Comp 460 - Algorithms & Complexity

Spring Semester 2020 - Week 9

Dr Nick Hayward

DEV Week Assessment

Course total = 15%

- continue development of application
    - *built from scratch*
    - *continue design and development of initial project outline and design*
    - *working app (as close as possible...)*
    - *NO blogs, to-do lists, note-taking...*
    - *...*
- outline research conducted
- describe data chosen for application
- define algorithms and data structures used in app
    - *why choose these options?*
    - *how have they been used?*
    - *define current performance &c.?*
    - *define testing of implementation & usage*
- show any prototypes, patterns, and designs

DEV Week Demo

DEV week assessment will include the following:

- brief presentation or demonstration of current project work
  - *~ 5 to 10 minutes per group*
  - *analysis of work conducted so far*
    - e.g. during semester & DEV week
  - *presentation and demonstration*
    - outline current state of app
    - explain what works & does not work
    - show implemented designs since project outline & mockup
    - show latest designs and updates
  - *due Tuesday 17th March 2020 @ 4.15pm*

Video - Algorithms and Data Structures

quicksort - part 3

Quicksort - UP TO 8:42

Source - Quicksort - Java - YouTube

binary search tree - intro - part 1

- binary search tree (BST) is a binary tree
- each node has a Comparable key (and an associated value)
  - *satisfies a defined restriction*
- e.g. key in any node is
  - *larger than the keys in all nodes in that node's left subtree*
  - *smaller than the keys in all nodes in that node's right subtree*
- comparison is context specific relative to the current node
- binary search - need to work with sorted data sets
- when we add or update the data
  - *need to re-sort list before using binary search*
- if we're working with sorted lists of data
  - *e.g. an array of books*
- quickly encounter a problem
  - *list may need to be updated - item in the array is deleted*
  - *then need to add a value to index where last deleted item stored...*

Algorithms and Data Structures

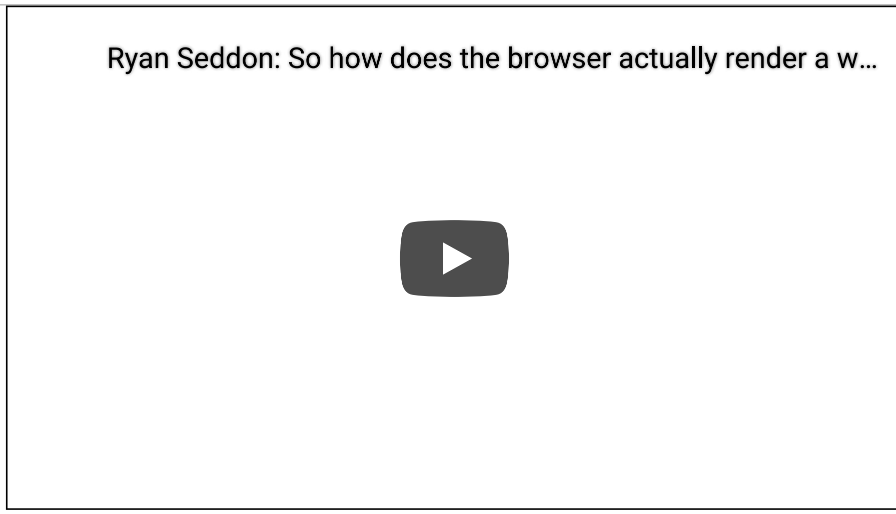
binary search tree - intro - part 2

- may now update list to meet specific criteria
  - *removing need to repeatedly sort dataset*
- binary search tree data structure
  - *basic tree data structure*

```
          node
           |
          / \
         /   \
      node   node
       |        |
      / \      / \
     /   \    /   \
  node  node  node   node
```

Video

Trees and parsing - part 1



How the browser renders a website - UP TO 7:40

Source - So how does the browser actually render a website - YouTube

Algorithms and Data Structures

binary search tree - intro - part 3

- sample binary search tree may be structured as follows

```
              daisy
                |
               / \
              /   \
           bea     emma
            |       |
           / \       \
          /   \       \
       anna   cat    rose
                       |
                      / \
                     /   \
              ophelia    yvaine
                           |
                          /
                         /
                    violet
```

- for every node in this tree
  - *nodes to left of current node are smaller*
  - *nodes to right of current node are larger*
- e.g. search for `violet` - begin at root then use following path
  - *V is after D - traverse right side of tree*
    - current node = emma
  - *V is after E - continue down right side*
    - current node = rose
  - *V is before Y - continue down left side*
    - `violet` node found...
- searching for a node in a binary search tree takes `O(log n)` on average
  - *`O(n)` for worst cases*
- sorted array, by contrast, takes `O(log n)` in worst case scenarios

