CS 260: Assignment #4
String Constant

Due on Tuesday, November 24, 2015

Prof. Hardekopf

Chad Spensky

	$\perp$	$y \in S$	$\top$
$\perp$	$\perp$	y	$\top$
$x \in S$	x	$\top$	$\top$
$\top$	$\top$	$\top$	$\top$

(a) Concatenation (+)

	$\perp$	$y \in S$	$\top$
$\perp$	T	T	T
$x \in S$	F	$\mathsf{substring}(x, y)$	T, F
$\top$	F	T, F	T, F

(b) Comparator ($\leq$)

Figure 1: Arithmetic Tables

1 Abstraction Domain Lattice

Let our original lattice $L = (\mathbb{P}(S), \leq)$ and our abstract-domain lattice $L^{\#} = (S \cup \{\top, \perp\}, \sqsubseteq)$ where S is the set of all strings (including the empty string) where $\top = S$ and $\perp$ is *undefined*. Let $\perp \sqsubset s \sqsubset \top, \forall s \in S$, i.e. the lattice of height 3, infinite width.

$$\alpha(x) : L \to L^{\#} = \begin{cases} \perp & \text{if } x \text{ is } \{\} \\ s & \text{if } x \text{ is } \{\text{s}\} \\ \top & \text{otherwise} \end{cases}$$

Meet ($x \in L^{\#}$)

- $\perp \sqcap x = \perp, \forall x$
- $\top \sqcap x = x, \forall x$
- $x \sqcap y = \perp, \forall x, y \in S, x \neq y$

Join ($x \in L^{\#}$)

- $\perp \sqcup x = x, \forall x$
- $\top \sqcup x = \top, \forall x$
- $x \sqcup y = \top, \forall x, y \in S, x \neq y$

The lattice is infinite, but of of finite height, therefore it **is noetherian**.

1.1 Monotone Operators

Concatenation (+) For + to be monotone the following must hold: $x + y \leq x' + y' \Rightarrow \alpha(x) + \alpha(y) \sqsubseteq \alpha(x') + \alpha(y')$, where $x, y, x', y' \in \mathbb{P}(S)$ and $a \leq b \Rightarrow \textbf{substring}(a, b)$ for $a, b \in S$ and $x + y \Rightarrow \{x_i + y_i\} \forall x_i \in x, y_i \in y$. Similarly $x \leq y \Rightarrow \textbf{substring}(x_i, y_i) \forall x_i \in x, y_i \in y$. In all cases $|x| = |y|$ and $|x'| = |y'|$.

- For case where $x = y = \{\}$, this holds trivially.
- Similarly for $|x| = |y| \geq 1$ and $|x'| = |y'| \geq 1$, since $\top \sqsubseteq \top$ will always be true.

$\square$

Comparison ($\leq$) The same logic follows for $\leq$. For $\leq$ to be monotone the following must hold: $x \leq y \Rightarrow \alpha(x) \sqsubseteq \alpha(y)$, where $x, y \in \mathbb{P}(S)$ and $x \leq y \Rightarrow$ **substring**$(x_i, y_i) \forall x_i \in x, y_i \in y$.

- For case where $x = y = \{\}$, this holds trivially.
- Similarly for $|x| = |y| = 1$, then the definitions are identical.
- Similarly for $|x| = |y| \geq 1$, since $\top \sqsubseteq \top$ will always be true.

$\square$

1.2 Galois Connection

$$\gamma(\hat{x}) : L^{\#} \rightarrow L = \begin{cases} \{\} & \text{if } \hat{x} \text{ is } \bot \\ \{s\} & \text{if } \hat{x} \text{ is s} \\ S & \text{if } \hat{x} \text{ is } \top \end{cases}$$

We must show that $\alpha(x) \sqsubseteq \hat{x} \iff x \subseteq \gamma(\hat{x})$.

$\alpha(x) \sqsubseteq \hat{x} \Rightarrow x \subseteq \gamma(\hat{x})$:

- If $x = \{\}$, this holds trivially.
- If $|x| = 1$, then either $\hat{x} \in S$, which holds trivially, or $\hat{x} = \top$, and $x \subseteq S, \forall x$
- For $|x| \geq 1$, $\hat{x} = \top$ must be true, and $x \subseteq S, \forall x$

$x \subseteq \gamma(\hat{x}) \Rightarrow \alpha(x) \sqsubseteq \hat{x}$:

- If $x = \{\}$, this holds trivially.
- If $|x| = 1$, then either $\hat{x} \in S$, which holds trivially, or $\hat{x} = \top$, and $\alpha(x) \sqsubseteq \top, \forall x$
- For $|x| \geq 1$, $\hat{x} = \top$ must be true, and $\alpha(x) \subseteq \top, \forall x$

$\square$

1.3 Soundness

Because a Galois connection exists, our approximation is both sound and precise.