

# Binomial Queue

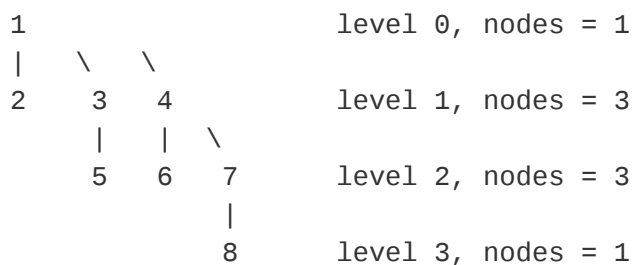
Binomial Queue is a forest of heap-ordered Binomial Tree. A Binomial Tree  $B_h$  of height  $h$  is recursively defined as:

- $B_0$  is a single node tree
- $B_h$  is a tree formed by attaching a  $B_{h-1}$  to the root of another  $B_{h-1}$

For a given number of elements  $N$ , there are a unique sets of binomial trees on the Binomial Queue.

## Why the name *Binomial*?

Consider a Binomial Tree  $B_3$  of height 3:



The sequence 1, 3, 3, 1 that represents the number of nodes at each level, are exactly the binomial coefficients. As a recall, binomial coefficients are coefficients of power expansion of the binomial  $(x + y)^n$ .

The coefficients and thus the number of nodes at each level of a Binomial Tree is given by:

$$\frac{n!}{i! \times (n - i)!} = \frac{height!}{level! \times (height - level)!} = \frac{h!}{i! \times (h - i)!}$$

Hence, the name *Binomial*!!!

Furthermore, the total number of nodes for each Binomial Tree is a function of its height.

Nodes in  $B_0 = 2^0 = 1$

Nodes in  $B_1 = 2^1 = 2$

Nodes in  $B_h = 2^h$

**Number of Binomial Trees in a Binomial Queue for a given  $N$  is  $O(\log N)$**

Let  $k$  be the number of Binomial Trees in a Binomial Queue  $B$  containing  $N$  elements. When all trees  $B_0, B_1, \dots, B_h$  are fully occupied we get maximum  $k$ . Thus,

$$2^0 + 2^1 + 2^2 + \dots + 2^h = N$$

$$2^{h+1} - 1 = N$$

$$h = \log(N + 1) - 1$$

Height of Binomial Trees in a Binomial Queue is  $O(\log N)$

Since  $k = h + 1$ , we get an upper bound on  $k$ .

$$k \leq (\log(N + 1) - 1) + 1$$

As earlier noticed, number of nodes in each binomial tree is a power of 2. It follows,  $k$  is also equal to 1's in binary representation of  $N$

Examples of Binomial queues:

B\_0   B\_1   B\_2

1	2	-1		N = 7 elements	bin(7) = 111 = 2 <sup>2</sup> + 2 <sup>1</sup> + 2 <sup>0</sup>
	\		\		
	3	5	10		
		11			

None	None	-1		N = 4 elements	bin(4) = 100 = 2 <sup>2</sup> + 0 + 0
			\		
		5	10		
			11		