Project #1

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Problem 1

1. Figure 1 and 2 (blue line) shows the gradient histogram of natural images. The histogram has a high and sharp peak which is not like a gaussian.

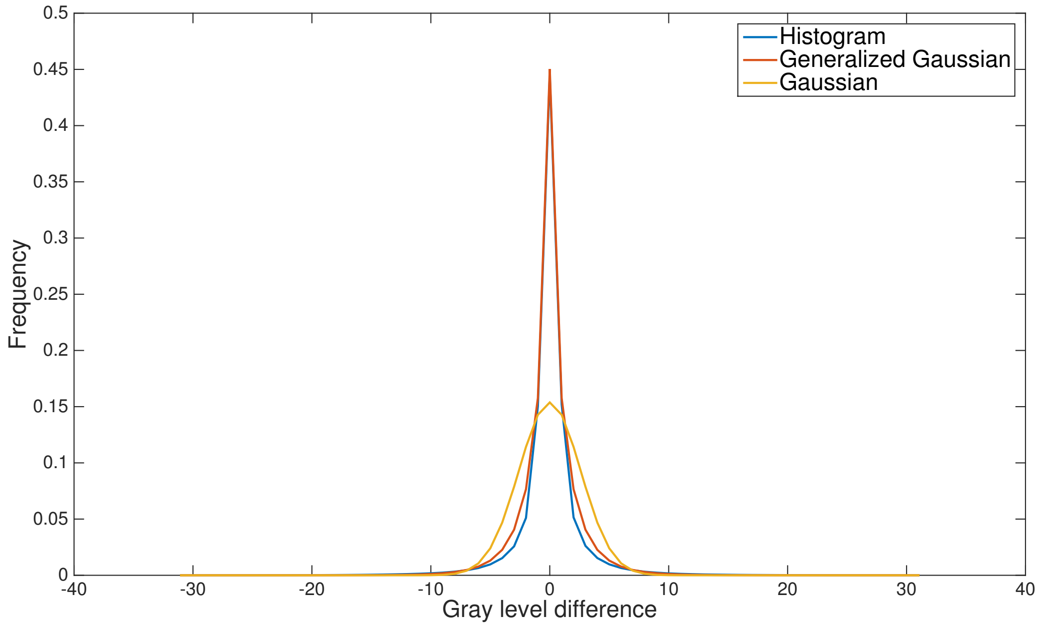

Figure 1: Gradient histogram of natural images

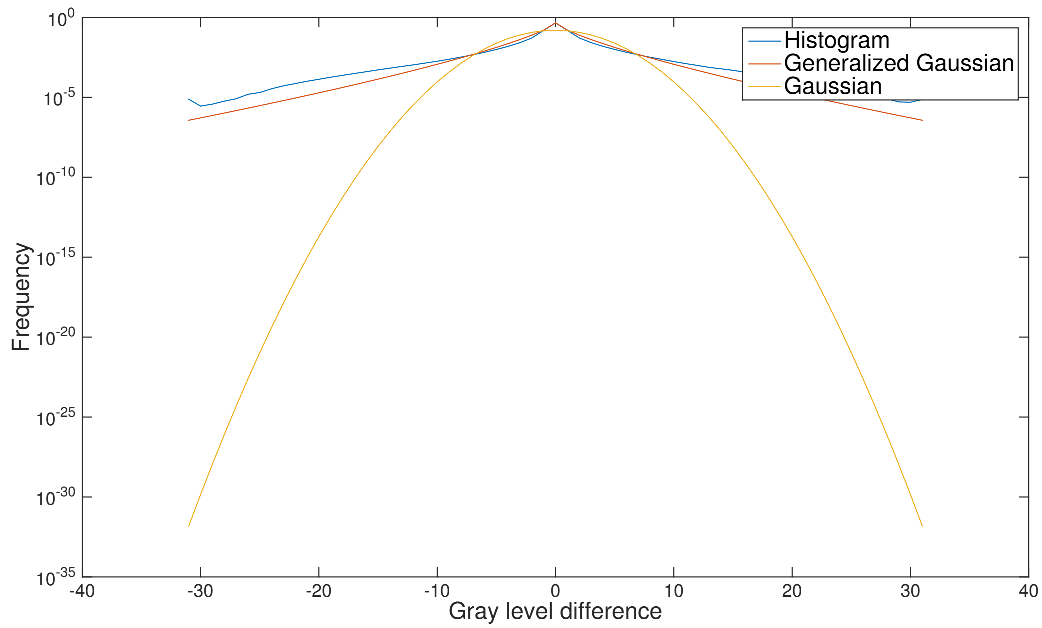

Figure 2: Gradient histogram of natural images (log-plot)

