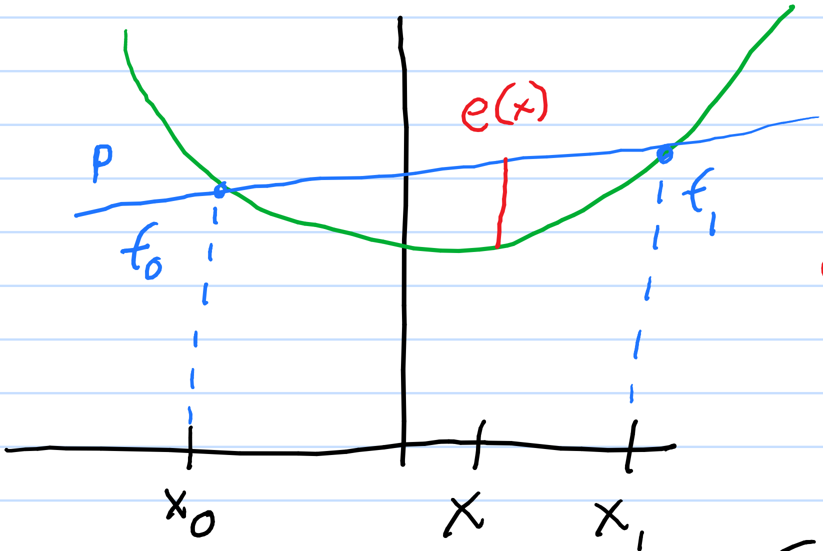
Interpolation



$$e(x) = f(x) - p_1(x)$$

n+1 th derivative

$$e(x) = f(x) - p_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!} \prod_{i=0}^{n} (x - x_i)$$

That leads to...

$$f(x) - p_1(x) = \frac{f^{2}(\xi(x))}{2!} (x - x_0)(x - x_1)$$

... for linear interpolation error

we know (from previous discussions) that...

$$\underbrace{p_1(x) = \frac{f_1 - f_0}{x_1 - x_0}(x - x_0) + f_0}_{\text{Newton's Form}}$$

$$p_1(x) = \frac{f_1 - f_0}{x_1 - x_0}(x - x_0) + f_0$$ Newton's Form

$$= \frac{(x_1 - x_0) f_0 - f_0 (x - x_0) + f_1 (x - x_0)}{(x_1 - x_0)}$$

$$= \frac{\big((x_1 - x_0) - (x - x_0)\big) f_0 + f_1 (x - x_0)}{(x_1 - x_0)}$$

$$= \frac{(x_1 - x_0 - x + x_0) f_0}{(x_1 - x_0)} + \frac{f_1 (x - x_0)}{x_1 - x_0}$$

$$= \frac{(x_1 - x) f_0}{(x_1 - x_0)} + \frac{(x - x_0)}{x_1 - x_0} f_1$$

$$= \left.\frac{x - x_1}{x_0 - x_1} f_0 + \frac{x - x_0}{x_1 - x_0} f_1 \right]$$ Lagrange Form

Polynomial Forms

$$p_n = \sum_{i=0}^{n} c_i x^i$$ **Power Form** **why?**

$$= c_0 x^0 + c_1 x^1 \dots c_n x^n$$

Shifted Power

$$p(x) = c_0 + c_1(x-a) + c_2(x-a)^2 \dots + c_n(x-a)^n$$

Nested Form

$$p(x) = c_0 + (x-a_1)\{c_1 + (x-a_2)\{\dots$$