), the assignment scores of all the six approaches increase, when the value range of maximum moving distance gets larger. The reason is that, the increase of d enlarges the access range of workers. The scores of four proposed algorithms are much better than those of two baseline algorithms. It is reasonable because the baseline algorithms do not consider tasks' dependency constraints when they allocate tasks and many of assignments are invalid. In addition, the *G-G*, *Game*, and *Game-5%* achieve higher scores than the *Greedy Algorithm*. The reason is that, each worker has multiple skills and for each skill, the number of workers who has the skill is different. When the *Greedy* assigns a task with a worker, it just selects a worker who has this skill and can reach the task on time. In other words, the *Greedy* does not guarantee that the assigned worker is the optimal choice for this task. Maybe the optimal choice is assigning another worker to do this task and assigning this worker to another task whose required skill can only be satisfied by this worker. However, the *G-G*, *Game*, and *Game-5%* can avoid this case. By checking all the strategies that each worker can take, the *G-G*, *Game*, and *Game-5%* can assign the worker to the task whose required skill is the skill that few workers have. Besides, *G-G* achieve the highest score among the six algorithms. The reason is that, compared with the random initialization in *Game* and *Game-5%*, the *G-G* can reach a Nash equilibrium with better quality.



(a) Score (b) Running Time

Figure 3: Effect of the Moving Distance Range $[d^-, d^+]$ (Real)



(a) Score (b) Running Time

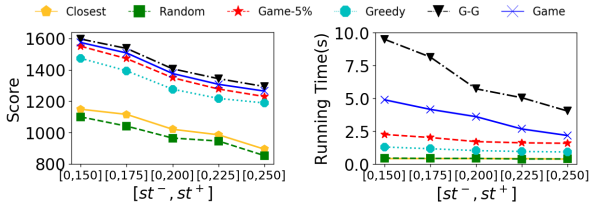
Figure 4: Effect of the Velocity Range $[v^-, v^+]$ (Real)

As shown in Figure 3(b), the running time of our four DA-SC approaches increase obviously, when the range of maximum moving distance gets larger. The reason is that, when the maximum moving distance increases, each worker has more valid tasks which thus leads to higher complexity of the *DA-SC* problem and the increase of the running time. Specifically, the *Greedy* achieves much lower running time than the *G-G*, *Game*, and *Game-5%*. The reason is that the *G-G*, *Game*, and *Game-5%* has a large search space and in each round it searches the whole search space while in each round the *Greedy* just searches a task combination with the largest cardinality and can be fully assigned with a group of workers. In addition, the running time of *G-G* is the highest. The reason is that, compared with the random initialization in *Game*, and *Game-5%*, its initialization costs much more time.

Effect of the Range of the Velocity $[v^-, v^+]$. Figure 4 illustrates the experimental results on different ranges, $[v^-, v^+]$, of each worker's velocity v from $[0.001, 0.005]$ to $[0.02, 0.025]$. In Figure 4(a), when the value range of velocity gets larger from $[0.001, 0.005]$ to $[0.01, 0.015]$, the assignment scores of all the six approaches first increase; then, they almost keep stable for the velocity range varying from $[0.01, 0.015]$ to $[0.02, 0.025]$. The reason is that, at beginning, with the increase of v, workers can reach more tasks on time. Nevertheless, workers are also constrained by other constraints, e.g., maximum moving distance. As shown in Figure 4(b), the running time of our four approaches increase, when the range of velocity gets larger. The reason is that, when the velocity increases, each worker has more valid tasks which thus leads to higher complexity of the *DA-SC* problem and the increase of the running time. However, when the constraint of v is relaxed, the constraints from other parameters prevent the running time keeping growing.