2. Mean = 0.00083   Variance = 6.73   Kurtosis = 18.86
   We can see the histogram of natural images gradients has high kurtosis. Zero mean is obvious because the expectation of the right edge minus the left edge should be zero.

3. The red line in Figure 1 and 2 represents the fitted curve of generalized gaussian distribution. ($\gamma = 0.755$, $\sigma = 2.39$) The generalized gaussian distribution fits the histogram well. And we can notice that the shape parameter γ is much less than 2.

4. The yellow line in Figure 1 and 2 represents the estimated normal distribution, which cannot fit the histogram. The real histogram has a fatter tail than a gaussian distribution.

5. Figure 3 and 4 shows the gradient histograms of natural images after down-sampling by 2×2 average. From the plot and the log plot, the histograms are almost the same. This demonstrates the scale invariance of natural images.

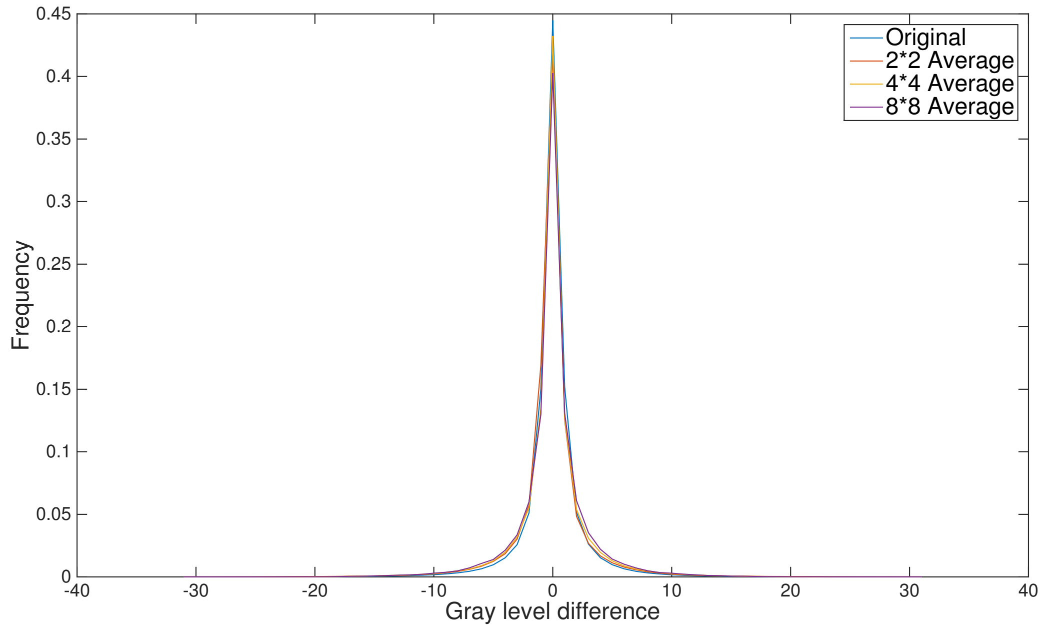

Figure 3: Gradient histogram of down-sampled natural images

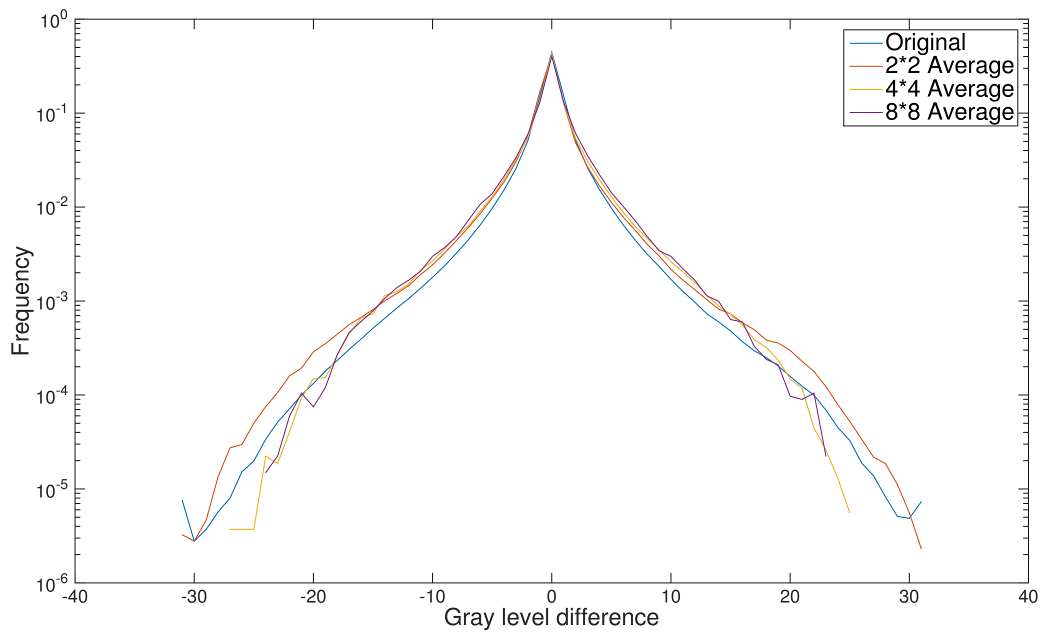

Figure 4: Gradient histogram of down-sampled natural images (log-plot)

6. Figure 5 and 6 shows the gradient histogram of noise images. Since the noise is sampled from uniform distribution, the histogram of gradients should be two lines.