- might initially consider arrays as preferable option
  - *sorted binary search tree is, on average, faster for insertions and deletions…*

binary search tree - issues

- binary search trees do not provide random access
- performance times are averages
  - *rely on a balanced tree*
- specialist trees may provide self-balancing mechanisms
- e.g. might use a *red-black* tree

Video - Algorithms and Data Structures

binary search trees - part 1

Binary Search Trees - UP TO 1:36

Source - Trees - Java - YouTube

Algorithms and Data Structures

binary search tree - basic logic implementation - part 1

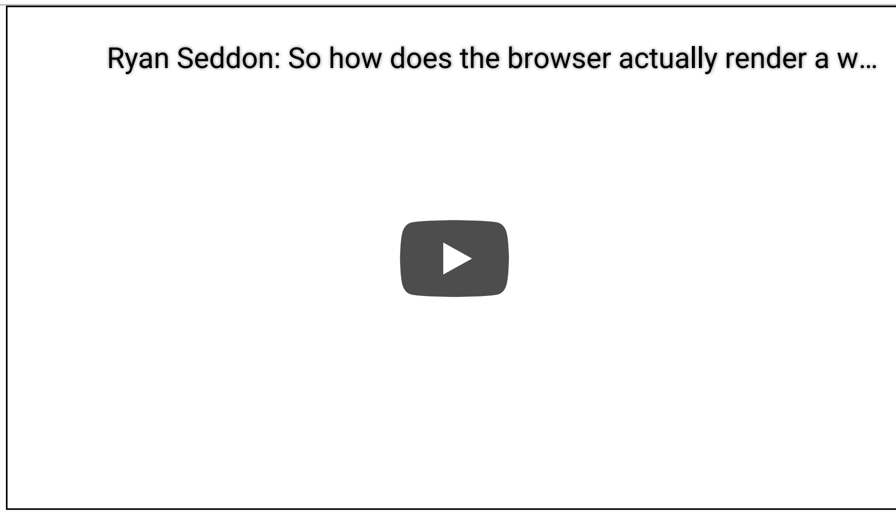
- *symbol-table API* for a binary search tree
  - *common option for implementing this type of traversal and search*
- symbol-table is also known as a map, dictionary, associative array &c.
  - *may vary by programming language*
- general concept is as follows
  - *abstraction of key/value pairs*
    - e.g. insert a value with a specified key
    - given key, search for corresponding value
- sample usage may include the following

application	search	key	value
dictionary	search for a definition	word	definition
index	search for a given reference, e.g book page	term	e.g. list of page numbers for a book
compiler	search for props of variables	variable name	type and value

- for binary search tree logic - may begin with custom function to define nodes
- function includes props required for a node, e.g.
  - *key*
  - *value*
  - *left link*
  - *right link*
  - *node count*

Video

Trees and rendering - part 2



A common working example - How the browser renders a website - UP TO 17:17

Source - So how does the browser actually render a website - YouTube

Algorithms and Data Structures

binary search tree - order-based methods - part 1

- common reason for working with binary search trees (BST)
  - *they keep the keys in order*
- may use BSTs in many disparate API contexts
  - *ensure consistent I/O structure*
- e.g. might consider a custom *symbol-table* API
- some sample methods and usage
  - *`min & max`*
    - if left link of root is `null`
      - smallest key in BST must be root node
    - if left link is not `null`
      - smallest key is in subtree referenced by left link
      - repeats for each left link in each subtree...
  - *`floor`*
    - if a given key is less than key at root of BST
      - floor of key must now be added to left subtree...
    - if key is greater than root
    - floor can be in right link but only if there is a smaller or equal existing key
      - if not, floor of key is root...
  - *`ceiling`*
    - same pattern as floor
      - except check relative to right link
  - *`selection`*
    - e.g. seek key of rank `k`
      - key such that precisely `k` other keys in BST are smaller
    - if number of keys `t` in left subtree is larger than `k`
      - look (recursively) for key of rank `k` in left subtree
    - if `t` is equal to `k`
      - return key at root
    - if `t` is smaller than `k`
      - look (recursively) for key of rank `k - t - 1` in right subtree

Algorithms and Data Structures

binary search tree - order-based methods - part 2

- `range search`
  - *to implement `keys()` method - returns all keys in a given range*
  - *begin with basic recursive BST traversal method - known as `inorder traversal`*
  - *e.g. to show this order traversal*
    - first print all keys in left side of BST - all less than root
    - then print root key
    - then print all keys in right side of BST
  - *`keys()` method*
    - define code to add each key that is in range to a *Queue*
    - skip recursive calls for subtrees that cannot contain keys in range
- `rank`
  - *if given key is equal to key at root*
    - return number of keys `t` in left subtree
  - *if given key is less than key at root*
    - return rank of key in left subtree
  - *if given key is larger than key at root*
    - return `t` plus one (to count key at root) plus rank of key in right subtree

