Infinite Stream Estimation under Personalized w-Event Privacy

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ABSTRACT

Streaming data collection is indispensable for stream data analysis, such as event monitoring. However, publishing these data directly leads to privacy leaks. w-event privacy is a valuable tool to protect individual privacy within a given window size while maintaining high accuracy in data collection. Most existing w-event privacy research on infinite data stream only focuses on homogeneous privacy requirements for all users. In this paper, we design personalized w-event privacy mechanisms that allows different users to have different privacy requirements in private data stream estimation. Specifically, we design a mechanism that allows users to maintain constant privacy requirements at each time slot, namely Personalized Window Size Mechanism (PWSM). Then, we propose two solutions to accurately estimate stream data statistics while achieving $\boldsymbol{w}$-event level $\boldsymbol{\Phi}$ personalized differential privacy, namely Personalized Budget Distribution (PBD) and Peronalized Budget Absorption (PBA). PBD always provides at least the same privacy budget for the next time step as the amount consumed in the previous release. PBA fully absorbs the privacy budget from the previous k time periods, while also borrowing from the privacy budget of the next k time periods, to increase the privacy budget for the current time period. We prove that both PBD and PBA outperform state-of-the-art private stream estimation methods while meeting the privacy requirements of all users. We demonstrate the efficiency and effectiveness of our PBD and PBA on both real and synthetic data sets, compared with the recent uniformity w-event approaches, Budget Distribution (BD) and Budget Absorption (BA). Our PBD achieves a 68% lower error than BD on average in real data sets. Besides, our PBA achieves a 24.9% lower error than BA on average in synthetic data sets.

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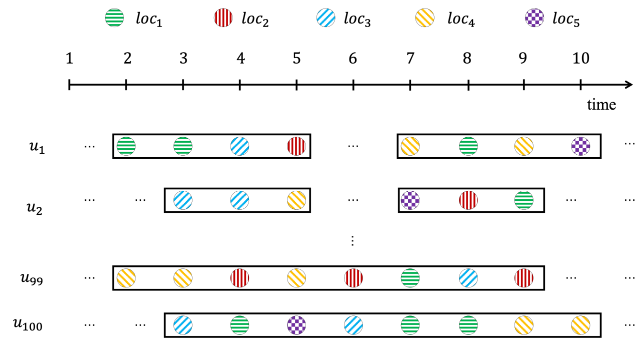


Figure 1: Different event window sizes for different time slots.

1 INTRODUCTION

With the popularity of smart devices and high-quality wireless networks, people can easily access the internet and communicate with online services. They continuously report data to platforms and receive services like log stream analysis [34], event monitoring [19], and video querying [27]. To provide better services, these platforms collect data and conduct real-time analysis over aggregated data streams.

However, collecting stream data directly poses severe privacy risks, causing users to refuse communication with platforms. For instance, an AIDS patient may decline to participate in an investigation due to privacy concerns [18]. To resolve this conflict, differential privacy (DP) is proposed [11]. It protects individual privacy while ensuring accurate data estimation.

Recently, w-event privacy based on DP has emerged for private stream data collection and analysis [29, 30, 33]. It effectively protects the privacy of w consecutive related events while offering accurate stream statistics. However, different users may have different privacy requirements for their data. For instance, entertainers may be reluctant to reveal too much about their locations (large w-event size), while outdoor anchors may be willing to expose more of their positions (small w-event size) to gain more attention. Thus, if we fix the window size w for all users, some users' privacy levels will be excessively high, while others' will be excessively low. Both of these cases lead to low efficiency in private stream estimation.

We illustrate an example of online car-hailing shown in Figure 1 to elaborate further.

Example 1. *Suppose there are* 100 *drivers* $U = \{u_1, ..., u_{100}\}$ *who provide their locations within* $\{loc_1, ..., loc_8\}$ *at each time slot. Each driver* u_i *has a privacy window size requirement* w_i*, meaning* u_i *requires privacy protection of events within at least a* w_i *time window (achieving* ϵ*-DP in any* w *windows). For instance in Figure 1,* u_1 *wants to protect any location sequence within any* 4 *time window. Besides,* u_{99} *and* u_{100} *want to protect any location sequence within* *any 8 time window. Suppose for each $u_i \in U \backslash \{u_{99}, u_{100}\}$, the window size is no more than 4. To satisfy all drivers' privacy needs, according to traditional w-event privacy, we need to set the event window size as the maximal value (i.e., $w = 8$), and make full use of the privacy budget to achieve high utility while satisfying 8-event privacy. Let AE_{avg} denote the average square error at each timestamp, defined as the variance when adding Laplace noise (i.e., $AE_{avg} = 2b^2 = 2 \times \left(\frac{1}{\epsilon/w}\right)^2$). Suppose he total privacy budget ϵ is 1 and the platform adopts the Uniform method [22]. Then for this example, under 8-event privacy, the average square error at each timestamp is $AE_{avg} = 2 \times (\frac{w}{\epsilon})^2 = 128$. However, it is not necessary for the first 98 drivers to set the window size as 8 (only achieving 8-event privacy). If we set the window size as $w = 4$ and use the threshold method [20] (or the sample method [20]), then we can get $AE_{avg} \approx 2 \times (\frac{w}{\epsilon})^2 = 32$, which is much less than the error of traditional 8-event privacy.*

In this paper, we define the Personalized w-event Private Release for Infinite Data Streams (PWPR-IDS) problem to model personalized requirements in stream data publication. To solve PWPR-IDS, we extend the recent private stream methods Budget Distribution (BD) and Budget Absorption (BA) to personalized situations. There are two challenges: 1) effectively unifying the privacy budget across all users into a single value that maximizes publication utility, and 2) effectively dividing each user's personalized privacy budget within their personalized window size to maximize publication utility.

The first challenge arises from the fact that traditional DP mechanisms require a single privacy budget for publishing statistics at each time slot. When users specify different privacy budgets, we must determine a unified budget that balances competing needs. Using the minimum budget ensures all privacy requirements are met, yet leads to poor utility. Conversely, using the maximum budget optimizes utility, yet compromises privacy for users with lower budget requirements. The challenge lies in finding an optimal privacy budget that both satisfies all users' privacy requirements and maximizes publication utility. To address this challenge, we propose Personalized Window Size Mechanism (PWSM, a novel personalized private data stream publication framework. For each time slot and privacy budget ϵ, PWSM approximates all users' privacy budget requirements as ϵ and employs the sampling mechanism [20] to prevent privacy leakage for users with lower budget requirements. PWSM then applies a DP mechanism with privacy budget ϵ to either publish a new obfuscated statistic or skip the current time slot. While both the approximation and DP mechanism steps introduce error, PWSM precisely defines this error err and achieves optimal utility by selecting the ϵ value that minimizes err.

For the second challenge, we need to divide the privacy budget into multiple parts while ensuring each part remains large enough to generate meaningful obfuscated values. The privacy budget functions as a resource that must be divided into finite portions. Data at different time slots carry varying levels of importance, depending on their distribution patterns. Intuitively, time slots with higher rates of change contain more information and thus hold greater importance. To maximize utility, we must allocate larger privacy budgets to publications at these important time slots while approximating others. Simply allocating the privacy budget uniformly (within a window) or selecting publication time slots at equal intervals results in low utility. The key challenge is determining how to identify important time slots and allocate privacy budgets to achieve maximum publication utility. To address this, we categorize original data distributions into two broad types: those with relatively high change rates and those with relatively low change rates. Building on PWSM, we develop two methods to handle these types: Personalized Budget Distribution (PBD) and Peronalized Budget Absorption (PBA). PBD takes an optimistic approach, assuming few publications per window and thus allocating larger budget portions to each publication. PBA, in contrast, assumes that successive statistics will closely resemble the publication timestamp. It maximizes current publication accuracy by utilizing unused budget from skipped publications while nullifying future timestamp budgets, enabling effective approximation of subsequent publications. We also demonstrate that both PBD and PBA satisfy $(\boldsymbol{w}, \Phi)$-EPDP and provide their average error upper bounds. We summarize our contributions as follows.

- We formally define personalized $\boldsymbol{w}$-event level Φ-Personalized Differential Privacy for PWPR-IDS in Section 3.
- We propose a basic mechanism, Personalized Window Size Mechanism (PWSM), and two methods, namely Personalized Budget Distribution (PBD) and Peronalized Budget Absorption (PBA), to support personalized $\boldsymbol{w}$-event privacy. These methods achieve accurate stream estimation while satisfying personalized privacy requirements (a fixed time window size and privacy budget for each user) as described in Section 4.
- We test our methods on both real and synthetic data sets to demonstrate their efficiency and effectiveness in Section 5.

2 RELATED WORK

We classify the related work in the area of data stream estimation under differential privacy and non-uniformity differential privacy.

2.1 Data Stream Estimation under Differential Privacy

Based on the privacy model, we categorize data stream estimation studies into two types: those employing centralized differential privacy [11] (CDP) and those utilizing local differential privacy [4] (LDP).

2.1.1 Data Stream Estimation under CDP.

Dwork et al. [13] first address the problem of Differential Privacy (DP) on data streams. They define two types of DP levels: *event-level differential privacy* (event DP) and *user-level differential privacy* (user DP).

In event DP, each single event is hidden in statistic queries. Dwork et al. [13] focus on the finite event scenarios and propose a binary tree method to achieve high statistical utility while maintaining event DP. Chan et al. [7] extend this method to infinite case, producing partial sums for binary counting. Dwork et al. [12] introduce a cascade buffer counter that updates adaptively based on stream density. Bolot et al. [6] propose *decayed privacy* which reduces the privacy costs for past data. Chen et al. [8] develop PeGaSus, a perturb-group-smooth framework for multiple queries under event-level DP. However, event DP supposes all element in a stream are independent, making it unsuitable for correlated data stream publishing.

In user DP, all events for each user are hidden in statistic queries. Fan et al. [17] propose the FAST algorithm with a sampling-and-filtering framework, counting finite stream data under user-level DP. Cummings et al. [9] address heterogeneous user data, estimating population-level means while achieving user-level DP. However, they only consider finite data. Offering user DP for infinite data requires infinite perturbation [22], leading to poor long-term utility.

To bridge the gap between event DP and user DP, Kellaris et al. [22] propose w-event DP for infinite streams. This ensures ϵ-DP for any group of events within a time window of size w. They introduce two methods, *Budget Distribution* and *Budget Absorption*, to optimize privacy budget use and estimate statistics effectively. However, neither method handles stream data with significant changes. Wang et al. [30] apply the w-event concept to FAST, proposing a multi-dimensional stream release mechanism called *ResueDP*, which achieves accurate estimation for both rapid and slow data stream changes. A limitation of all these methods is their reliance on a trusted server to ensure privacy.

2.1.2 Data Stream Estimation under LDP. To overcome the dependence on a trusted server, Local Differential Privacy (LDP) [4] has recently been proposed. It has been adopted by major companies such as Microsoft, Apple and Google. Erlingsson et al. [16] introduce RAPPOR to estimate finite streams under LDP. They design a two-layer randomized response mechanism (i.e., permanent randomized response and instantaneous randomized response) to protect individual data. However, RAPPOR is limited to uncorrelated stream data. To address correlated time series data, Erlingsson et al. develop a new privacy model that introduces *shuffling* to amplify the LDP privacy level [15]. However, this model only suits finite stream data. Joseph et al. [21] propose THRESH for evolving data under LDP. THRESH consumes privacy budget at global update times selected by users' LDP voting. However, it is not applicable to infinite streams as it assumes a fixed number of global updates. Wang et al. [31] extend event-level privacy from CDP to LDP and design the efficient ToPL method under event LDP. Yet, event-level LDP focuses solely on event-level privacy, lacking privacy protection for correlated data in streams. Bao et al. [3] propose an (ϵ, δ)-LDP method (called CGM) for finite streaming data collection using the analytic Guassian mechanism. However, this method requires periodic privacy budget renewal. Ren et al. [29] introduce LDP-IDS for infinite streaming data collection and analysis under w-event LDP. They propose two budget allocation methods and two population allocation methods, bridging the gap between event LDP and user LDP while improving estimation accuracy. However, all these methods cannot be applied to settings with personalized event window sizes.

2.2 Non-Uniformity Differential Privacy

In contrast to traditional uniformity DP, some studies address the non-uniform privacy requirements among items (table columns) or records (table rows) [28].

Alaggan et al. [1] first examine scenarios where each database instance comprises a single user's profile. They focus on varying privacy requirements for different items and formally define Heterogeneous Differential Privacy (HDP).

Table 1: Summary for related work.

