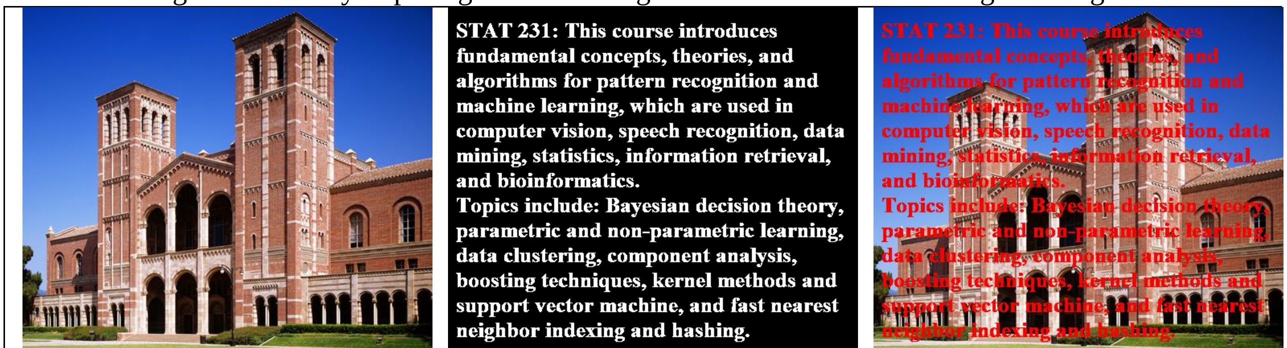
Stat 232A–CS266A : Matlab Exercise / project #2 (total 10 points)

Due Feb 10 (Wednesday) at Class meeting

Objective: This project is a warm up exercise for dealing with Gibbs/MRF models and the basics of Gibbs sampler and PDE, in a task of image restoration and inpainting. In the figure below, we show three images which are included in the zip file. The images are in bmp format so that you will not have the distortions caused by image compression. The original image has (Red, Green, Blue) bands, and the distorted image is created by imposing the mask image on the Red-band of the original image.



(a) Original image | (b) Mask image | (c) Distorted image

Now, pretend that you are only given the distorted image I in (c) and the mask image M in (b), the goal of this experiment is to recover the original image O by filling in the masked pixels in the R-band. You don't need to do anything for the Green and Blue bands. Since information has been lost in the masked pixels, you may only restore an image X, which is a good approximation to O. Therefore you need to measure the per pixel error (for all the masked pixels) between the X and O to evaluate the results.

Method 1: Gibbs sampler Let Λ be the white pixels in the mask image M (distorted in I), and $\partial\Lambda$ the boundary condition (i.e. the undistorted pixels) which will stay unchanged. We compute

$$\mathrm{X}_\Lambda | X_{\partial\Lambda} \sim \mathrm{p}(\mathrm{X}_\Lambda | X_{\partial\Lambda})$$

by sampling from a Gibbs or MRF model of the following form.

$$\mathrm{p}(\mathrm{X}_\Lambda | X_{\partial\Lambda}) = \frac{1}{\mathrm{Z}} \exp\{-\beta \textstyle\sum_{\{(x,y)\in\Lambda\}} \mathrm{E}\big(\nabla_x \mathrm{X}(x,y)\big) + \mathrm{E}\Big(\nabla_y \mathrm{X}(x,y)\Big)\ \}.$$

In the above Gibbs model, E() is a potential energy, we need to try two choices of functions

$$\mathrm{E}\big(\nabla_x \mathrm{X}(x,y)\big) = |\ \nabla_x \mathrm{X}(x,y)\ | ------ (1) \text{ i.e. the } \mathrm{L}._1 - \text{norm}$$

$$\mathrm{E}\big(\nabla_x \mathrm{X}(x,y)\big) = \big(\nabla_x \mathrm{X}(x,y)\big)^2 ------ (2) \text{ i.e. the } \mathrm{L}._2 - \text{norm}$$

From this Gibbs models, we can derive the local characteristics, given al

$$\mathrm{X}_s | X_{\partial s} \sim \mathrm{p}(\mathrm{X}_s | X_{\partial s}), \qquad \forall\ s \in \Lambda$$

By drawing from this 1D probability (you draw random numbers in [0,1] and then use it to draw from this 1D probability), we can iteratively get the values of the distorted pixels. Visiting all distorted pixels once is called 1-sweep (in whatever order, does not matter). You need to experiment with an annealing scheme by increasing the parameter β slowly from 0.1 to 1 (or more).

Method 2: PDE. For $\mathrm{L}._2 -$ norm energy functions, you can minimize the energy by the heat-diffusion equation.

Hand-in: i) Plot the per pixel error $\mathrm{Err} = \frac{1}{|\Lambda|}\sum\big(\mathrm{X}(x,y) - \mathrm{O}(x,y)\big)^2$ over the number of sweeps t (up to 20 sweeps) in method 1 and number of iterations in method 2. ii) Show the restored results.

Remark: The green, blue bands may be distorted when they are integrated with the R band (through a renormalization or color map), try to resolve this if you know how.