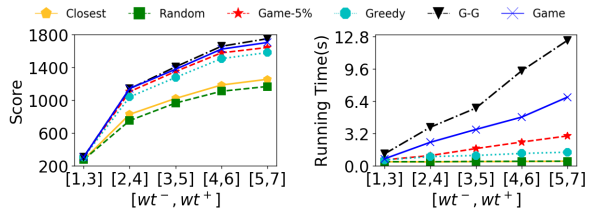
Effect of the Range of the Start Timestamp $[st^-, st^+]$. Figure 5 illustrates the experimental results on different ranges, $[st^-, st^+]$, of each worker's/task's start timestamp st from $[0, 150]$ to $[0, 250]$. In Figure 5(a), the assignment scores of all the six approaches decrease, when the value range of start timestamp gets larger. The reason is that, the increase of st's range disperses the tasks/workers over time and thus each task has less valid workers who can reach the task timely. The *G-G* achieves the highest score and the scores of four proposed algorithms are much better than those of two baseline algorithms. As shown in Figure 5(b), the running time of our six approaches decrease, when the range of start timestamp gets larger. The reason is that, when the range increase, each worker has fewer valid tasks which thus leads to lower complexity of the *DA-SC* problem and the decrease of the running time.

Effect of the Range of the Waiting Time $[wt^-, wt^+]$. Figure 6 illustrates the experimental results on different ranges, $[wt^-, wt^+]$,



(a) Score (b) Running Time

Figure 5: Effect of the Start Time Range $[st^-, st^+]$ (Real)



(a) Score (b) Running Time

Figure 6: Effect of the Waiting Time Range $[wt^-, wt^+]$ (Real)

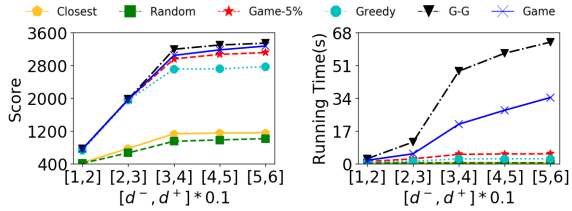
of each worker's/task's waiting time wt from $[1, 3]$ to $[5, 7]$. In Figure 6(a), the assignment scores of all the six approaches increase, when the value range of waiting time gets larger. The reason is that, the increase of wt let each worker can reach more tasks timely. The *G-G* achieves the highest score and the scores of four proposed algorithms are much better than those of two baseline algorithms. As shown in Figure 6(b), the running time of our four approaches increase, when the range of waiting time gets larger. The reason is that, when the waiting time increase, each worker has more valid tasks which thus leads to higher complexity of the *DA-SC* problem and the increase of the running time.

5.5 Results on Synthetic Datasets

In order to examine the effects of the cardinality of the skill universe, the number of workers, the number of tasks, the cardinality of each task's dependency set, and the cardinality of each worker's skill set, we generate the synthetic dataset and run the experiments on it.

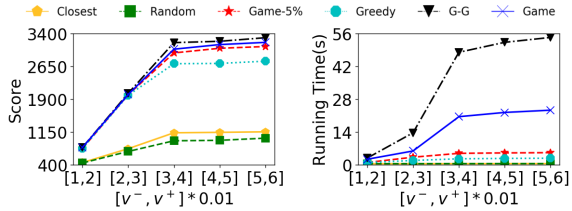
Effect of the Range of the Maximum Moving Distance $[d^-, d^+]$. Figure 7 illustrates the experimental results on different ranges, $[d^-, d^+]$, of each worker's maximum moving distance d from $[0.1, 0.2]$ to $[0.5, 0.6]$. In Figure 4(a), when the value range of maximum moving distance gets larger from $[0.1, 0.2]$ to $[0.3, 0.4]$, the assignment scores of all the six approaches first increase; then, they almost keep stable for the velocity range varying from $[0.3, 0.4]$ to $[0.5, 0.6]$. The reason is that, at beginning, with the increase of d, workers can reach more tasks. Nevertheless, workers are also constrained by other constraints, e.g., deadline constraint. As shown in Figure 7(b), the running time of our four approaches increase, when the range of maximum moving distance gets larger. The reason is that, when the maximum moving distance increase, each worker has more valid tasks which thus leads to higher complexity of the *DA-SC* problem and the increase of the running time. However, when the constraint of d is relaxed, the constraints from other parameters prevent the running time keeping growing.

