ECE408 / CS483 / CSE408
Summer 2025

Applied Parallel Programming

Lecture 14: Parallel Sparse Methods

What Will You Learn Today?

parallel sparse matrix methods

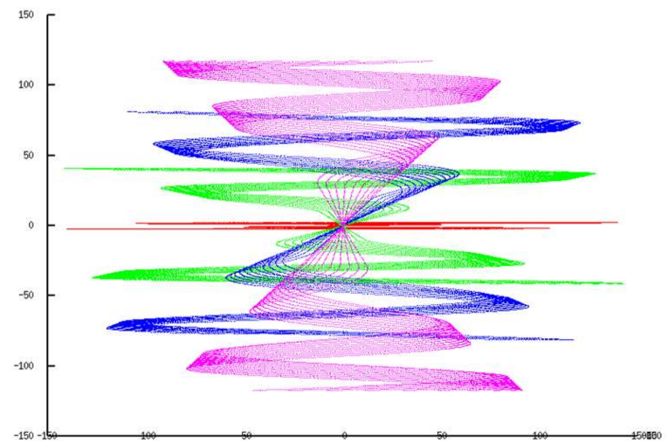
- key techniques for compacting input data
- benefits of sparse methods
  - reducing consumption of memory bandwidth
  - better utilization of on-chip memory
  - fewer bytes transferred to on-chip memory
  - better utilization of global memory
- challenge: retaining regularity

Sparse Matrix

- Many real-world systems are sparse in nature
  - Linear systems described as sparse matrices
- Solving sparse linear systems
  - Traditional inversion algorithms such as Gaussian elimination can create too many “fill-in” elements and explode the size of the matrix
  - Iterative Conjugate Gradient solvers based on sparse matrix-vector multiplication is preferred
- Solution of PDE systems can be formulated into linear operations expressed as sparse matrix-vector multiplication

Sparse Data
Motivation for Compaction

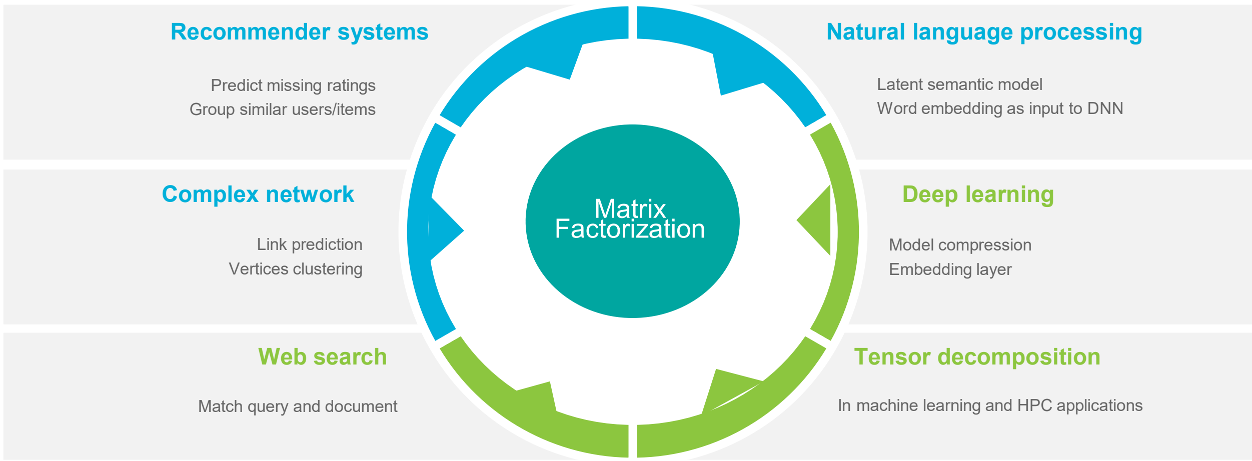
- Many real-world inputs are sparse/non-uniform
- Signal samples, mesh models, transportation networks, communication networks, etc.

