

$$X = \underbrace{(0.10)_2}_{\text{base 2}} \cdot \underbrace{10^1}_{\text{base 10}}$$

2 - mantissa bits
~~3~~ - exponent bits
 2

Converted to base 10

$$0.5 \cdot 10^1$$

$$\frac{1}{2} \cdot 10^1$$

$$2^{-1} \cdot 10^1$$

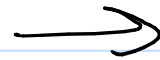
$$\frac{1}{2}(10) = 5 \rightarrow \underbrace{(101)_2}_{\text{must be normalized}} \cdot 2^0 \quad 8 = 2^3$$

$$\underbrace{(0.101)_2}_{\text{scientific notation in base 2}} \cdot 2^3$$

scientific notation in base 2

2^3 indicates that I need to move
 the decimal 3 places to the right

Now... for the actual math!



We have $x = 0.101_2 \cdot 2^3$

$$x^* = \text{fl}(x) = 0.\underline{11} \cdot 2^3$$

round to 2
base 2 decimal
places

$$x^* = ((0.11)_2 \cdot 2^3)^{10}$$

If we look at the
exponent 2^3 ... we can note
that $\pm 3_{10}$ is $\pm 11_2$

In this example
we have

2-mantissa bits

2-exponent bits

If we enumerate all possible 2-bit
exponents... we get:

we are treating sign
bits as separate

$$-11 = -3$$

$$-10 = -2$$

$$-01 = -1$$

$$00 = 0$$

$$01 = 1$$

$$10 = 2$$

$$11 = 3$$

So... "3" is at the upper limit of
what we can store



$$((0.11)_2 \cdot 2^3)^{10}$$



$$\left(\frac{3}{4} \cdot 2^3\right)^{10}$$



$$\left(\frac{3}{4}\right)^{10} \cdot (2^3)^{10} = \frac{3^{10}}{4^{10}} \cdot 2^{30}$$

let us get
this completely in
base 10

and do some algebra/
arithmetic

30 requires
5 bits

$$30_{10} = 11110_2$$

we only have 2
exponent bits

This is a textbook
example of overflow
"too big"

review
properties
of exponents

$$= \frac{3^{10}}{2^{20}} \cdot 2^{10}$$

$$= 3^{10} \cdot \frac{2^{10}}{2^{20}}$$

$$= 3^{10} \cdot 2^{-20}$$

If we had $-11110_2 \dots$ it would be
underflow
"too small"