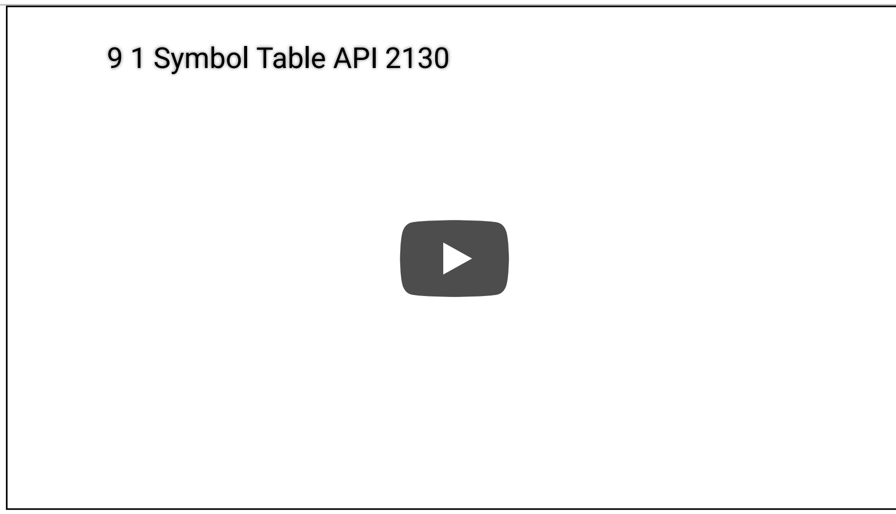
Algorithms and Data Structures

binary search tree - order-based methods - part 3

- `delete min & max`
  - *delete minimum*
  - *go left until finding a node that has a null left link*
  - *then replace link to that node by its right link*
  - *symmetric method works for delete maximum*
- `delete`
  - *proceed in a similar manner to delete a node with one or null childrem*
  - *for two or more - start by replacing current node with its successor*
  - *successor is node with smallest key in its right subtree*
- accomplish task of replacing x by its successor in four easy steps
  - *1. save a link to node to be deleted in `t`*
  - *2. set `x` to point to its successor `min(t.right)`*
  - *3. set right link of `x`*
    - supposed to point to BST containing all keys larger than `x.key`
    - to `deleteMin(t.right)`
      - the link to BST containing all keys that are larger than `x.key` after deletion
  - *4. set left link of `x` to `t.left`*
    - all keys that are less than both deleted key and its successor...

Video

symbol table API



Symbol table API - UP TO 5:40

Source - Symbol Table API - YouTube

binary search tree - usage - intro

- binary search tree (BST) has a non-linear insertion algorithm
- BST is similar in nature to a doubly-linked list
  - *a linked data structure*
  - *includes set of sequentially linked records, commonly known as nodes*
- each node defines three fields,
  - *link field - `previous` node in sequence of nodes*
  - *link field - `next` node in sequence of nodes*
  - *one `data` field*
- link fields may be represented using a common example of a convoy, a train &c.
  - *e.g. linked ships sailing in a convoy...*

binary search tree - usage - links and usage - part 1

- binary search tree defines its pointers as `left` and `right` to help indicate any duplication of logic &c.
- algorithm may be detected
    - *providing support for left and right traversal*
- binary search tree node’s pointers are typically called *left* and *right*
    - *indicate subtrees of values relating to current value*
- simple JavaScript implementation of such a node is as follows

```
const node = {
    value: 123,
    left: null,
    right: null
}
```

- BST is a unique tree due to its inherent ordering of nodes based on value *binary search tree - usage - links and usage - part 2*

- any child nodes in a left subtree are always less than parent node's value
- converse holds for a right subtree
  - *values in subtree will always be greater*
- e.g.

```
          7
         / \
        5   9
       /     \
      2       11
     / \        \
    1   3        12
                  \
                   14
                  /
                 13
```

- traverse the tree and check key of current node
- if search key is less than current node's value
  - *follow left link*
  - *otherwise, follow right link*
- position of node values is based on a few factors
  - *value of node*
  - *value of root*
  - *order of insertion*
- e.g.
  - *root is set to 7*
  - *5 is less than root - insert as left link*
  - *9 is greater than root - insert as right link*
  - *2 is less than root - follow left link*
    - 2 is less than 5 - left link is null - insert as left link

