# ECE408 / CS483 / CSE408
# Summer 2025

# Applied Parallel Programming

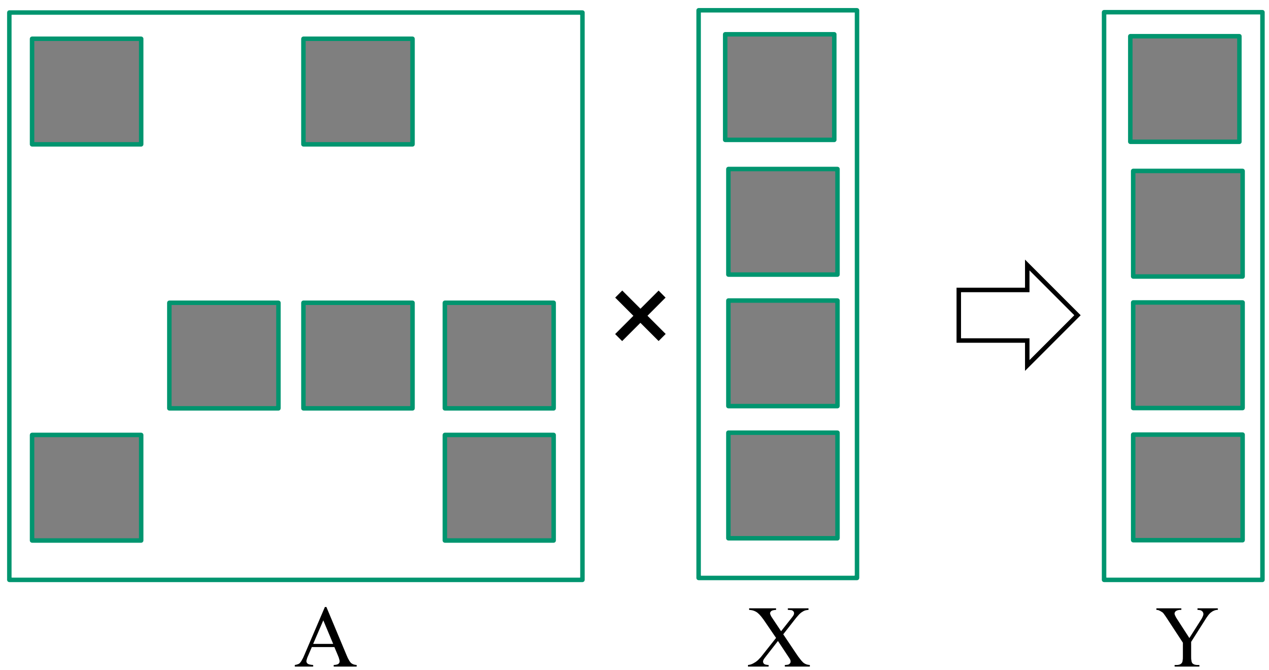
# Lecture 15: Parallel Sparse Methods (Part 2)

# What Will You Learn Today?

to regularize irregular data by
- limiting variations with clamping,
- sorting, and
- transposition

to write
- a high-performance SpMV kernel
- based on JDS transposed format

# Review: Sparse Matrix-Vector Multiplication (SpMV)



# Review: Compressed Sparse Row (CSR) Format

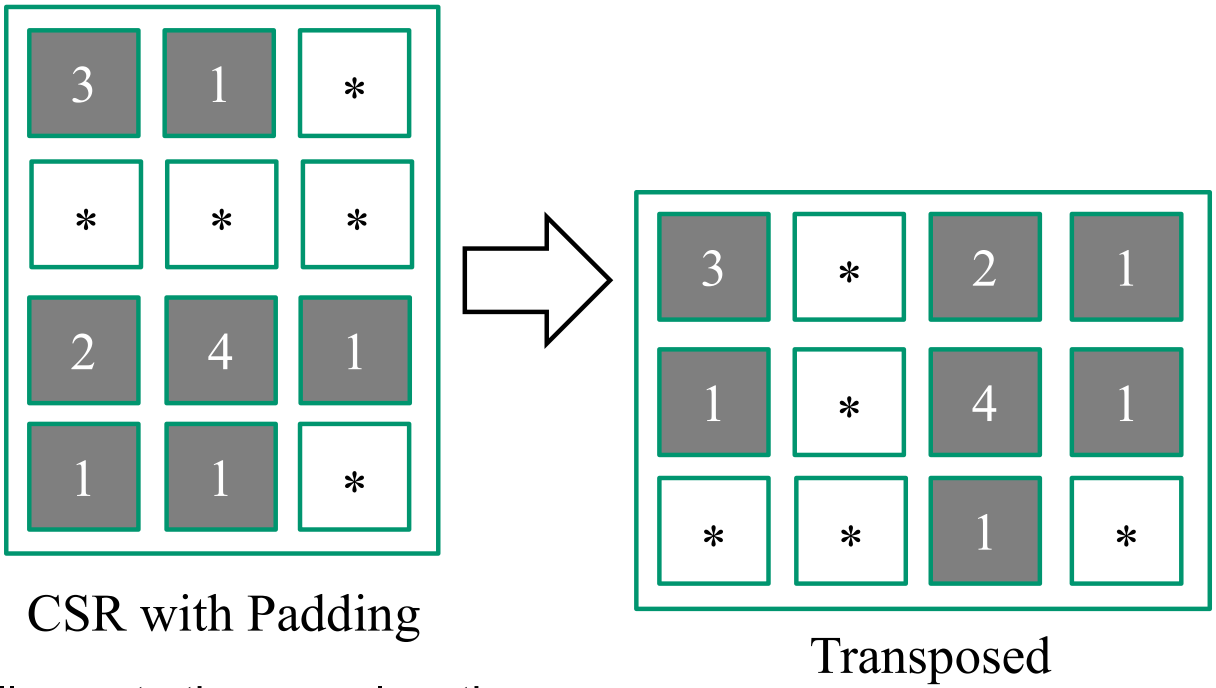
**CSR Representation**

| | | Row 0 | | Row 2 | | | Row 3 | |
|---|---|---|---|---|---|---|---|---|
| Nonzero values | `data[7]` | { 3, | 1, | 2, | 4, | 1, | 1, | 1 } |
| Column indices | `col_index[7]` | { 0, | 2, | 1, | 2, | 3, | 0, | 3 } |
| Row Pointers | `row_ptr[5]` | { 0, | 2, | 2, | 5, | 7 } | | |

**Dense representation**

| | | | | | |
|---|---|---|---|---|---|
| Row 0 | 3 | 0 | 1 | 0 | Thread 0 |
| Row 1 | 0 | 0 | 0 | 0 | Thread 1 |
| Row 2 | 0 | 2 | 4 | 1 | Thread 2 |
| Row 3 | 1 | 0 | 0 | 1 | Thread 3 |

# Review: Regularizing SpMV with ELL(PACK) Format




- Pad all rows to the same length
  - Inefficient if a few rows are much longer than others
- Transpose (Column Major) for DRAM efficiency
- Both data and col_index padded/transposed

# Review: Coordinate (COO) format

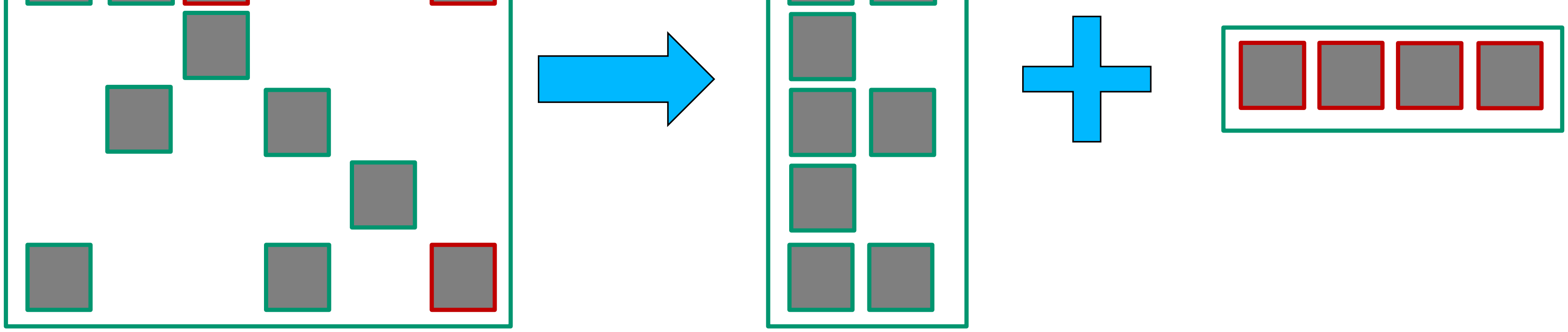
- Explicitly list the column and row indices for every non-zero element

| | | Row 0 | | Row 2 | | | Row 3 | |
|---|---|---|---|---|---|---|---|---|
| Nonzero values | data[7] | { 3, | 1, | 2, | 4, | 1, | 1, | 1 } |
| Column indices | col_index[7] | { 0, | 2, | 1, | 2, | 3, | 0, | 3 } |
| Row indices | row_index[7] | { 0, | 0, | 2, | 2, | 2, | 3, | 3 } |

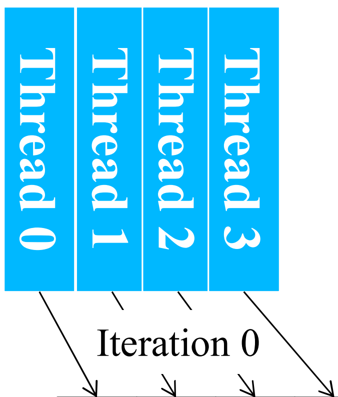
# Review: Hybrid Format (ELL + COO)

- ELL handles *typical* entries
- COO handles *exceptional* entries
  - Implemented with segmented reduction