Effect of the range of the velocity, $[v^-, v^+]$. Figure 8 illustrates the experimental results on different ranges, $[v^-, v^+]$, of each worker's velocity v from $[0.01, 0.02]$ to $[0.05, 0.06]$. In Figure 8(a), when the value range of velocity gets larger from $[0.01, 0.02]$ to $[0.03, 0.04]$, the assignment scores of all the six approaches first increase; then, they almost keep stable for the velocity range varying from $[0.03, 0.04]$ to $[0.05, 0.06]$. The reason is that, at beginning, with the increase of v, workers can reach more tasks on time. Nevertheless, workers are also constrained by other constraints, e.g., maximum moving distance constraint. As shown in Figure 8(b), the running time of our



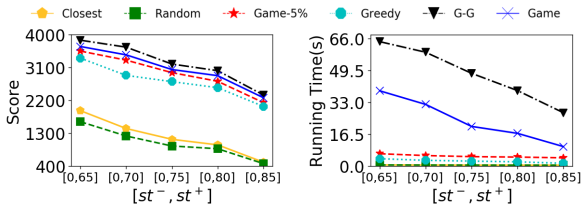
(a) Score (b) Running Time

Figure 7: Effect of the Moving Distance Range $[d^-, d^+]$ (Synthetic)



(a) Score (b) Running Time

Figure 8: Effect of the Velocity Range $[v^-, v^+]$ (Synthetic)



(a) Score (b) Running Time

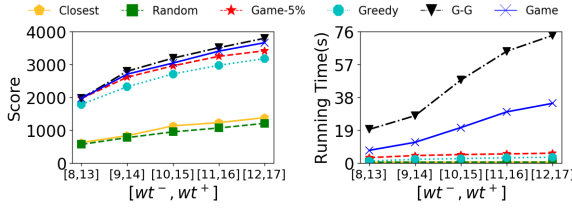
Figure 9: Effect of the Start Time Range $[st^-, st^+]$ (Synthetic)

four approaches increase, when the range of velocity gets larger. The reason is that, when the velocity increase, each worker has more valid tasks which thus leads to higher complexity of the *DA-SC* problem and the increase of the running time. However, when the constraint of v is relaxed, the constraints from other parameters prevent the running time keeping growing.

Effect of the range of the start timestamp, $[st^-, st^+]$. Figure 9 illustrates the experimental results on different ranges, $[st^-, st^+]$, of each worker's/task's start timestamp st from $[0, 65]$ to $[0, 85]$. In Figure 9(a), the assignment scores of all the six approaches decrease, when the value range of start timestamp gets larger. The reason is that, the increase of st's range disperses the tasks/workers over time and thus each task has less valid workers. The *G-G* achieves the highest score and the scores of four proposed algorithms are much better than those of two baseline algorithms. As shown in Figure 9(b), the running time of our four approaches decreases, when the range of start timestamp gets larger. The reason is that, when the range increase, each worker has less valid tasks which thus leads to lower complexity of the *DA-SC* problem and the decrease of the running time.