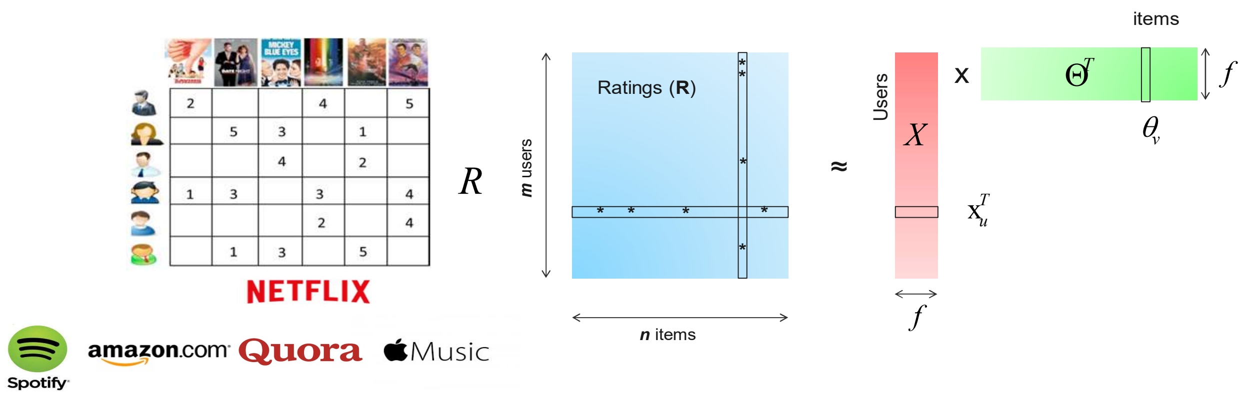


Sparse Matrix in Scientific Computing

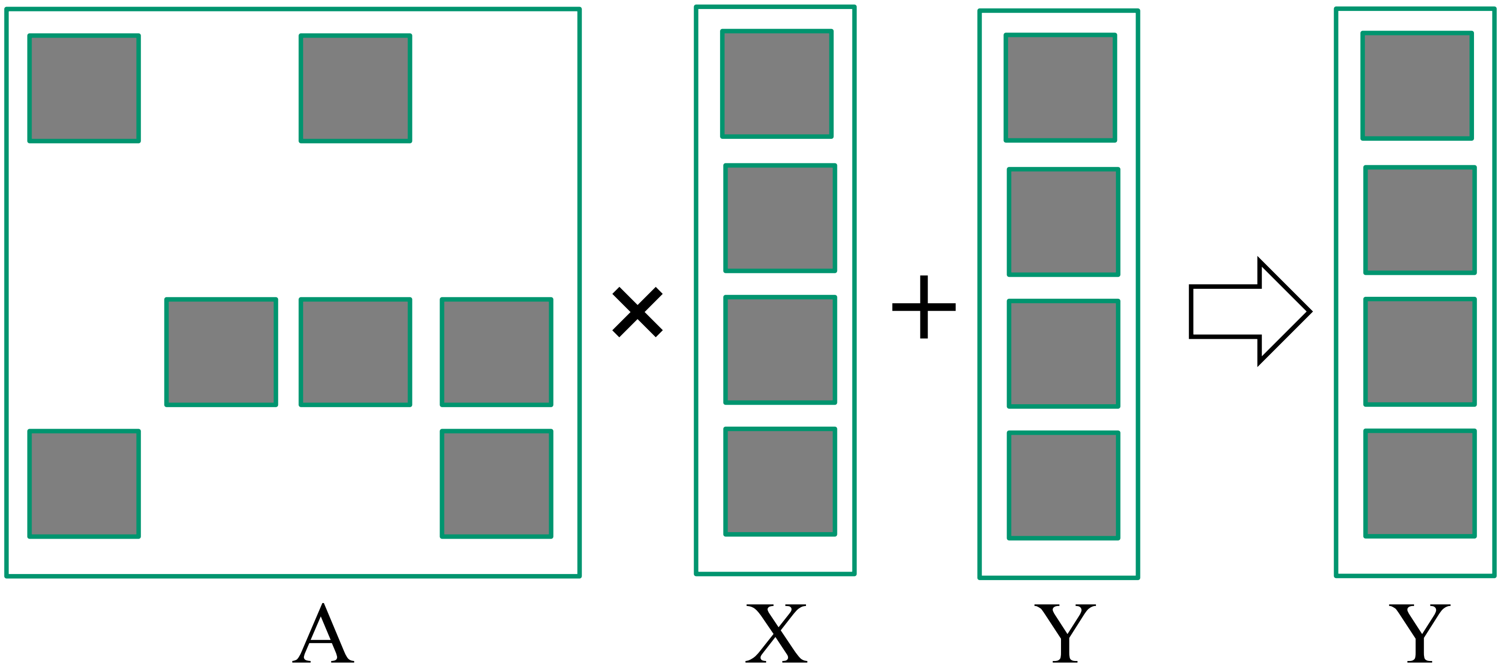
Science Area	Number of Teams	Codes	Struct Grids	Unstruct Grids	Dense Matrix	Sparse Matrix	N-Body	Monte Carlo	FFT	PIC	Sig I/O
Climate and Weather	3	CESM, GCRM, CM1/WRF, HOMME	X	X		X		X			X
Plasmas/Magnetosphere	2	H3D(M),VPIC, OSIRIS, Magtail/UPIC	X				X		X		X
Stellar Atmospheres and Supernovae	5	PPM, MAESTRO, CASTRO, SEDONA, ChaNGa, MS-FLUKSS	X			X	X	X		X	X
Cosmology	2	Enzo, pGADGET	X			X	X				
Combustion/Turbulence	2	PSDNS, DISTUF	X						X		
General Relativity	2	Cactus, Harm3D, LazEV	X			X					
Molecular Dynamics	4	AMBER, Gromacs, NAMD, LAMMPS				X	X		X		
Quantum Chemistry	2	SIAL, GAMESS, NWChem			X	X	X	X			X
Material Science	3	NEMOS, OMEN, GW, QMCPACK			X	X	X	X			
Earthquakes/Seismology	2	AWP-ODC, HERCULES, PLSQR, SPECFEM3D	X	X			X				X
Quantum Chromo Dynamics	1	Chroma, MILC, USQCD	X		X	X					
Social Networks	1	EPISIMDEMICS									
Evolution	1	Eve									
Engineering/System of Systems	1	GRIPS,Revisit						X			
Computer Science	1			X	X	X			X		X

Sparse Matrix in Analytics and AI





Sparse Matrix-Vector Multiplication (SpMV)



Challenges

- Compared to dense matrix multiplication, SpMV
  - Is irregular/unstructured
  - Has little input data reuse
  - Benefits little from compiler transformation tools
- Key to maximal performance
  - Maximize regularity (by reducing divergence and load imbalance)
  - Maximize DRAM burst utilization (layout arrangement)

A Simple Parallel SpMV

Row 0	3	0	1	0	Thread 0
Row 1	0	0	0	0	Thread 1
Row 2	0	2	4	1	Thread 2
Row 3	1	0	0	1	Thread 3