    - …
  - repeat for additional inserts…

Video - Algorithms and Data Structures

binary search trees - part 2

Binary Search Trees - Insert - UP TO 3:00

Source - Trees - Java - YouTube

binary search tree - usage - search pattern

- clearly defined pattern - searching a BST becomes easier to reduce to a repeatable pattern
- traverse left for a value less than current node, and right for a greater value
- simple pattern is helped by exclusion of duplicates
- in last example - currently have a depth of 5 means furthest we need to travel is 5 nodes from root to find a value
- BSTs also have a natural sorted order
  - *due to the insertion algorithm*
- makes BSTs particularly useful for quick searching
  - *eliminate options at each node*

binary search tree - usage - BST with JavaScript - part 1

- a few options for implementing BSTs in JavaScript
- e.g. extend an object's prototype to include various custom functions to manage a BST
- a non-map implementation might include the following
  - *define initial data and props for constructor*
  - *extend Prototype - add custom methods*
    - add
    - has
    - delete
    - size
  - *expose universal interface*

binary search tree - usage - BST with JavaScript - part 2

- initial coded example

```
/*
* Constructor BST
*/
function BinarySearchTree() {
    // instantiated object - private prop - root default...
    this._root = null;
}

/*
* Prototype
* -extend with custom functions
* - methods
*/
BinarySearchTree.prototype = {
    // extend - custom functions
    add: function(value) {

    },
    has: function(value) {

    },
    delete: function(value) {

    },
    size: function(

    }
};
```

binary search tree - usage - BST with JavaScript - part 3

- easiest to begin such custom methods with an outline of a `has()` method
- may define a general structure for querying a BST
- method defines single parameter for value
  - *returns `true` if value is found and `false` if null found*
- logic follows a basic binary search algorithm
  - *to determine presence of a value*

binary search tree - usage - BST with JavaScript - part 4

- coded example for `has()` method
- e.g. consider following initial outline for querying keys

```
has: function(value){
    const found = false,
        current = this._root;

    // check node is available for search...
    while(!found && current){

        // check node - if value less than current node's, go LEFT
        if (value < current.value){
            // update 'current' prop
            current = current.left;
        // check node - if value greater than current node's, go RIGHT
        } else if (value > current.value){
            // update 'current' prop
            current = current.right;
        //check node - values are equal, found node...
        } else {
            // update boolean...
            found = true;
        }
    }
    // return search status...
    return found;
},
```

Algorithms and Data Structures

binary search tree - usage - BST with JavaScript - part 5

- search starts at root
  - *initially checking there is a `root` key and key has not been found*
  - *`current` is initially set to `root` to begin BST traversal*
- sets `while` loop running
- allows following checks to be executed
  - *check search value against current node value*
    - if less than - set current to left link
  - *check search value again...*
    - if greater than - set current to right link
  - *otherwise - value has been `found`*
    - `found` updated to true
    - `while` loop broken
  - *`contains` now complete...*

Video - Algorithms and Data Structures

binary search trees - part 3

Binary Search Trees - has/add - UP TO 7:24

Source - Trees - Java - YouTube

binary search tree - usage - BST with JavaScript - part 6

- may also use same underlying pattern for node insertion
  - *defining* `add()` *method*
- need to modify search for a place to insert a node
  - *instead of returning an existing value*

```
add: function(value){
    // define a new node - placeholder object & props...
    const node = {
            value: value,
            left: null,
            right: null
        },
        // variable for current node - use during BST traversal...
        current;

    // CHECK -  no items yet in the BST
    if (this._root === null){
        // ROOT - BST empty - set root to current node -
        this._root = node;
    } else {
        // update current prop - set to root node
        current = this._root;

        // TRAVERSE - begin traversal of BST from current node - start at root
        while(true){

            // check node - if value less than current node's, go LEFT
            if (value < current.value){
                // check node - if no node in left link
                if (current.left === null){
                    // update current prop - set `left` to new node
                    current.left = node;
                    // EXIT - node inserted as `left` link
                    break;
                } else {
                    // node set to existing left link...
                    current = current.left;
                }
```

```
            // check node - if value greater than current node's, go RIGHT
            } else if (value > current.value){
                // chevk node = if no node in right link
                if (current.right === null){
                    // update current prop - set `right` to new node
                    current.right = node;
                    // EXIT - node inserted as `right` link
                    break;
                } else {
                    // node set to existing right link...
                    current = current.right;
                }

            // if new value = current one - ignore
            } else {
                break;
            }
        }
    }
},
```

binary search tree - usage - BST with JavaScript - part 7

- to determine just size of BST
  - *traverse tree in any order*
- may also need to flatten tree to an array, map &c.
  - *e.g. abstract a `traverse()` function to ensure ordered traversal*

```
traverse: function(process){
    // inner scope helper function - pass node...call recursively
    function inOrder(node){
        // check node exists
        if (node) {
            // check node - if left link exists
            if (node.left !== null){
                // call recursively - pass current node from subtree - checks extent of
         subtree...
                inOrder(node.left);
            }
            // call the passed process method on this node
            process.call(this, node);

            // traverse the right subtree
            if (node.right !== null){
                inOrder(node.right);
            }
        }
    }

    // define start node - pass root
    inOrder(this._root);
},
```

