Rest of BDA3 and other reading

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- Gaussian process course in spring
- Regression and Other Stories
- Bayesian Workflow

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Highly recommended to read. Very informative, but also a dense chapter.

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- Data collection
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  - Designed experiments
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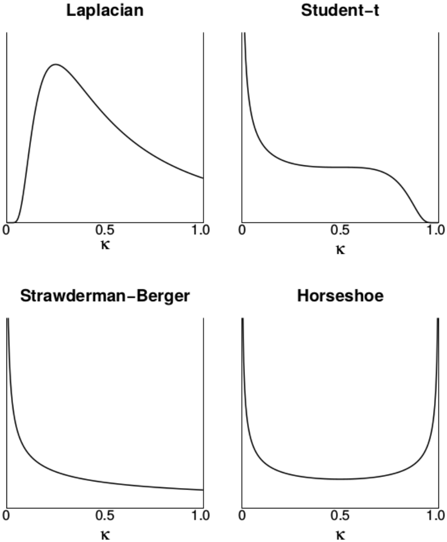
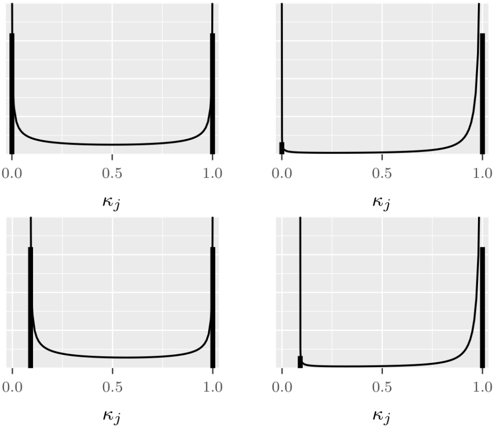
  - empirically better results obtained with more sparse priors
  - it's best to separate selection of sensible prior, good posterior inference, and the decision analysis of which variables are important

Sparse priors



from Carvalho, Polson, Scott (2009).

Regularized horseshoe

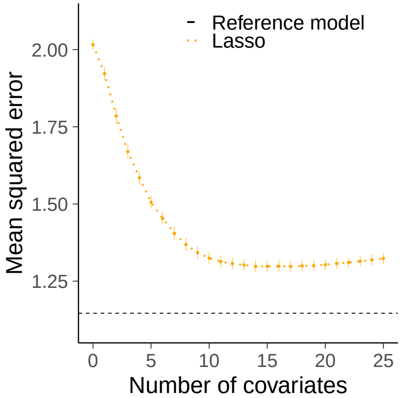


- Piironen and Vehtari (2017). Sparsity information and regularization in the horseshoe and other shrinkage priors. In Electronic Journal of Statistics, 11(2):5018-5051. Online
- `rstanarm`: `prior=hs()`
- `brms`: `prior=horseshoe()`

Projpred selection vs. Lasso

See projpred in an extra lecture

Simulated regression data
$n = 50$, $p = 500$, $p_{\text{rel}} = 150$, $\rho = 0.5$



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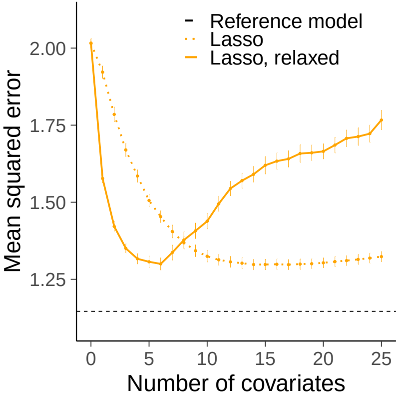