Often implemented in sequential host code in practice



# Review: Reduced Padding with Hybrid Format




| | data | | col_index | |
|---|---|---|---|---|
| Thread 0 | 3 | 1 | 0 | 2 |
| Thread 1 | * | * | * | * |
| Thread 2 | 2 | 4 | 1 | 2 |
| Thread 3 | 1 | 1 | 0 | 3 |

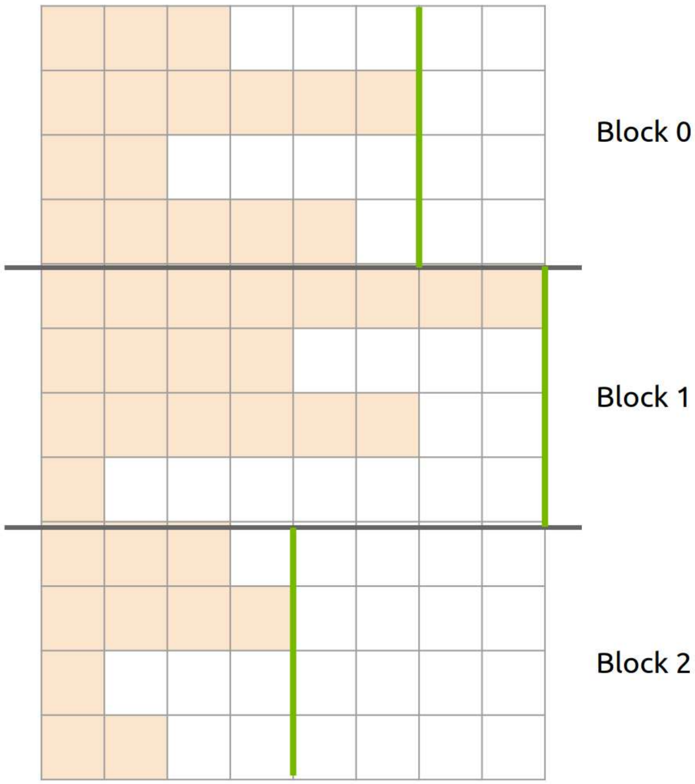
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| data | 3 | * | 2 | 1 | 1 | * | 4 | 1 |
| index | 0 | * | 1 | 0 | 2 | * | 2 | 3 |

**ELL**

| | |
|---|---|
| data | 1 |
| col_index | 3 |
| row_index | 2 |

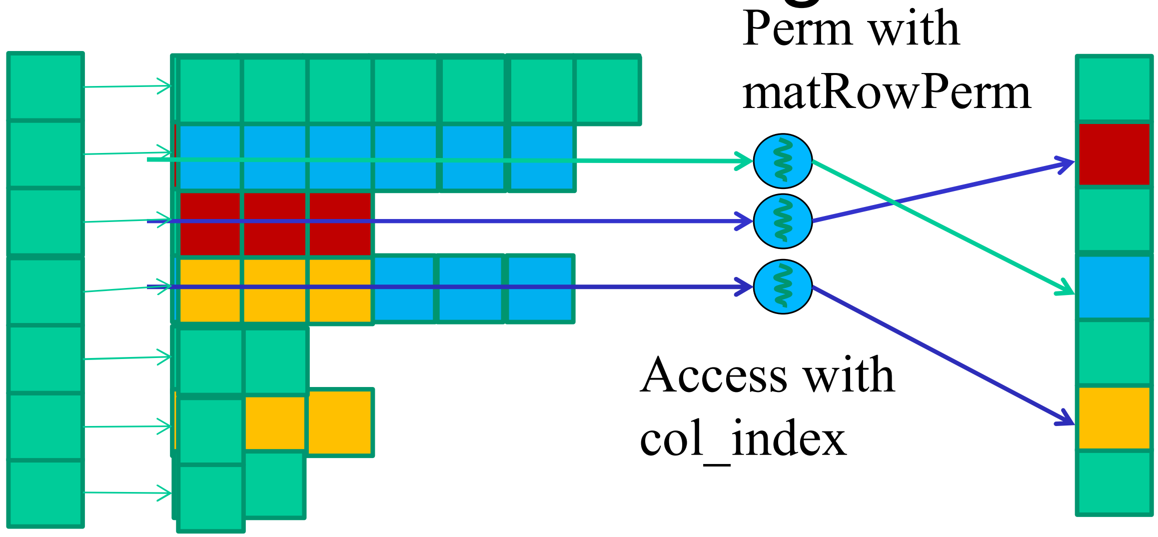
**COO**

# CSR Run-time



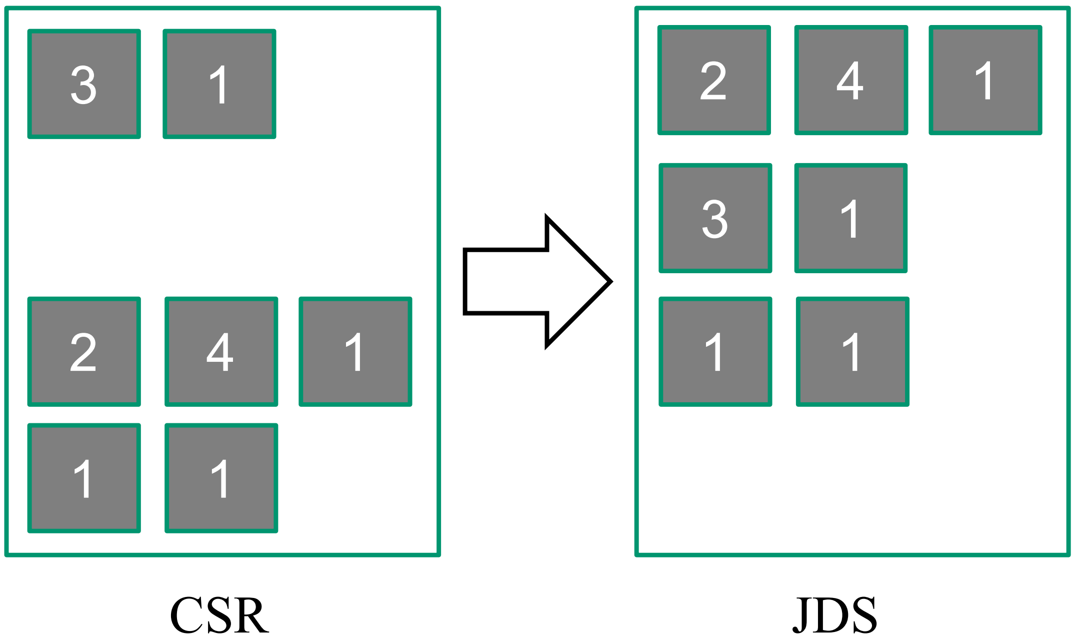

Block performance is determined by longest row

# JDS (Jagged Diagonal Sparse) Kernel Design for Load Balancing




Sort rows into descending order according to number of non-zero. Keep track of the original row numbers so that the output vector can be generated correctly.

# Sorting Rows According to Length (Regularization)



# CSR to JDS Conversion

| | | Row 0 | Row 2 | Row 3 | |
|---|---|---|---|---|---|
| Nonzero values | data[7] | { 3, 1, | 2, 4, 1, | 1, 1 | } |
| Column indices | col_index[7] | { 0, 2, | 1, 2, 3, | 0, 3 | } |
| Row Pointers | row_ptr[5] | { 0, 2, | 2, | 5, | 7 } |

| | | Row 2 | Row 0 | Row 3 | |
|---|---|---|---|---|---|
| Nonzero values | data[7] | { 2, 4, 1, | 3, 1, | 1 1 | } |
| Column indices | col_index[7] | { 1, 2, 3, | 0, 2, | 0, 3 | } |
| JDS Row Pointers | jds_row_ptr[5] | {0, | 3, | 5, | 7,7 } |
| JDS Row Indices | jds_row_perm[4] | {2, | 0, | 3, | 1 } |

# JDS Summary

| | | |
|---|---|---|
| Nonzero values | data[7] | { 2, 4, 1, 3, 1, 1, 1 } |
| Column indices | Jds_col_index[7] | { 1, 2, 3, 0, 2, 0, 3 } |
| JDS row indices | Jds_row_perm[4] | { 2, 0, 3, 1 } |
| JDS Row Ptrs | Jds_row_ptr[5] | { 0, 3, 5, 7, 7 } |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 4 | 1 | 3 | 1 | 1 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 0 | 0 | 2 | 3 |

# A Parallel SpMV/JDS Kernel

```
__global__ void SpMV_JDS(int num_rows, float *data,
       int *col_index, int *jds_row_ptr,int *jds_row_perm,
       float *x, float *y) {
      int row = blockIdx.x * blockDim.x + threadIdx.x;
      if (row < num_rows) {
        float dot = 0;
        int row_start = jds_row_ptr[row];
        int row_end =   jds_row_ptr[row+1];
        for (int elem = row_start; elem < row_end; elem++) {
          dot += data[elem] * x[col_index[elem]];
        }
        y[jds_row_perm[row]] = dot;
      }
    }
```

| | | Row 2 | | | Row 0 | | Row 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| Nonzero values | data[7] | { 2, | 4, | 1, | 3, | 1, | 1 | 1 | } |
| Column indices | col_index[7] | { 1, | 2, | 3, | 0, | 2, | 0, | 3 | } |
| JDS Row Pointers | jds_row_ptr[5] | {0, | | | 3, | | 5, | 7,7 | } |
| JDS Row Indices | jds_row_perm[4] | {2, | | | 0, | | 3, | 1 | } |

# JDS vs. CSR - Control Divergence

- Threads still execute different number of iterations in the JDS kernel for-loop
  - However, neighboring threads tend to execute similar number of iterations because of sorting.
  - Better thread utilization, less control divergence