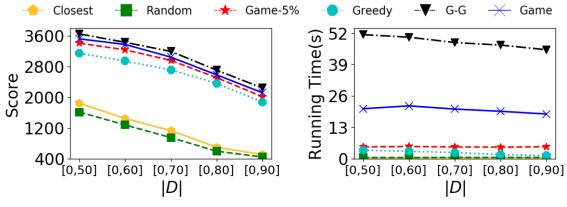
Effect of the range of the waiting time, $[wt^-, wt^+]$. Figure 10 illustrates the experimental results on different ranges, $[wt^-, wt^+]$, of each worker's/task's waiting time wt from $[8, 13]$ to $[12, 17]$. In Figure 10(a), the assignment scores of all the six approaches increase, when the value range of waiting time gets larger. The reason is that, the increase of wt let each worker can reach more tasks timely. The *G-G* achieves the highest score and the scores of four proposed algorithms are much better than those of two baseline algorithms. As shown in Figure 10(b), the running time of our four approaches increase, when the range of waiting time gets larger. The reason is that, when the waiting time increase, each worker has more valid tasks which thus leads to higher complexity of the *DA-SC* problem and the increase of the running time.

Effect of the range of dependency set size, $|D|$. Figure 11 illustrates the experimental results on different ranges, of each task's



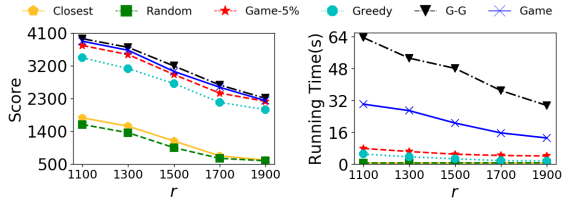
(a) Score (b) Running Time

Figure 10: Effect of the Waiting Time Range $[wt^-, wt^+]$ (Synthetic)



(a) Score (b) Running Time

Figure 11: Effect of the Dependency Size Range (Synthetic)



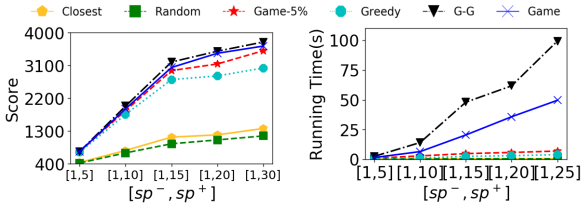
(a) Score (b) Running Time

Figure 12: Effect of the Cardinality of the Skill Universe (Synthetic)

dependency set size $|D|$ from $[0, 50]$ to $[0, 90]$. In Figure 11(a), the assignment scores of all the six approaches decrease, when the value range of each task's dependency gets larger. The reason is that, the increase of $|D|$ let the tasks' dependency constraints are more difficult to satisfy. Specifically, the two baseline algorithms is more sensitive to the increase and their score decrease sharply with the increase of $|D|$. The reason is that, with the increase of $|D|$, the tasks' dependency constraints are more difficult to satisfy and the assignments made by the two baseline algorithms which do not consider the dependency constraints are more likely be invalid. As shown in Figure 11(b), the running time of *G-G*, *Game*, and *Game-5%* keeps stable when the value range of each task's dependency gets larger. The reason is that, the change of dependency's size does not influence these algorithms' search space.

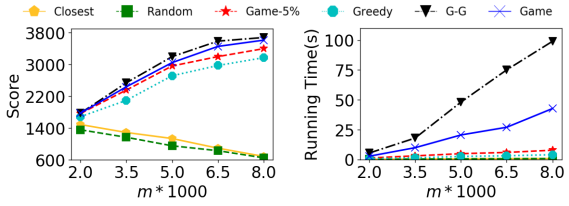
Effect of the cardinality of the skill universe r. Figure 12 illustrates the experimental results on different cardinality, r, of the skill universe Ψ from 1100 to 1900. In Figure 12(a), the assignment scores of all the six approaches decrease, when the cardinality of the skill universe gets larger. The reason is that, the increase of r disperses the tasks/workers over skills. The *G-G* achieves the highest score and the scores of four proposed algorithms are much better than those of two baseline algorithms. As shown in Figure 12(b), the running time of our four approaches decrease, when the cardinality of the skill universe gets larger. The reason is that, when the cardinality of the skill universe gets larger, the number of tasks that each worker can serve decreases which thus leads to lower complexity of the *DA-SC* problem and the decrease of the running time.

