Homework 1, CS225 | ECE205A, UCSB, 2016 Winter

Deadline, Copyright, and Academic Honesty:

¶ *Your answers to this assignment should be uploaded as a pdf to the Gauchospace of the course no later than Thursday Week II (2016-01-14, 23:59). The pdf should be produced using an appropriate typesetting program; it is not allowed to simply scan in your handwritten notes. Include your name and, if applicable, your perm. number on the pdf. Always explain your answer, but keep things succinct.*

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¶ *You should complete these specific assignments by yourself. What you submit should be your own work; you are not allowed to copy or quote work by others. Students violating these or other rules of Academic Honesty will receive an "F" for the course and will be reported to the Dean of Students Office.*

Question 1
Prove that for each discrete random variable $X \sim p(x)$ it holds that

$$H(X) \leq 2\log\Big(\sum_{x\in\mathcal{X}}\sqrt{p(x)}\Big).$$

Question 2
Consider two random variables X and Y with alphabets $\mathcal{X} = \{0,1\}$ and $\mathcal{Y} = \{a,b,c\}$ and the following joint distribution:

$$\begin{aligned}
&\Pr[X=0,Y=a]=0.15, \quad &&\Pr[X=0,Y=b]=0.3, \quad &&\Pr[X=0,Y=c]=0.05,\\
&\Pr[X=1,Y=a]=0.25, \quad &&\Pr[X=1,Y=b]=0.15, \quad &&\Pr[X=1,Y=c]=0.1.
\end{aligned}$$

Determine the entropic quantities $H(X)$, $I(X;Y)$, et cetera, involved in the Venn diagram of the random variables X and Y.

Question 3
Let p_0 and p_1 be probabilities distributions over a finite alphabet $\mathcal{X} = \{1,\ldots,D\}$. Consider a 'mixture' of p_0 and p_1 described by $p_\lambda = (1-\lambda)\cdot p_0 + \lambda \cdot p_1$ with $0 \leq \lambda \leq 1$, such that indeed for $\lambda = 0$ one gets p_0, for $\lambda = 1$ we get p_1, for $\lambda = 1/2$ we have the 50/50 mixture of the two probability distributions and so on. What can you prove about the entropy $H(p_\lambda)$ in terms of $H(p_0)$, $H(p_1)$, λ, and D? Prove your statements.

Question 4
Prove or disprove the triangle inequality for the relative entropy function D. In other words, does it hold that for all probability distributions p, q, r over the alphabet $\mathcal{X}$ we have $D(p \parallel q) + D(q \parallel r) \geq D(p \parallel r)$?

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