- Each thread processes one row

Compressed Sparse Row (CSR) Format

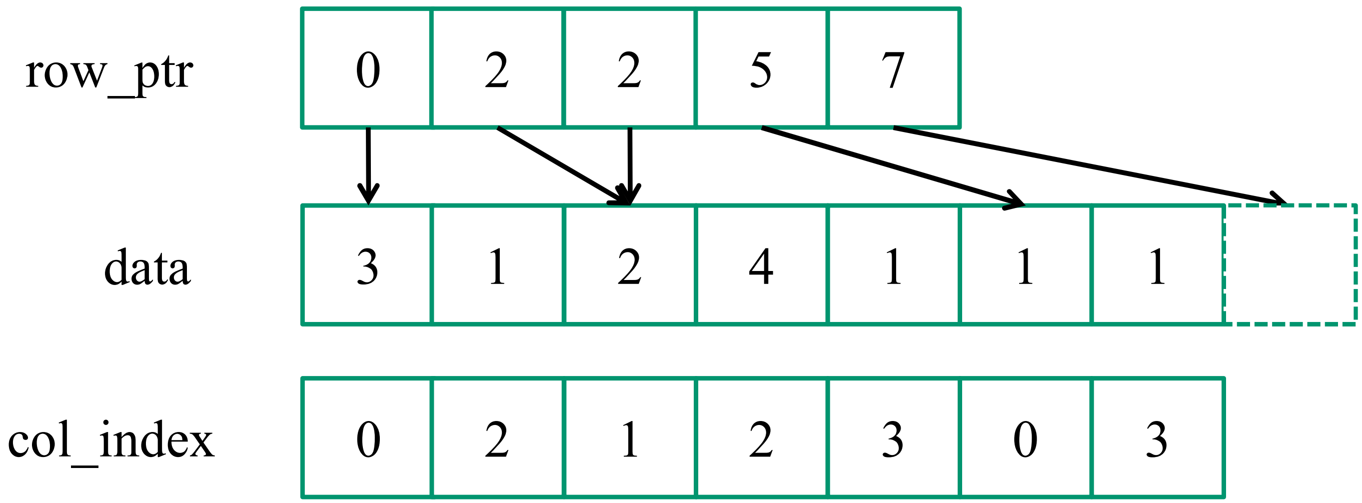
CSR Representation

		Row 0		Row 2			Row 3	
Nonzero values data[7]	{	3, 1,		2, 4, 1,			1, 1	}
Column indices col_index[7]	{	0, 2,		1, 2, 3,			0, 3	}
Row Pointers ptr[5]	{	0, 2,		2, 5, 7			}	

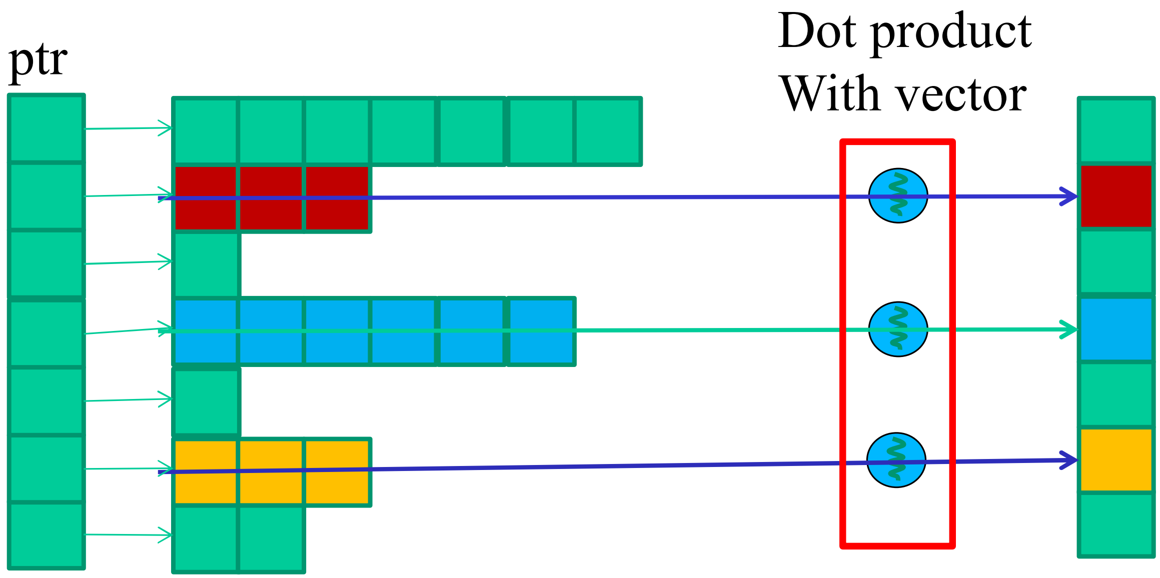
Dense representation

Row 0	3	0	1	0	Thread 0
Row 1	0	0	0	0	Thread 1
Row 2	0	2	4	1	Thread 2
Row 3	1	0	0	1	Thread 3