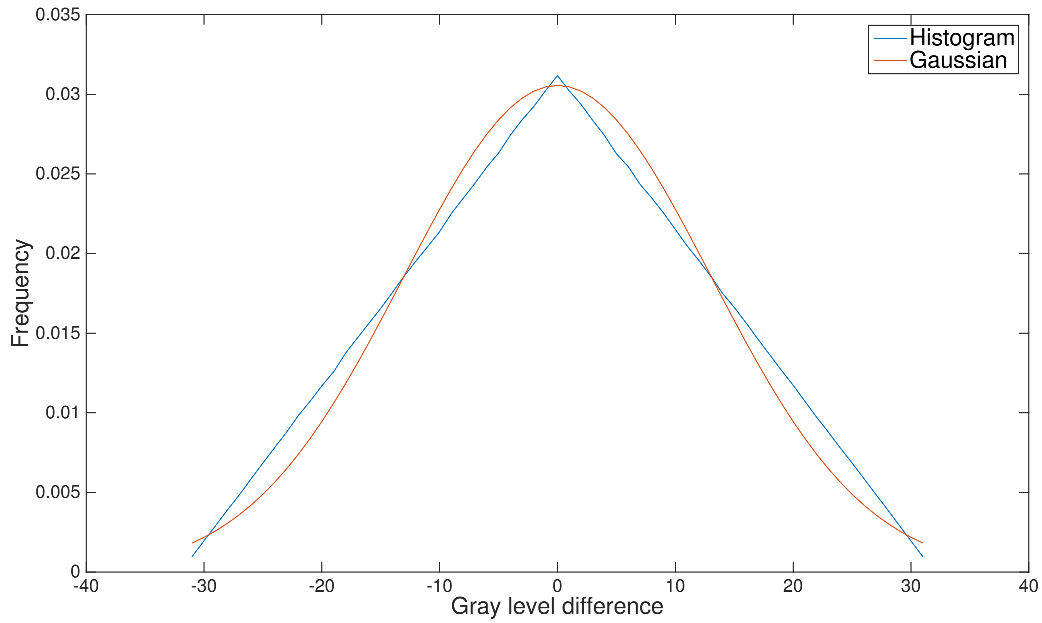

Figure 5: Gradient histogram of natural images

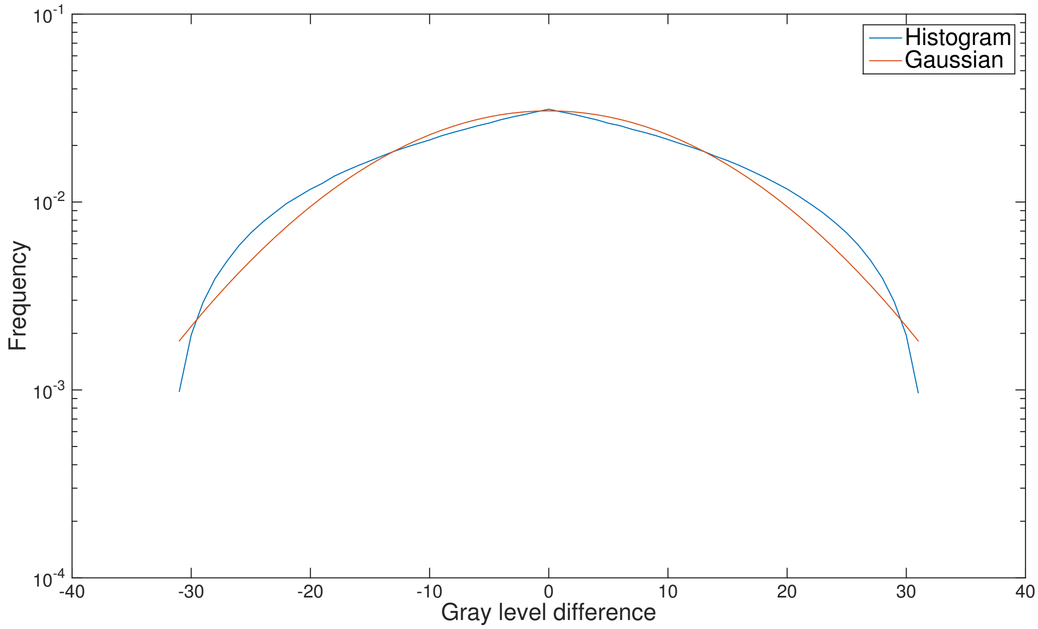

Figure 6: Gradient histogram of natural images (log-plot)

Figure 7 and 8 shows the gradient histograms of noise images after down-sampling by 2×2 average. The histograms differ a lot. So scale invariance doesn't hold for noise images.

