Area Estimation Using the Monte Carlo Method

Exercise 8

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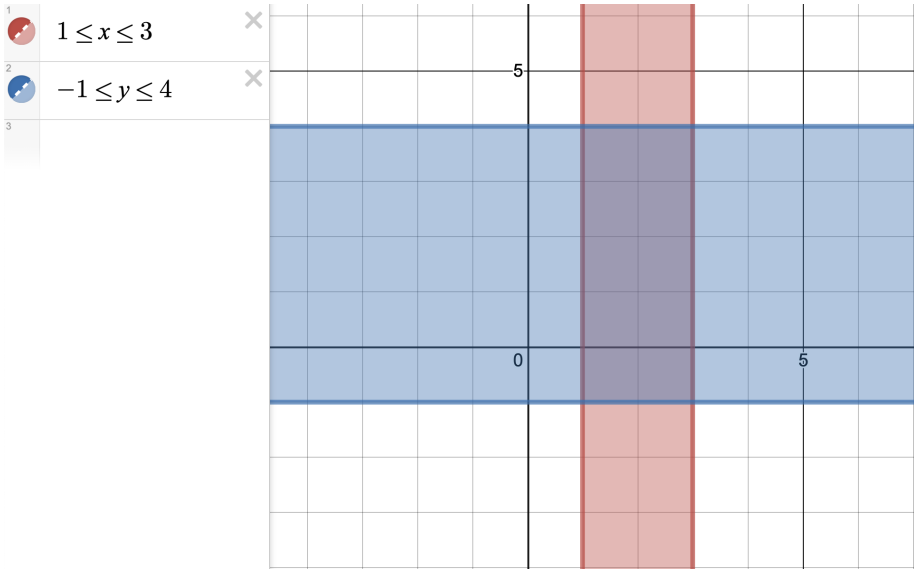
Problem Statement

Use the Monte Carlo method to approximate the area of the figure defined by

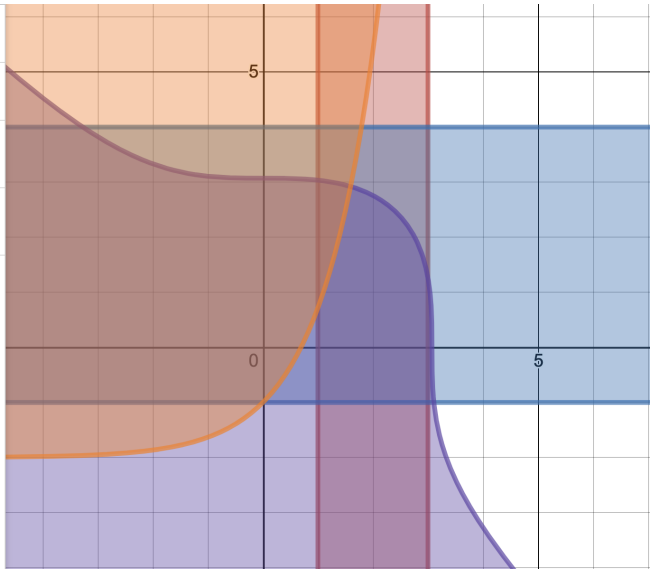
$$\begin{cases} 1 \leq x \leq 3 \\ -1 \leq y \leq 4 \\ x^3 + y^3 \leq 29 \\ y \geq e^x - 2 \end{cases}$$

Bounding Box



Plot

1. $1 \leq x \leq 3$
2. $-1 \leq y \leq 4$
3. $x^3 + y^3 \leq 29$
4. $y \geq e^x - 2$
5.



Approach

Generate N random points at random locations inside the rectangle. The area is given by:

Area

$$A_{\text{figure}} = A_{\text{rectangle}} \times \frac{\text{Points inside figure}}{\text{Total points generated (N)}}$$

Average of Areas

This process is repeated LOOPS times and at the end of the N iterations the average of the areas is calculated.

Result

For N = 30000, LOOPS = 30000, and seed equal to -87654321 the area is $7.581675111111076 \times 10^{-1}$.

Tools

- Two independent streams of random numbers with Intel MKL
- OpenMP for multithreading
- C Math Library for basic math functions

Mathematical Solution

Mathematically,

$$\int_1^a (\sqrt[3]{29 - x^3} - e^x + 2)dx \approx 7.581218821150386e - 01$$

with $a = 1.593743361313601$, point of intersection between the two curves defined by $y \geq e^x - 2$ and $y \leq \sqrt[3]{29 - x^3}$.

Observations

Given the absolute error $4.562899606896931 \times 10^{-5}$, the Monte Carlo method can be used to estimate areas with a good level of precision. Finally, OpenMP can make this computation much more efficient.