CSR Data Layout



CSR Kernel Design



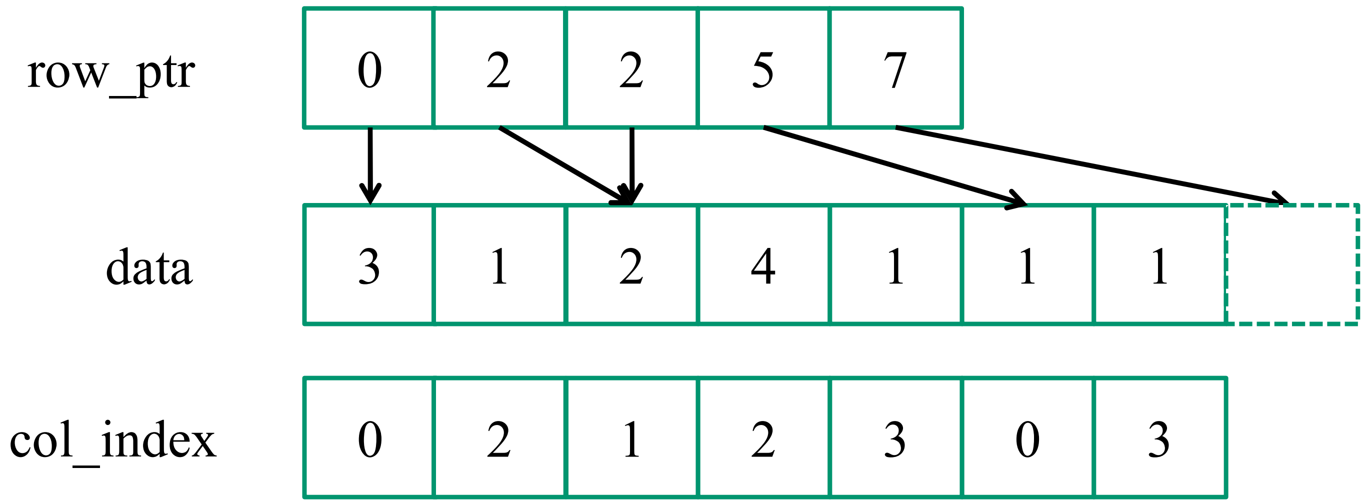
A Parallel SpMV/CSR Kernel (CUDA)

```
__global__ void SpMV_CSR(int num_rows, float *data,
    int *col_index, int *row_ptr, float *x, float *y) {
     int row = blockIdx.x * blockDim.x + threadIdx.x;
     if (row < num_rows) {
       float dot = 0;
       int row_start = row_ptr[row];
       int row_end =   row_ptr[row+1];
       for (int elem = row_start; elem < row_end; elem++) {
         dot += data[elem] * x[col_index[elem]];
       }
       y[row] = dot;
     }
   }
```

		Row 0	Row 2	Row 3
Nonzero values	data[7]	{ 3, 1,	2, 4, 1,	1, 1 }
Column indices	col_index[7]	{ 0, 2,	1, 2, 3,	0, 3 }
Row Pointers	row_ptr[5]	{ 0, 2, 2, 5, 7 }		