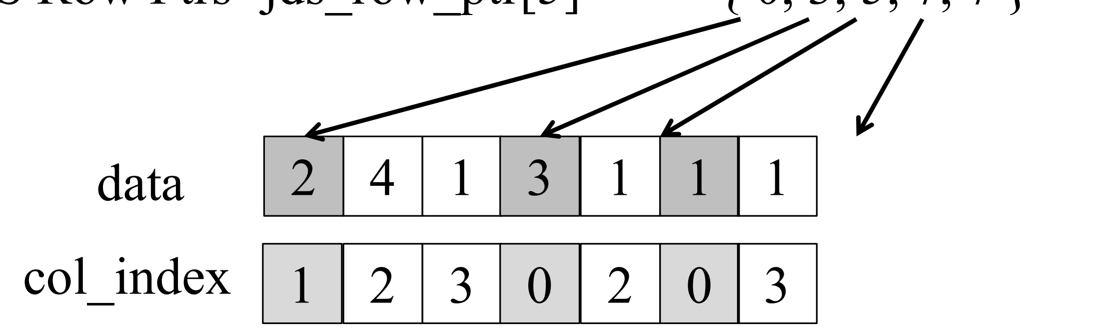
| | | |
|---|---|---|
| Nonzero values | data[7] | { 2, 4, 1, 3, 1, 1, 1 } |
| Column indices | col_index[7] | { 1, 2, 3, 0, 2, 0, 3 } |
| JDS row indices | Jds_row_perm[4] | { 2, 0, 3, 1 } |
| JDS Row Ptrs | Jds_row_ptr[5] | { 0, 3, 5, 7, 7 } |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| data | 2 | 4 | 1 | 3 | 1 | 1 | 1 |
| col_index | 1 | 2 | 3 | 0 | 2 | 0 | 3 |

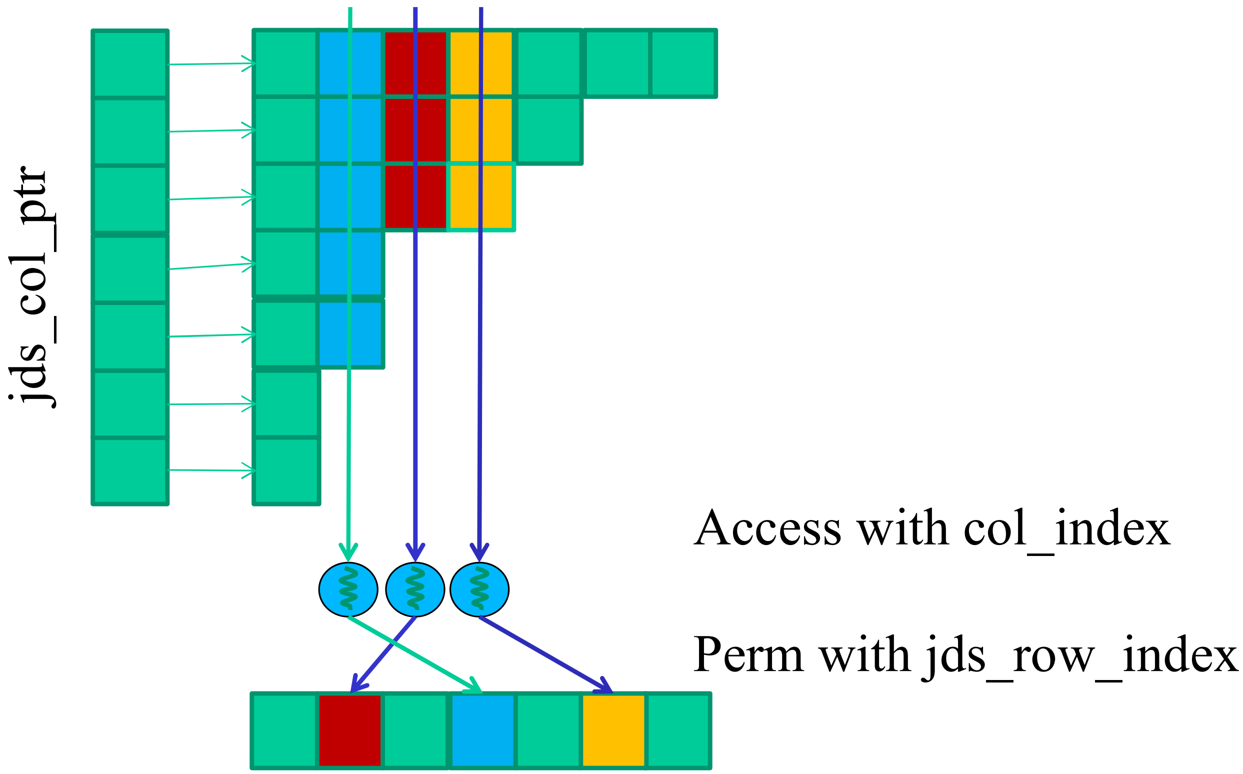
# JDS vs. CSR Memory Divergence

- Adjacent threads still access non-adjacent memory locations

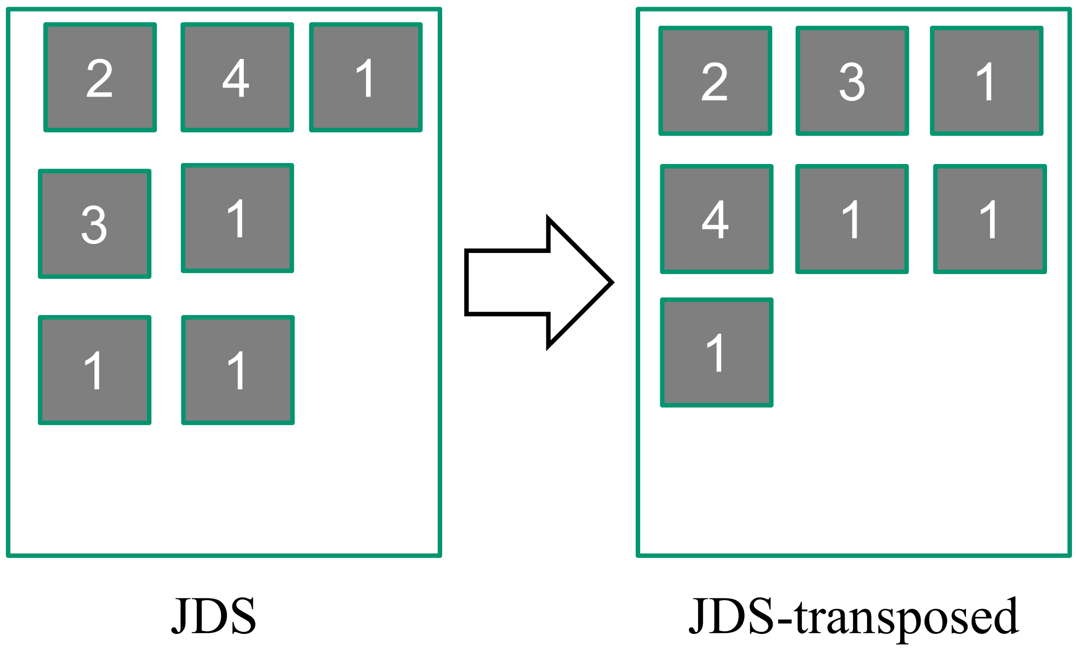
| | | |
|---|---|---|
| Nonzero values | data[7] | { 2, 4, 1, 3, 1, 1, 1 } |
| Column indices | col_index[7] | { 1, 2, 3, 0, 2, 0, 3 } |
| JDS row indices | jds_row_perm[4] | { 2, 0, 3, 1 } |
| JDS Row Ptrs | jds_row_ptr[5] | { 0, 3, 5, 7, 7 } |



# JDS with Transposition



# Transposition for Memory Coalescing



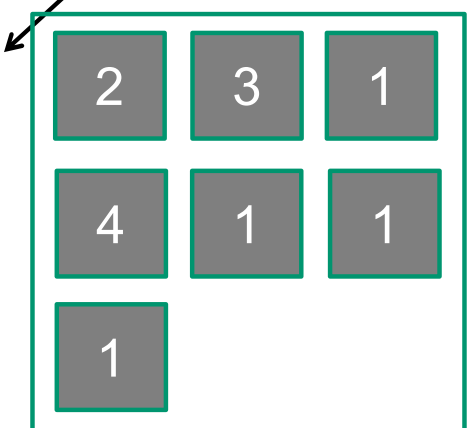
# JDS Format with Transposed Layout

| | | | | | |
|---|---|---|---|---|---|
| Row 0 | 3 | 0 | 1 | 0 | Thread 0 |
| Row 1 | 0 | 0 | 0 | 0 | Thread 1 |
| Row 2 | 0 | 2 | 4 | 1 | Thread 2 |
| Row 3 | 1 | 0 | 0 | 1 | Thread 3 |