Video - Algorithms and Data Structures

binary search trees - part 4

Binary Search Trees - Traverse - UP TO 3:55

Source - Trees - Java - YouTube

Algorithms and Data Structures

binary search tree - usage - BST with JavaScript - part 8

- `traverse()` function defines single parameter, `process`
  - *passed argument should be a function*
  - *may be executed on every node in tree*
- function `inOrder`
  - *used to recursively traverse tree*
  - *n.b. recursion only works when left and right links exist*
- rule is designed to reduce reference to `null` nodes to a bare minimum
- `traverse()` function begins traversal from root
  - *passed `process()` function handles each node…*

Video - Algorithms and Data Structures

binary search trees - part 5

Binary Search Trees - traversal function - UP TO 9:27

Source - Trees - Java - YouTube

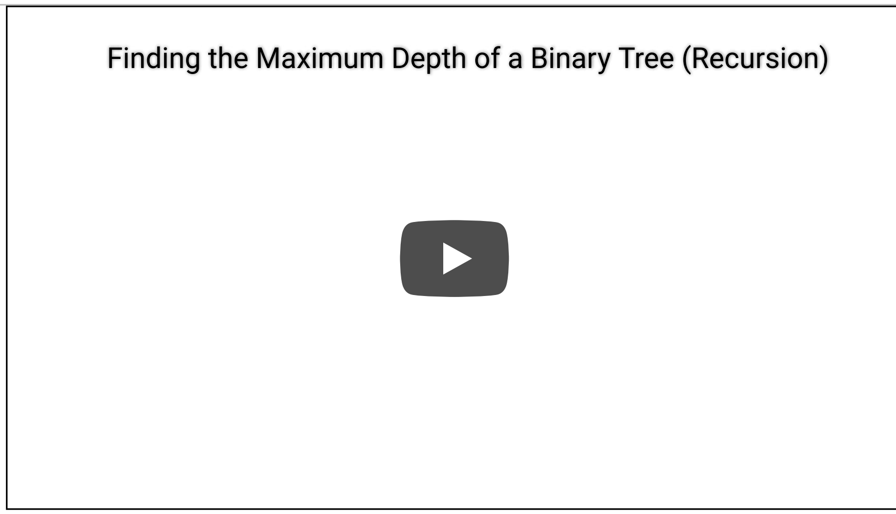
binary search tree - usage - BST with JavaScript - part 9

- now use abstracted function, `traverse()`
  - *e.g. use with a custom `size()` function*

```
size: function(){
    const length = 0;

    this.traverse(function(node){
        length++;
    });

    return length;
},
```

- `size()` function calls above abstracted `traverse()` function
  - *passing a custom function for counting nodes*

binary search trees - maximum depth



Maximum depth of binary tree with recursion - UP TO 2:34

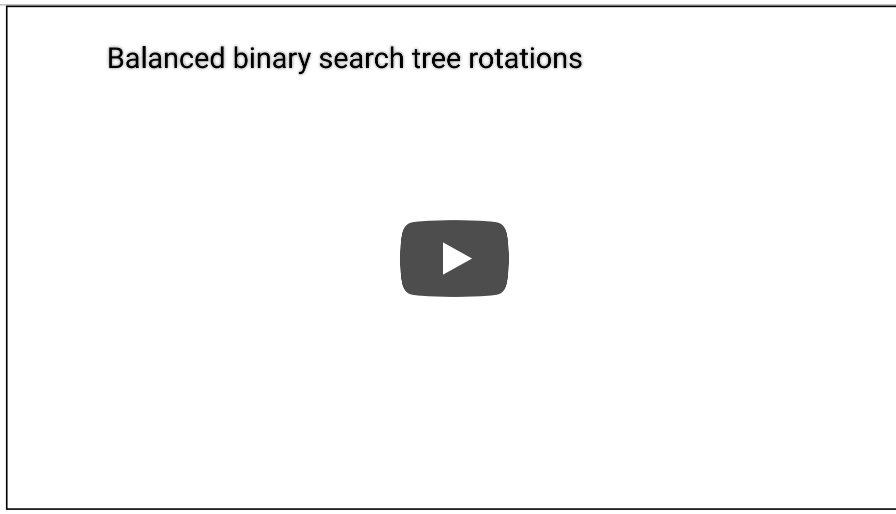
Source - Trees - Max height using recursion - YouTube

binary search tree - usage - BST with JavaScript - part 10

- removing or deleting nodes from a BST can become complex
  - *due to necessary balancing of tree*
- for each node removed
  - *need to check if it's root*
- removal of root node is handled in a similar manner to other nodes
  - *except it will also need to be replaced*
- simple matter of tree integrity
  - *may be handled as a special case in the logic*