Effect of the Range of each Worker's Skill Size $[sp^-, sp^+]$. Figure 13 illustrates the experimental results on different ranges, $[sp^-, sp^+]$, of each worker's skill set $|WS|$ from $[1, 5]$ to $[1, 30]$. In Figure 13(a), the assignment scores of all the six approaches increase, when the value range of each worker's skill size gets larger. The reason is that, the increase of $|WS|$ let each task has more valid workers. The *G-G* achieves the highest score and the scores of four proposed algorithms are much better than those of two baseline al-



(a) Score (b) Running Time

Figure 13: Effect of each Worker's Skill Size Range (Synthetic)



(a) Score (b) Running Time

Figure 14: Effect of the Number of Tasks (Synthetic)

gorithms. As shown in Figure 13(b), the running time of our four approaches increases, when the range of each worker's skill size gets larger.

Effect of the Number of Tasks m. Figure 14 illustrates the experimental results on different number, m, of tasks from $2K$ to $8K$. In Figure 14(a), when the number of tasks gets larger, the assignment scores of our proposed approaches increase while the scores of two baseline algorithms decrease. The reason is that, the increase of m let each worker has more valid tasks. Since the two baseline algorithm does not consider the dependency constraint when they allocate the tasks, the increase of the number of tasks that each worker can do decreases the probability that the assigned task's dependency constraint is satisfied. As shown in Figure 14(b), the running time of our four approaches increase, when the number of tasks gets larger. The reason is that, when the number of tasks increase, each worker has more valid tasks which thus leads to higher complexity of the *DA-SC* problem and the increase of the running time.

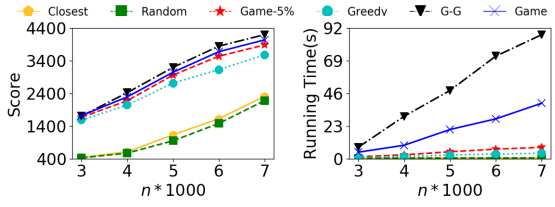
Effect of the Number of Workers n. Figure 15 illustrates the experimental results on different number, n, of workers from $3K$ to $7K$. In Figure 15(a), the assignment scores of all the six approaches increase, when the number of workers gets larger. The reason is that, the increase of n let each task has more valid workers. As shown in Figure 15(b), the running time of our two approaches increase, when the number of workers gets larger. The reason is that, when the number of workers increases, each task has more valid workers which thus leads to higher complexity of the *DA-SC* problem and the increase of the running time.

In summary, on both real and synthetic data sets, the four approximate algorithm (*G-G*, *Game*, *Game-5%* and *Greedy*) can achieve results with much more valid worker-and-task pairs compared with that of baseline algorithms. In addition, Greedy runs much faster than another three approximate algorithms while can achieve results with close number of valid worker-and-task pairs compared with them. Besides, the heuristic that running *DASC_Greedy* for *DASC_Game*'s initialization is effective while it cost much time.

6. RELATED WORK

Crowdsourcing has attracted considerable attention due to its high practicality for real-world applications. Without considering the location constraint, previous work [20, 21, 22] studied the task assignment in crowdsourcing to finish the tasks more efficiently and accurately.

In spatial crowdsourcing [1], workers are requested to physically move to specific locations to conduct tasks on sites. Based on



(a) Score (b) Running Time

Figure 15: Effect of the Number of Workers (Synthetic)

the ontology [26], from the perspective of the publishing models, task assignment in spatial crowdsourcing can be classified into two groups: worker selected tasks (WST) mode and server assigned tasks (SAT) mode. Specifically, for the WST mode, spatial tasks are broadcast to all the workers or group of close workers, then the workers select preferred tasks by themselves. In prior work [11] in task assignment in SAT mode, the author proposed methods to design a travel route for each worker such that the worker can finish as many tasks as possible before the deadlines. In the contrast, in WST mode, the workers reveal their real time locations to the server, then the server will assign the suitable tasks to workers. Since the server has the control of the task assignment in WST mode, it is more convenient to optimize the targeted goal and many existing studies [1, 26, 27, 8, 28] in task assignment in spatial crowdsourcing are using the WST mode.