Model Types		Methods	Is infinite and correlated	Is personalized privacy
Centralized DP	event-level privacy	Finite B-tree [13]	✗	✗
		Infinite B-tree [7]	✗	✗
		Adaptive-density Counter [12]	✗	✗
		Decayed Privacy [6]	✗	✗
		PeGaSus [8]	✗	✗
	user-level privacy	FAST [17]	✔	✗
		Private heterogeneous mean estimation [9]	✔	✗
	w-event privacy	BD & BA [22]	✔	✗
		ResuseDP [30]	✔	✗
Local DP	event-level privacy	RAPPOR [16]	✗	✗
		ToPL [31]	✗	✗
	user-level privacy	Shuffling LDP [15]	✔	✗
		THRESH [21]	✔	✗
		CGM [3]	✔	✗
	w-event privacy	LDP-IDS [29]	✔	✗
Item heterogeneous		HDP [1]	✗	✗
Record heterogenous		PDP [20]	✗	✔
		OSDP [23]	✗	✔
		Geo-I [2]	✗	✔
		PWSM, VPDM [32]	✗	✔
		PUCE, PGT [10]	✗	✔
		PFA, PFA+ [26]	✗	✔
Our mechanisms			✔	✔

Jorgensen et al. [20] investigate the privacy preservation for individual rows, introducing Personalized Differential Privacy (PDP). They design two mechanisms leveraging non-uniform privacy requirements to achieve better utility than standard DP. Kotsogiannis et al. [23] recognize that different data have different sensitivity. They define One-side Differential Privacy (OSPD) and propose algorithms that truthfully release non-sensitive record samples to enhance accuracy in DP-solutions. Andrés et al. [2] introduce a novel non-uniform privacy concept called Geo-Indistinguishability (Geo-I). In Geo-I, the privacy level for any point increases as the distance to this point decreases. Wang et al. [32] and Du et al. [10] explore personalized differential privacy in spatial crowdsourcing. They develop highly effective private task assignment methods to satisfy diverse workers' privacy and utility requirements. Liu et al. [26] investigate heterogeneous differential privacy in federated learning. They assume different clients hold different privacy budget and divide them into private and public parts. They propose two methods for projecting the "public" clients' model into "private" clients' models, improving the joint model's utility. However, all above studies are not suitable for stream data.

In this paper, we propose Personalized Window Size Mechanism (PWSM) with two implementation methods: Personalized Budget Distribution (PBD) and Peronalized Budget Absorption (PBA). Our approach extends traditional w-event privacy mechanisms by introducing Φ-Personalized Differential Privacy methods to support personalized privacy requirements. This enhancement enables our mechanism and methods to handle both infinite correlated data streams and personalized privacy requirements, building upon the foundations of traditional w-event privacy mechanisms.

3 PROBLEM SETTINGS

In this section, we first introduce key concepts, including data streams. Next, we present the new definition of $\boldsymbol{w}$-event $\boldsymbol{\Phi}$ personalized DP. Finally, we provide the problem definition: Personalized

w-event Private Release for Infinite Data Streams (PWPR-IDS). Table 2 outlines the notations used throughout this paper.

3.1 Data Stream

Let $\mathcal{D}$ represent the database domain with d columns. Next, we define the concepts of data stream and stream prefix.

Definition 1. (Data Stream). Let $D_t \in \mathcal{D}$ be a database with d columns and n rows (each row representing a user) at t-th time slot. The infinite database tuple $S = (D_1, D_2, \ldots)$ is called a data stream, where $S[t]$ is the t-th element in S (i.e., $S[t] = D_t$).

Definition 2. (Stream Prefix). For any data stream S, its substream between time slot t_l and t_r (where $t_l < t_r$) is noted as $S_{t_l,t_r} = (D_{t_l}, D_{t_l+1}, \ldots, D_{t_r})$. When $t_l = 1$, we refer to this as the stream prefix of S and denote it as $S_t = (D_1, D_2, \ldots, D_t)$.

For any data stream, it is valuable to obtain count statistic at each timestamp (for example, in real-time traffic flow prediction). We provide the definition of data stream count release in Definition 3.

Definition 3. (Data Stream Count Release). Let $Q : \mathcal{D} \to \mathbb{R}^d$ be a count query. Then $Q(S[t]) = Q(D_t) = \boldsymbol{c}_t$ is the count data to be released at time slot t, where $\boldsymbol{c}_t(j)$ represents the count of the j-th column of D_t. The infinite count data series $(\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots)$ is called a data stream count release.

3.2 Differential Privacy

In this subsection, we introduce the definitions of w-neighboring in data streams and Φ-personalized differential privacy. Then, we present a new concept: $\boldsymbol{w}$-event level Φ-Personalized Differential Privacy, which is the primary focus of this paper.

Definition 4. (w-neighboring [7, 22]). Let w be a positive integer. Two stream prefixes S_t, S'_t are w-neighboring (denoted as $S_t \sim_w S'_t$), if

(1) for each $S_t[k], S'_t[k]$ such that $k \in [t]$ and $S_t[k] \neq S'_t[k]$, it holds that $S_t[k]$ and $S'_t[k]$ are neighboring, and
(2) for each $S_t[k_1], S_t[k_2], S'_t[k_1], S'_t[k_2]$ with $k_1 < k_2$, $S_t[k_1] \neq S'_t[k_1]$ and $S_t[k_2] \neq S'_t[k_2]$, it holds that $k_2 - k_1 + 1 \leq w$.

We show the definition and the corresponding mechanism of personalized differential privacy [20] as follows.

Definition 5. (Φ-Personalized Differential Privacy, Φ-PDP [20]). Given a universe of users $U = \{u_1, u_2, \ldots, u_{|U|}\}$, the context of privacy specification $\Phi = \{(u_1, \epsilon_1), (u_2, \epsilon_2), \ldots (u_{|U|}, \epsilon_{|U|})\}$, a randomized mechanism $\mathcal{M} : \mathcal{D} \to R$ satisfies Φ-personalized differential privacy (or Φ-PDP), if for every pair of neighboring datasets $D, D' \subset \mathcal{D}$, with $D \overset{tp_i}{\sim} D'$, and for all sets $O \subseteq R$ of possible outputs,

$$\Pr[\mathcal{M}(D) \in O] \leq e^{\Phi_{u_i}} \cdot \Pr[\mathcal{M}(D') \in O],$$

where $u_i \in U$ is the user associated with tuple tp_i, and Φ_{u_i} denotes u_i's privacy preference (i.e., $\Phi_{u_i} = \epsilon_i$).

The Sampling Mechanism (SM) [20] is a standard mechanism that satisfies Φ-PDP. SM consists of two steps: *sample* (SM_s) and *disturb* (SM_d). In SM_s, the server first sets a privacy budget threshold ϵ_θ. It then constructs a sampling subset D_S by appending items x_i with $\epsilon_i \geq \epsilon_\theta$ to D_S, while sampling other items x_j with $\epsilon_j < \epsilon_\theta$ at a probability of $p_j = \frac{e^{\epsilon_j}-1}{e^{\epsilon_\theta}-1}$. In SM_d, the server employs a DP mechanism (e.g., the Laplace Mechanism) to report an obfuscated result that achieves ϵ_θ-DP.

With the two definitions above, we now present our new concept of $\boldsymbol{w}$-event level Φ-Personalized Differential Privacy.

Definition 6. ($\boldsymbol{w}$-event level Φ-Personalized Differential Privacy, $(\boldsymbol{w}, \Phi)$-EPDP). Let $\mathcal{M}$ be a mechanism that takes a stream prefix of arbitrary size as input. Let O be the set of all possible outputs of $\mathcal{M}$. Given a universe of users $U = \{u_1, u_2, \ldots, u_{|U|}\}$, the context of privacy specification $\Phi = \{(u_1, \mathcal{E}_1), (u_2, \mathcal{E}_2), \ldots (u_{|U|}, \mathcal{E}_{|U|})\}$, then $\mathcal{M}$ is $\boldsymbol{w}$-event level Φ-personalized differential privacy if $\forall O \subseteq O$, $\forall w_i \in \boldsymbol{w}$ and $\forall S_t, S'_t$ satisfying $S_t \sim_{w_i} S'_t$,

$$\Pr[M(S_t) \in O] \leq e^{\Phi_{u_i}} \Pr[M(S'_t) \in O],$$

where $u_i \in U$ requires w_i-event level window size, and Φ_{u_i} denotes u_i's privacy budget requirement (i.e., $\Phi_{u_i} = \mathcal{E}_i$).

In this paper, we define u_i's *privacy requirement* as the pair $(\mathcal{E}_i, w_i)$. Specifically, $\mathcal{E}_i$ represents u_i's privacy budget requirement, while w_i denotes u_i's privacy window size requirement (also referred to as window size requirement).

3.3 Definition of PWPR-IDS

Given a data stream S, the server obtains the data stream count release (statistical histogram at each time slot) as $\boldsymbol{c} = (\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots)$. To protect user privacy, however, the server only receives an obfuscated version of S and publishes the obfuscated data stream count (also called estimation count) $\boldsymbol{r} = (\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots)$. We first analyze the errors in reporting obfuscated data stream counts and then present our problem definition.

For each time slot, adopting the Sampling Mechanism (SM) [20] introduces two types of errors: *sampling error* and *noise error*. Specially, given the privacy budget threshold ϵ_θ at time slot t, the data reporting error is $err(\epsilon_\theta) = err_s(\epsilon_\theta) + err_{dp}(\epsilon_\theta)$. Here, $err_s(\epsilon_\theta)$ represents the *sampling error* resulting from sampling users with privacy budgets lower than ϵ_θ. Additionally, $err_{dp}(\epsilon_\theta)$ denotes the *noise error* caused by adding noise to achieve ϵ_θ-differential privacy (ϵ_θ-DP). We will now define these sampling and noise errors in more detail.

Definition 7. (Sampling Error [20]). Given a privacy budget threshold ϵ_θ and m kinds of privacy budget requirements $\tilde{\epsilon}_1, \tilde{\epsilon}_2, \ldots, \tilde{\epsilon}_m$ from n users with $\tilde{\epsilon}_i < \tilde{\epsilon}_j$ for $i < j$ and $i, j \in [m]$ where $\tilde{\epsilon}_i$ is declared by n_i users ($\sum_{i=1}^{m} n_i = n$), the sampling error $err_s(\epsilon_\theta)$ is defined as

$$\begin{aligned} err_s(\epsilon_\theta) &= Var(count(\boldsymbol{r}_t)) + bias(\boldsymbol{r}_t)^2 \\ &= \sum_{\tilde{\epsilon}_i < \epsilon_\theta} n_i p_i (1 - p_i) + \left(\sum_{\tilde{\epsilon}_i < \epsilon_\theta} n_i (1 - p_i) \right)^2, \end{aligned} \tag{1}$$

where $p_i = \frac{e^{\tilde{\epsilon}_i}-1}{e^{\epsilon_\theta}-1}$.

Definition 8. (Noise Error). The noise error $err_{dp}(\epsilon_\theta)$ is defined as the error of the Laplace mechanism, namely,

$$err_{dp}(\epsilon_\theta) = \frac{2}{\epsilon_\theta^2}. \tag{2}$$

Various metrics exist to measure the errors of Laplace mechanisms for noise error, including variance [20, 29], scale [14, 22], and (α, β)-usefulness [5, 14]. In this work, we employ variance as our metric, consistent with the approach in Reference [20].

Based on Equation (1) and (2), we can observe that err_s depends only on n_i, $\tilde{\epsilon}_i$ and ϵ_θ, and is independent of $\boldsymbol{r}_t$. Similarly, err_{dp} depends only on ϵ_θ, and is independent of $\boldsymbol{r}_t$.

We now present the problem definition for Personalized w-event Private Release for Infinite Data Streams (PWPR-IDS).

Definition 9. (PWPR-IDS). Given a user set $U = \{u_1, u_2, ..., u_n\}$ where each u_i holds a privacy requirement pair $(\mathcal{E}_i, w_i)$ and a series data $\boldsymbol{x}_{i,t}$ for $t \in \mathbb{N}^+$. All $\boldsymbol{x}_{i,t}$ for $u_i \in U$ at time slot t form D_t. All D_t form an infinite data stream $S = (D_1, D_2, \ldots)$. PWPR-IDS is to release an obfuscated histogram $\boldsymbol{r} = (\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots)$ of S in each timestamp t achieving $(\boldsymbol{w}, \Phi)$-EPDP with the error between $\boldsymbol{r}$ and $\boldsymbol{c}$ minimized, namely $\forall T \in \mathbb{N}^+$,

$$\min_{\epsilon_\theta} \quad \sum_{t \in [T]} \|\boldsymbol{r}_t - \boldsymbol{c}_t\|_2^2$$

$$s.t. \quad \sum_{k=\min(t-w_i+1,1)}^{t} \epsilon_{i,k} \leq \mathcal{E}_i, \quad \forall u_i \in U$$

Table 2: Notations.