Video - Algorithms and Data Structures

binary search trees - integrity and balance - part 1



Trees - Balancing - UP TO 1:22

Source - Trees - Balancing - YouTube

binary search tree - usage - BST with JavaScript - part 11

delete() - part 1

- first part of a node's removal is checking it exists in defined BST

```
delete: function(value){

    let found = false,
        parent = null,
        current = this._root,
        childCount,
        replacement,
        replacementParent;

    // check node - if not found & node still exists
    while(!found && current){

        // check value - if less than current - traverse left
        if (value < current.value){
            parent = current;
            current = current.left;
        }
        // check value - if greater than current - travers right
        else if (value > current.value){
            parent = current;
            current = current.right;
        }
        // value found...
        else {
            found = true;
        }
    }

    // continue - value found...
    if (found){
        // continue
    }

},
```

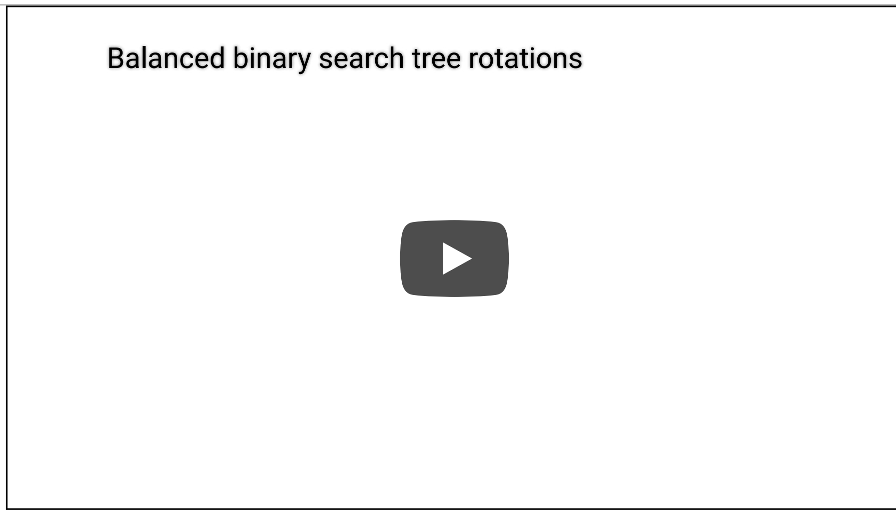
Algorithms and Data Structures

binary search tree - usage - BST with JavaScript - part 12

- may check required node using a standard *binary search*
- traverse left if value is less than current node
- traverse right if value is greater
- as part of traversal
  - *monitor parent node as passed node will need to be removed from its parent*
- when requested node is found
  - `current` *defines node to remove*
  - *initial part of defining a remove or delete option for binary search trees*

Video - Algorithms and Data Structures

binary search trees - integrity and balance - part 2



Trees - Balancing and Rotation - UP TO 4:21

Source - Trees - Balancing & Rotation- YouTube

binary search tree - usage - BST with JavaScript - part 13

delete() - part 2

- node in BST is fooound
  - *consider options for removing node*
- i.e. three applicable conditions we need to consider
  - *a leaf node*
  - *a node with one child*
  - *a node with two children*
- first two cases are easy to implement
  - *leaf node may simply be deleted from tree*
  - *a child may replace parent leaf node upon deletion*
- third condition is more involved in its modification of the tree
  - *need to determine if selected node has children, how many, and if it's root...*
- if leaf node selected for deletion is root
  - *updated decision tree is relatively simple*

binary search tree - usage - BST with JavaScript - part 14

- for example,

```
// root node - special case
if (current === this._root){
    // check no. of child nodes - execute matching case
    switch(childCount){

        // no children - erase root
        case 0:
            this._root = null;
            break;

        // one child - child  is now root
        case 1:
            this._root = (current.right === null ?
                            current.left : current.right);
            break;

        // two children -
        case 2:

            //TODO

        // no default - one of above cases always matched...

    }
}
```

- for a root leaf node
  - *deletion is simple to handle and implement*

Algorithms and Data Structures

binary search tree - usage - BST with JavaScript - part 15

- for a child leaf node
  - *need to check and update tree*
- e.g.
  - *value lower than parent*
    - left pointer must be reset
    - reset to `null` (no children) or node’s left child pointer
  - *value higher than parent*
    - right pointer must be reset
    - reset to `null` (no children) or node’s right child pointer

