Data 605 - Assignment 3

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1. Problem Set 1

(1) What is the rank of Matrix A?

$$A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{vmatrix}$$

The rank is 4.

(2) Given $m \times n$ where m > n what is max rank what is min rank (assuming non-zero)?

The max rank is n and the min rank is 1 assuming non-zero.

(3) What is the rank of B?

$$B = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{vmatrix}$$

The rank is 1 because $r_2 = 2 \times r_1$ and $r_3 = 3 \times r_1$.

2. Problem Set 2

Compute the eigenvalues and eigenvectors of the matrix A.

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{vmatrix}$$

First we solve for

$$det |A - \lambda I|$$

$$\begin{vmatrix}
1 - \lambda & 2 & 3 \\
0 & 4 - \lambda & 5 \\
0 & 0 & 6 - \lambda
\end{vmatrix}$$

Reducing the matrix gives us:

$$1 - \lambda \begin{vmatrix} 4 - \lambda & 5 \\ 0 & 6 - \lambda \end{vmatrix} - 2 \begin{vmatrix} 0 & 5 \\ 0 & 6 - \lambda \end{vmatrix} + 3 \begin{vmatrix} 0 & 4 - \lambda \\ 0 & 0 \end{vmatrix}$$

This gives us when fully reduced $0=(1-\lambda)(6-\lambda)(4-\lambda)$ which gives us the eigenvalues = 1,6,4 with eigenvectors

$$\begin{array}{c|c|c|c}
\lambda & 0 & 0 & 0.83 \\
\lambda & 0 & 0.37 & -0.55 \\
1 & -0.93 & 0.06
\end{array}$$