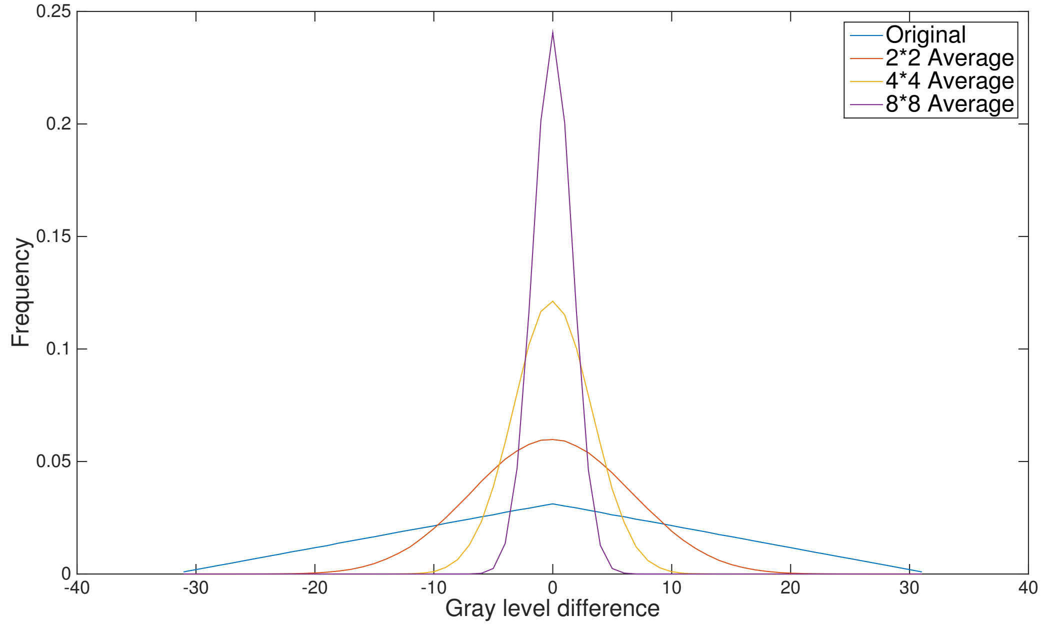

Figure 7: Gradient histogram of down-sampled noise images

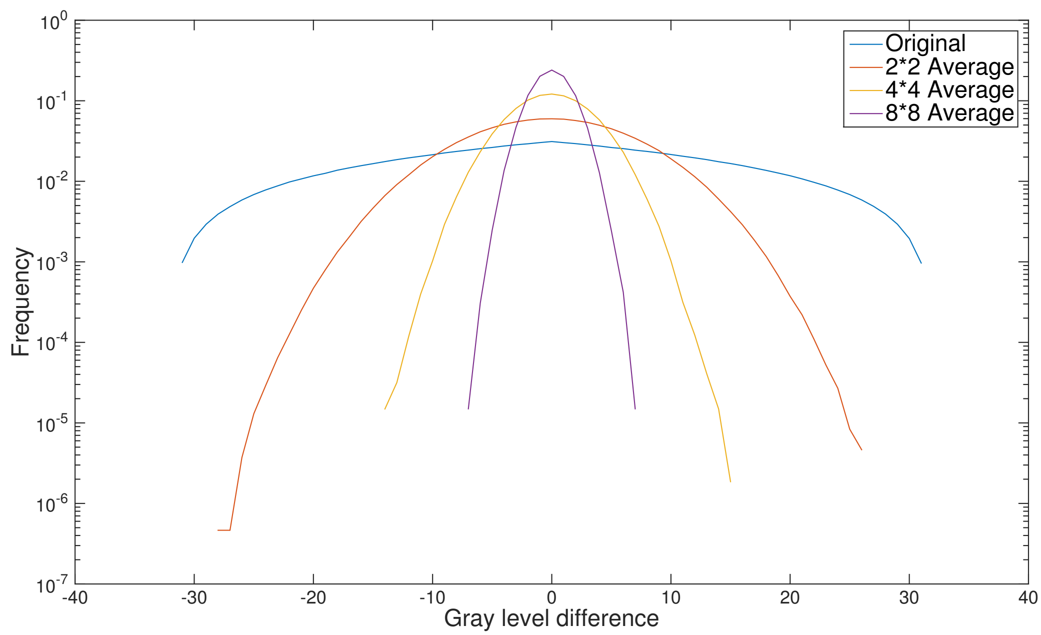

Figure 8: Gradient histogram of down-sampled noise images (log-plot)

7. Additional comparison: In Figure 9 and 10, the gradient histogram of 8×8 averaged noise is compared with the fitted gaussian distribution. This shows the law of large numbers, and further demonstrates the difference between noise images and natural images.