Notations	Description
$\mathcal{D}$	the database domain
D_t	a database at time slot t
S	a data stream
n	the quantity of users
d	the quantity of attributes
u_i	the i-th user
$\boldsymbol{x}_{i,t}$	u_i's data at time slot t
$\boldsymbol{c}_t$	a real statistical histogram at time slot t
$\boldsymbol{r}_i$	an estimation statistic histogram at time slot t
w_i	u_i's (privacy) window size requirement
$\mathcal{E}_i$	u_i's privacy budget requirement

4 PERSONALIZED WINDOW SIZE MECHANISM

In this section, we first introduce an optimal privacy budget selection method (OBS) that minimizes data reporting error, then propose Personalized Window Size Mechanism (PWSM) to address PWPR-IDS. The core idea of PWSM is to select the optimal $\epsilon_{opt}(t)$ at each time slot t and report the obfuscated count result satisfying $\epsilon_{opt}(t)$-DP.

4.1 Optimal Budget Selection

Given the privacy budget requirements $(\epsilon_{1,t}, \epsilon_{2,t}, \ldots, \epsilon_{n,t})$ of n users, we can determine the frequency of each privacy budget requirement and select the optimal ϵ_θ that minimizes the data reporting error err. This process is detailed in Algorithm 1.

Taking n privacy budgets as input, the Optimal Budget Selection (OBS) algorithm counts the different privacy budgets. Assume there are $\tilde{n}$ distinct privacy budgets, with n_k users requiring $\tilde{\epsilon}_k$ for $k \in [\tilde{n}]$. OBS records the different privacy budget as $\tilde{\mathcal{E}}$ and their relative frequencies N, shown in Lines 1 and 2. The process of finding the minimum reporting error err_{min} is detailed from Line 4 to Line 8. Specifically, it iterates over all $\tilde{\epsilon}_k$ in $\tilde{\mathcal{E}}$ and selects the value $\tilde{\epsilon}$ with the smallest $err_k = err_s(\tilde{\epsilon}_k) + err_{dp}(\tilde{\epsilon}_k)$.

Algorithm 1: Optimal Budget Selection

Input: $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)$
Output: ϵ_{opt}, err_{min}

```
1 Set ε̃ = (ε̃_1, ε̃_2, ..., ε̃_ñ) as the different ε ∈ ε;
2 Set N = (n_1, n_2, ..., n_ñ) as the frequency of ε̃_k ∈ ε̃;
3 Initialize err_min as the upper bound of error value;
4 for ε̃_k ∈ ε̃ do
5     Set err_k = err_s(ε̃_k) + err_dp(ε̃_k);
6     if err_k < err_min then
7         Set err_min = err;
8         Set ε_opt as ε̃_k;
9 return ε̃_opt, err_min
```

Let's illustrate the OBS process with an example: Suppose we have 10 privacy budgets as input: $\boldsymbol{\epsilon}$ = (0.1, 0.4, 0.4, 0.1, 0.4, 0.4, 0.8, 0.8, 0.8, 0.4). OBS first determines $\tilde{\boldsymbol{\epsilon}}$ = (0.1, 0.4, 0.8) and N = (2, 5, 3). Based on these statistics, OBS iterates through the 3 privacy budgets in $\tilde{\boldsymbol{\epsilon}}$ and calculates their relative errors: $err_1 = 0 + \frac{2}{0.1^2} = 200$, $err_2 = 2 \times \frac{e^{0.1}-1}{e^{0.4}-1} \times (1 - \frac{e^{0.1}-1}{e^{0.4}-1}) + (2 \times (1 - \frac{e^{0.1}-1}{e^{0.4}-1}))^2 + \frac{2}{0.4^2} = 15.31$ and $err_3 = 2 \times \frac{e^{0.1}-1}{e^{0.8}-1} \times (1 - \frac{e^{0.1}-1}{e^{0.8}-1}) + 5 \times \frac{e^{0.4}-1}{e^{0.8}-1} \times (1 - \frac{e^{0.4}-1}{e^{0.8}-1}) + (2 \times (1 - \frac{e^{0.1}-1}{e^{0.4}-1}) + 5 \times (1 - \frac{e^{0.4}-1}{e^{0.8}-1}))^2 + \frac{2}{0.8^2} = 27.73$. Finally, OBS returns 0.4 with the minimal error 15.31.

4.2 Personalized Window Size Mechanism

Budget division [22, 29] is a traditional framework for publishing private stream data under w-event privacy. It comprises two basic methods, namely *Uniform* and *Sampling* and two adaptive methods, namely *Budget Distribution* (BD) and *Budget Absorption* (BA). The adaptive methods leverage the stream's variation tendency, resulting in more accurate obfuscated estimations compared to the basic methods.

In this subsection, we extend the adaptive budget division framework to a personalized context and introduce our Personalized Window Size Mechanism (PWSM). Building on PWSM, we propose two methods: namely Personalized Budget Distribution (PBD) and Peronalized Budget Absorption (PBA).

Before PWSM, users must specify their privacy budget and window size. System administrators first define a discretized privacy budget range (e.g., $\{0.1, 0.5, 0.9\}$) and a window size range (e.g., $\{40, 80, 120\}$). They map ascending privacy budget values to descending privacy budget levels (e.g., High, Medium, Low) and ascending window size values to ascending window size levels (e.g., Low, Medium, High). Users can then select both a privacy budget level and a window size level based on their needs and past experience. After users submit these selections to the server, it converts them into the corresponding privacy budgets and window sizes.

Algorithm 2 outlines the process of PWSM. The algorithm takes three inputs: the historical estimation *His*, personalized privacy budget $\boldsymbol{\mathcal{E}}$, and personalized window size set $\boldsymbol{w}$. Both $\boldsymbol{\mathcal{E}}$ and $\boldsymbol{w}$ are fixed values collected from all users during system initialization. The algorithm starts by calculating all users' privacy budget resources $\boldsymbol{\epsilon}$ at the current timestamp to satisfy $(\boldsymbol{w}, \Phi)$-PDP (Line 1). It then divides $\boldsymbol{\epsilon}$ into two parts: $\boldsymbol{\epsilon}^{(1)}$ and $\boldsymbol{\epsilon}^{(2)}$ (Line 2). Using $\boldsymbol{\epsilon}^{(1)}$, PWSM calculates the dissimilarity *dis* between the current count value and the last reported one by invoking the Sampling Mechanism [20] (Line 3). Next, it sets the change threshold as the reporting error *err*, calculated using $\boldsymbol{\epsilon}^{(2)}$ (Line 4). Finally, PWSM adaptively decides whether to release a new obfuscated estimation or skip (approximating to the last released one) by comparing *dis* to the square root of *err* (from Line 5 to Line 9).

Algorithm 2: PWSM

Input: Historical estimation *His*, Personalized privacy budget set $\mathcal{E}$, Personalized window size set $\boldsymbol{w}$
Output: $\boldsymbol{r}$

1 Get the current privacy budgets $\boldsymbol{\epsilon}$ of all users by $\mathcal{E}$ and $\boldsymbol{w}$;
   // Satisfying $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^{(1)} + \boldsymbol{\epsilon}^{(2)}$
2 Divide $\boldsymbol{\epsilon}$ into two parts $\boldsymbol{\epsilon}^{(1)}$ and $\boldsymbol{\epsilon}^{(2)}$;
   // Sub mechanism $\mathcal{M}_{t,1}$
3 Get dissimilarity *dis* between current estimation and the last estimation by $SM(\boldsymbol{\epsilon}^{(1)})$;
   // Sub mechanism $\mathcal{M}_{t,2}$
4 Get reporting error *err* of current estimation by $OBS(\boldsymbol{\epsilon}^{(2)})$;
5 **if** $dis > \sqrt{err}$ **then**
6   Get current estimation $\boldsymbol{r}$ by $SM(\boldsymbol{\epsilon}^{(2)})$;
7 **else**
8   Set current estimation $\boldsymbol{r}$ as the last reporting value;
9 **return** $\boldsymbol{r}$;

To determine whether to choose a new obfuscated estimation or skipping, we need to introduce a judgment measure called the *personalized private dissimilarity measure*.

Personalized Private Dissimilarity Measure. The personalized dissimilarity measure dis^* is defined as the absolute error between the true statistic $\tilde{\boldsymbol{c}}_t$ under SM_s at current time slot and the previous release $\boldsymbol{r}_l$, namely,

$$dis^* = \frac{1}{d}\sum_{k=1}^{d} |\tilde{c}_t[k] - \boldsymbol{r}_l[k]|. \tag{3}$$

Our goal is to privately obtain the personalized dissimilarity dis^* using privacy budget ϵ_{opt}. The personalized private dissimilarity measure *dis* is then defined as:

$$dis = dis^* + Lap\left(\frac{1}{d \cdot \epsilon_{opt}}\right). \tag{4}$$

Here, *Lap* represents the Laplace mechanism, which adds noise to ensure privacy.

Next, we introduce two methods to implement our PWSM: Personalized Budget Distribution (PBD) and Peronalized Budget Absorption (PBA).

Personalized Budget Distribution (PBD). Algorithm 3 outlines the process of PBD. This method takes as input the current user data D_i, historical data publication, and all users' privacy budget and window size requirements. The privacy budget requirements, denoted as $\mathcal{E}_i$ for each user u_i, are divided into two parts: 1) The first part is used by $\mathcal{M}_{t,1}$ to calculate the dissimilarity between the current data distribution and the last published obfuscated data distribution; 2) The second part is utilized by $\mathcal{M}_{t,2}$ for obfuscated distribution publication at the current time slot.

Algorithm 3: Personalized Budget Distribution

Input: D_t, $(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{t-1})$, $(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n)$, $(w_1, w_2, \ldots, w_n)$
Output: $\boldsymbol{r}_t$

1 Get the current window average budget $\epsilon_i = \mathcal{E}_i / w_i$ for each $i \in [n]$;
   // Sub mechanism $\mathcal{M}_{t,1}$
2 Set $\boldsymbol{\epsilon}_t^{(1)} = (\epsilon_1/2, \epsilon_2/2, \ldots, \epsilon_n/2)$;
3 Get the optimal budget threshold $\epsilon_{opt}^{(1)}$ by Algorithm 1;
4 Get $\tilde{D}_t^{(1)} = SM_s(D_t, \boldsymbol{\epsilon}_t^{(1)}, \epsilon_{opt}^{(1)})$;
5 Calculate $\tilde{\boldsymbol{c}}_t^{(1)} = Q(\tilde{D}_t^{(1)})$;
6 Get the last non-null release $\boldsymbol{r}_l$ from $(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{t-1})$;
7 Set $dis = \frac{1}{d}\sum_{j=1}^{d} |\tilde{\boldsymbol{c}}_t^{(1)}[j] - \boldsymbol{r}_l[j]| + Lap(1/(d \cdot \epsilon_{opt}^{(1)}))$;
   // Sub mechanism $\mathcal{M}_{t,2}$
8 Get remaining budget $\epsilon_{rm,i} = \mathcal{E}_i/2 - \sum_{k=t-w_i+1}^{t-1} \epsilon_{t,2}$;
9 Set $\boldsymbol{\epsilon}_t^{(2)} = (\epsilon_{rm,1}/2, \epsilon_{rm,2}/2, \ldots, \epsilon_{rm,n}/2)$;
10 Get the optimal budget threshold $\epsilon_{opt}^{(2)}$ and error $err_{opt}^{(2)}$ by Algorithm 1;
11 **if** $dis > \sqrt{err_{opt}^{(2)}}$ **then**
12   Get $\tilde{D}_t^{(2)} = SM_s(D_t, \boldsymbol{\epsilon}_t^{(2)}, \epsilon_{opt}^{(2)})$;
13   Calculate $\tilde{\boldsymbol{c}}_t^{(2)} = Q(\tilde{D}_t^{(2)})$;
14   **return** $\boldsymbol{r}_t = SM_d(\tilde{\boldsymbol{c}}_t^{(2)}, \epsilon_{opt}^{(2)})$;
15 **else**
16   **return** $\boldsymbol{r}_t = \boldsymbol{r}_{t-1}$;

In $\mathcal{M}_{t,1}$, the privacy budget for dissimilarity calculation is set as half of the average privacy budget among all time slots (i.e., $\mathcal{E}_i/(2w_i)$ for u_i). It then applies Optimal Budget Selection (Algorithm 3) to choose the best budget threshold $\epsilon_{opt}^{(1)}$. Subsequently, $\mathcal{M}_{t,1}$ employs the Sample Mechanism [20] of PDP to obtain the dissimilarity *dis*, as shown from Line 4 to Line 7.

