

Define  $\mathbb{R}^2$  as the set of 2D points. We will represent a point  $(x, y)$  as a 2D vector. Parameterized by a 2D vector  $\mathbf{w}$ , a linear classifier  $h$  maps a point  $\mathbf{p}$  to 1 if  $\mathbf{w} \cdot \mathbf{p} \geq 0$ , or  $-1$  otherwise.

**Problem 1.** In the seminar, we introduced the Perceptron algorithm for *online* learning. Suppose that the algorithm initially holds a linear classifier (i.e.,  $h_{\text{now}}$ ) parameterized by  $\mathbf{w} = (0, 0)$ . Use the algorithm to process (in the online model) the following sequence of points:

point  $\mathbf{a} = (1, 2)$ , label  $-1$   
point  $\mathbf{b} = (-2, 3)$ , label  $-1$   
point  $\mathbf{c} = (2, 4)$ , label  $1$

Given the value of  $\mathbf{w}$  after processing each point.

**Problem 2.** What is the worst-permutation mistake bound of the Perceptron algorithm on the point set  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ ? Here,  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  are the same as given in Problem 1. You should assume that the algorithm *always* starts with  $\mathbf{w} = (0, 0)$ .

**Problem 3.** We learned a method to convert Perceptron to a batch learning algorithm. Let us run the converted Perceptron algorithm on the set  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ . Recall that the method runs in iterations. Assume that, in each iteration, the points remaining in the set are processed in alphabetic order. What is the final  $\mathbf{w}$  output by the algorithm?