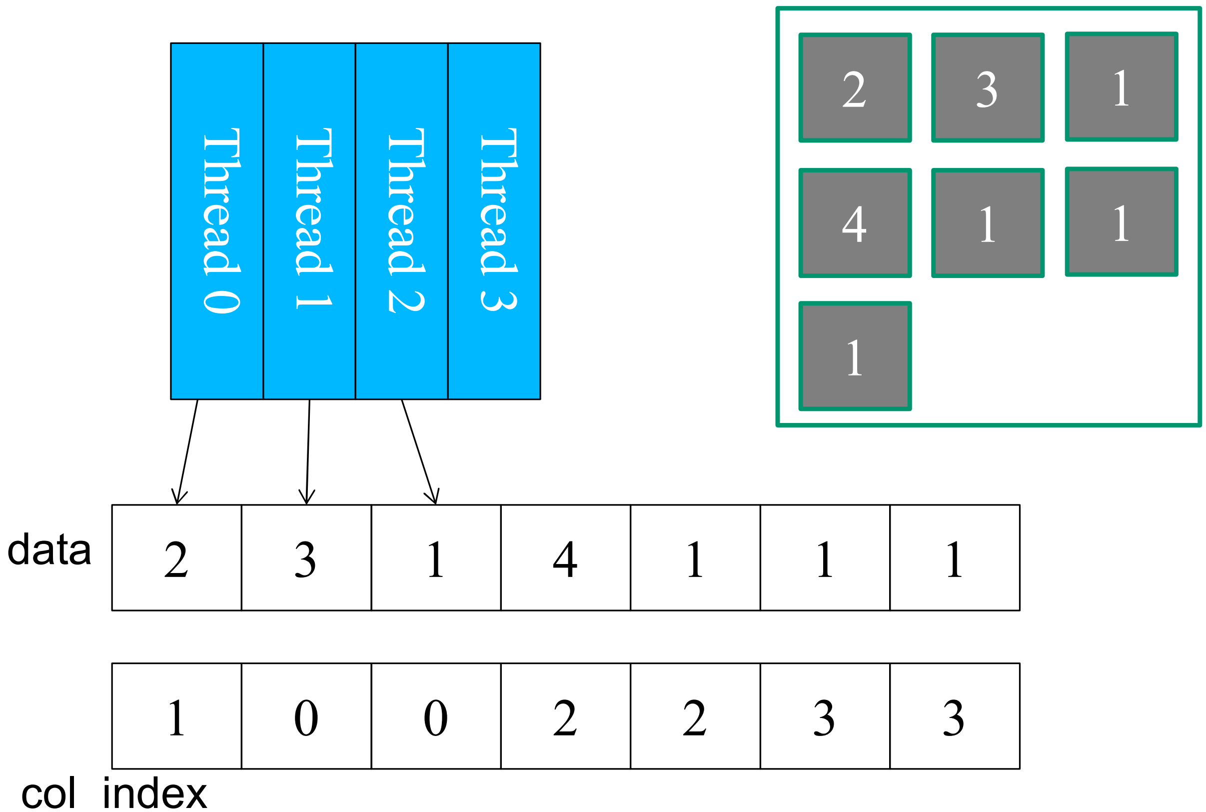
JDS row indices Jds_row_perm[4] { 2, 0, 3, 1 }

JDS column pointers jds_t_col_ptr[4] { 0, 3, 6, 7 }

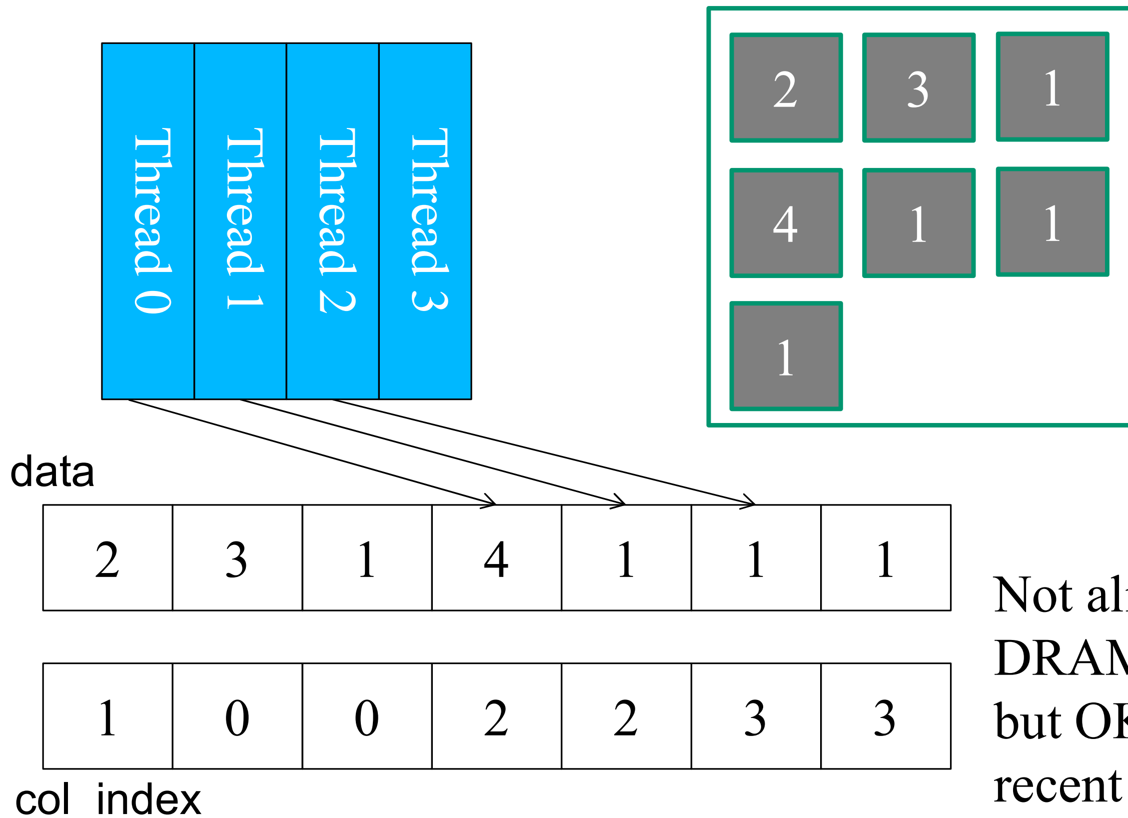
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| data | 2 | 3 | 1 | 4 | 1 | 1 | 1 |
| col_index | 1 | 0 | 0 | 2 | 2 | 3 | 3 |



# JDS with Transposition Memory Coalescing

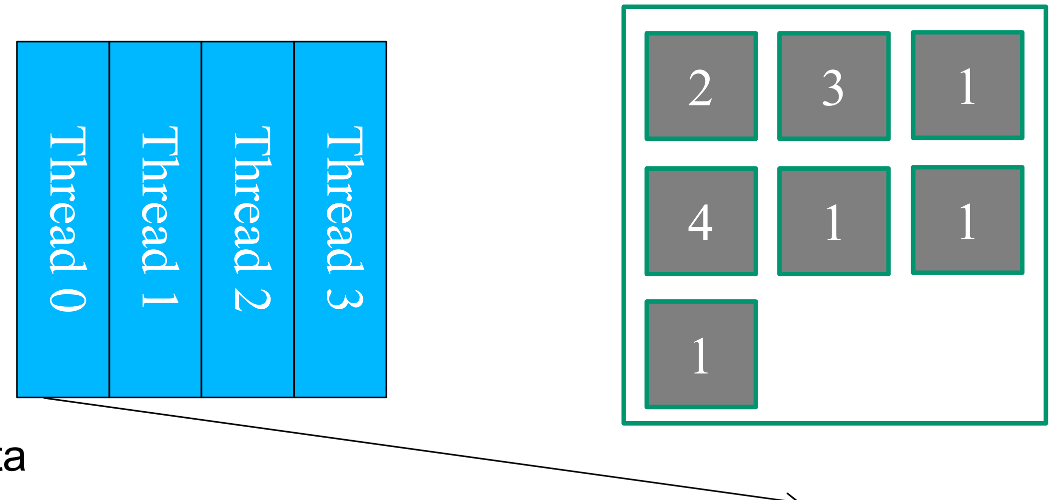


# JDS with Transposition Memory Coalescing




Not aligned with DRAM bursts but OK with recent GPUS

# JDS with Transposition Memory Coalescing




data

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 3 | 1 | 4 | 1 | 1 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 2 | 2 | 3 | 3 |

col_index

# A Parallel SpMV/JDS_T Kernel

```
__global__ void SpMV_JDS_T(int num_rows, float *data,
     int *col_index, int *jds_t_col_ptr, int *jds_row_perm,
     float *x, float *y) {
    int row = blockIdx.x * blockDim.x + threadIdx.x;
    if (row < num_rows) {
      float dot = 0;
      unsigned int sec = 0;
      while (jds_t_col_ptr[sec+1]-jds_t_col_ptr[sec] > row){
         dot += data[jds_t_col_ptr[sec]+row] *
                x[col_index[jds_t_col_ptr[sec]+row]];
         sec++;
      }
      y[jds_row_perm[row]] = dot;
    }
}
```