In particular, Kazemi et al. [1] studied the problem of maximizing the number of assigned problem under the constraint of the working areas and the capacities of workers and the deadlines of tasks. With considering the reliability of workers, Cheng et al. [27] tackle the problem of reliable diversity-based spatial crowdsourcing problem, which assigns a set of workers to each task such that the spatial/temporal diversity and the reliability score of the answers to the task is optimized. Tong et al. [28] studied the online task assignment for spatial crowdsourcing, in which the server needs to assign the most suitable worker to each task when the workers and tasks are coming one-by-one and the platform has no information about the future tasks or workers. Previous studies on multi-skill oriented spatial crowdsourcing [7, 8] tackle the problem through finding a set of workers and the union of their skill sets can fully support the requirement of the assigned complex task. However, the existing methods do not consider the dependencies between the subtasks such that some assigned workers need to wait until their subtasks are ready to conduct, which makes that the existing methods are not efficient. In this paper, we take the dependencies between tasks into consideration and propose tailored approximation algorithms to efficiently and effectively solve the DA-SC problems with theory bounds of the number of the assigned worker-and-task pairs for each batch process, which had not been studied before.

7. CONCLUSION

In this paper, we proposed *Dependnecy-Aware Spatial Crowdsourcing* (DA-SC) problem, a new problem of batch-based task allocation in spatial crowdsourcing, where tasks have been allocated following the dependency constraints. To address the DA-SC problem, we proposed two approximation algorithms, including greedy and game-theoretic approaches. Specifically, we defined the *task combination* and design a submodule function in the greedy approach, which greedily allocates a task combination with the largest cardinality and guarantees the approximate bounds of the results in each batch process. In addition, we propose a game-theoretic approach to further increase the number of the assigned worker-and-tasks. We conclude that our proposed two solutions are effective and efficient in extensive experiments on both real and synthetic data sets.

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APPENDIX

A. PROOF OF THEORE 4.1

Theorem 4.1. DA-SC problem constitutes an exact potential game.

Proof. Recall from Section 4.1, that it suffices to show that for every worker w, who changes his strategy from the current one s_w to the best-response s'_w, and for all possible combinations of the other players' strategies $\overline{s_w}$ it holds that:

$$U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w}) = \Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w})$$

Suppose np_{s_w} and $np_{s'_w}$ are the numbers of workers who are assigned to the tasks s_w and s'_w in the assignment $(s_w, \overline{s_w})$, respectively. Similarly, $\overline{np_{s_w}}$ and $\overline{np_{s'_w}}$ are the numbers of workers who are assigned to the tasks s_w and s'_w in the assignment $(s'_w, \overline{s_w})$, respectively. Note that, $np_{s_w} = \overline{np_{s_w}} + 1$ and $\overline{np_{s'_w}} = np_{s'_w} + 1$. Indeed, when $D_{s_w} \neq \emptyset$ and $D_{s'_w} \neq \emptyset$, we have:

$$\begin{aligned}
&\Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w}) \\
=&-\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s'_w}} a_f}{\alpha\cdot(np_{s'_w}+1)} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(np_{s'_w}+1)\cdot|D_l|}\right) \\
&+\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s_w}} a_f}{\alpha\cdot(\overline{np_{s_w}}+1)} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(\overline{np_{s_w}}+1)\cdot|D_{s_w}|}\right) \\
=&\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s_w}} a_f}{\alpha\cdot np_{s_w}} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot np_{s_w}\cdot|D_{s_w}|}\right) \\
&-\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s'_w}} a_f}{\alpha\cdot\overline{np_{s'_w}}} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot\overline{np_{s'_w}}\cdot|D_{s'_w}|}\right) \\
=&U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w})
\end{aligned}$$

Similarly, when (1) $D_{s_w} = \emptyset$ and $D_{s'_w} \neq \emptyset$; (2) $D_{s_w} \neq \emptyset$ and $D_{s'_w} = \emptyset$; (3) $D_{s_w} = \emptyset$ and $D_{s'_w} = \emptyset$, we can have the same

result: $\Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w}) = U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w})$. Due to the space limitation, we do not show the details here. For the full proof, please refer to Appendix A of our technical report [18].