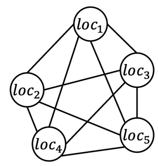
In $\mathcal{M}_{t,2}$, the remaining privacy budget $\epsilon_{rm,i}$ for each u_i is first calculated. The publication privacy budget for each u_i is then set as half of $\epsilon_{rm,i}$. Similar to $\mathcal{M}_{t,1}$, it determines the optimal privacy budget $\epsilon_{opt}^{(2)}$ and the corresponding error using the Optimal Budget Selection algorithm.

After $\mathcal{M}_{t,1}$ and $\mathcal{M}_{t,2}$, we obtain two measurements: the dissimilarity *dis* and the error *err*. We compare these two measurement to determine whether to publish a new obfuscated statistic result or approximate the current result with the last publication. If the *dis* is greater than *err*, it indicates that the difference between the current data and the last published data exceeds the error of noise. In this case, we need to republish a new obfuscated statistic result. Otherwise, we can take the last published result instead.

We illustrate the process of Personalized Budget Distribution with an example as follows:

Example 2. *Suppose there are n users distributed across 5 locations, forming a complete graph. Figure 2 illustrates the privacy budget requirements, window size requirements and locations for the first three users across time slots 1 to 5. Figure 3 demonstrates*

	u_1	u_2	u_3	...
$\mathcal{E}$	$\mathcal{E}_1$	$\mathcal{E}_2$	$\mathcal{E}_3$	...
W	4	2	3	...



	1	2	3	4	5
u_1	loc_2	loc_1	loc_1	loc_3	loc_2
u_2	loc_1	loc_1	loc_3	loc_3	loc_4
u_3	loc_5	loc_4	loc_4	loc_2	loc_4
...	...	...	...	...	...

Figure 2: An Information example for PBD.

the estimation process of PBD. The total privacy budget for each user u_i is evenly split into two parts, each containing $\mathcal{E}_i/2$. The first part is allocated for dissimilarity calculation, while the second is for publication noise calculation. For instance, $\mathcal{E}_1$ is divided into $\boldsymbol{\epsilon}_1^{(1)}(u_1) = \mathcal{E}_1/2$ and $\boldsymbol{\epsilon}_1^{(2)}(u_1) = \mathcal{E}_1/2$. We compute the privacy budget usage $\epsilon_{i,t}^{(1)}$ for dissimilarity and $\epsilon_{i,t}^{(2)}$ for noise statistic publication for each user at each timestamp. These values are recorded in an $n \times 2$ matrix at each timestamp in Figure 3. Using u_1 as an example, $\epsilon_{1,t}^{(1)} = \boldsymbol{\epsilon}_1^{(1)}(u_1)/w_1 = \mathcal{E}_1/8$. At timestamp 1, $\epsilon_{1,1}^{(2)} = \boldsymbol{\epsilon}_1^{(2)}(u_1)/2 = \mathcal{E}_1/4$. The algorithm calculates the dissimilarity dis at timestamp 1 using all $\epsilon_{i,1}^{(1)}$, and the error $err_{opt}^{(2)}$ using all $\epsilon_{1,t}^{(2)}$. Assume dis $> \sqrt{err_{opt}^{(2)}}$, then a new obfuscated statistic $\boldsymbol{r}_1$ is published at timestamp 1. At timestamp 2, assume dis $\leq \sqrt{err_{opt}^{(2)}}$, then $\epsilon_{i,2}^{(2)}$ is not used to publish a new obfuscated statistic result, and its usage is set to zeros for all users. At timestamp 3, $\epsilon_{1,3}^{(2)} = (\mathcal{E}_1/2 - \epsilon_{1,1}^{(2)})/2 = \mathcal{E}_1/8$. The vector below each matrix in Figure 3 represents the total privacy budget used at the current timestamp for each user. For example, at timestamp 1, the total privacy budget usage for u_1 is $\epsilon_{1,1}^{(1)} + \epsilon_{1,1}^{(2)} = 3\mathcal{E}_1/8$.

Peronalized Budget Absorption (PBA). Algorithm 4 outlines the process of PBA. The dissimilarity calculation in PBA is identical to that of PBD. However, these methods differ significantly in their strategies for allocating the publication privacy budget.

For $\mathcal{M}_{t,2}$ in PBA, we assume an average privacy budget of $\frac{\mathcal{E}_i}{2w_i}$ (one share) for each u_i at each time slot t. A publication at time slot t can use more than one share by borrowing from its successor time slots. The variable $t_{N,i}$ in Line 8 represents the number of successor time slots occupied by the last publication. We calculate the maximal $\tilde{t}_N$ of all $t_{N,i}$ and determine whether the current time has been occupied ($t - L \leq \tilde{t}_N$). If so, we approximate the publication using the last published result. Otherwise, we calculate the remaining budget shares from the precursor time slots (i.e., $t_{A,i}$ in Line 13) and set the current publication budget as the total absorbed shares

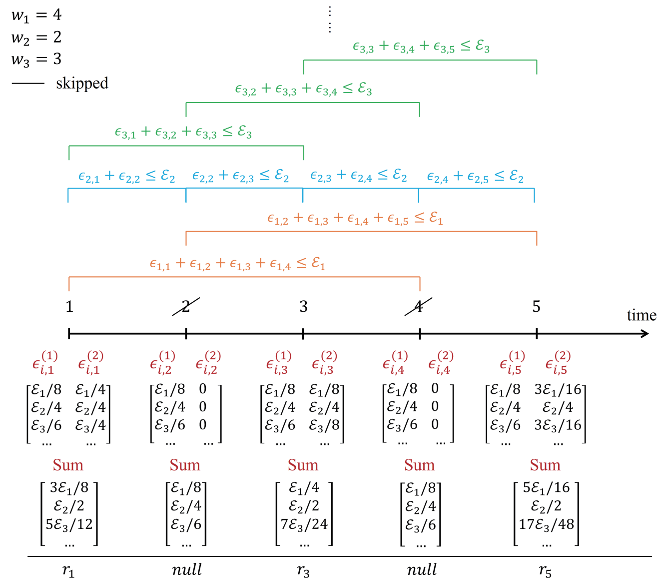


Figure 3: A process example for PBD.

(Line 14). The subsequent steps follow the same process as outlined in Algorithm 3. Figure 4 illustrates this process with an example.

Example 3. *Figure 2 displays the user information. The process of $M_{t,1}$ remains identical to that in Example 2. For $M_{t,2}$, at timestamp 1, with no budget to absorb, all users utilize one share (i.e., $\mathcal{E}_i/(2w_i)$) to publish a new obfuscated statistic result. Assume timestamp 2 is skipped (dis $\leq \sqrt{err_{opt}^{(2)}}$). At timestamp 3, $t_{N,1} = 1$, $t_{N,2} = 0$, and $t_{N,3} = 1.5$. Assuming the nullified bound $\tilde{t}_N$ is 1.8. Since $t - L = 3 - 1 = 2 > \tilde{t}_N$, a new obfuscated statistic result is reported. The publication budget set is calculated as $\boldsymbol{\epsilon}_3^{(2)} = (\mathcal{E}_1/4, \mathcal{E}_2/2, \mathcal{E}_3/3, \ldots)$. At timestamp 4, $t_{N,1} = 1$, $t_{N,2} = 1$ and $t_{N,3} = 1$ (Actually, all $t_{N,i} = 1$). As $t - L = 4 - 3 = 1 \leq \tilde{t}_N$, no output is produced. At timestamp 5, all $t_{N,i}$ remain 1, and $t - L = 5 - 3 = 2 > \tilde{t}_N$. The absorbed timestamps $t_{A,i}$ all equal 1. The resulting publication budget set is $\boldsymbol{\epsilon}_5^{(2)} = (\mathcal{E}_1/8, \mathcal{E}_2/4, \mathcal{E}_3/6, \ldots)$.*

4.3 Analysis

In this subsection, we analyze the time cost and privacy aspects of our PBD and PBA.

Time Cost Analysis. Let m be the number of distinct privacy requirement pairs $(\mathcal{E}_i, w_i)$, where $m \leq n$. The time complexity of Optimal Budget Selection is $O(m)$ for both PBD and PBA. The Sample Mechanism and Query operations each have a time complexity of $O(n)$. Assume T is the upper bound of data series, the overall time complexities of PBD and PBA are $O(nT)$.

Privacy Analysis. We present the privacy analysis for PBD and PBA in Theorem 4.1.

Theorem 4.1. *PBD and PBA satisfy $(\boldsymbol{w}, \Phi)$-EPDP.*

Proof. We separately analyze the privacy guarantees of PBD and PBA.

(1) PBD satisfies $(\boldsymbol{w}, \Phi)$-EPDP.

Algorithm 4: Personalized Budget Absorption

Input: D_t, $(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{t-1})$, $(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n)$, $(w_1, w_2, \ldots, w_n)$
Output: $\boldsymbol{r}_t$

```
Get the current window average budget ϵ_i = ℰ_i/w_i for each i ∈ [n];
// Sub mechanism M_{t,1}
Set ϵ_t^(1) = (ϵ_1/2, ϵ_2/2, ..., ϵ_n/2);
Get the optimal budget threshold ϵ_opt^(1) by Algorithm 1;
Get D̃_t^(1) = SM_s(D_t, ϵ_t^(1), ϵ_opt^(1));
Calculate c̃_t^(1) = Q(D̃_t^(1));
Get the last non-null release r_l from (r_1, r_2, ..., r_{t-1});
Set dis = (1/d) Σ_{j=1}^{d} |c̃_t^(1)[j] − r_l[j]| + Lap(1/(d · ϵ_opt^(1)));
// Sub mechanism M_{t,2}
Set nullified timestamps t_{N,i} = ϵ_{i,L}^(2) / (ℰ_i/(2w_i)) − 1 for i ∈ [n];
Set nullified timestamp bound t̃_N = max_{i∈[n]} t_{N,i};
if t − L ≤ t̃_N then
    return r_t = r_{t-1};
else
    Set absorbed timestamps t_{A,i} = max(t − L − t_{N,i}, 0) for i ∈ [n];
    Set publication budget ϵ_{i,t}^(2) = (ℰ_i/(2w_i)) · min(t_{A,i}, w_i) for i ∈ [n];
    Set ϵ_t^(2) = (ϵ_{1,t}^(2), ϵ_{2,t}^(2), ..., ϵ_{n,t}^(2));
    Get remaining budget ϵ_{rm,i} = ℰ_i/2 − Σ_{k=t−w_i+1}^{t−1} ϵ_{t,2};
    Set ϵ_t^(2) = (ϵ_{rm,1}/2, ϵ_{rm,2}/2, ..., ϵ_{rm,n}/2);
    Get the optimal budget threshold ϵ_opt^(2) and error err_opt^(2) by Algorithm 1;
    if dis > sqrt(err_opt^(2)) then
        Get D̃_t^(2) = SM_s(D_t, ϵ_t^(2), ϵ_opt^(2));
        Calculate c̃_t^(2) = Q(D̃_t^(2));
        return r_t = SM_d(c̃_t^(2), ϵ_opt^(2));
    else
        return r_t = r_{t-1};
```

In sub-mechanism $M_{t,1}$, for each user u_i, the dissimilarity budget at each timestamp is $\mathcal{E}_i/(2w_i)$. Then for each timestamp t, we have

$$\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(1)} = \mathcal{E}_i/2. \tag{5}$$

In sub-mechanism $M_{t,2}$, for each user u_i at timestamp t, only half of the publication budget is used when publication occurs: $\epsilon_{i,t}^{(2)} = (\mathcal{E}_i/2 - \sum_{k=\max(t-w_i+1,1)}^{t-1} \epsilon_{i,k}^{(2)})/2$. For any timestamp $t \in [1, w_i]$, the summation publication budgets used for u_i is at most $\sum_{k=1}^{w_i} \mathcal{E}_i/(2 \cdot 2^k) \leq (\mathcal{E}_i/2) \cdot (1 - \frac{1}{2^{w_i}}) \leq \mathcal{E}_i/2$. Suppose $\sum\limits_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} \leq \mathcal{E}_i/2$ for $t = w_i + s$ (i.e., $\sum\limits_{k=\max(s+1,1)}^{w_i+s} \epsilon_{i,k}^{(2)} \leq \mathcal{E}_i/2$). Then for $t = w_i + s + 1$,