Algorithms and Data Structures

binary search tree - usage - BST with JavaScript - part 16

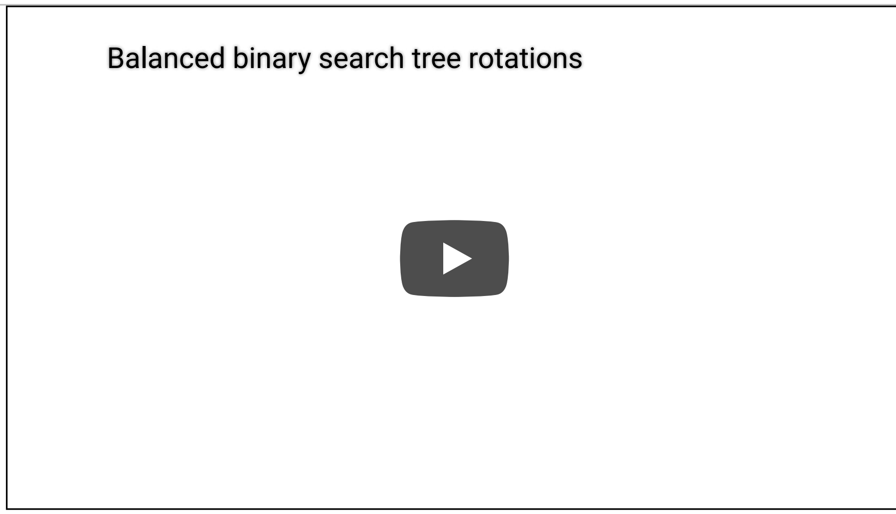
- for a child node
  - *initially check for 0 or 1 children of current selected node*

```
switch (childCount){

    // no children - delete from tree
    case 0:// check delete value relative to parent
        if (current.value < parent.value){
            // value < parent - null parent's left pointer
            parent.left = null;
        } else {
            // else - null parent's right pointer
            parent.right = null;
        }
        break;

    // one child - replace deleted parent node
    case 1: // check value relative to parent
        if (current.value < parent.value){
            // value < parent - reset left pointer
            parent.left = (current.left === null ?
                            current.right : current.left);
        } else {
            // value > parent - reset right pointer
            parent.right = (current.left === null ?
                            current.right : current.left);
        }
        break;

}
```

- need to update pointer on parent based on value of node to delete
- if deleted node's value was less than parent
  - *reset left pointer either to `null` or to left pointer of deleted node*
- if deleted node's value was greater than parent
  - *need to reset right pointer*

Video - Algorithms and Data Structures

binary search trees - integrity and balance - part 3



Trees - Balancing and Rotation - UP TO 6:08

Source - Trees - Balancing & Rotation- YouTube

binary search tree - usage - BST with JavaScript - part 17

delete() - part 3

- most complex deletion is a node with two children
- e.g. consider following tree - issue with deletion of node 2

```
        8
       / \
      6   9
     /     \
    2       11
   / \       \
  1   4       12
     / \       \
    3   5       14
               /
              13
```

- issue is how we now update tree based on child nodes of deleted node

binary search tree - usage - BST with JavaScript - part 18

- two common options to consider for such trees relative to deleted node
  - *in-order predecessor - left child*
  - *in-order successor - left-most child of sright subtree*
- may end up with either 1 or 3 replacing 2 in tree
- either option is acceptable
  - *i.e. may be used to update tree*
- e.g. consider the following
  - ***in-order predecessor** = value before deleted value*
    - examine left subtree of deleted node and select right-most descendant
  - ***in-order successor** = value immediately after deleted value*
    - examine right subtree of deleted node and select left-most descendant
- n.b. each option also requires traversal of tree to find required node

Video - Algorithms and Data Structures

binary search trees - in-order traversal

Binary Search Trees - In-Order Traversal - UP TO 2:48

Source - Trees - In-Order Traversal - YouTube

binary search tree - usage - BST with JavaScript - part 19

- for a root node with two children

```
/* root - two children
* - in-order predecessor
*   - check left subtree
*       - select right most descendant
*
*             8
*            / \
*           6   9
*          / \   \
*         2   7   11
*        / \       \
*       1   4       12
*          / \       \
*         3   5      14
*                    /
*                   13
*/
case 2: // e.g. delete root node - 8
        // check left subtree - get left of root (6)
    replacement = this._root.left;

    // check right-most child node - if not null
    while (replacement.right !== null){ // (7)
        replacementParent = replacement; // (6)
        replacement = replacement.right; // (7)
    }

    // check replacement parent
    if (replacementParent !== null){ // (6)

        // check for left node of replace - if exists, move to right of parent
        replacementParent.right = replacement.left; // (null)

        // new root - update with child nodes from existing root node
        replacement.right = this._root.right;
        replacement.left = this._root.left;
    } else {
        // new root - assign existing root's child nodes
        replacement.right = this._root.right;
```

```
    }

    // new root - UPDATE root value after deletion of root...
    this._root = replacement;
```