Column indices col_index[7] { 1, 0, 3, 2, 2, 3 3 }

JDS_T Column Pointers jds_t_col_ptr[5] {0, 3, 6, 7,7 }

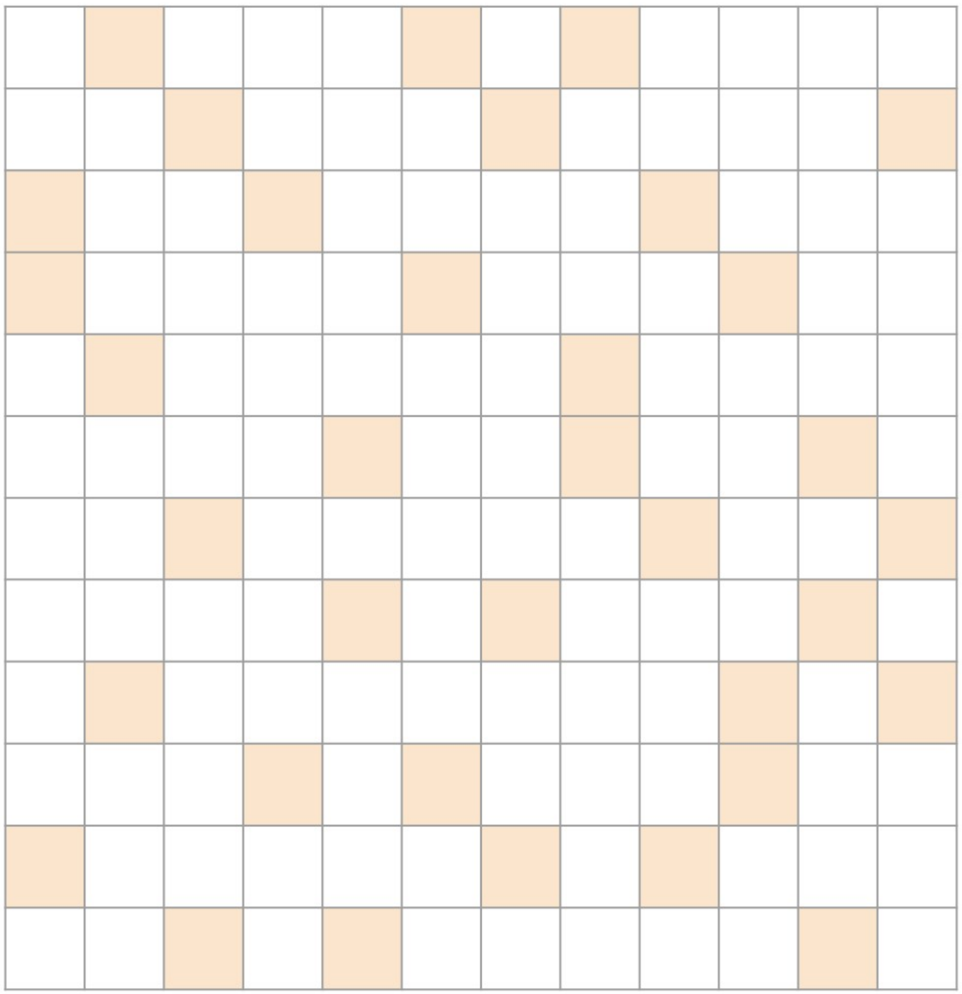
JDS Row Indices jds_row_perm[4] {2, 0, 3, 1 }

# Lab 8 Variable Names

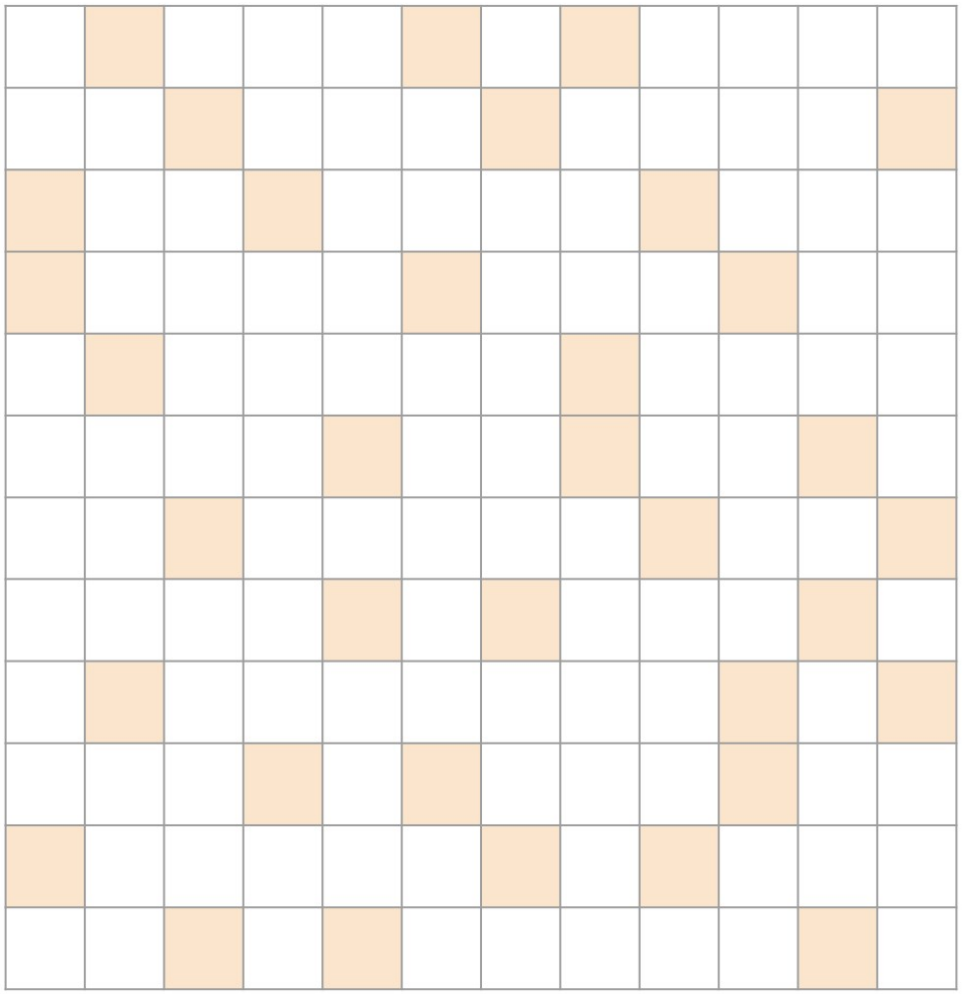
| | | | | |
|---|---|---|---|---|
| JDS_T Length of Cols matRows[4] | {3, | 2, | 2, | 0 } |

| | Sec 0 | Sec 1 | Sec 2 | |
|---|---|---|---|---|
| Nonzero values matData[7] | { 2, 3, 1, | 4, 1, 1 | 1 | } |
| Column indices matCols[7] | { 1, 0, 0, | 2, 2, 3 | 3 | } |

| | | | | |
|---|---|---|---|---|
| JDS_T Column Pointers matColStart[4] | {0, | 3, | 6, | 7 } |
| JDS Row Indices matRowPerm[4] | {2, | 0, | 3, | 1 } |