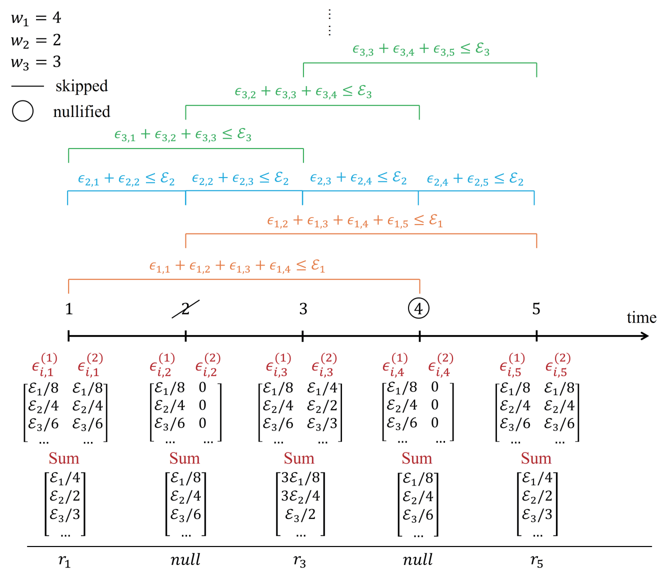


Figure 4: A process example for PBA.

we have:

$$\sum_{k=\max(s+2,1)}^{w_i+s+1} \epsilon_{i,k}^{(2)} = \sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)} + \epsilon_{i,w_i+s+1}^{(2)}. \tag{6}$$

Since $\epsilon_{i,w_i+s+1}^{(2)}$ is at most half of the remaining publication budget at timestamp $w_i + s$:

$$\epsilon_{i,w_i+s+1}^{(2)} \leq (\mathcal{E}_i/2 - \sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)})/2. \tag{7}$$

According to Equations (6) and (7), we have:

$$\begin{aligned} \sum_{k=\max(s+2,1)}^{w_i+s+1} \epsilon_{i,k}^{(2)} &\leq \sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)} + (\mathcal{E}_i/2 - \sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)})/2 \\ &= \mathcal{E}_i/4 + (\sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)})/2 \\ &\leq \mathcal{E}_i/4 + \mathcal{E}_i/4 \\ &= \mathcal{E}_i/2. \end{aligned} \tag{8}$$

Therefore, for any $t \geq 1$, we have:

$$\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} \leq \mathcal{E}_i/2. \tag{9}$$

According to the Composition Theorems [14], we have:

$$\begin{aligned} \sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k} &= \sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(1)} + \sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} \\ &\leq \mathcal{E}_i. \end{aligned} \tag{10}$$

For any user u_i and any two w_i-neighboring stream prefixes S_t and S'_t (i.e., $S_t \sim_{w_i} S'_t$), let t_s be the earliest timestamp where $S_t[t_s] \neq S'_t[t_s]$ and t_e be the latest timestamp where $S_t[t_e] \neq S'_t[t_e]$.

Then we have $t_e - t_s + 1 \le w_i$. Denoting the output of our PBD as $PBD(S_t[t]) = o_t \in \mathcal{O}$, for any $O \subseteq \mathcal{O}$, we have:

$$\begin{aligned}\frac{\Pr[PBD(S_t)] \in O}{\Pr[PBD(S_t')] \in O} &\le \Pi_{k=t_s}^{t_e} \frac{\Pr[PBD(S_t[k]) = o_k]}{\Pr[PBD(S_t'[k]) = o_k]}\\ &\le e^{\sum_{k=t_s}^{t_e} \epsilon_{i,k}}\\ &\le e^{\sum_{k=\max(t_e - w_i+1,1)}^{t_e} \epsilon_{i,k}}\\ &\le e^{\mathcal{E}_i}.\end{aligned} \tag{11}$$

Therefore, PBD satisfies $(\boldsymbol{w}, \Phi)$-EPDP where $\boldsymbol{w} = (w_1, w_2, \dots, w_n)$ and $\Phi = ((u_1, \mathcal{E}_1), (u_2, \mathcal{E}_2), \dots, (u_n \mathcal{E}_n))$.

(2) PBA satisfies $(\boldsymbol{w}, \Phi)$-EPDP.

The sub-mechanism $M_{t,1}$ in PBA is identical to that that in PBD. Consequently, for each timestamp t, we have:

$$\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(1)} = \mathcal{E}_i/2. \tag{12}$$

In sub-mechanism $M_{t,2}$, for any user u_i and any window of size w_i, there are s_i publication timestamps in the window. We denote these publication timestamps as $(k_1, k_2, \dots, k_{s_i})$. For any publication timestamp k_j ($j \in [s_i]$), the quantity of its absorbing unused budgets is denoted as α_{i,k_j}. Figure 5 illustrates an example where $s_i = 3$ and $w_i = 9$.

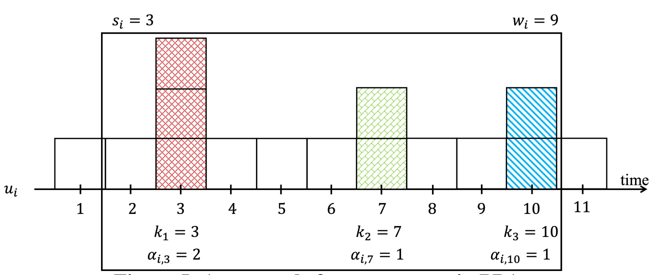


Figure 5: An example for parameters in PBA.

Based on Algorithm 4, we have:

$$w_i \ge \sum_{j=1}^{s_i} (1 + 2\alpha_{i,k_j}) - \alpha_{i,k_1} - \alpha_{i,k_{s_i}}. \tag{13}$$

Then, for the total publication budgets used in any window, we have

$$\begin{aligned}\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} &\le \frac{\mathcal{E}_i}{2w_i} \cdot \sum_{j=1}^{s_i} (1 + \alpha_{i,k_j})\\ &\le \frac{\mathcal{E}_i \cdot \sum_{j=1}^{s_i} (1 + \alpha_{i,k_j})}{2\sum_{j=1}^{s_i}(1 + 2\alpha_{i,k_j}) - 2\alpha_{i,k_1} - 2\alpha_{i,k_{s_i}}}\\ &= \frac{\mathcal{E}_i \cdot \sum_{j=1}^{s_i} (1 + \alpha_{i,k_j})}{2\sum_{j=1}^{s_i}(1 + \alpha_{i,k_j}) + 2\sum_{j=2}^{s_i-1} \alpha_{i,k_j}}\\ &\le \mathcal{E}_i/2.\end{aligned} \tag{14}$$

Based on Equations (12) and (14), and applying the Composition Theorems [14], we obtain:

$$\begin{aligned}\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k} &= \sum_{\substack{k=\max(t\\ -w_i+1,1)}}^{t} \epsilon_{i,k}^{(1)} + \sum_{\substack{k=\max(t\\ -w_i+1,1)}}^{t} \epsilon_{i,k}^{(2)}\\ &\le \mathcal{E}_i.\end{aligned} \tag{15}$$

The subsequent proof process follows the same steps as in PBD. Ultimately, we demonstrate that PBA also satisfies $(\boldsymbol{w}, \Phi)$-EPDP. □

Utility Analysis. For each user u_i in PBD and PBA, we define w_L as the smallest window size among all users. For each u_i, given $(\mathcal{E}_i, w_i)$, let $\epsilon_L = \min_{i\in[n]} \frac{\mathcal{E}_i}{w_i}$ and $\epsilon_R = \max_{i\in[n]} \frac{\mathcal{E}_i}{w_i}$ be the minimum and maximum values of $\frac{\mathcal{E}_i}{w_i}$, respectively. Let n_A be the number of times ϵ_R appears among all users. We assume that at most $\tilde{s} \le w_L$ publications occur at timestamps q_1, q_2,..., $q_{\tilde{s}}$ in the window of size w_L. We also assume there is no budget absorption from past timestamps outside the window. Furthermore, for each user, each publication approximates the same number of skipped or nullified publications. For PBD we present Theorem 4.2 as follows.

Theorem 4.2. *The average error per timestamp in PBD is at most* $\min\left(\frac{8}{d^2\epsilon_L}, Z + \frac{8}{d^2\epsilon_R}\right) + \min\left(\frac{32\cdot(4^{\tilde{s}}-1)}{3\tilde{s}\epsilon_L}, Z + \frac{32\cdot(4^{\tilde{s}}-1)}{3\tilde{s}\epsilon_R}\right)$ *where* $Z = (n - n_A)(n - n_A + \frac{1}{4})$, *if at most* $\tilde{s}$ *publications occur in any window with size* w_L.

Before proving Theorem 4.2, we present a crucial lemma.

Lemma 4.1. *Given m distinct privacy budget-quantity pairs* $P = \{(\epsilon_j, n_j) | j \in [m], \sum_{j\in[m]} n_j = n\}$ *where pair* (ϵ_j, n_j) *indicates that* ϵ_j *appears* n_j *times in the user privacy requirement, and a query with sensitivity I, the error upper bound* $\widetilde{err}_O(P)$ *of the Sample Mechanism (SM) process with privacy budget chosen from Optimal Budget Selection (OBS) is:*

$$\min\left(\frac{2I^2}{\min_j \epsilon_j^2}, (n - n_M)(n - n_M + \frac{1}{4}) + \frac{2I^2}{\max_j \epsilon_j^2}\right),$$

where $n_M = n_k$ *with* $k = \arg\max_{j\in[m]} \epsilon_j$.

Proof. Let M_L be the Sample Mechanism with privacy budget chosen as $\min_j \epsilon_j$. According to the SM process, all budget types will be selected. In this case, the sampling error err_s is 0 and the noise error err_{dp} is $2 \cdot (\frac{I}{\min_j \epsilon_j})^2 = \frac{2I^2}{\min_j \epsilon_j^2}$. Thus, the total error of M_L is $err_{M_L} = \frac{2I^2}{\min_j \epsilon_j^2}$. Let M_R be the Sample Mechanism with privacy budget chosen as $\max_j \epsilon_j$. In this case, $(m-1)$ types of privacy budget are chosen with probability $p_k = \frac{e^{\epsilon_k}-1}{e^{\max_j \epsilon_j}-1}$ less than

1 ($k \in [m]$). For the sampling error, we have:

$$
\begin{aligned}
err_s &= \sum_{\epsilon_k < \max_j \epsilon_j} n_k p_k (1-p_k) + \left(\sum_{\epsilon_k < \max_j \epsilon_j} n_k (1-p_k) \right)^2 \\
&< \sum_{\epsilon_k < \max_j \epsilon_j} n_k \left(\frac{p_k + 1 - p_k}{2} \right)^2 + \left(\sum_{\epsilon_k < \max_j \epsilon_j} n_k \right)^2 \\
&= \frac{1}{4}(n - n_M) + (n - n_M)^2 \\
&= (n - n_M)(n - n_M + \frac{1}{4}).
\end{aligned}
$$

The noise error err_{dp} in this case is $2 \cdot (\frac{I}{\max_j \epsilon_j})^2 = \frac{2I^2}{\max_j \epsilon_j^2}$. Thus, the total error of M_R is $err_{M_R} = (n - n_M)(n - n_M + \frac{1}{4}) + \frac{2I^2}{\max_j \epsilon_j^2}$. According to the OBS process, we have $\widetilde{err}_O(P) \le err_{M_L}$ and $\widetilde{err}_O(P) \le err_{M_R}$. Therefore,

$$
\begin{aligned}
\widetilde{err}_O(P) &\le \min(err_{M_L}, err_{M_R}) \\
&= \min\left(\frac{2I^2}{\min_j \epsilon_j^2}, (n - n_M)(n - n_M + \frac{1}{4}) + \frac{2I^2}{\max_j \epsilon_j^2} \right)
\end{aligned}
$$

□

We now present the proof of Theorem 4.2.

PROOF. Given a privacy budget-quantity pair set P, let $EOPT(P)$ be the optimal privacy budget chosen from OBS. Given a positive number β, we define $\beta \cdot P = \{(\beta \cdot \epsilon_j, n_j) | (\epsilon_j, n_j) \in P\}$. For each user u_i with privacy requirement pair $(\mathcal{E}_i, w_i)$, we calculate their average budget per window as $\frac{\mathcal{E}_i}{w_i}$. We denote the set of all average budgets as $\overline{\mathcal{E}} = \{\frac{\mathcal{E}_i}{w_i} | i \in [n]\}$. We then construct the privacy budget-quantity pair set of each type of average budget as $P_A = \{(\epsilon_j, n_j) | \epsilon_j \in \overline{\mathcal{E}}\}$. Let $Z = (n - n_A)(n - n_A + \frac{1}{4})$ be the sampling error upper bound, where n_A is the quantity of $\max_{i \in [n]} \frac{\mathcal{E}_i}{w_i}$ in $\overline{\mathcal{E}}$.