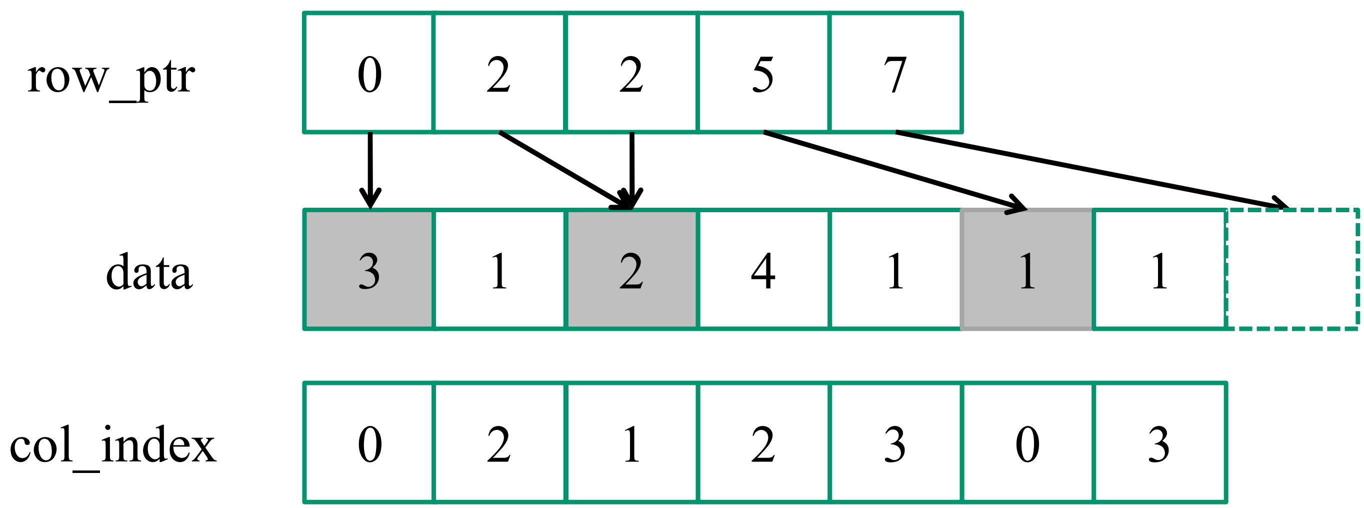
CSR Kernel Control Divergence

- Threads execute different number of iterations in the kernel for-loop

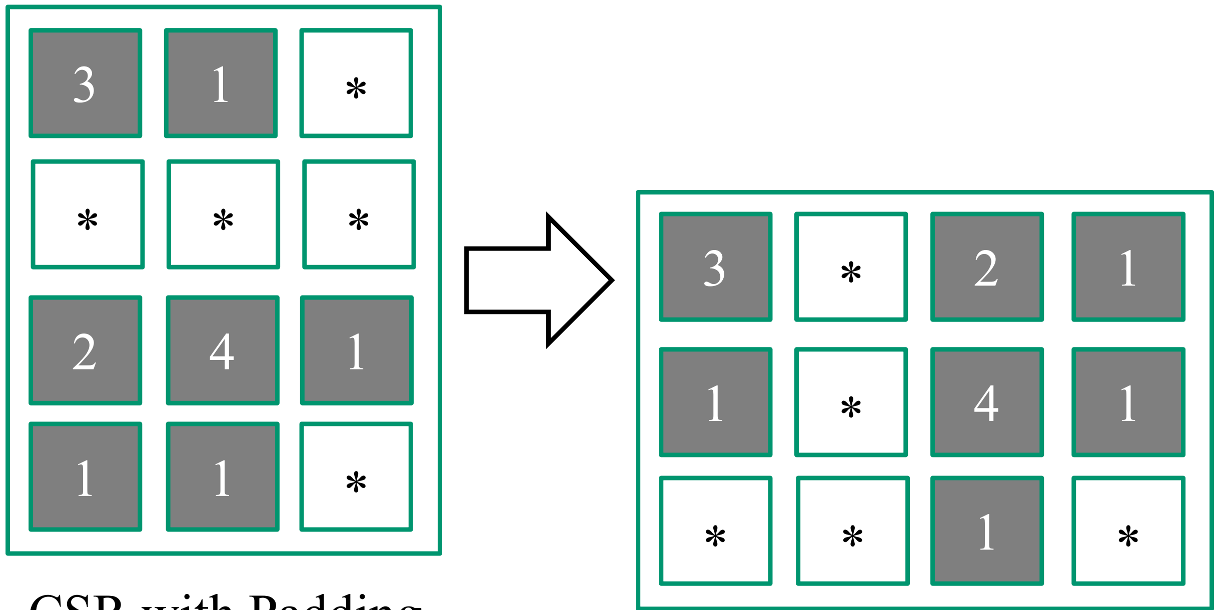


CSR Kernel Memory Divergence (Uncoalesced Accesses)

- Adjacent threads access non-adjacent memory locations
  - Grey elements are accessed by all threads in iteration 0



Regularizing SpMV with ELL(PACK) Format

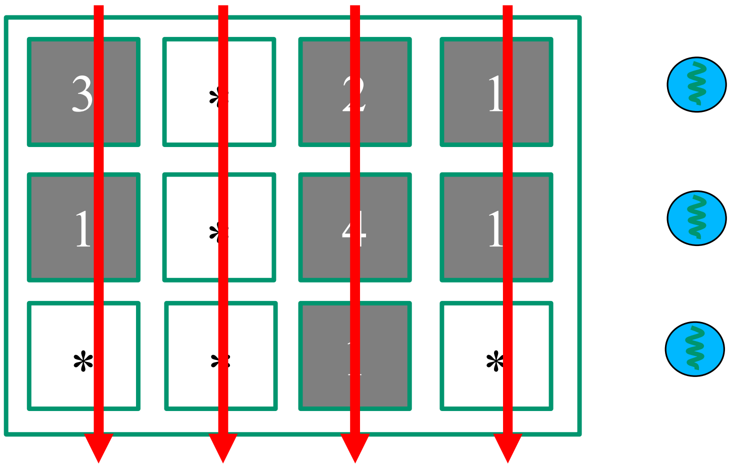


CSR with Padding

Transposed

- Pad all rows to the same length
  - Inefficient if a few rows are much longer than others
- Transpose (Column Major) for DRAM efficiency
- Both data and col_index padded/transposed

