

Assign9

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Question 1

The price of one share of stock in the Pilsdorff Beer Company (see Exercise 8.2.12) is given by Y_n on the n th day of the year. Finn observes that the differences $X_n = Y_{n+1} - Y_n$ appear to be independent random variables with a common distribution having mean $\mu = 0$ and variance $\sigma^2 = 1/4$. If $Y_1 = 100$, estimate the probability that Y_{365} is (a) > 100 . (b) > 110 . (c) > 120 .

Answer

Using CLT we can assume that the $Y_{365} - Y_1$ is normally distributed. We know that $E[Y_{365} - Y_1] = 0$ and that the $\text{var}(Y_{365} - Y_1) = 364 \cdot 1/4$.

$$\text{STD} = \sqrt{365 \cdot .25} = 9.55$$

a. $P\{Y_{365} - Y_1 > 0\} = P(Y_{365} > 0 / 9.55)$ since $\mu = 0$ and the $E=0$ then probability = 0.5

```
pnorm(0, mean = 0, sd = sqrt(365*.25), lower.tail = FALSE)
```

```
## [1] 0.5
```

b. $P\{Y_{365} - Y_1 > 10\} = P(Y_{365} > 10 / 9.55)$

```
pnorm(10, mean = 0, sd = sqrt(365*.25), lower.tail = FALSE)
```

```
## [1] 0.1475849
```

c. $P\{Y_{365} - Y_1 > 20\} = P(Y_{365} > 20 / 9.55)$

```
pnorm(20, mean = 0, sd = sqrt(365*.25), lower.tail = FALSE)
```

```
## [1] 0.01814355
```

Question 2

Calculate the expected value and variance of the binomial distribution using the moment generating function.

Answer

To find $E(X)$ and $V(X)$ we need to start with the moment of the binomial distribution which is as follows: $M(t) = [(1-p) + pe^t]^n$

In order to find the values we need to take the first and the second derivative and solve them for at time $t=0$.

Using derivate we get: $M'(t) = n(pe^t)^{[(1-p)+pe^t]}(n-1)$ $M'(0) = n \cdot 1 \cdot [(1-p)+p]^{(n-1)}$ $E(X) = np$

$M''(t) = n(n-1)(pe^t)^2[(1-p)+pe^t]^{(n-2)} + n(pe^t)[(1-p)+pe^t]^{(n-1)}$ $M''(0) = np(1-p) + n(n-1)p^2 = np(1-p) + n^2p^2 - np^2 = np(1-p) + np^2 = np$

Question 3

Calculate the expected value and variance of the exponential distribution using the moment generating function.

Answer

Using the same method above:

$M(t) = 1 / (1-t)$

Using derivate we get: $M'(t) = 1 / (1-t)^2$ $M'(0) = 1 / 1$ $E(X) = 1 / 1$

$M''(t) = 2 / (1-t)^3$ $M''(0) = 2 / 1^3 = 2$ $\sigma^2 = M''(0) - [M'(0)]^2 = 2 - 1 = 1$