When $D_{s_w} = \emptyset$ and $D_{s'_w} \neq \emptyset$, we have:

$$
\begin{aligned}
&\Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w}) \\
=&-\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s'_w}} a_f}{\alpha\cdot(np_{s'_w}+1)} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(np_{s'_w}+1)\cdot|D_{s'_w}|}\right) \\
&+\left(\frac{\prod_{f\in D_{s_w}} a_f}{\overline{np_{s_w}}+1} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(\overline{np_{s_w}}+1)\cdot|D_{s_w}|}\right) \\
=&\left(\frac{\prod_{f\in D_{s_w}} a_f}{np_{s_w}} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot np_{s_w}\cdot|D_{s_w}|}\right) \\
&-\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s'_w}} a_f}{\alpha\cdot\overline{np_{s'_w}}} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot\overline{np_{s'_w}}\cdot|D_{s'_w}|}\right) \\
=&U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w})
\end{aligned}
$$

When $D_{s_w} \neq \emptyset$ and $D_{s'_w} = \emptyset$, we have:

$$
\begin{aligned}
&\Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w}) \\
=&-\left(\frac{\prod_{f\in D_{s'_w}} a_f}{np_{s'_w}+1} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(np_{s'_w}+1)\cdot|D_{s'_w}|}\right) \\
&+\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s_w}} a_f}{\alpha\cdot(\overline{np_{s_w}}+1)} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(\overline{np_{s_w}}+1)\cdot|D_{s_w}|}\right) \\
=&\left(\frac{(\alpha-1)\cdot\prod_{f\in D_{s_w}} a_f}{\alpha\cdot np_{s_w}} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot np_{s_w}\cdot|D_{s_w}|}\right) \\
&-\left(\frac{\prod_{f\in D_{s'_w}} a_f}{\overline{np_{s'_w}}} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot\overline{np_{s'_w}}\cdot|D_{s'_w}|}\right) \\
=&U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w})
\end{aligned}
$$

When $D_{s_w} = \emptyset$ and $D_{s'_w} = \emptyset$, we have:

$$
\begin{aligned}
&\Phi(s_w, \overline{s_w}) - \Phi(s'_w, \overline{s_w}) \\
=&-\left(\frac{\prod_{f\in D_{s'_w}} a_f}{np_{s'_w}+1} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(np_{s'_w}+1)\cdot|D_{s'_w}|}\right) \\
&+\left(\frac{\prod_{f\in D_{s_w}} a_f}{\overline{np_{s_w}}+1} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot(\overline{np_{s_w}}+1)\cdot|D_{s_w}|}\right) \\
=&\left(\frac{\prod_{f\in D_{s_w}} a_f}{np_{s_w}} + \sum_{s_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot np_{s_w}\cdot|D_{s_w}|}\right) \\
&-\left(\frac{\prod_{f\in D_{s'_w}} a_f}{\overline{np_{s'_w}}} + \sum_{s'_w\in D_l}\frac{\prod_{f\in D_l\cup\{l\}} a_f}{\alpha\cdot\overline{np_{s'_w}}\cdot|D_{s'_w}|}\right) \\
=&U_w(s_w, \overline{s_w}) - U_w(s'_w, \overline{s_w})
\end{aligned}
$$

Then, we proved DA-SC problem is an exact potential game. □