ELL Kernel Design

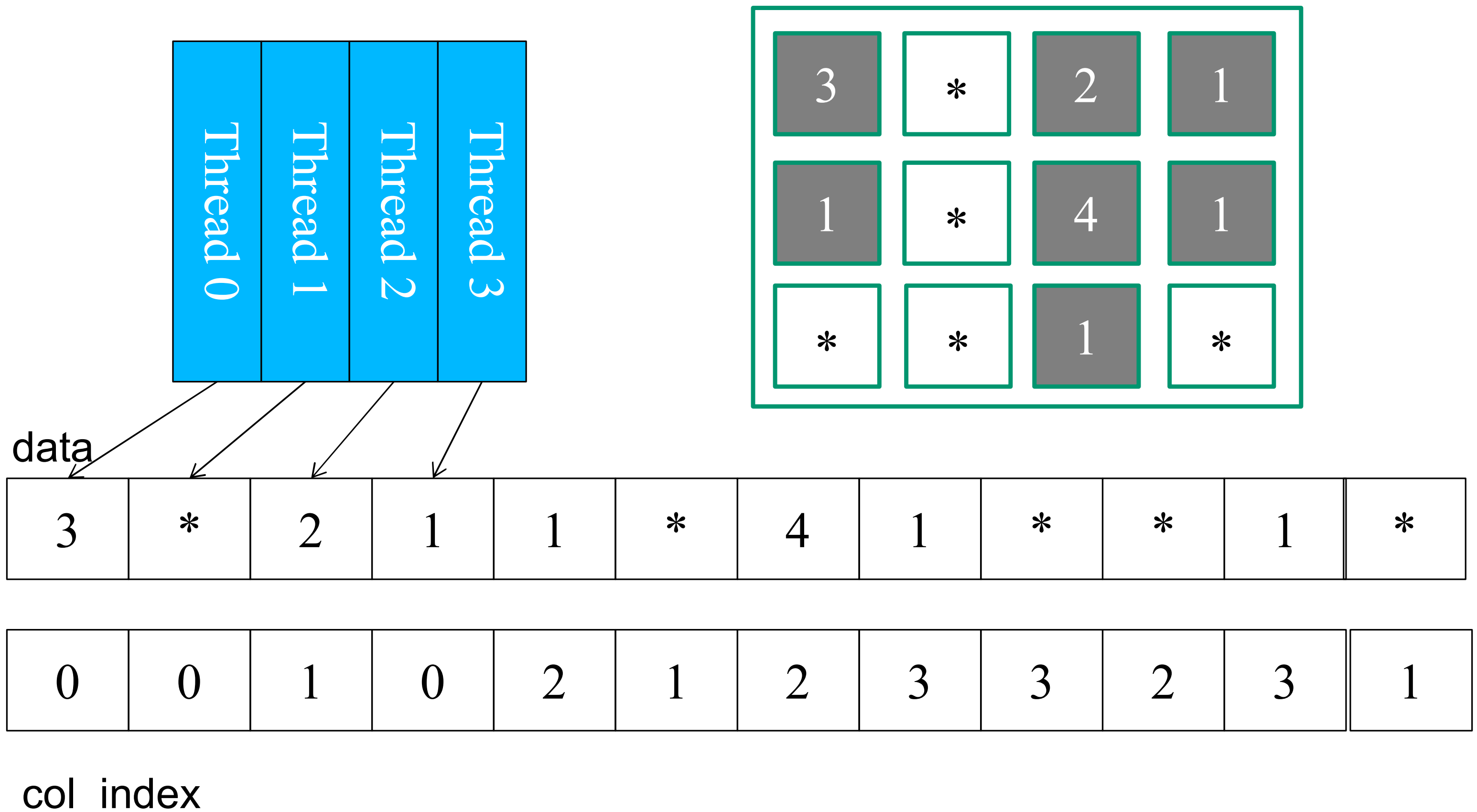


A parallel SpMV/ELL kernel

```
__global__ void SpMV_ELL(int num_rows, float *data,
     int *col_index, int num_elem, float *x, float *y) {

  int row = blockIdx.x * blockDim.x + threadIdx.x;
  if (row < num_rows) {
    float dot = 0;
    for (int i = 0; i < num_elem; i++) {
      dot += data[row+i*num_rows]*x[col_index[row+i*num_rows]];
    }
    y[row] = dot;
  }
}
```

Memory Coalescing with ELL



Coordinate (COO) format

- Explicitly list the column and row indices for every non-zero element

		Row 0		Row 2			Row 3	
Nonzero values	data[7]	{ 3,	1,	2,	4,	1,	1,	1 }
Column indices	col_index[7]	{ 0,	2,	1,	2,	3,	0,	3 }
Row indices	row_index[7]	{ 0,	0,	2,	2,	2,	3,	3 }

COO Allows Reordering of Elements

		Row 0		Row 2			Row 3		
Nonzero values	data[7] {	3,	1,	2,	4,	1,	1,	1	}
Column indices	col_index[7] {	0,	2,	1,	2,	3,	0,	3	}
Row indices	row_index[7] {	0,	0,	2,	2,	2,	3,	3	}

Nonzero values	data[7] {	1	1,	2,	4,	3,	1	1	}
Column indices	col_index[7] {	0	2,	1,	2,	0,	3,	3	}
Row indices	row_index[7] {	3	0,	2,	2,	0,	2,	3	}

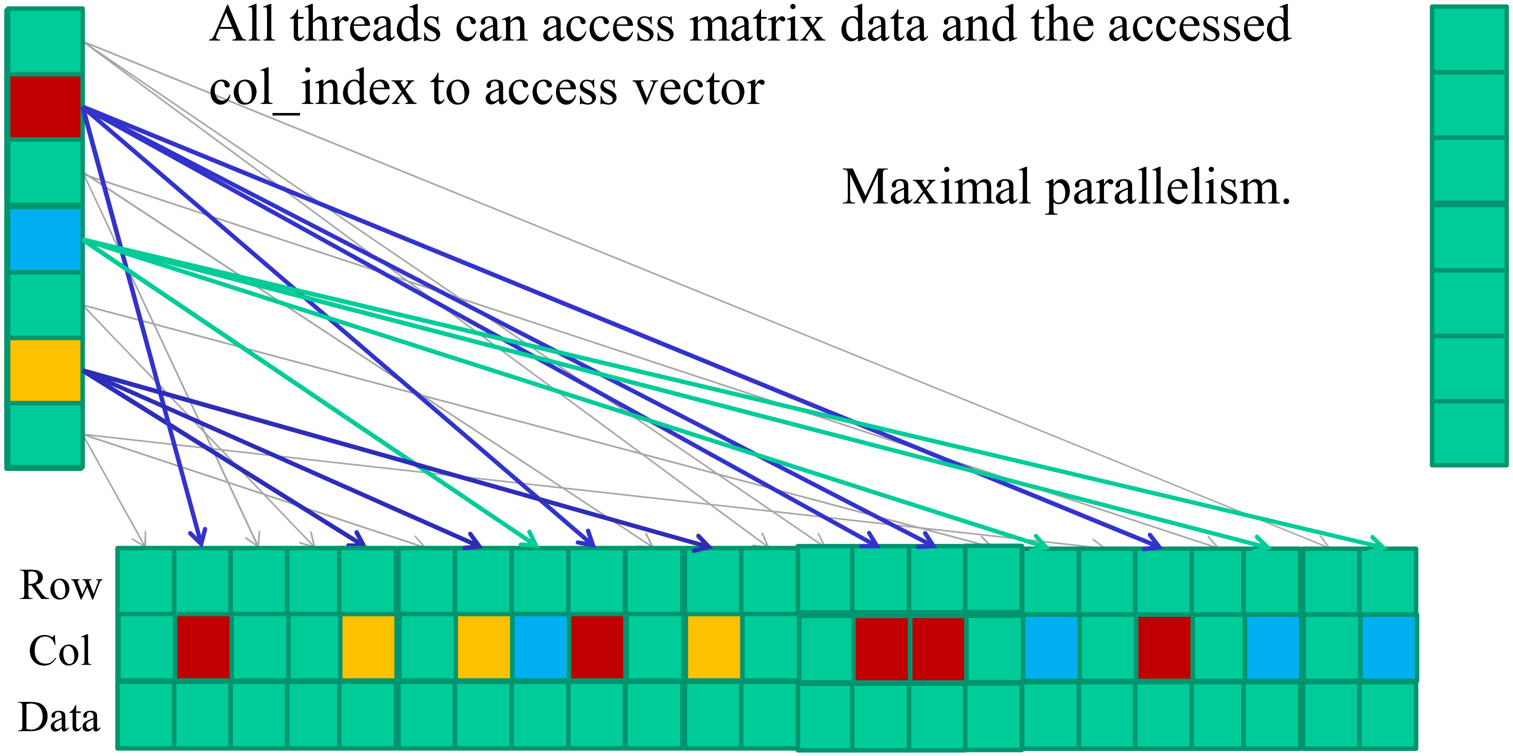
```
for (int i = 0; i < num_elem; row++)
  y[row_index[i]] += data[i] * x[col_index[i]];
```