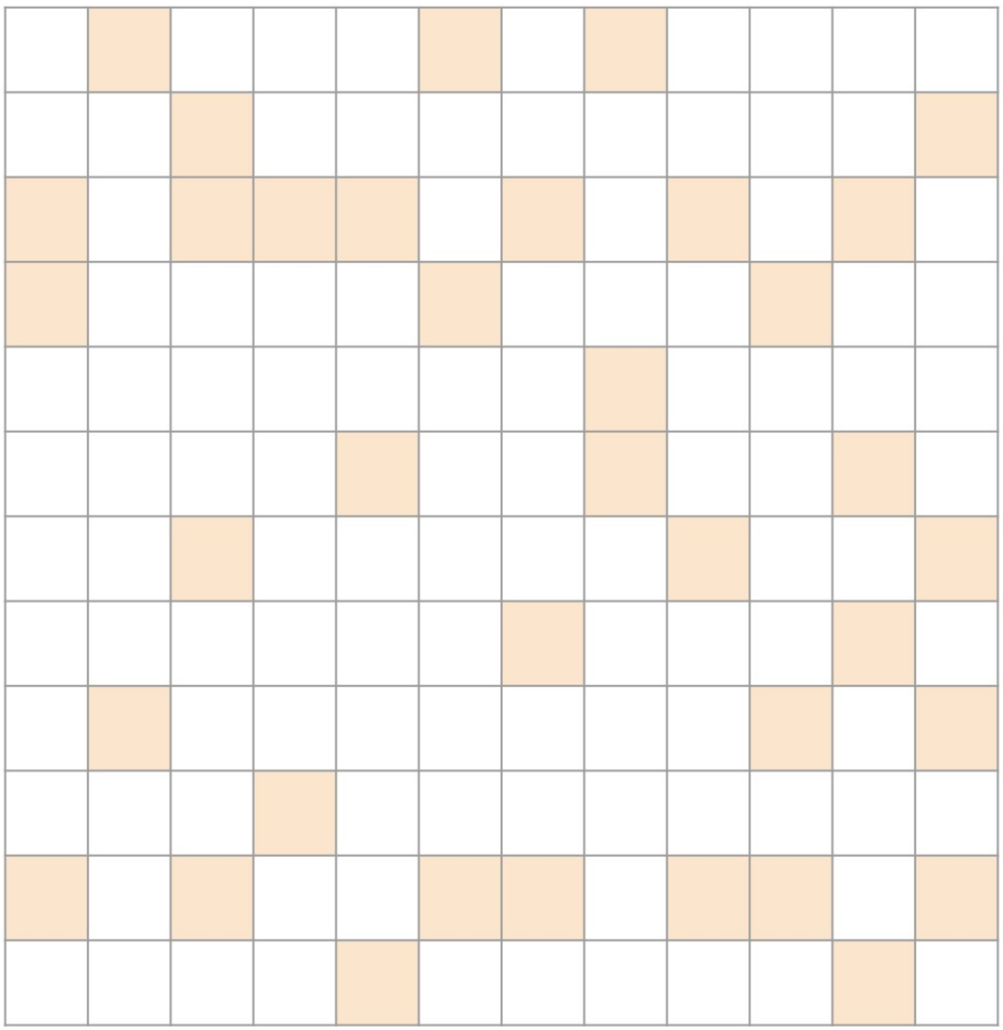
Roughly Random…



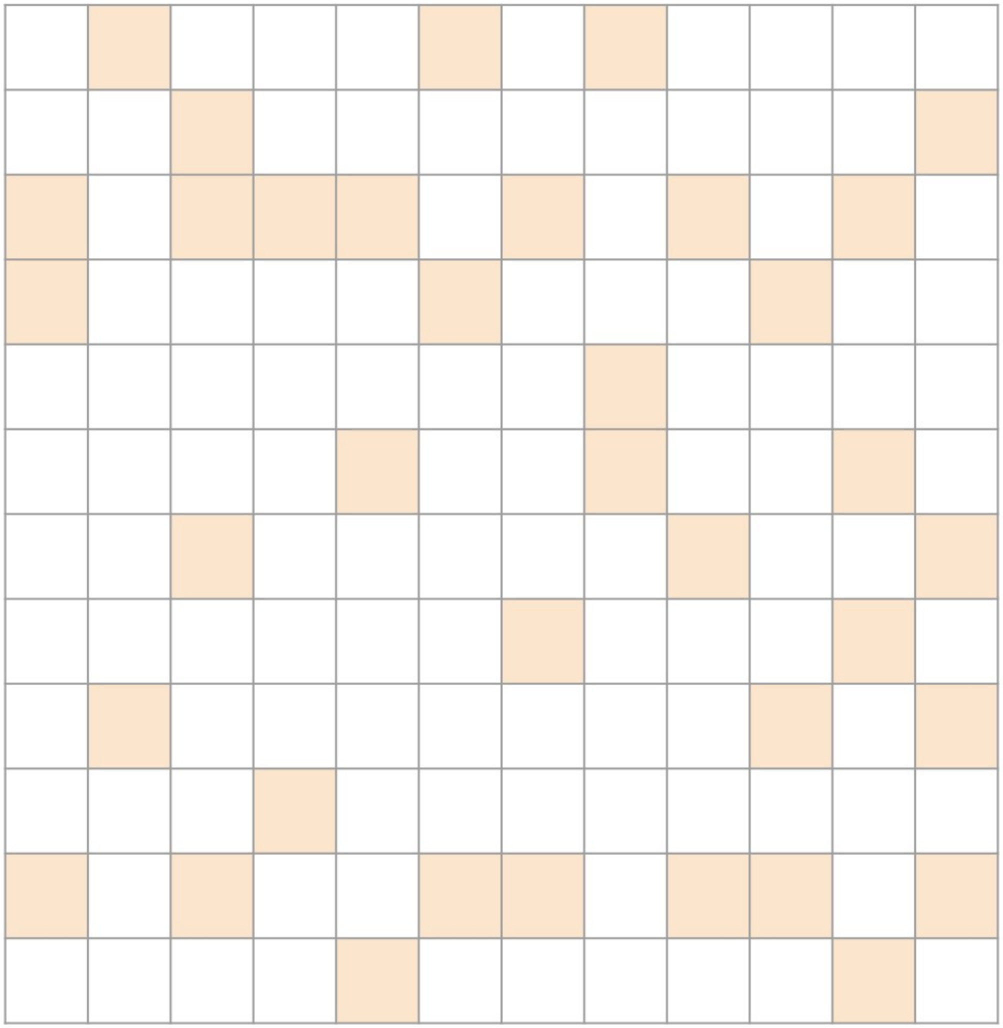
Roughly Random…

Probably best with ELL.