- this example always looka for *in-order predecessor*
  - *check left subtree*
  - *select right most descendant*

Algorithms and Data Structures

binary search tree - usage - BST with JavaScript - part 20

- if previous tree was modified as follows

```
          8
         / \
        6   9
       / \   \
      2   7   11
     / \        \
    1   4        12
       / \         \
      3   5         14
                   /
                 13
```

- if we deleted root node 8
  - *update the root to the node 7*
- follows the pattern
  - *check left subtree = 6*
  - *select right most descendant = 7*

binary search tree - usage - BST with JavaScript - part 21

- if we deleted 6 node using *in-order predecessor*
  - *check left subtree, node 2*
  - *then traverse following right pointers*
- now replace deleted node 6 with node 5
- n.b. if we used *in-order successor*
  - *end up replacing deleted node in this tree with node 7*

Video - Algorithms and Data Structures

binary search trees - review of deletion

Trees - Review of Deletion - UP TO 7:56

Source - Trees - Deletion - YouTube

binary search tree - usage - BST with JavaScript - part 22

- for a child node with two children
  - *add following case to existing `switch` statement*

```
/* child node - two children
* - in-order predecessor
*   - check left subtree
*   - select right most descendant
*/
case 2:
    // two children - reset pointers for new traversal
    replacement = current.left;
    replacementParent = current;

    //find the right-most node
    while(replacement.right !== null){
        replacementParent = replacement;
        replacement = replacement.right;
    }

    replacementParent.right = replacement.left;

    // assign - children to replacement
    replacement.right = current.right;
    replacement.left = current.left;

    // add replacement to correct node in tree
    if (current.value < parent.value){
        // current < parent - add replacement to parent's left pointer
        parent.left = replacement;
    } else {
        // current > parent - add replacement to parent's right pointer
        parent.right = replacement;
    }
```

binary search tree - usage - BST with JavaScript - part 23

- either of these options will work to update tree
- e.g. initial tree may be represented as follows

```
initial BST = {
  "_root": {
    "value": 8,
    "left": {
      "value": 6,
      "left": {
        "value": 2,
        "left": {
          "value": 1,
          "left": null,
          "right": null
        },
        "right": {
          "value": 4,
          "left": {
            "value": 3,
            "left": null,
            "right": null
          },
          "right": {
            "value": 5,
            "left": null,
            "right": null
          }
        }
      },
      "right": {
        "value": 7,
        "left": null,
        "right": null
      }
    },
    "right": {
      "value": 9,
      "left": null,
      "right": {
        "value": 11,
        "left": null,
```

```
        "right": {
          "value": 12,
          "left": null,
          "right": {
            "value": 14,
            "left": {
              "value": 13,
              "left": null,
              "right": null
            },
            "right": null
          }
        }
      }
    }
  }
}
```

binary search tree - usage - BST with JavaScript - part 24

- now delete node 6
  - *using in-order predecessor*
- updated tree may be rendered as follows

```
after node deletion = {
  "_root": {
    "value": 8,
    "left": {
      "value": 5,
      "left": {
        "value": 2,
        "left": {
          "value": 1,
          "left": null,
          "right": null
        },
        "right": {
          "value": 4,
          "left": {
            "value": 3,
            "left": null,
            "right": null
          },
          "right": null
        }
      },
      "right": {
        "value": 7,
        "left": null,
        "right": null
      }
    },
    "right": {
      "value": 9,
      "left": null,
      "right": {
        "value": 11,
        "left": null,
        "right": {
          "value": 12,
```

```
          "left": null,
          "right": {
            "value": 14,
            "left": {
              "value": 13,
              "left": null,
              "right": null
            },
            "right": null
          }
        }
      }
    }
  }
}
```

- n.b. by just using one option exclusively, we may end up with an unbalanced tree
  - *may consider modifying logic to ensure monitor of tree to maintain a self-balancing search tree...*

Resources

- Quicksort - Java - YouTube
- So how does the browser actually render a website - YouTube
- Symbol Table API - YouTube
- Trees - Balancing - YouTube
- Trees - Deletion - YouTube
- Trees - In-Order Traversal - YouTube
- Trees - Java - YouTube
- Trees - Max height using recursion - YouTube