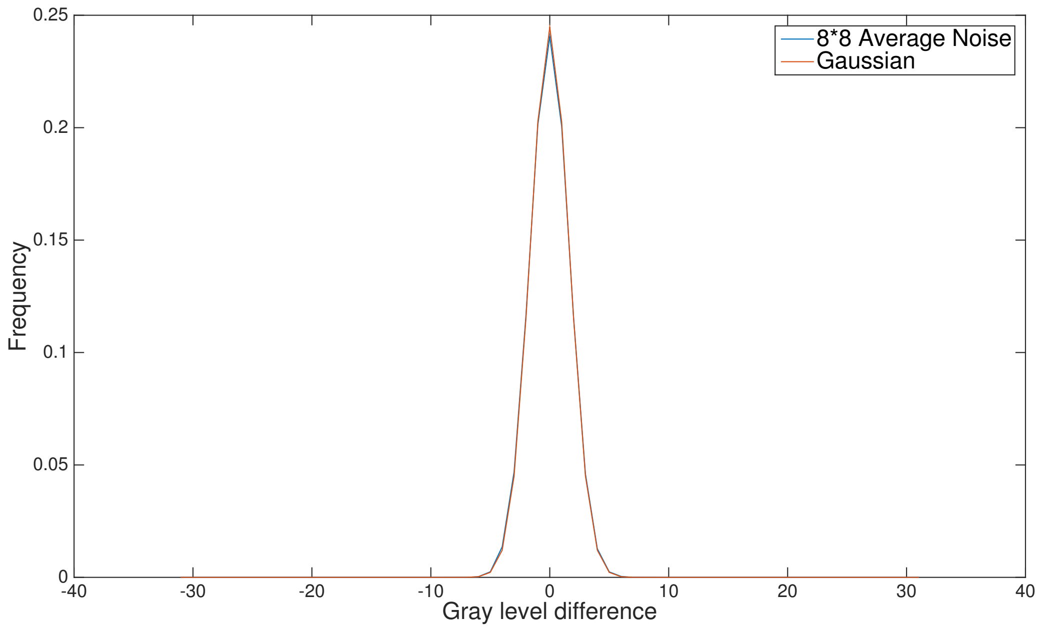

Figure 9: Gradient histogram of down-sampled noise images and fitted gaussian

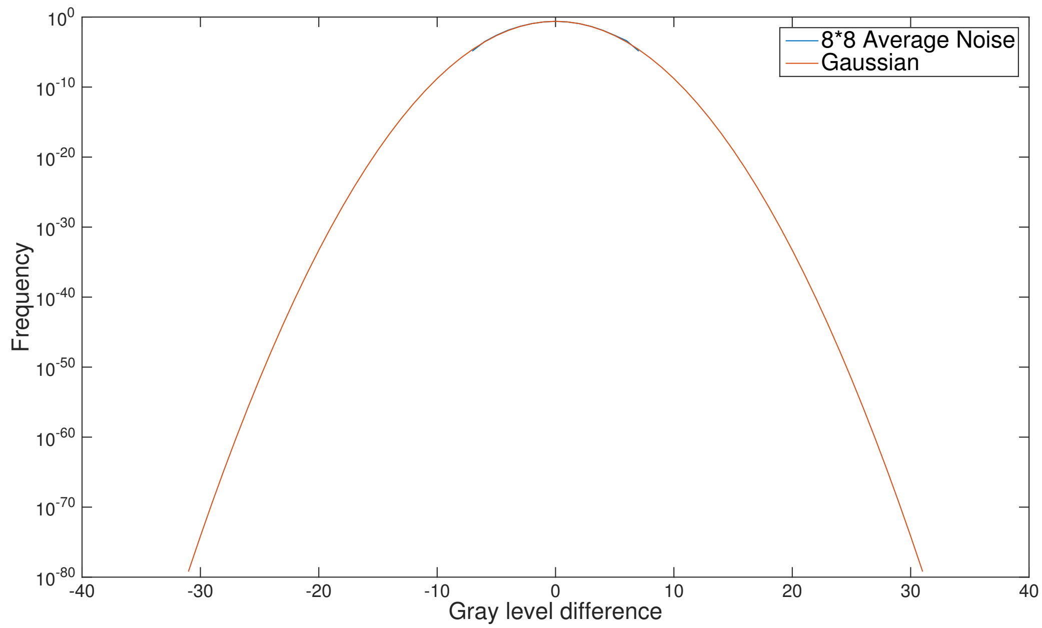

Figure 10: Gradient histogram of down-sampled noise images and fitted gaussian (log-plot)

Problem 2

1. In figure 11, $logA(f)$ against $logf$ is close to a straight line. The slope clusters to $-\frac{1}{2}$ as expected.

$$\Longrightarrow A(f) \sim \frac{1}{f^{0.5}}$$
$$\Longrightarrow A^2(f) \sim \frac{1}{f}$$
$$\Longrightarrow A^2(\xi, \eta) \sim \frac{1}{\xi^2 + \eta^2}$$
$$\Longrightarrow A(\xi, \eta) \sim \frac{1}{\sqrt{\xi^2 + \eta^2}}$$

This implies the FFT amplitude of natural images satisfies 1/f power law. Intuitively, natural images contain items of different scales (power law). If we zoom in or zoom out, the spectrum should not change.