a sequential loop that implements SpMV/COO

COO Kernel Design
Accessing Input Matrix and Vector

All threads can access matrix data and the accessed col_index to access vector

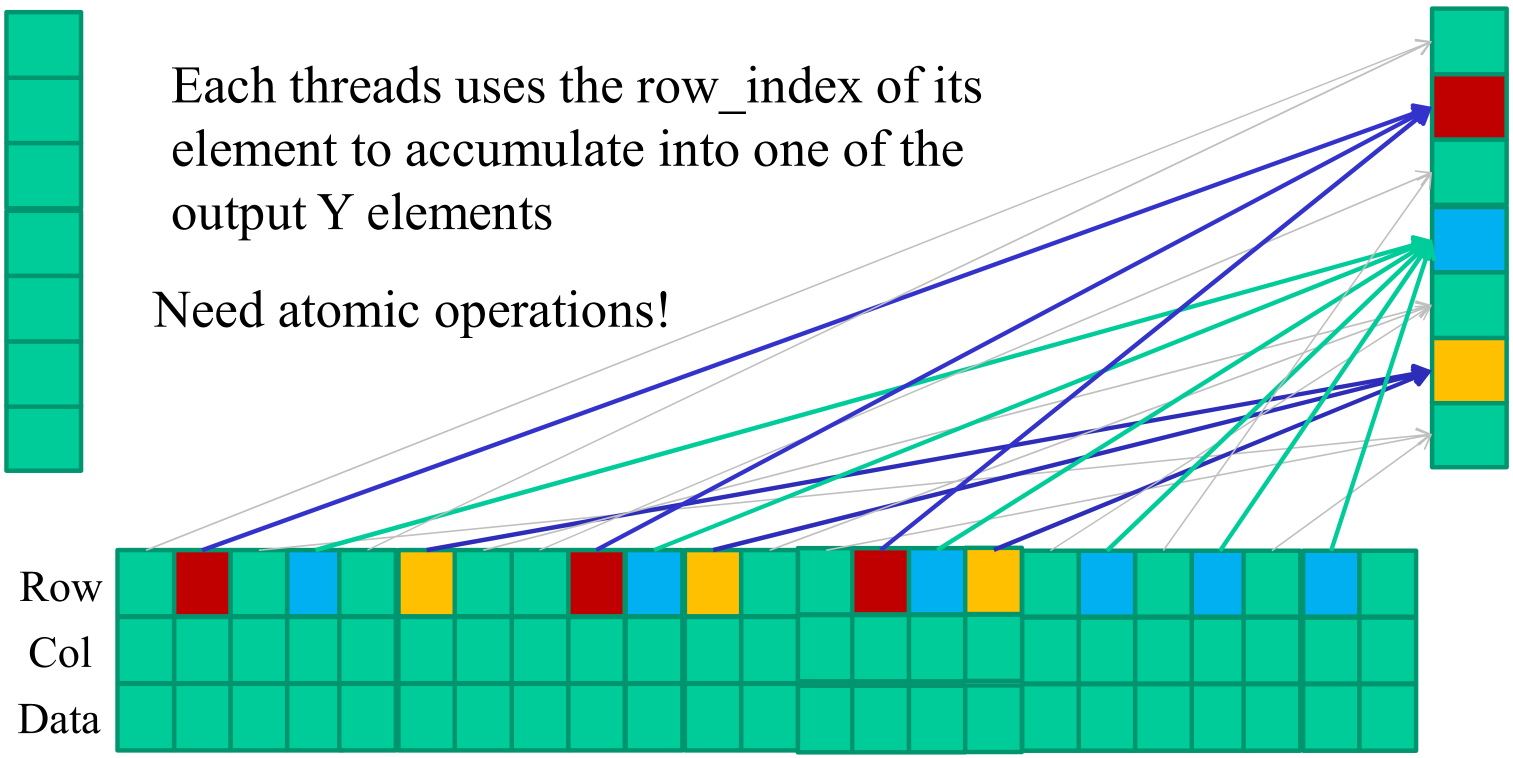
Maximal parallelism.