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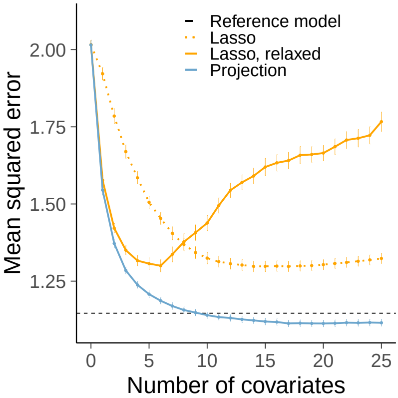

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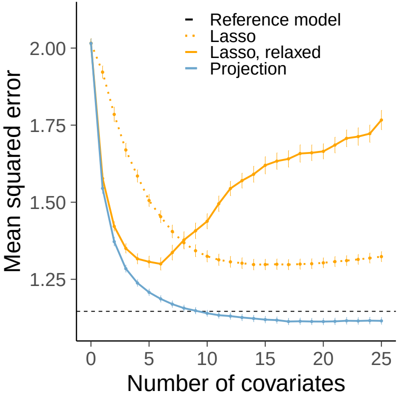

Chapter 15: Hierarchical linear models

- Since you know hierarchical models, theory is easy
- With probabilistic programming computation is also easy
  - BDA3 discusses some other computational issues
  - section on transformations for HMC is relevant
    (see also Stan user guide 21.7 Reparameterization)

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`y ~ 1 + x`	fixed / population effect; pooled model
`y ~ 1 + (0 + x | g)`	random / group effects
`y ~ 1 + x + (1 + x | g)`	mixed effects; hierarchical model

- ANOVA in section 15.6 (see also `stan_aov`)

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- Hierarchical GLM natural extension
- 16.3 Weakly informative priors section is excellent although the recommendation on using Cauchy has changed (see https://github.com/stan-dev/stan/wiki/Prior-Choice-Recommendations)

Chapter 17: Models for robust inference

- For example (see also ROS Ch 15)

normal	$\rightarrow$	*t*-distribution
Poisson	$\rightarrow$	negative-binomial
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  - rstanarm doesn't have *t*-distribution for outcome, but brms has

Chapter 18: Models for missing data

- Extends the data collection modelling from Chapter 8
- Useful terms

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- `brms` can handle some missing data

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  - useful prior for non-linear functions
  - for any finite number of variables, the marginal is multivariate normal $f_1, \ldots, f_n \sim \mathrm{N}\left(\mu(x_1, \ldots, x_n), K(x_1, \ldots, x_n)\right)$

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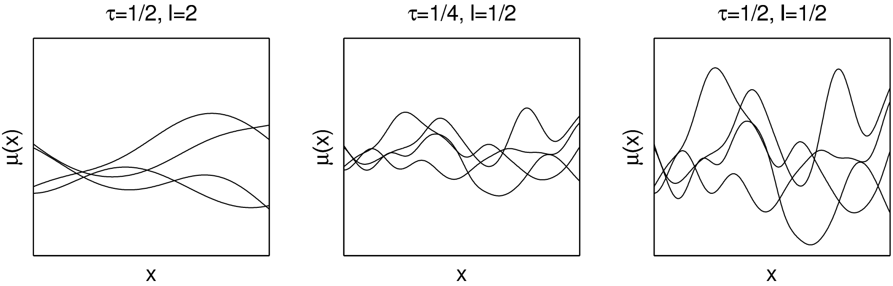