2. Figure 12 plots the integration of Fourier power over different rings. They are close to horizontal lines as expected.

$$A^2(f) \sim \frac{1}{f} \implies S(f_0) \sim [log(2f_0) - log(f_0)] \implies S(f_0) \sim log(2)$$

which means $S(f_0)$ is a constant over f_0. This result further verifies the 1/f power law. Note that the plot is truncated to show the constant property clearly, because when f_0 is small $S(F_0)$ can be very large.

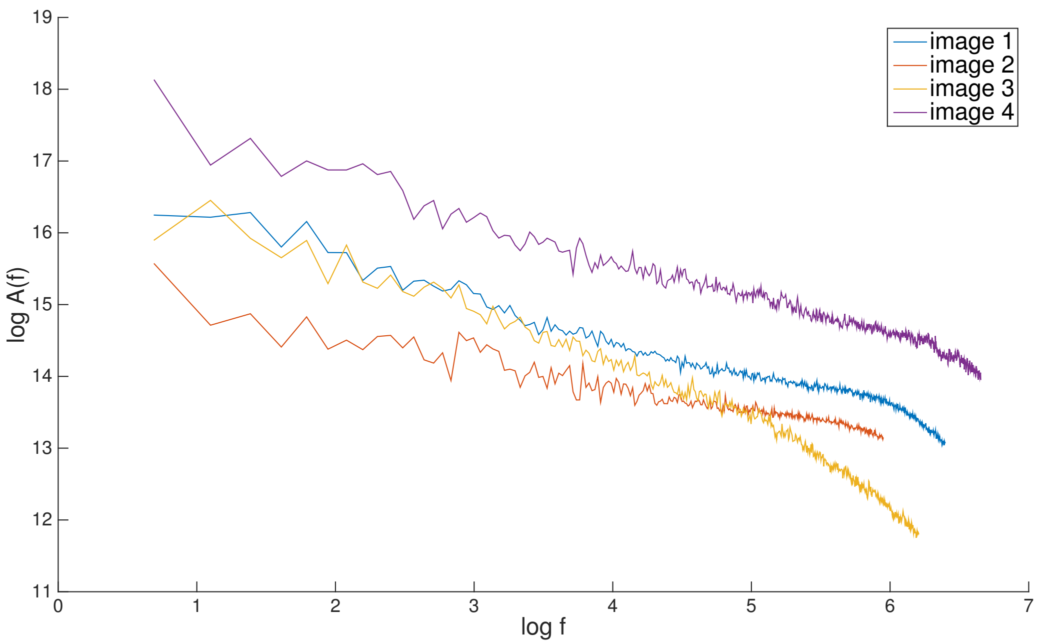

Figure 11: $log A(f)$ against $log f$.