COO kernel Design
Accumulating into Output Vector

Each threads uses the row_index of its element to accumulate into one of the output Y elements

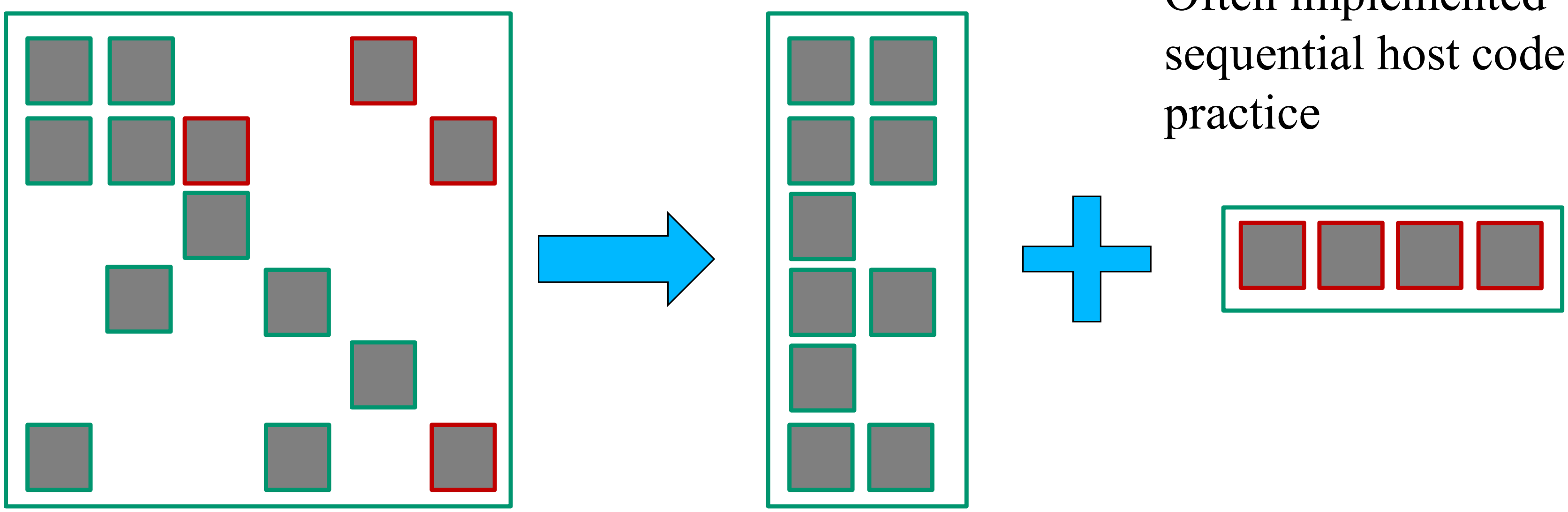
Need atomic operations!



Hybrid Format (ELL + COO)

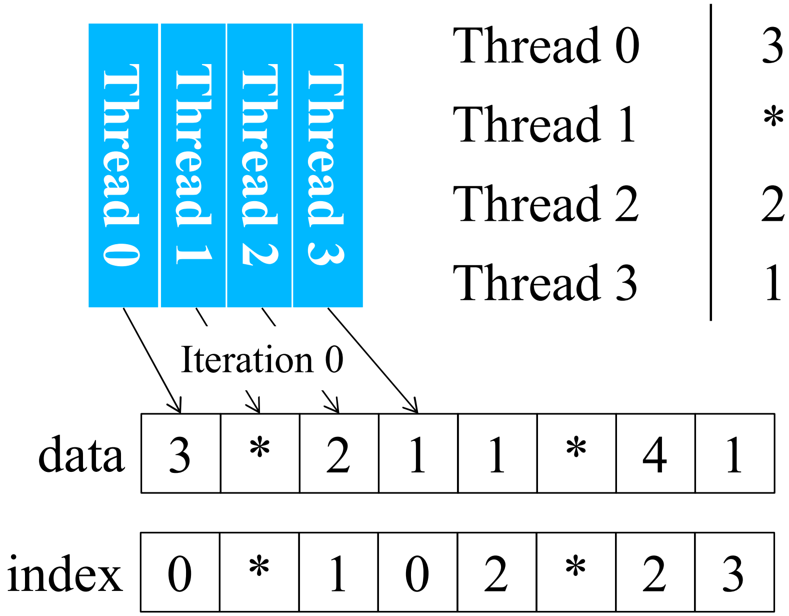
- ELL handles *typical* entries
- COO handles *exceptional* entries
  - Implemented with segmented reduction

Often implemented in sequential host code in practice



Reduced Padding with Hybrid Format

	data		col_index	
Thread 0	3	1	0	2
Thread 1	*	*	*	*
Thread 2	2	4	1	2
Thread 3	1	1	0	3



ELL

data	1
col_index	3
row_index	2

COO

QUESTIONS?

READ CHAPTER 10!

Problem Solving

- Q: Given matrix A,
  which of the following are correct?

$$A = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 1 \end{vmatrix}$$

CSR representation:

Data = [1,2,1,1,2,3,4,1,1]

Col_idx = [0,2,3,0,1,2,3,0,3]

Row_ptr = [0,1,3,7,9]

COO representation

Data = [1,2,1,1,2,3,4,1,1]

Col_idx = [0,2,3,0,1,2,3,0,3]

Row_idx = [0,1,1,3,3,3,3,7,7]

- A: only CSR (COO row indices are incorrect)