How to estimate a population proportion if data are possibly subject to misclassification error?

The case of estimating contraceptive prevalence based on self-reported usage.

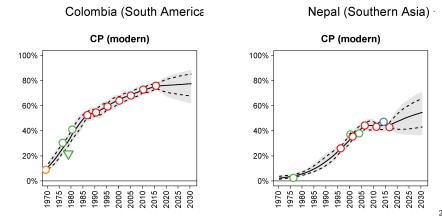
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Introduction

- Motivating question:
 How to estimate contraceptive prevalence using self-reported data collection, i.e. demographic health surveys (DHS)
- Approach: Family Planning Estimation Model (FPEM, Cahill et al., 2018)



How is the data used in FPEM?

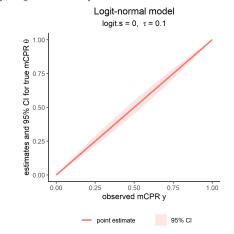
- ► FPEM: data model + process model
 - data model: describe how observed data relate to true modern contraceptive prevalence (mCPR)

$$logit(y) \sim N(logit(\theta), logit.s^2 + \tau^2)$$

- Notation
 - v observed mCPR.
 - \bullet true mCPR.
 - ► logit.s sampling error,
 - ightharpoonup au non-sampling error
- Visualization: posterior median and 95% CIs based on the posterior:

$$p(\theta|y) \propto p(\theta)p(y|\theta)$$

- ▶ prior $\theta \sim U(0,1)$
- $p(y|\theta)$ given by logit-normal data model



Data on non-sampling error in self-reported modern contraceptive use

- Two post-survey studies of DHS provides data on non-sampling errors in the form of misclassification
 - sensitivity (se) = proportion of modern users who reported themselves as such
 - specificity (sp) = proportion of non-modern users who reported themselves as such
- Findings:
 - ▶ 2014 Ghana DHS (Staveteig, 2017) (sample size = 48) $se = 0.857 \pm 0.106, sp = 0.875 \pm 0.094$
 - ▶ 2016 Nepal DHS (Staveteig et al., 2018) (sample size = 194) $se = 0.925 \pm 0.037, sp = 0.968 \pm 0.025$

Visualization of the relation between true prevalence θ and self-reported use y in presence of misclassification

True data model when data are subject to misclassification (simplified, s refers to sampling error):

$$y \sim N(se \cdot \theta + (1-sp)(1-\theta), s^2) T[0, 1],$$
Ghana
$$se = 0.857, sp = 0.875$$
Repal
$$se = 0.925, sp = 0.968$$

$$0.50$$
Repal
$$0.75$$
Repal
$$0$$

point estimate of logit-normal model

95% CI of logit-normal model

95% CI of true data model

How to estimate a population proportion if data are possibly subject to misclassification error?

- Conclusion so far:
 - ► Two small post-DHS studies suggest that self-reported mCPR is subject to misclassification
 - ► The additional uncertainty related to non-sampling error in FPEM does not capture the relationship implied by the studies
- ▶ Generalizability problem: only two studies in specific settings ⇒ do not apply bias-adjustments to self-reported use for all DHS data points based on two studies only
- What we can do: Update the data model to better reflect uncertainty associated with potential misclassification errors

Proposed new data model based on a Normal-Laplace distribution

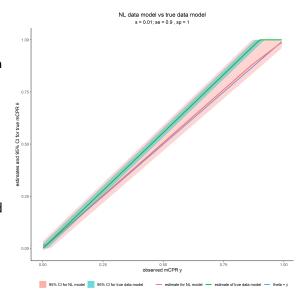
- ► Goal: data model to better reflect uncertainty associated with potential misclassification errors
- Aims for the posterior associated with the new data model with assumed sensitivity se^a and specificity sp^a (and U(0,1) prior):
 - $\hat{\theta} = y$
 - ightharpoonup If no misclassification: 95% CI determined by sampling error s
 - ▶ If misclassification: 95% CI determined by s, se^a, sp^a
- Model specification:
 - based on Normal-Laplace (NL) distribution (Reed, 2006) for the normalized likelihood,
 - **P** posterior under U(0,1) prior given by

$$p(\theta|y) \sim NL(\mu, \sigma, \alpha, \beta)T(0, 1)$$

Parameters fixed to meet aims

Illustration of the new data model

- Visualization: true data model and NL data model
- True relation: s = 0.01, se = 0.9, sp = 1
- Settings NL: $s = 0.01, se^a = 0.9, sp^a = 1$, such that $\hat{\theta} = y$ and upper bound of 95% CI increases with y



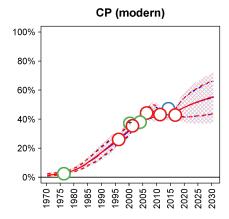
Simulation study

- We fixed $\theta^{\text{true}} = 0.3, 0.5, 0.7$, use different combinations of true misclassification se, sp and assumed misclassification se^a, sp^a , and generate 100 data sets per setting.
- Compare posterior estimates from FPEM logit-normal data model vs proposed NL model
- Findings (as expected):
 - 1. (bias in) point estimates are comparable between the logit-normal and NL model;
 - 2. 95% Cls are conservative with coverage exceeding 95%, when assumed misclassification > true misclassification;
 - NL model improves upon logit-normal model in terms of coverage of 95% CIs when misclassification is present and accounted for.

New data model in FPEM

Nepal (Southern Asia)

- NL model used in FPEM, applied to DHS data only
- ► Two settings:
 - $ightharpoonup se^a = 0.9, sp^a = 1 \text{ (Nepal)}$
 - $ightharpoonup se^a = sp^a = 0.9 \text{ (Ghana)}$
- ► Findings:
- (1) Differences in FPEM fit are more variable as compared to the simulations

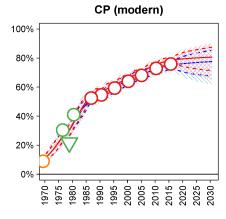


Blue = logit-normal, Red = NL with $se^a = 0.9, sp^a = 1$

New data model in FPEM (ctd)

Colombia (South America

- Findings (ctd):
- (1) Differences in FPEM fits are more variable as compared to the simulations
- (2) When there are differences,
 (i) estimated bounds vary in expected direction based on se^a, sp^a;
 - (ii) some updating of point estimates as well
 - Example: Colombia with $se^a = 0.9, sp^a = 1$



Blue = logit-normal, Red = NL with $se^a = 0.9$, $sp^a = 1$

Summary

- We investigated how to estimate a population proportion if data are possibly subject to misclassification error, motivated by reported evidence on misclassification in self-reported modern contraceptive use.
- We propose a new normal-laplace data model to account for increased asymmetric uncertainty associated with potential misclassification errors.
- Simulation study shows improvement in coverage of credible intervals when data are subject to misclassification.
- Ongoing: assessing the effect a change in data model from logit-normal to the normal-laplace in FPEM; findings so far: differences are typically smaller but more variable as compared to what we expect, and include changes in point estimates.

Thanks!

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