When $M_{t,1}$ is not private, the error stems from $M_{t,2}$. In $M_{t,2}$, errors arise from both publications and approximations. According to the $M_{t,2}$, an approximation error does not exceed the publication error at the most recent publication timestamp. For the average error $\overline{err}_{M_{t,2}}$ of all timestamps within the window of size w_L, based on the PBD process, we have:

$$
\begin{aligned}
\overline{err}_{M_{t,2}} &= \frac{1}{w_L} \sum_{k \in [\tilde{s}]} \frac{w_L}{\tilde{s}} \cdot \widetilde{err}_O \left(\frac{1}{2^{k+1}} P_A \right) \\
&< \frac{1}{\tilde{s}} \sum_{k \in [\tilde{s}]} \min \left(\frac{2}{(\frac{\epsilon_L}{2^{k+1}})^2}, Z + \frac{2}{(\frac{\epsilon_R}{2^{k+1}})^2} \right) \\
&< \frac{1}{\tilde{s}} \min \left(\sum_{k \in [\tilde{s}]} \frac{8 \cdot 4^k}{\epsilon_L^2}, \tilde{s} \cdot Z + \sum_{k \in [\tilde{s}]} \frac{8 \cdot 4^k}{\epsilon_R^2} \right) \\
&= \min \left(\frac{32 \cdot (4^{\tilde{s}} - 1)}{3\tilde{s}\epsilon_L^2}, Z + \frac{32 \cdot (4^{\tilde{s}} - 1)}{3\tilde{s}\epsilon_R^2} \right).
\end{aligned}
\tag{16}
$$

When $M_{t,1}$ is private, the error from $M_{t,1}$ can lead to two scenarios: (1) falsely skipping a publication or (2) falsely performs a publication. Both cases are bounded by the error in $M_{t,1}$. In $M_{t,1}$, we execute the Sample Mechanism with Optimal Budget Selection. The sensitivity of dis is $1/d$. For the average error $\overline{err}_{M_{t,1}}$ of each timestamp in window size w_L, according to Lemma 4.1, we have:

$$
\begin{aligned}
\overline{err}_{M_{t,1}} &< \min \left(\frac{2}{d^2 \min_{i \in [n]} (\frac{\mathcal{E}_i}{2w_i})^2}, Z + \frac{2}{d^2 \max_{i \in [n]} (\frac{\mathcal{E}_i}{2w_i})^2} \right) \\
&= \min \left(\frac{8}{d^2 \epsilon_L^2}, Z + \frac{8}{d^2 \epsilon_R^2} \right).
\end{aligned}
\tag{17}
$$

Based on Equation (17) and (16), we can get the average error upper bound as $\overline{err}_{M_{t,1}} + \overline{err}_{M_{t,2}}$.

□

PBD achieves low error when the number of publications $\tilde{s}$ per window is small. However, the error increases exponentially with $\tilde{s}$. Additionally, the error in $M_{t,1}$ (the first part of the error upper bound in PBD) rises as w_L increases, however, it diminishes as d increases. This is because a large d reduces sensitivity leading to smaller noise error.

For PBA, assume α skipped publications occur before a publication. Let $\epsilon_{\tilde{L}}$ and $\epsilon_{\tilde{R}}$ be the minimum and maximum publication privacy budget among all users at timestamps $t = w_L$ and $t = (\alpha+1)$, respectively. According to the PBA process, there will be α nullified publications after the publication. These nullified publications are filled by the last timestamp's publication without comparison. Consequently, the nullified publication error depends on the data distribution at nullified timestamps. We denote the average error of each nullified publication in PBA as $\overline{err}_{nlf}$. For PBA, we have Theorem 4.3 as follows.

Theorem 4.3. *The average error per timestamp in PBA is at most* $\min(\frac{8}{d^2\epsilon_L}, Z + \frac{8}{d^2\epsilon_R}) + \frac{1}{2\alpha+1}(\widetilde{err}^{(s,p)}_{M_{t,2}} + \alpha \cdot \overline{err}_{nlf})$ *where* $\widetilde{err}^{(s,p)}_{M_{t,2}}$ *is* $\min(\frac{2}{\epsilon_{\tilde{L}}^2} H^2_{\alpha+1}, (\alpha+1)Z + \frac{2}{\epsilon_{\tilde{R}}^2} H^2_{\alpha+1})$ *when* $\alpha \le w_L$ *and* $\min(\frac{2}{\epsilon_{\tilde{L}}^2} H^2_{w_L}, w_L Z + \frac{2}{\epsilon_{\tilde{R}}^2} H^2_{w_L}) + (\alpha - w_L + 1)\min(\frac{2}{\epsilon_{\tilde{L}}^2}, Z + \frac{2}{\epsilon_{\tilde{R}}^2})$ *when* $\alpha > w_L$ *and* $Z = (n - n_A)(n - n_A + \frac{1}{4})$ *and* H^2_x *is the x-th square harmonic number, if there are* α *skipped publications occur in average before each publication.*

PROOF. Similar to PBD, we first analyze the error of $M_{t,2}$ in PBA by assuming $M_{t,1}$ is not private. We then add the error of $M_{t,1}$, which is identical to that in PBD, to obtain the final total error. When $M_{t,1}$ is not private, the error stems from $M_{t,2}$. In $M_{t,2}$, each publication corresponds to α skipped publications preceding it and α nullified publications succeeding it.

For each user u_i's skipped publication, the publication privacy budget lower bound doubles with each timestamp increase until it reaches $\mathcal{E}_i/2$ or a publication occurs. For example, in Figure 6, where $\alpha = 5$, the publication timestamp is t_6. At timestamp t_1, each u_i's publication budget lower bound is $\mathcal{E}_i/(2w_i)$. Take u_1 as an example: it reaches $\mathcal{E}_1/2$ at timestamp t_4. The publication lower bound for u_1 remains at $\mathcal{E}_1/2$ until timestamp t_6. Let the publication budget lower bound set for all users at skipped timestamps (spanning α timestamp) be $\hat{\mathcal{E}} = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_\alpha\}$. Then, the error upper bound of each skipped publication is the error of publishing new data using ϵ_k ($k \in [\alpha]$). For example in Figure 6, the error upper bound at

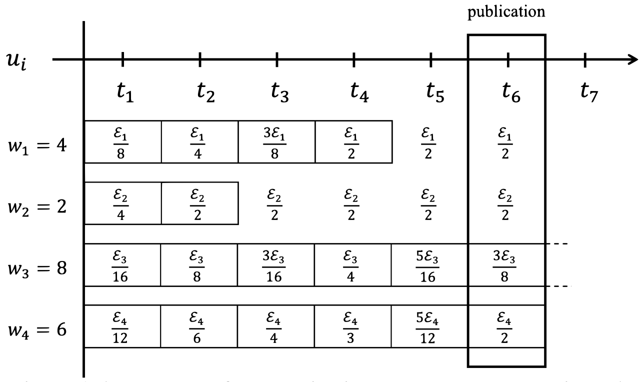


Figure 6: An example of the publication budget lower bound in PBA.

t_3 is the error of publication a new obfuscated statistic result using $\{\frac{3\mathcal{E}_1}{2}, \frac{\mathcal{E}_2}{2}, \frac{3\mathcal{E}_3}{16}, \frac{\mathcal{E}_4}{4}\}$.

Let $Z = (n - n_A)(n - n_A + \frac{1}{4})$ be the sampling error upper bound, where n_A is the number of users with maximum value of $\frac{\mathcal{E}_i}{w_i}$. We now consider two cases: $\alpha \le w_L$ and $\alpha > w_L$.

(1) case 1: $\alpha \le w_L$.

In this case, the publication budget lower bound doubles with each timestamp increase. Let $err^{(sk)}_{M_{t,2}}(\alpha)$ and $err^{(pb)}_{M_{t,2}}$ be the total error upper bounds of the α skipped publications and the publication in $M_{t,2}$, respectively. Let $err^{(s,p)}_{M_{t,2}}$ be the error of all skipped publications and the publication in $M_{t,2}$. According to Lemma 4.1, we have

$$\begin{aligned} err^{(sk)}_{M_{t,2}}(\alpha) &< \sum_{k\in[\alpha]} \min\left(\frac{2}{(k\epsilon_L)^2}, Z + \frac{2}{(k\epsilon_R)^2}\right) \\ &\le \min\left(\frac{2}{\epsilon_L^2} H_\alpha^2, \alpha Z + \frac{2}{\epsilon_R^2} H_\alpha^2\right) \end{aligned} \tag{18}$$

and

$$\begin{aligned} err^{(s,p)}_{M_{t,2}} &< err^{(sk)}_{M_{t,2}}(\alpha) + err^{(pb)}_{M_{t,2}} \\ &= err^{(sk)}_{M_{t,2}}(\alpha + 1) \\ &= \min\left(\frac{2}{\epsilon_L^2} H^2_{\alpha+1}, (\alpha+1)Z + \frac{2}{\epsilon_R^2} H^2_{\alpha+1}\right). \end{aligned} \tag{19}$$

Thus, we derive the average error upper bound $\overline{err}_{M_{t,2}}$ of each timestamp in $M_{t,2}$ as

$$\overline{err}_{M_{t,2}} < \frac{1}{2\alpha+1}(\widetilde{err}^{(s,p)}_{M_{t,2}} + \alpha \cdot \overline{err}_{nlf}) \tag{20}$$

where $\widetilde{err}^{(s,p)}_{M_{t,2}}$ is the final value in Equation (19).

(2) case 2: $\alpha > w_L$.

In this case, we have

$$\begin{aligned} err^{(s,p)}_{M_{t,2}} &< err^{(sk)}_{M_{t,2}}(w_L) + \sum_{k=w_L+1}^{\alpha+1} \min\left(\frac{2}{\epsilon_{\tilde{L}}^2}, Z + \frac{2}{\epsilon_{\tilde{R}}^2}\right) \\ &= err^{(sk)}_{M_{t,2}}(w_L) + (\alpha - w_L + 1)\min\left(\frac{2}{\epsilon_{\tilde{L}}^2}, Z + \frac{2}{\epsilon_{\tilde{R}}^2}\right) \\ &< \min\left(\frac{2}{\epsilon_L^2} H^2_{w_L}, w_L Z + \frac{2}{\epsilon_R^2} H^2_{w_L}\right) \\ &\quad + (\alpha - w_L + 1)\min\left(\frac{2}{\epsilon_{\tilde{L}}^2}, Z + \frac{2}{\epsilon_{\tilde{R}}^2}\right). \end{aligned} \tag{21}$$

Therefore, we obtain the average error upper bound $\overline{err}_{M_{t,2}}$ for each timestamp in $M_{t,2}$ as

$$\overline{err}_{M_{t,2}} < \frac{1}{2\alpha+1}(\widetilde{err}^{(s,p)}_{M_{t,2}} + \alpha \cdot \overline{err}_{nlf}) \tag{22}$$

where $\widetilde{err}^{(s,p)}_{M_{t,2}}$ is the value derived in Equation (21).

When $M_{t,1}$ is private, its error is identical to that in PBD:

$$\overline{err}_{M_{t,1}} < \min\left(\frac{8}{d^2\epsilon_L^2}, Z + \frac{8}{d^2\epsilon_R^2}\right). \tag{23}$$

Based on Equation (23), (20) and (22), we can derive the average error upper bound for each timestamp in PBA as:

$$\min\left(\frac{8}{d^2\epsilon_L^2}, Z + \frac{8}{d^2\epsilon_R^2}\right) + \frac{1}{2\alpha+1}(\widetilde{err}^{(s,p)}_{M_{t,2}} + \alpha \cdot \overline{err}_{nlf}), \tag{24}$$

where $\widetilde{err}^{(s,p)}_{M_{t,2}}$ is the final result from Equation (19) when $\alpha \le w_L$, and from Equation (21) when $\alpha > w_L$. □

5 EXPERIMENTS

This section compares our methods with state-of-the-art approaches using both real and synthetic datasets.

