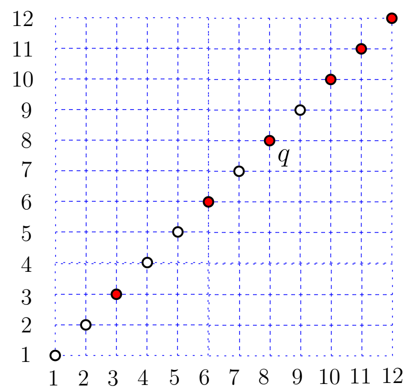
Exercises

By Yufei Tao

Problem 1. Consider the following dataset, where a red point has label 1 and a white point has label 0:



- What is the width of the dataset?
- What is the value of k? Recall that k is the smallest error of a monotone classifier.

Answer: The width is 1, and $k = 3$.

Problem 2. Prove: the fractional edge covering number of a triangle must be at least 1.5.

Answer: Let A, B, C be the three vertices of a triangle, and let w_A, w_B, w_C be their weights, respectively. We know:

$$\begin{aligned} w_A + w_B &\geq 1.5 \\ w_B + w_C &\geq 1.5 \\ w_A + w_C &\geq 1.5. \end{aligned}$$

Hence:

$$2(w_A + w_B + w_C) \geq 3$$

which means that $w_A + w_B + w_C$ must be at least 1.5.

Problem 3. Prove: a graph G with m edges can contain $O(m^{1.5})$ triangles.

Answer: Call a vertex of G *small* if its degree is at most $\sqrt{m}$; otherwise, call it *large*. The number of large vertices is at most $2m/\sqrt{m} = 2\sqrt{m}$. We divide the triangles into two types:

1. At least one vertex u is small, whereas the other vertices v and w can be small or large. There are at most $m^{1.5}$ triangles of this type. First, choose an edge $\{u, v\}$ with (at least) one small vertex u; there are m ways to do so. For each $\{u, v\}$, there are at most $\sqrt{m}$ choices of w because u has at most $\sqrt{m}$ neighbors.
2. All vertices are large. The number of triangles of this type is clearly $(2\sqrt{m})^3 = 8m^{1.5}$.