Temporal Walk Based Centrality Metric for Graph Streams

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DYNAMIC GRAPHS

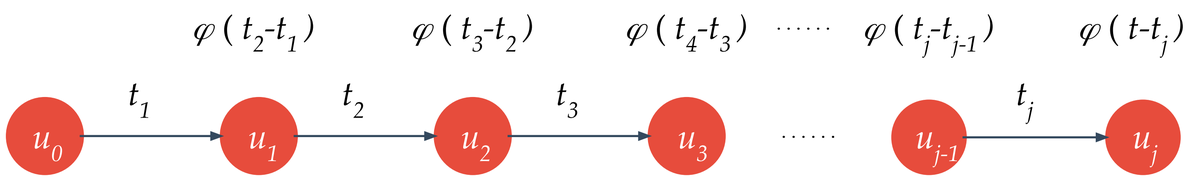
- Edge stream: (u,v,t) triplets, link occured from u to v at time t
- Directed edges can be activated multiple times, no deletion

TEMPORAL KATZ CENTRALITY

- Centrality measure for dynamic graphs
- Extension of Katz-index to temporally changing networks
- Online updateable from the edge stream

DEFINITION

- $\varphi(\tau)$: time decay function, e.g. $\varphi(\tau) = \beta\, exp(-c\tau)$
- Time-respective walk: $t_1 < t_2 < t_3 < \ldots < t_j \leq t$



$$\Phi(z,t) := \prod_{i=1}^{j} \varphi(t_{i+1} - t_i);\ t_{j+1} = t$$

- $r_u(t)$: temporal Katz centrality score of node u at time t
- Weighted sum of time-respective walks ending at node u

$$r_u(t) := \sum_{v} \sum_{\text{temporal paths } z \text{ from } v \text{ to } u} \Phi(z,t)$$

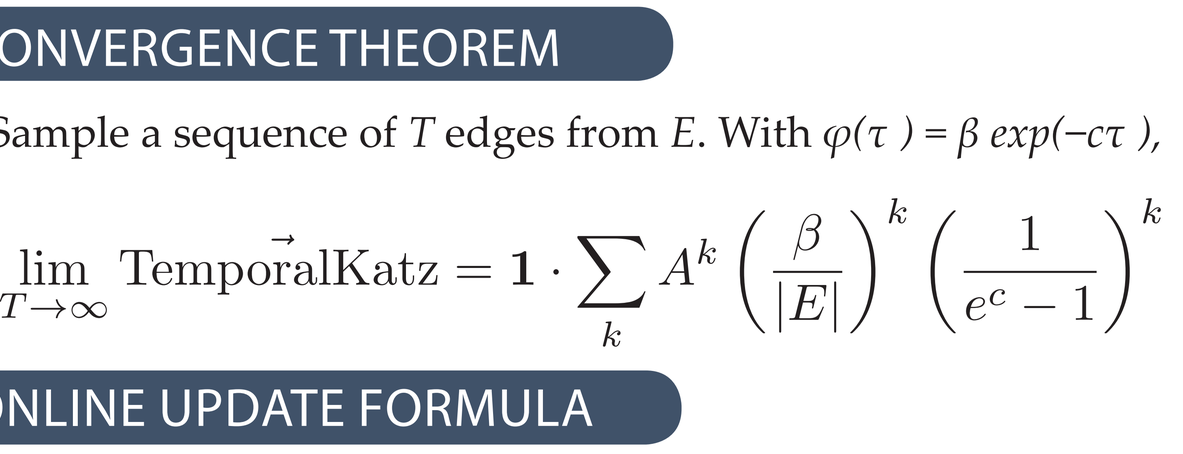
CONVERGENCE THEOREM

Sample a sequence of T edges from E. With $\varphi(\tau) = \beta\, exp(-c\tau)$,

$$\lim_{T\to\infty} \vec{\text{TemporalKatz}} = \mathbf{1} \cdot \sum_k A^k \left(\frac{\beta}{|E|}\right)^k \left(\frac{1}{e^c - 1}\right)^k$$

ONLINE UPDATE FORMULA

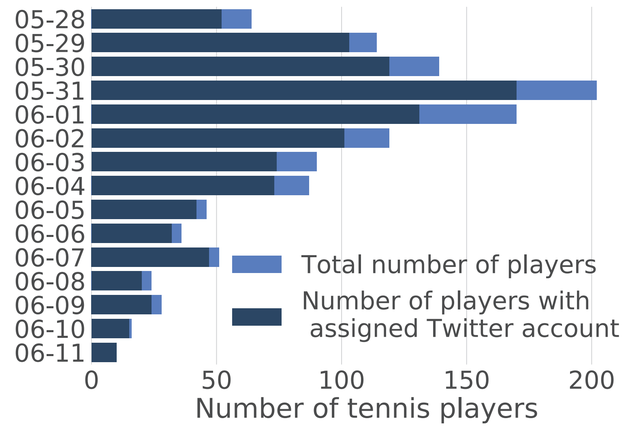
- $E(t)$: multi set of edges activated before time t
- r_v : temporal Katz centrality score of node v
- w_{vu} : weight over edge vu
- t_{vu} : last edge activation time of edge vu
- r,w and t are online updated from the edge stream