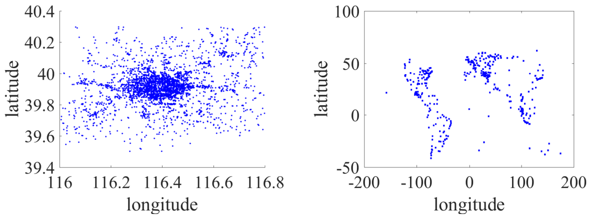
5.1 Datasets

We evaluate our solutions on both real and synthetic datasets.

Real datasets. We use two real-world datasets, *Taxi* [37, 38] and *Foursquare* [35, 36], to evaluate the practical performance of our algorithms.

Taxi. This dataset contains real-time trajectories of 10,357 taxis' in Beijing from February 2 to February 8, 2008. Each taxi has between 0 to 154,699 records, with each record comprising *taxi id*, *data time*, *longitude* and *latitude*. For the spatial dimension, we first remove all duplicate records, then extract records with longitude between 116 and 116.8 and latitude between 39.5 and [40.3], resulting in 14,859,377 records. We denote this area ($[116, 116.8] \times [39.5, 40.3]$) as A_E. Figure 7(a) shows 50% of random points in A_E. We further divide A_E uniformly into a 10×10 grids, designating these 100 cells as the location space. For the time dimension, we sample records every minute and get 8,889 records.

Foursquare. This dataset contains 33,278,683 Foursquare check-ins from 266,909 users, spanning April 2012 to September 2013. Each record consists of user id, venue id (place), and time. We convert the venue id to the country where the venue is located. After removing invalid records, we randomly extract 5% of users' data.



(a) Taxi

(b) Foursquare

Figure 7: Real datasets.

We set the publication time interval to 100 minutes, dividing the chick-ins period into 7,649 time stamps.

Synthetic datasets. We generate three binary stream datasets using different sequence models. Let $p_t = f(t)$ be the probability of setting the real value to 1 at timestamp t. We set the length of each binary stream as T and the number of users as N. For each stream, we first generate a probability sequence $(p_1, p_2, ..., p_T)$. At each timestamp t, each user's real value is set to 1 with probability p_t and 0 otherwise. The probability functions we use are as follows:

- TLNS function. In TLNS, $p_t = p_{t-1} + \mathcal{N}(0, Q)$, where $\mathcal{N}(0, Q)$ is Gaussian noise with standard variance $\sqrt{Q} = 0.0025$. We set $p_0 = 0.05$ as the initial value. If $p_t < 0$, we set $p_t = 0$; If $p_t > 1$, we set $p_t = 1$.
- Sin function. In Sin, $p_t = A \sin(\omega t) + h$, where $A = 0.05$, $\omega = 0.01$ and $h = 0.075$.
- Log function. In Log, $p_t = A/(1 + e^{-bt})$, where $A = 0.25$ and $b = 0.01$.

5.2 Experiment Setup

We split the total time series into two batches for all datasets, with each batch containing at most half of the total timestamps.

We compare our PBD and PBA with two non-personalized methods: Budget Distribution (BD) and Budget Absorption (BA) [22]. We also compare against a simple personalized LDP method, Personalized LDP Budget Uniform (PLBU), which extends LDP Budget Uniform (LBU) [29] by replacing the inner CDP mechanism with an LDP mechanism.

Let $\mathcal{E}$ and w be the privacy budget and window size in non-personalized static methods (BD and BA). For non-personalized static methods, we set the $\mathcal{E}$ to vary from 0.2 to 1.0 and w to vary from 40 to 200. To make our PBD and PBA comparable with BD and BA, we set the lower bound of each user's privacy budget as $\mathcal{E}$ and the upper bound of each user's window size as w in PBD and PBA to align the privacy level.

Given $\tilde{n}$ different privacy budgets $\tilde{\mathcal{E}} = \{\epsilon_1, ..., \epsilon_{\tilde{n}}\}$, let $\tilde{N}(\tilde{\mathcal{E}})$ be the total number of appearances of these budgets. For any $\epsilon_k \in \tilde{\mathcal{E}}$ with appearance count $\tilde{N}(\epsilon_k)$, we define the budget ratio of ϵ_k as $\frac{\tilde{N}(\epsilon_k)}{\tilde{N}(\tilde{\mathcal{E}})}$, which is the appearance ratio of ϵ_k among all $\tilde{\mathcal{E}}$. Similarly, we define the window size ratio of any w_k in different window sizes $\tilde{W} = \{w_1, ..., w_{\tilde{n}}\}$ as $\frac{\tilde{N}(w_k)}{\tilde{N}(\tilde{W})}$. We collectively term both the budget ratio and the window size ratio as the user ratio. To study the influence of different users' requirements, we also compare the impact of privacy budget ratio and window size ratio in PBD and PBA. We set the privacy domain as $\{0.5, 1.0\}$ and the window size domain as $\{10, 20\}$. We alter the user ratio of $\mathcal{E}_i = 0.5$ from 0.1 to 0.9 and $w_i = 10$ from 0.1 to 0.9. The parameters are shown in Table 3.

Table 3: Experimental settings.

Parameters	Values
static privacy budget $\mathcal{E}$	0.2, 0.4, **0.6**, 0.8, 1.0
static window size w	40, 80, **120**, 160, 200
personalized privacy budget $\mathcal{E}_i$	$\mathcal{E}, \ldots, 0.8, 1.0$
personalized window size w_i	$40, 80, \ldots, w$
user ratio	0.1, 0.3, **0.5**, 0.7, 0.9

We run our experiments on an Intel(R) Xeon(R) Silver 4210R CPU @ 2.4GHz with 128 RAM in Java. Each experiment is run 10 times, and we report the average value.

5.3 Measures

We evaluate the performance of different mechanisms based on their running time and data utility. Running time is measured by the duration needed to complete T time slots. We measure data utility as *Average Mean Relative Error* (*AMRE*) and *Average Jensen-Shannon Divergence* (*AJSD*, $\bar{D}_{JS}$).

AMRE is defined as the average value of Mean Relative Error (*MRE*), which is shown in Equation (25).

$$AMRE = \frac{1}{T}\sum_{\tau=1}^{T} MRE_\tau = \frac{1}{T}\sum_{\tau=1}^{T}\frac{1}{d}\|\boldsymbol{r}_\tau - \boldsymbol{c}_\tau\|_2^2, \tag{25}$$

where T represents the number of timestamps and d denotes the data dimension size.

AJSD is defined as the average value of Jensen-Shannon Divergence (*JSD*, D_{JS}) [25], which is based on Kullback-Leibler Divergence [24], as shown in Equation (26).

$$\begin{aligned}
\bar{D}_{JS}(\boldsymbol{r}\|\boldsymbol{c}) &= \frac{1}{T}\sum_{\tau=1}^{T} D_{JS}(\boldsymbol{r}\|\boldsymbol{c}) \\
&= \frac{1}{T}\sum_{\tau=1}^{T}\left(\frac{1}{2}D_{KL}(\boldsymbol{r}\|\boldsymbol{v}) + \frac{1}{2}D_{KL}(\boldsymbol{c}\|\boldsymbol{v})\right) \\
&= \frac{1}{2T}\sum_{\tau=1}^{T}\sum_{j=1}^{d}\left(\boldsymbol{r}_\tau(j)\log\left(\frac{\boldsymbol{r}_\tau(j)}{\boldsymbol{v}_\tau(j)}\right) + \boldsymbol{c}_\tau(j)\log\left(\frac{\boldsymbol{c}_\tau(j)}{\boldsymbol{v}_\tau(j)}\right)\right),
\end{aligned} \tag{26}$$

where $\boldsymbol{v}$ represents the average distribution of $\boldsymbol{r}$ and $\boldsymbol{c}$, defined as $\boldsymbol{v}(j) = \frac{1}{2}(\boldsymbol{r}(j) + \boldsymbol{c}(j))$. Here, T denotes the number of timestamps and d is the data dimension size. For timestamp τ, $r_\tau(j)$ and $c_\tau(j)$ represent the j-th dimensional values in the obfuscated and original data, respectively.

5.4 Overall Utility Analysis

Figure 8 displays the natural logarithm of *AMRE* as the privacy budget $\mathcal{E}$ varies. Across all datasets, *AMRE* decreases as $\mathcal{E}$ increases. This is because a larger $\mathcal{E}$ results in smaller noise variance, leading to a lower *AMRE*. The decrease in *AMRE* is more pronounced in real datasets compared to synthetic ones. This difference occurs because data density function changes rapidly in real datasets, while changing gradually in synthetic datasets. When the density function changes rapidly, the dissimilarity at each time slot becomes large. In this case, PBD publishes more new statistical results than PBA because PBD always reserves part of its privacy budget for the next time slot (though this budget decreases over time within a window). Thus, PBD leads to higher accuracy than PBA. When the density function changes gradually, the dissimilarity at each time slot remains small. In this case, publishing one highly accurate statistical result at a time slot is more important than publishing multiple new statistical results. Therefore, PBA performs significantly better than PBD. PLBU performs worse than other methods across all datasets except TLNS as LDP methods achieve lower accuracy than CDP methods under the same privacy budget. In real datasets, our PBD consistently outperforms other methods. The *AMRE* of PBD is on average 70.8% (17.5% in terms of $\ln(AMRE)$) lower than that of BD in Taxi and 69.6% (15.9% in terms of $\ln(AMRE)$) lower in Foursquare. Our PBA performs slightly worse than BA. This is because our PBA is more sensitive to noise in high-dimensional data. For synthetic datasets, our PBA consistently outperforms other methods. Compared to BA, the *AMRE* of PBA is lower by average of 36.9% (6.0% in terms of $\ln(AMRE)$) for TLNS, 27.7% (4.2% in terms of $\ln(AMRE)$) for Sin, and 28.9% (4.5% in terms of $\ln(AMRE)$) for Log. Moreover, our PBD consistently outperforms BD.

Figure 9 illustrate the natural logarithm of *AMRE* as the window size w varies. As w increases, *AMRE* rises gently, particularly in the synthetic datasets. This occurs because a large window size results in a small privacy budget at each time stamp, leading to increased error. PLBU shows lower performance than other methods in all datasets except TLNS, since LDP methods achieve lower accuracy than CDP methods under equivalent privacy budgets. For real datasets, our PBD achieves the lowest error compared to others methods. The *AMRE* of PBD is on average 63.1% (15.6% in terms of $\ln(AMRE)$) lower than that of BD in Taxi and 68.4% (16.5% in terms of $\ln(AMRE)$) in Foursquare. In synthetic datasets, our PBA demonstrates the lowest error among all methods. Compared to BA, the *AMRE* of PBA is lower by average of 35.1% (5.4% in terms of $\ln(AMRE)$) for TLNS, 4.2% (0.4% in terms of $\ln(AMRE)$) for Sin, and 16.6% (2.2% in terms of $\ln(AMRE)$) for Log. Moreover, our PBD consistently outperforms BD across all datasets.

In summary, our PBD demonstrates superior performance in real datasets, with an *AMRE* at least 63% lower than BD. For synthetic datasets, our PBA outperforms BA, achieving an *AMRE* reduction of at least 16%.

5.5 Impact of User Requirement Type

We define a set of users with privacy budget requirement $\mathcal{E}_k$ and window size requirement w_k as $(\mathcal{E}_k, w_k)$-*requirement type*. In this subsection, we examine the impact of user type on the utility. For our analysis, we set $\mathcal{E}_k$ candidate set as $\{0.6, 1.0\}$ with a default value of 0.6, and the w_k candidate set as $\{40, 120\}$ with a default value of 120. We first vary the users' quantity ratio of $\mathcal{E}_k = 1.0$ from 0.1 to 0.9 while keeping $w_k = 120$, and then vary the users' quantity ratio of $w_k = 40$ from 0.1 to 0.9 while keeping $\mathcal{E}_k = 0.6$. We analyze the impact of these ratio variations on *AMRE*.

Figure 10 illustrates the change in user ratio for $\mathcal{E}_k = 1.0$ from 0.1 to 0.9, with a fixed window size of $w_k = 120$. Figure 11 shows the change in user ratio for $w_k = 40$ from 0.1 to 0.9, with a fixed privacy budget of $\mathcal{E}_k = 0.6$. In both cases, we observe that as the user ratio increases, the *AMRE* remains relatively stable. However, when the user ratio of either $\mathcal{E}_k = 1.0$ or $w_k = 40$ exceeds 0.8, we can see a significant decrease in *AMRE* for PBD and PBA. This occurs because when the user ratio surpasses a certain threshold, the optimal budget from the *Optimal Budget Selection* in Algorithm 1 becomes dominated by a higher ϵ, resulting in lower error.

