Exercises

Problem 1. Which of the following can be a property of a minimal sorting algorithm?

A. It performs $O(\log n)$ supersteps, where n is the number of elements to sort.
B. It requires a machine to send $O(n/p^{0.99})$ words in some superstep.
C. It requires a machine to spend $O((n/p)\log(n/p))$ CPU time in some superstep.
D. It requires a machine to use $O(n/p^{0.99})$ space in some superstep.

Solution. C.

Problem 2. In the seminar, we introduced a minimal algorithm for sorting. Assuming that each machine has n/p elements in its local storage at the beginning of the algorithm, answer the following questions:

(a) In Phase 1, how many elements are sampled from each machine in expectation?

(b) Still in Phase 1, how many elements does each machine *receive* in expectation?

Solution. (a) Since each element is sampled with probability $(p/n)\ln(np)$, the expected number of elements sampled is $(n/p) \cdot (p/n)\ln(np) = \ln(np)$.

(b) Each machine receives the sample elements from all other machines. Therefore, it receives $(p-1)\ln(np)$ elements in expectation.

Problem 3. In our argument for proving the lower bound on the load of the cartesian product problem, we had the sentence: "Machine 1 sees $n + L$ elements overall $\Rightarrow$ it can produce at most $(\frac{n+L}{2})^2$ pairs." Give a proof of the sentence.

Solution. Let x and y be the number of red and blue elements Machine 1 sees, respectively. Thus, $x + y \le n + L$. The number of pairs that can be produced by Machine 1 is xy. We have:

$$\begin{aligned}
\sqrt{xy} &\le (x+y)/2 && \Rightarrow \\
xy &\le ((x+y)/2)^2 && \Rightarrow \\
&= \left(\frac{n+L}{2}\right)^2.
\end{aligned}$$