$X = \underbrace{(0.10)_{\underbrace{2}}}_{\text{base 2}} \cdot \underbrace{10}_{\text{base 10}}^{1}$

2 - mantissa bits
~~3~~ 2 - exponent bits

Converted to base 10

$0.5 \cdot 10^1$

$\frac{1}{2} \cdot 10^1$

$2^{-1} \cdot 10^1$

$\frac{1}{2}(10) = 5 \rightarrow \underbrace{(101)_2 \cdot 2^0}_{\text{must be normalized}}$ $\quad 8 = 2^3$

$\underbrace{(0.101)_2 \cdot 2^3}_{\text{scientific notation in base 2}}$

2^3 indicates that I need to move the decimal <u>3 places</u> to <u>the right</u>

Now... for the actual math! $\longrightarrow$

we have $x = 0.101_2 \cdot 2^3$

$x^* = fl(x) = 0.\underbrace{11} \cdot 2^3$

round to 2 base 2 decimal places

$x^* = ((0.11)_2 \cdot 2^3)^{10}$

If we look at the exponent 2^3... we can note that $\pm 3_{10}$ is $\pm 11_2$

In this example we have

2 - mantissa bits
2 - exponent bits

If we enumerate all possible 2-bit exponents... we get:

we are treating sign bits as seperate

$-11 = -3$
$-10 = -2$
$-01 = -1$
$00 = 0$
$01 = 1$
$10 = 2$
$11 = 3$

So... "3" is at the upper limit of what we can store

$\rightarrow$

$$\left((0.11)_2 \cdot 2^3\right)^{10}$$

let us get this completely in base 10

$$\left(\frac{3}{4} \cdot 2^3\right)^{10}$$

and do some algebra/ arithmetic

$$\left(\frac{3}{4}\right)^{10} \cdot \left(2^3\right)^{10} = \frac{3^{10}}{4^{10}} \cdot 2^{\underbrace{30}}$$

30 requires 5 bits

$30_{10} = 11110_2$

we only have $\underline{2}$ exponent bits

review properties of exponents

$$= \frac{3^{10}}{2^{20}} \cdot 2^{10}$$

$$= 3^{10} \cdot \frac{2^{10}}{2^{20}}$$

$$= 3^{10} \cdot 2^{-20}$$

This is a textbook example of $\underbrace{\text{overflow}}_{\text{"too big"}}$

If we had $\underbrace{-11110_2}_{\text{"too small"}}$ ... it would be underflow