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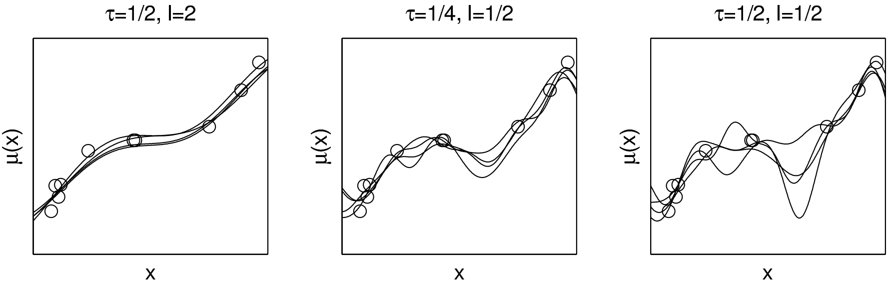
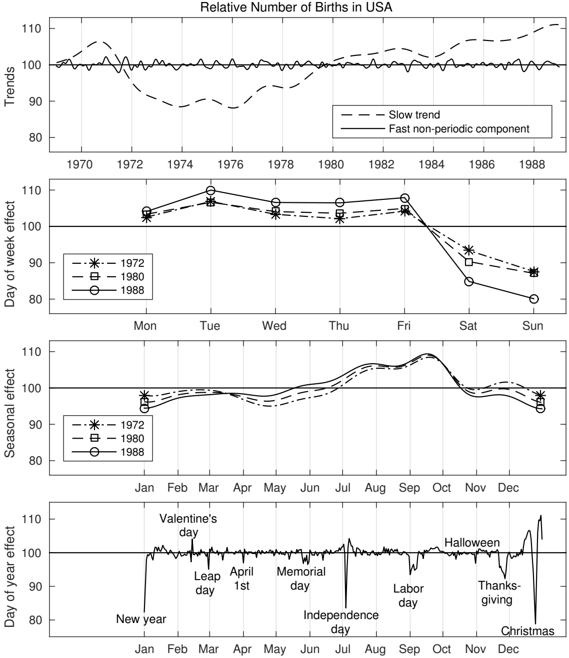


Chapter 21: Gaussian process models

- Conditional on covariance function parameter the posterior is just multivariate normal
  - need to make inference for covariance function parameters given the marginal likelihood
  - the exact computation of the marginal likelihood scales $O(N^3)$

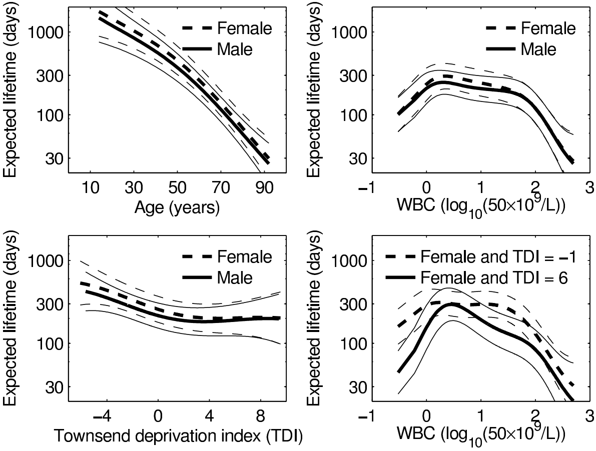
- Easy to make additive models

$$y_t(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t) + f_5(t) + \epsilon_t$$



Chapter 21: Gaussian process models

- For non-Gaussian outcome models similar extension as GLMs
- Survival model example:



GPs in Stan

- GP specific software (e.g. GPy, GPflow, GPyTorch) scale computationally better for GPs than Stan
- Stan has some built-in covariance functions
- Hilbert space basis function approximation of GPs is fast for 1D-3D (Riutort-Mayol et al., 2022)
  - Birthday example
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- `brms`:
  - covariance matrix based computation:
    ```
    y ~ gp(x)
    ```
  - Hilbert space basis function approximation:
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    y ~ gp(x, k=20)
    ```

Regression and Other Stories

- Gelman, Hill, and Vehtari (2020). Regression and Other Stories.
  - uses Bayesian inference, but maths and computation is minimal
  - focuses on different models and how think about modeling
  - a lot of different examples
  - https://avehtari.github.io/ROS-Examples/

Bayesian workflow

Gelman, Vehtari, Simpson, Margossian, Carpenter, Yao, Kennedy, Gabry, Bürkner, and Modrák (2020). Bayesian workflow. arXiv:2011.01808