- Padding will be uniformly distributed
- Sparse representation will be uniform



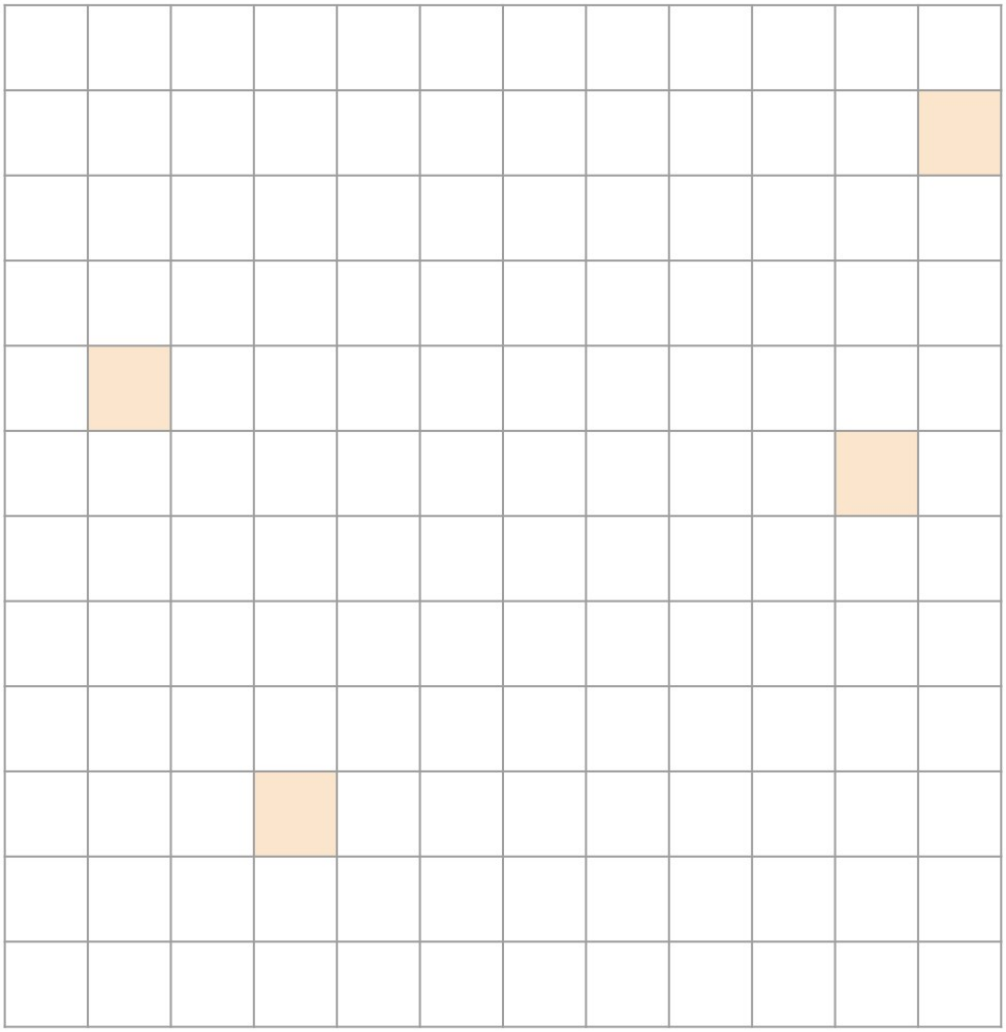
High variance in rows…



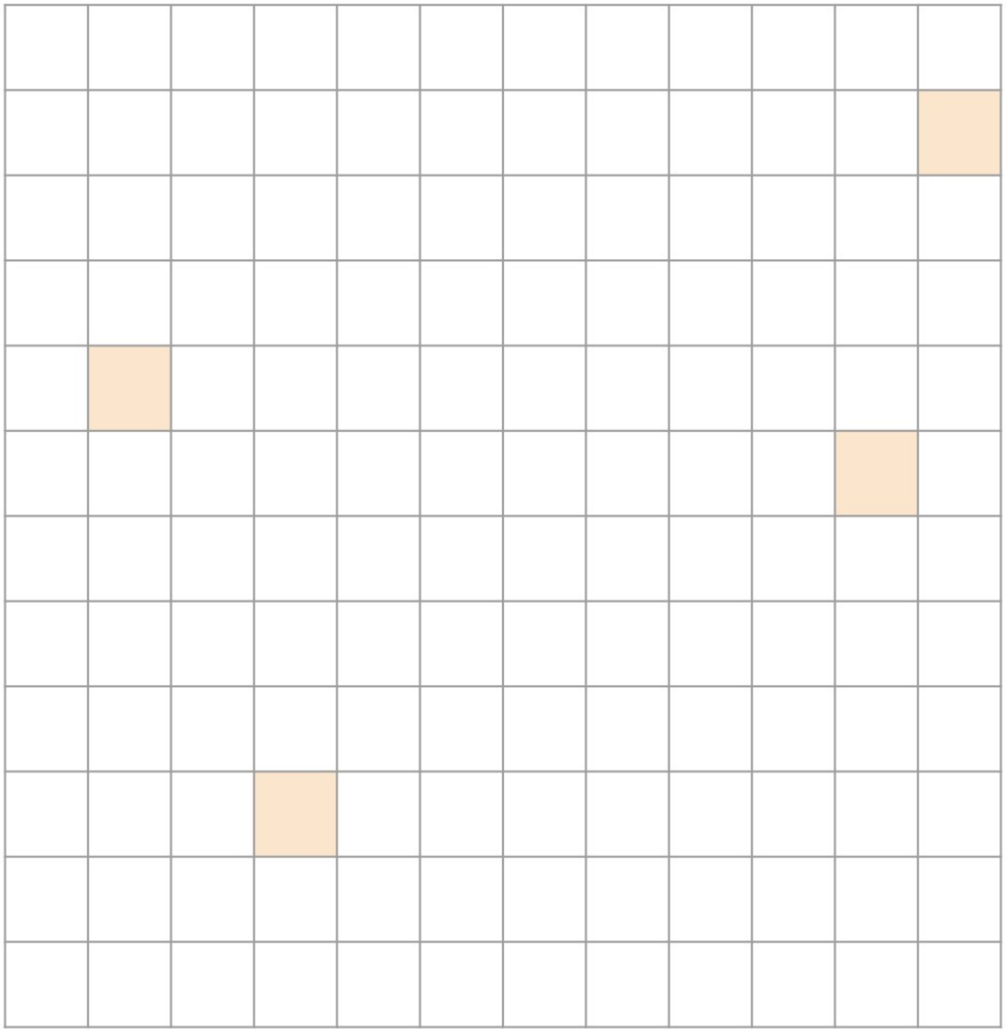
High variance in rows

Probably best with ELL/COO

- Benefit of ELL for most cases
- Outliers are captured with COO



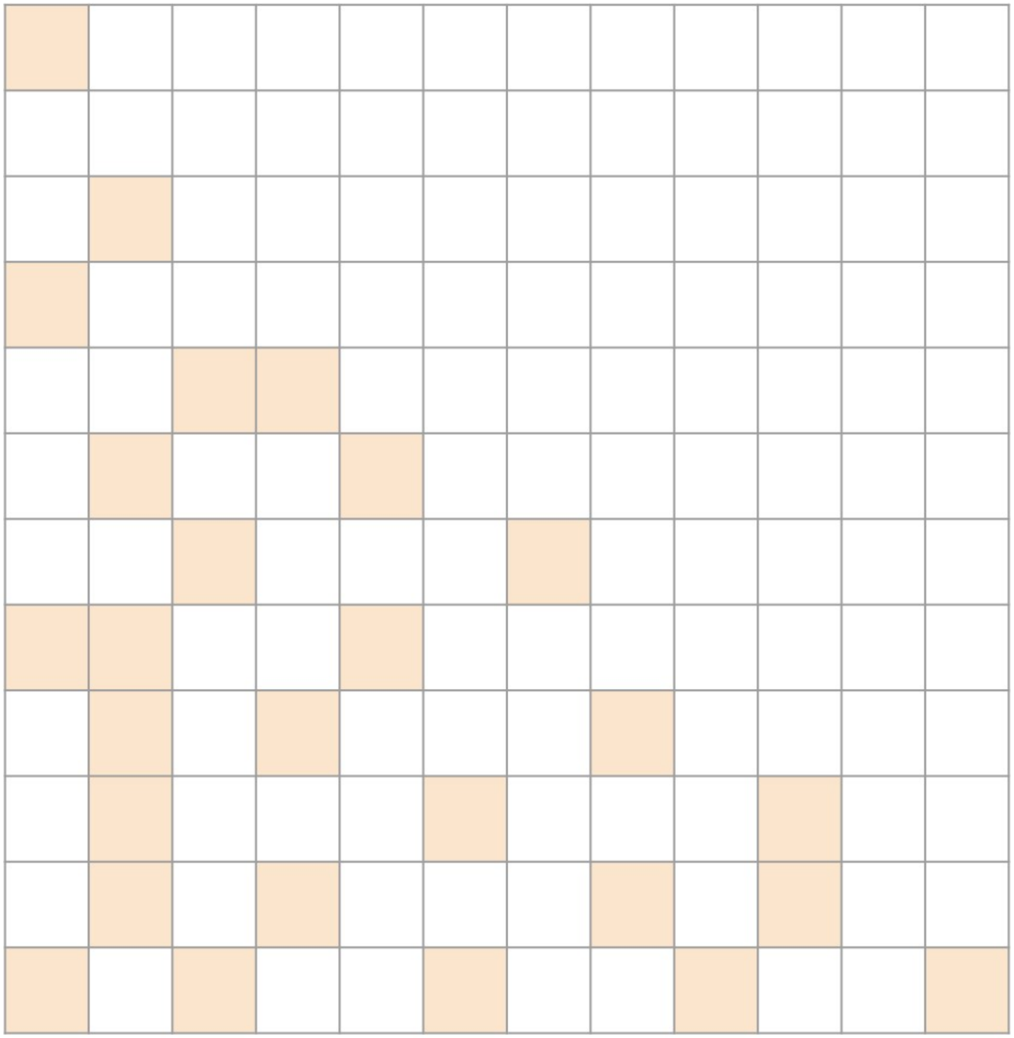
Very sparse…



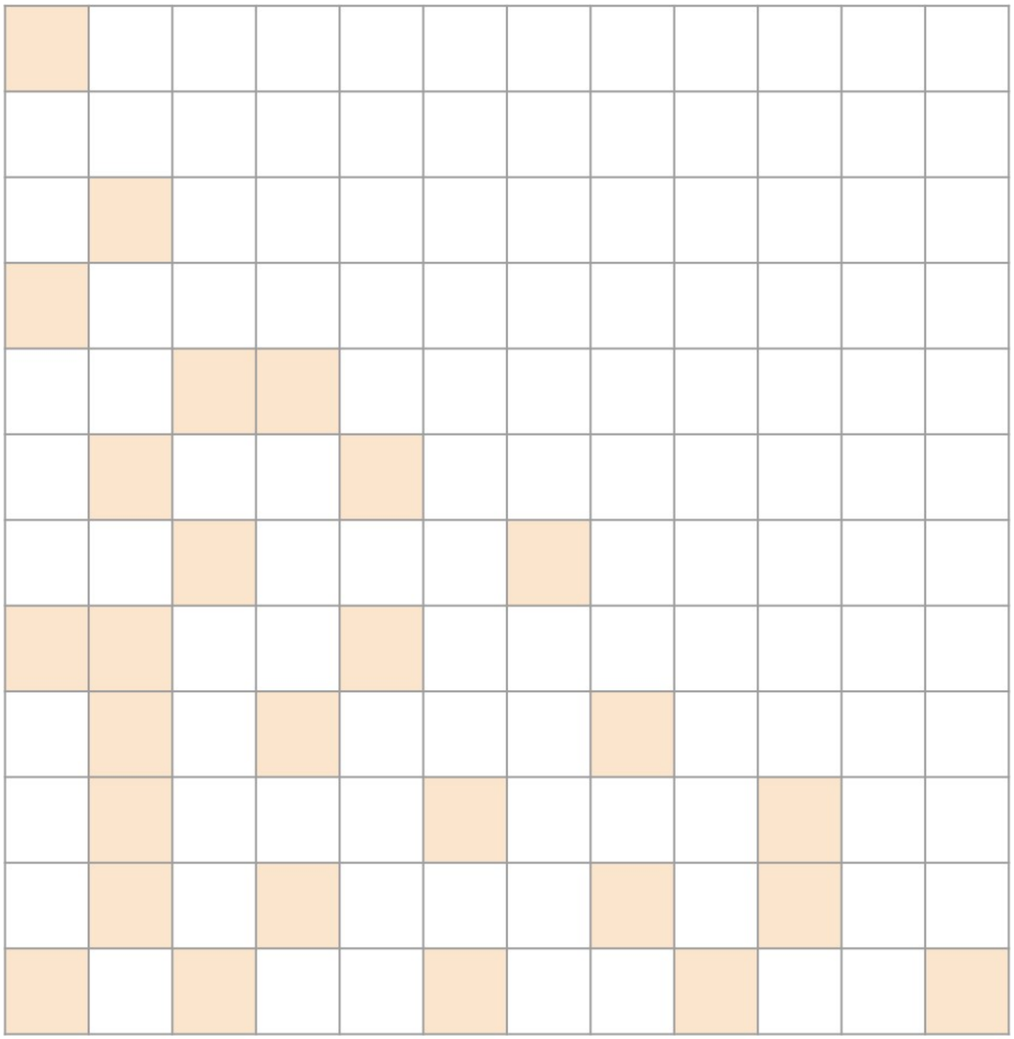
Very sparse

Probably best with COO

- Not a lot of data, compute is sparse



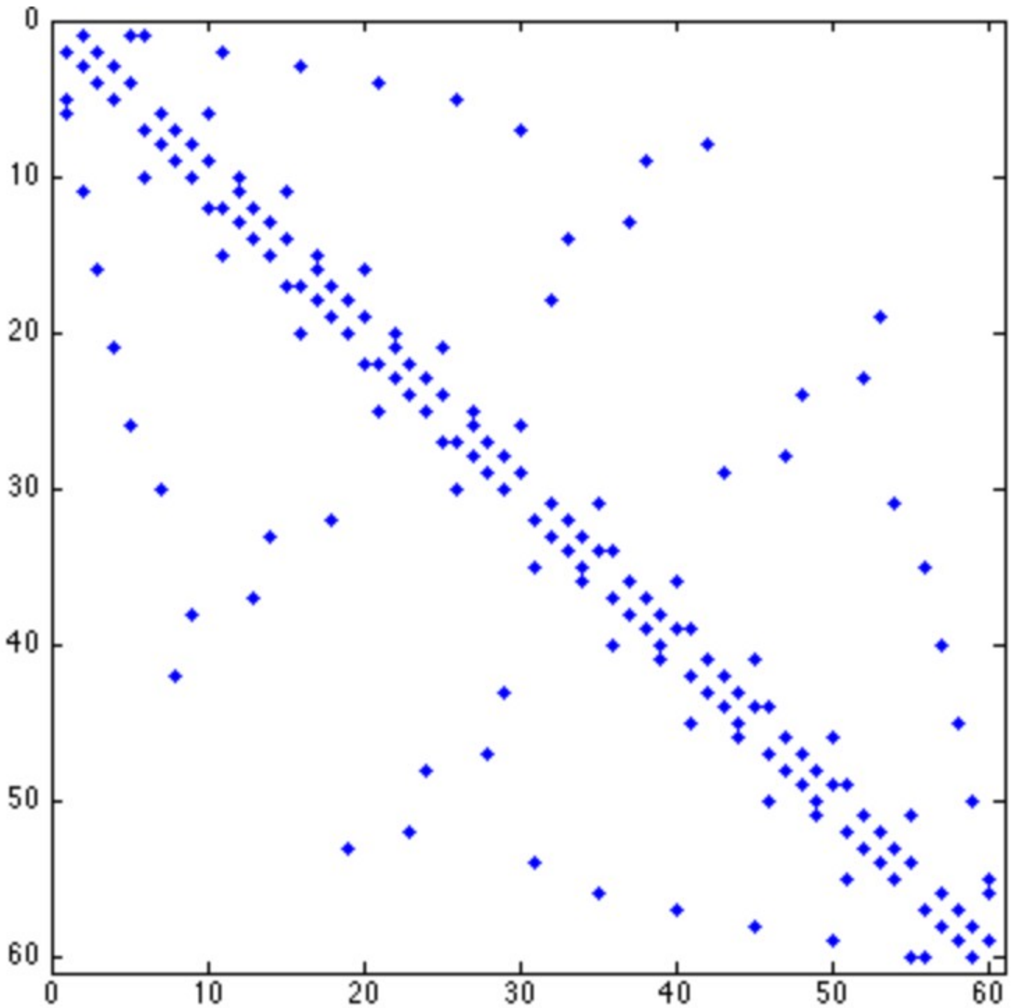
Roughly triangular…



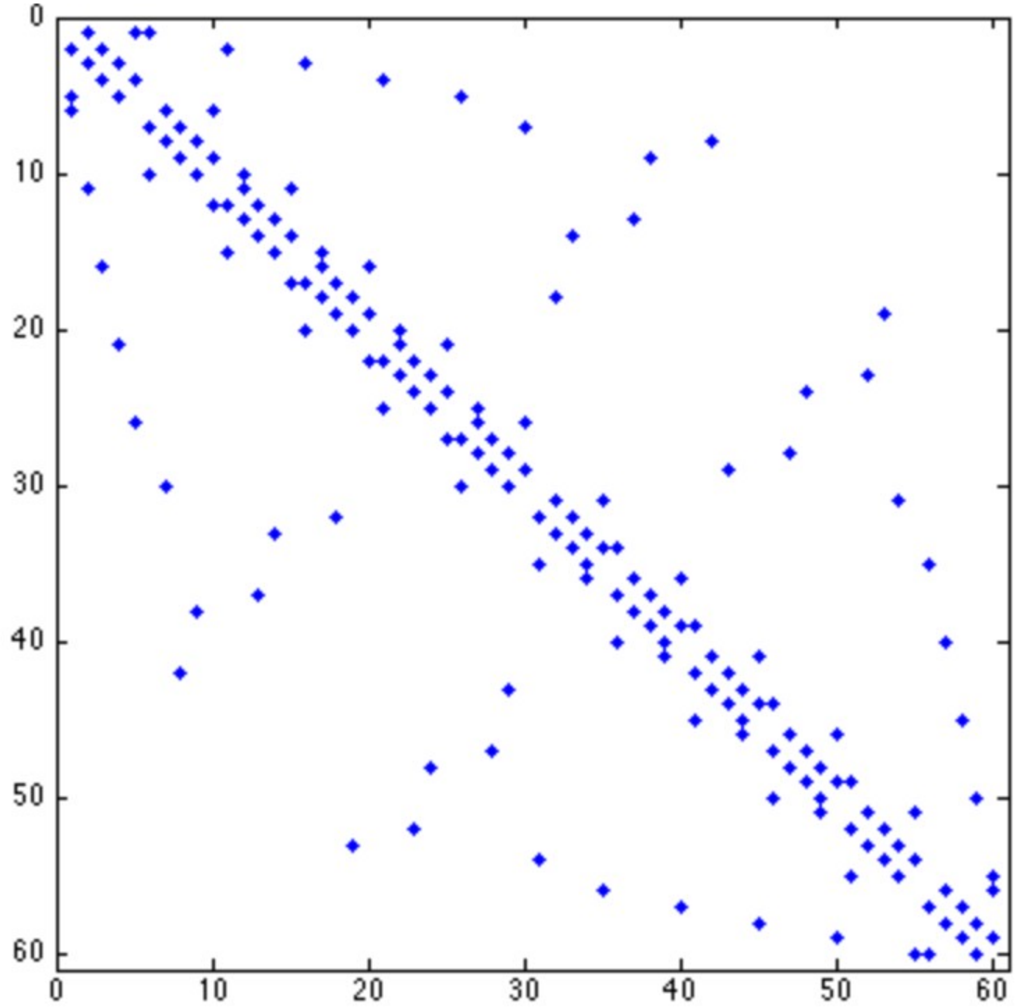
Roughly triangular…

Probably best with JDS

- Takes advantage of sparsity structure



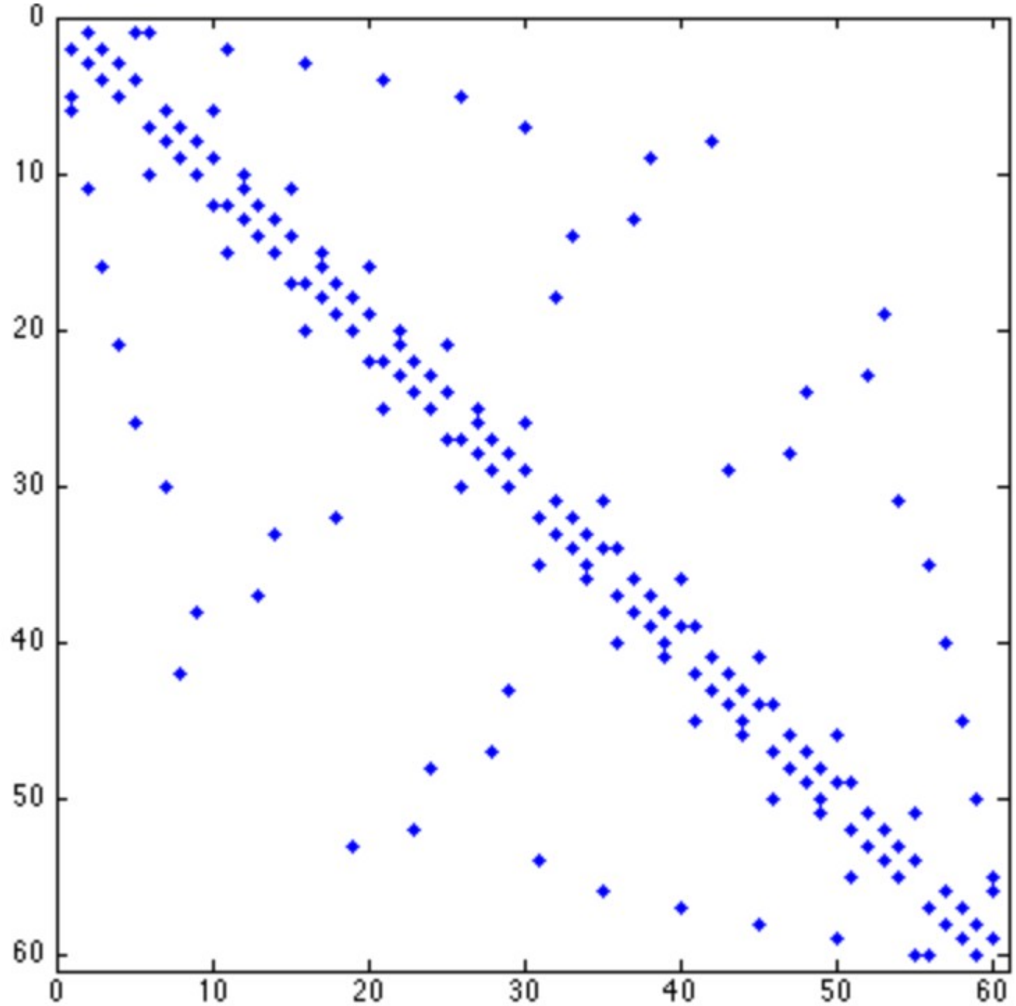

Banded Matrix…

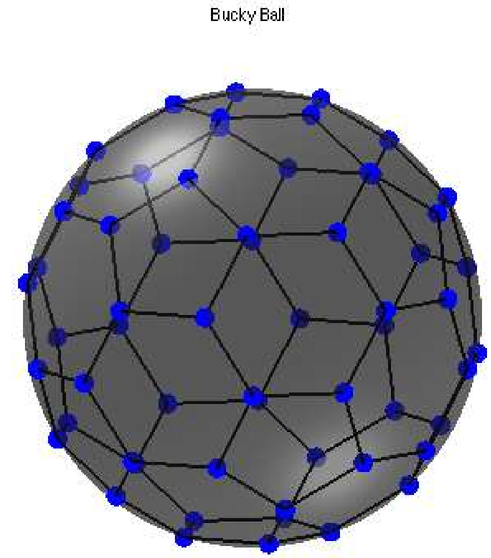



Banded Matrix…

Probably best with ELL

- Small amount of variance in rows


0
10
20
30
40
50
60
0
10
20
30
40
50
60


Bucky Ball

# Other formats

- Diagonal (DIA): for strictly banded/diagonal matrices
- Packet (PKT): create diagonal submatrices by reordering rows/cols
- Dictionary of Keys (DOK): map of (row/col) to data
- Compressed Sparse Column (CSC): when to use over CSR?
- Blocked CSR: useful for block sparse matrices
- Hybrids of these…

# Sparse Matrices as Foundation for Advanced Algorithm Techniques

- Graphs are often represented as sparse adjacency matrices
  - Used extensively in social network analytics, natural language processing, etc.