- More efficient update if $\varphi(\tau_1 + \tau_2) = \varphi(\tau_1)\,\varphi(\tau_2)$ e.g. exponential

TWITTER DATASETS

- Tweets from Roland-Garros and US Open 2017
- Mention network: nodes are accounts, edges are mentions
  - RG17: 336,234 edges; 78,095 nodes; 19 days
  - UO17: 482,061 edges; 106,920 nodes; 21 days
- We assigned tennis players to Twitter accounts
- Nodes of the network are temporally labeled
  - e.g. RG17 semi final day (June 9) at 12:00



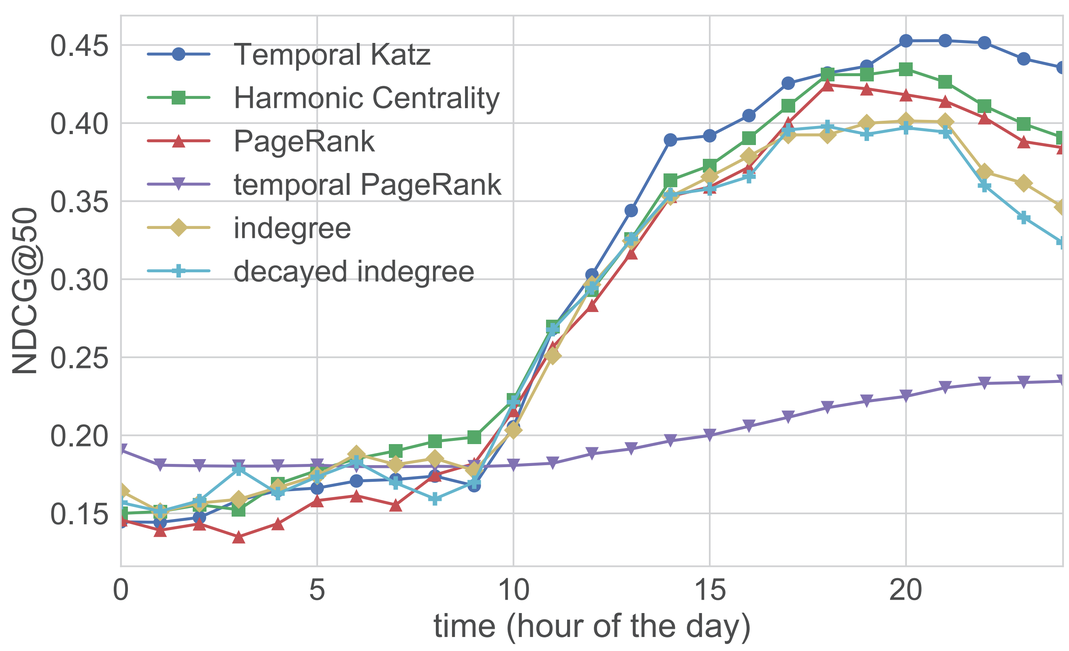
Rank	Node (Twitter account)	Label
1	@rolandgarros	0
2	@stanwawrinka	1
3	@andy_murray	1
4	@Simona_Halep	0
5	@RafaelNadal	1
6	@ThiemDomi	1
7	@TimeaOfficial	0
8	@AnaIvanovic	0
9	@rohanbopanna	0
10	@WTA	0

MEASUREMENT

- Central node prediction in temporally evolving networks
- **Goal:** predict list of players who play on a given day
- **Evaluation** by

$$\text{NDCG} = \frac{1}{\text{IDCG}} \sum_{i=1}^{\infty} \frac{\text{rel}(i)}{\log_2(i+1)},$$

- ranking is based on network centrality
- rel(i) = 1 if node i is related to a player who participated on the given day and *0* otherwise



- Temporal Katz centrality outperforms baselines on both data

NDCG@50	UO17	RG17
indegree	0.321	0.342
decayed indegree	0.321	0.346
negative beta measure	0.319	0.333
PageRank	0.325	0.349
temporal PageRank	0.187	0.195
harmonic centrality	0.353	0.359
temporal Katz centrality	0.370	0.368

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All datasets and results are available on GitHub
https://github.com/ferencberes/online-centrality