6 CONCLUSION

In this paper, we address the problem of Personalized w-event Private Release for Infinite Data Streams. We propose a mechanism called PWSM and two methods called PBD and PBA to solve this problem in scenarios with personalized privacy budget and window sizes for each users. We also compare our PBD and PBA with recent solutions to demonstrate their efficiency and effectiveness.

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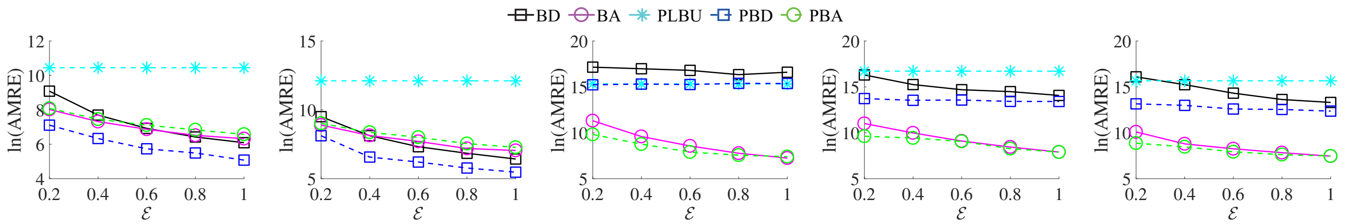
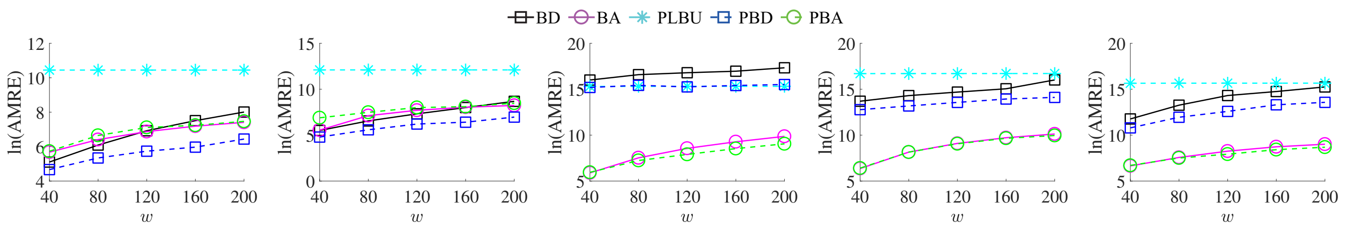


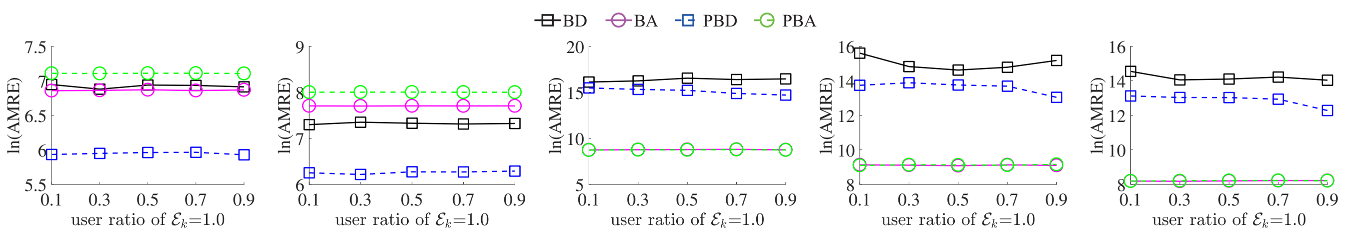
(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 8: ***AMRE*** **with ϵ varied.**



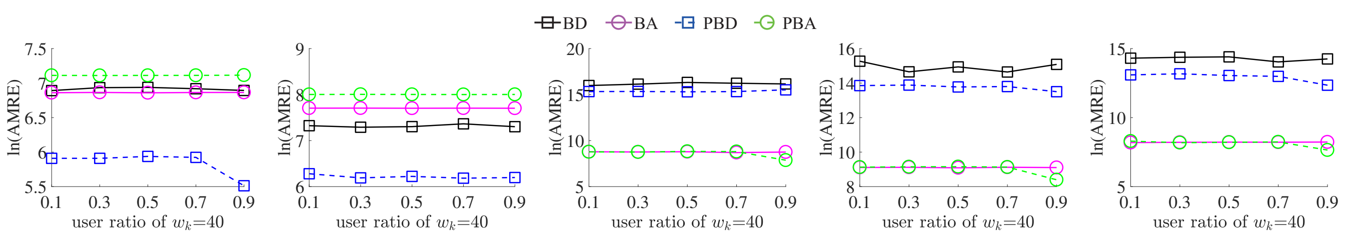
(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 9: ***AMRE*** **with w varied.**



(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 10: ***AMRE*** **with user ratio of privacy budget varied.**



(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 11: ***AMRE*** **with user ratio of window size varied.**

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7 APPENDIX

7.1 Running time Analysis

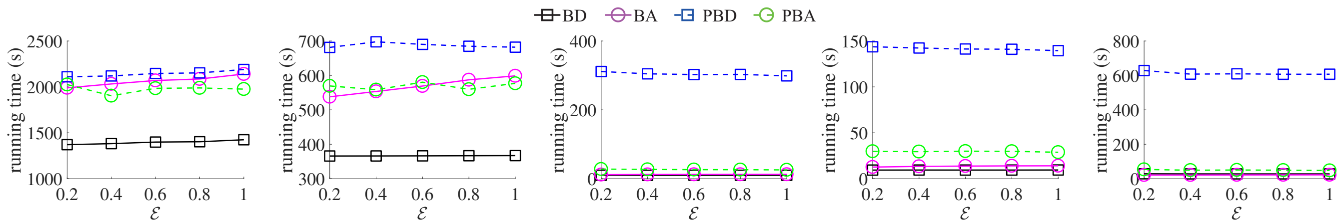
In this subsection, we compare the running time of BD, BA, PBD and PBA.

Figure 12 shows the running time as the privacy budget varies from 0.2 to 1. For synthetic datasets, the running time remains stable across different privacy budgets. This stability occurs because as datasets change gradually and skipped time slots increase, different privacy budgets have minimal impact on the number of new publications. In real datasets, the running time of BA increases slightly, likely due to larger privacy budgets requiring more comparisons between dissimilarity and error when datasets change rapidly. PBD requires the highest computation time among all methods, particularly with synthetic datasets, while BD requires the least time for real datasets and some synthetic datasets (i.e., TLNS and Sin). It is because non-personalized methods (BD and BA) have fewer steps in BD than in BA for publication judgments (which is denoted as algorithm complexity running time, TC_{ac}). As a result, BD requires less time than BA for most datasets. Personalized methods (PBD and PBA), however, require additional steps for optimal budget selection. These methods also have a higher probability of dissimilarity exceeding error (since they achieve lower error rates than non-personalized methods), resulting in fewer skips or nullifications compared to non-personalized methods. Fewer skips or nullifications leads to more comparisons in the total stream publication (which is defined as publication number running time, TC_{pn}). When time slots are sufficiently large, TC_{pn} has a greater impact than TC_{ac}. This effect becomes particularly noticeable with data changing slowly (as seen in synthetic datasets).

Figure 13 shows the running time as the window size changes from 40 to 200. All methods except PBD maintain stable running times as the window size increases. For PBD, its running time increases when the window sizes increase, because larger windows result in smaller per-user privacy budgets within each window. Reserving half of the privacy budget for future publications leads to larger dissimilarity and error. Since PBD's optimal budget selection step mitigates error's growth, the dissimilarity increases at a lower rate than the error, resulting in more frequent publications. Similar to Figure 12, BD requires the least running time among all methods on real datasets and most synthetic datasets (i.e., TLNS and Sin), while PBD requires the highest running time. It is because TC_{ac} is lower in BD than in BA. Additionally, personalized methods introduce an optimal budget selection step that increases the running time by TC_{pn}. Compared to PBA, the dissimilarity of PBD increases more rapidly than the error, resulting in fewer skips or nullifications and thus a larger TC_{pn} in PBD. When time slots are sufficiently large, TC_{pn} has a greater impact than TC_{ac}, causing PBD to have the longest running time.

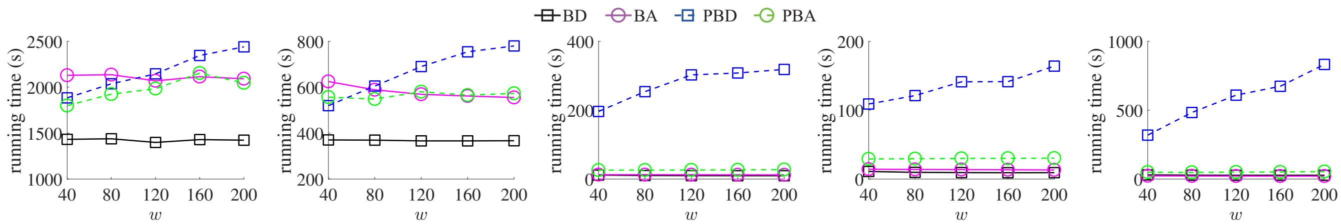
7.2 Experimental Result under $AJSD$ Metric

In this subsection, we compare the performance of BD, BA, PLBU, PBD and PBA using $AJSD$ metric.



(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 12: The running time with ϵ varied.

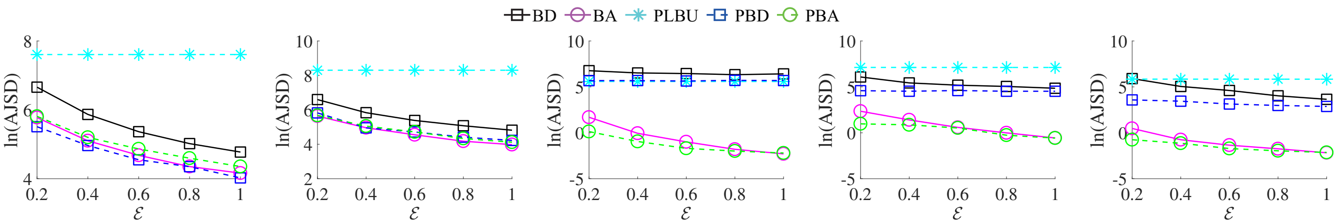


(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 13: The running time with w varied.

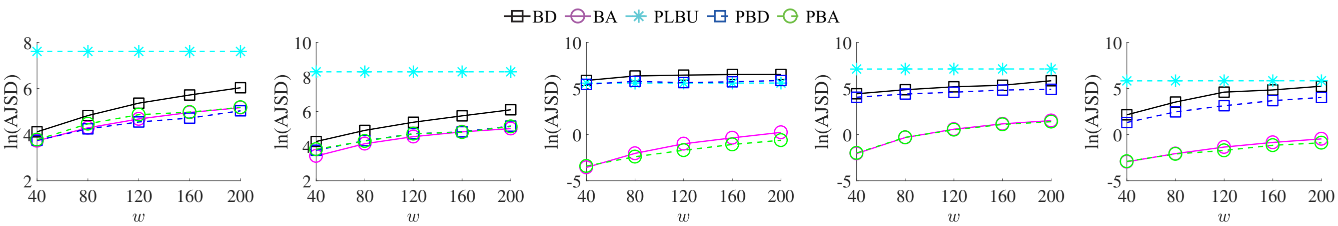
Figure 14 shows the results of $AJSD$ as the privacy budget $\mathcal{E}$ varies from 0.2 to 1. For all methods, $AJSD$ decreases as $\mathcal{E}$ increases, which aligns with the $AMRE$ results in Section 5.4. PLBU performs worse than other methods across all datasets except TLNS as LDP methods achieve lower accuracy than CDP methods under the same privacy budget. Both PBD and PBA consistently outperform BD. PBD achieves the best accuracy on the Taxi dataset, while PBA performs best with the three synthetic datasets. In the Foursquare dataset, BA outperforms other methods, due to the dataset's sparsity causing larger error in $AJSD$ calculation.

Figure 15 shows the results of $AJSD$ as the window size w varies from 20 to 200. $AJSD$ also increases with larger window sizes for all methods. PLBU shows lower utility than other methods in all datasets except TLNS, since LDP methods achieve lower accuracy than CDP methods under equivalent privacy budgets. Consistent with the results in Figure 14, both PBD and PBA outperform BD. PBD achieves the best performance in the Taxi dataset, while PBA leads in the three synthetic datasets. For the Foursquare dataset, BA achieves the lowest $AJSD$. However, this result may be unreliable due to the dataset's sparsity, which causes large calculation errors in the $AJSD$ measurements.



(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 14: The $AJSD$ with ϵ varied.



(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 15: The $AJSD$ with w varied.