  - Sparse Matrix-Matrix multiplication (SpMM) is a fundamental operator in GNNs, which performs a multiplication between a sparse matrix and a dense matrix.
- Binning techniques often use sparse matrices for data compaction
  - Used extensively in ray tracing, particle-based fluid dynamics methods, and games
- These will be covered in ECE508/CS508

# Sparse Matrices as Foundation for Advanced Algorithm Techniques

- Graphs are often represented as sparse adjacency matrices
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## QUESTIONS?

## READ CHAPTER 10!

# Problem Solving

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 7 & 0 & 9 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 6 & 5 & 0 & 0 & 8 \end{bmatrix}$$

- Q: Consider the following sparse Matrix:
- For each of the following **data** layouts in memory, select the option that best matches all the sparse matrix formats that can store the data in memory as depicted.
- A:
  - 1) CSR, COO
  - 2) ???
  - 3) JDS, COO
  - 4) COO
  - 5) JDS-Transposed, COO

Layout 1:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 4 | 2 | 7 | 9 | 3 | 6 | 5 | 8 |

Layout 2:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 7 | 6 | 4 | 0 | 9 | 5 | 0 | 0 | 3 | 8 |

Layout 3:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 9 | 3 | 6 | 5 | 8 | 1 | 4 | 2 |

Layout 4:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 7 | 1 | 2 | 4 | 3 | 5 | 8 | 6 |

Layout 5:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 6 | 1 | 2 | 9 | 5 | 4 | 3 | 8 |