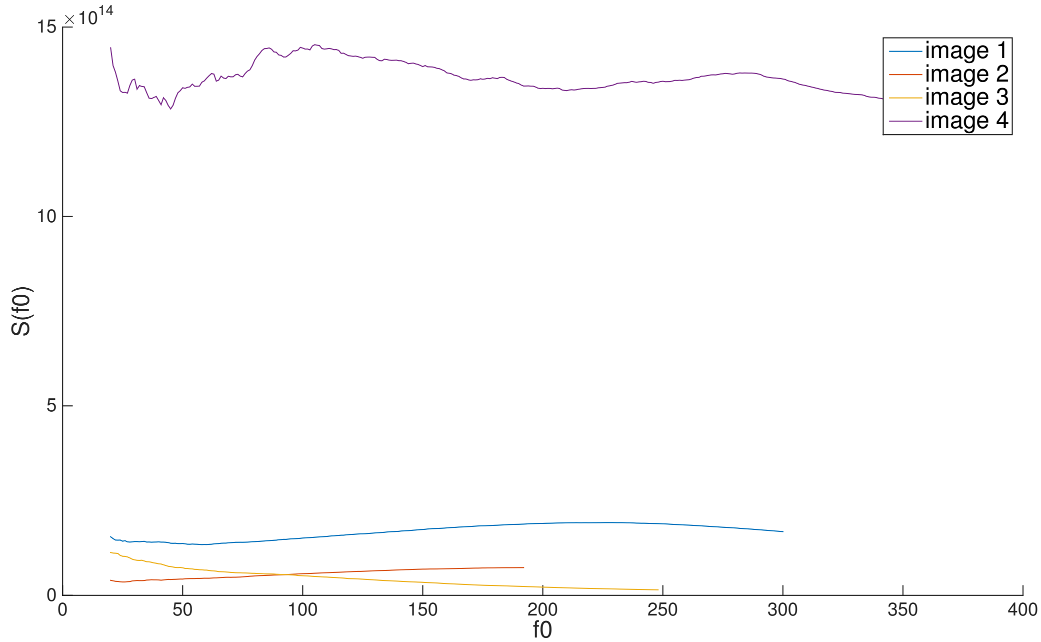

Figure 12: $S(f_0)$ against f_0.

Problem 3 From Figure 13, the 6 images look the same, which verifies the scale invariance of natural images. We cannot tell what scale the 6 images are cropped from. That is, if we zoom in and zoom out in this world, we cannot tell the difference, which is the essence of scale invariance. Note that during the implementation, down-sampling is done by changing the scale of line segments and reprinting the image. Down sampling the image directly will change the looking of the line segments.

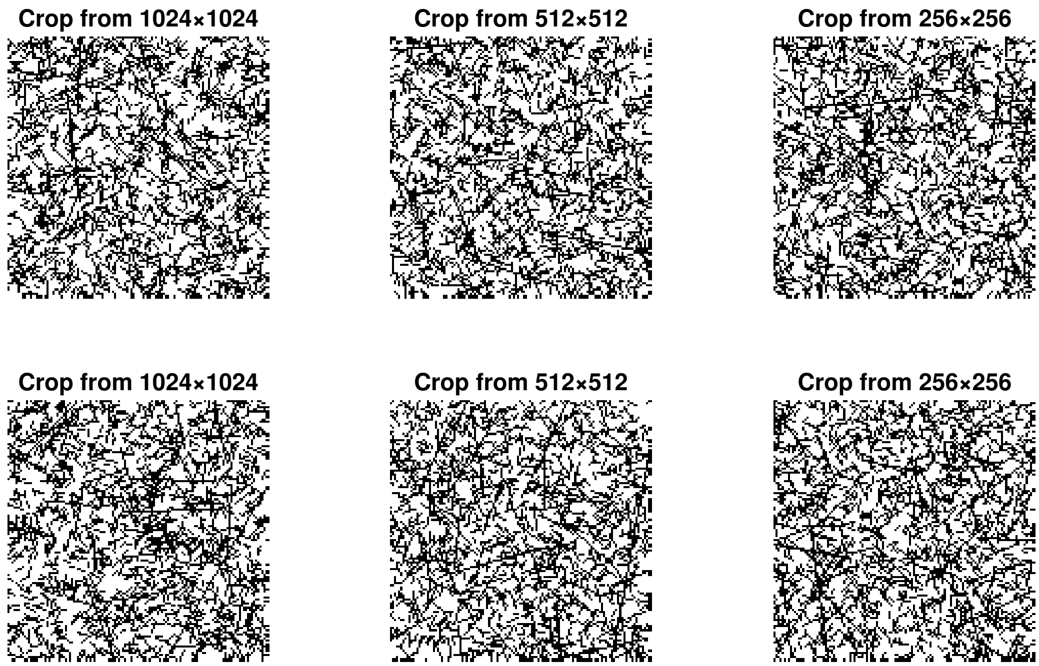

Figure 13: Cropped images from I_1, I_2, I_3