$\underbrace{\left| \epsilon_0 - \sum_{i=1}^{n-1} \epsilon_i \right|}$ vs $\left| \epsilon_0 - \sum_{i=1}^{n-1} \epsilon_i \right|$ multiplication

Division

or

$\left| \sum_{i=0}^{n-1} \epsilon_i \right|$

$|\epsilon_x - \epsilon_y|$

$|\epsilon_x - \epsilon_y| \le |\epsilon_x| + |\epsilon_y|$

$|\epsilon_x| \le eps \wedge |\epsilon_y| \le eps$

$|\epsilon_x - \epsilon_y| \le |\epsilon_x| + |\epsilon_y|$

$\le \underline{2eps}$

$|\epsilon_x + \epsilon_y|$

$|\epsilon_x + \epsilon_y| \le |\epsilon_x| + |\epsilon_y|$

$\le 2eps$

<u>is 2eps okay?</u>

<u>properties abs values</u>

a $0.\boxed{11010}\,1101$

a^* $0.\boxed{11010}$ truncate

a^* $0.\boxed{11011}$ round

$0.\boxed{00001}$

$0.1 \cdot 2^{-|k|}$

machine precision (<u>eps</u>)

~~O(n)~~

o(n) ← absolute worst-case error

<u>goal</u> asymptotically tight upper bound

$\underbrace{O}_{\text{big-O}}(n)$ not $\underbrace{o}_{\text{little-o}}(n)$

$$\left| \sum_{i=0}^{n-1} \epsilon_i \right| \leq \sum_{i=0}^{n-1} |\epsilon_i| \qquad \forall_i \quad |\epsilon_i| \leq |eps|$$

$$\leq \sum_{i=0}^{n-1} |eps|$$

$$\leq n\,|eps| \qquad\qquad o(n)$$

$$\left| \epsilon_0 - \sum_{i=1}^{n-1} \epsilon_i \right| \leq |\epsilon_0| + \sum_{i=1}^{n-1} |\epsilon_i|$$

$$\leq n\,|eps| \qquad\qquad o(n)$$

$$\left| \frac{x}{x+y}\epsilon_x + \frac{y}{x+y}\epsilon_y \right|$$

let $\epsilon = |eps|$

$$\leq \left| \frac{x}{x+y} eps + \frac{y}{x+y} eps \right|$$

$$\leq \left| \frac{x}{x+y}\epsilon + \frac{y}{x+y}\epsilon \right|$$

$$\leq \boxed{\left| \frac{x+y}{x+y} \right| \epsilon \leq eps}$$

$$\underbrace{fl\left(\overbrace{fl(x+y)}^{\epsilon_1}+z\right)}_{\epsilon_2} \quad \text{vs} \quad \overbrace{fl\left(x+\underbrace{fl(y+z)}_{\epsilon_3}\right)}^{\epsilon_4}$$

$\epsilon_{fl(x+y)}$

$$fl(fl(x+y)+z)$$

$$((x+y)(1+\epsilon_1)+z)(1+\epsilon_2)$$

$$(x+y)(1+\epsilon_1)(1+\epsilon_2)+z(1+\epsilon_2)$$

$$(x+y)(1+\epsilon_1+\epsilon_2)+z(1+\epsilon_2)$$

$$\underbrace{(x+y)(1+\epsilon_2)}+(x+y)\epsilon_1+\underbrace{z(1+\epsilon_2)}$$

rel error
$$\left|\frac{(x+y+z)(1+\epsilon_2)+(x+y)\epsilon_1-(x+y+z)}{(x+y+z)}\right|$$

$$\left|\frac{(x+y+z)(\epsilon_2)}{x+y+z}+\frac{x+y}{x+y+z}\epsilon_1\right|$$

$\hat{e} = c_0 + c_1 x + c_2 x^5$

$f(x) = a + bx + cx^5$

– Least Squares Approximation <u>then</u> interpolation

$\hat{e}^* = c_0^* + c_1^* x^* + c_2 (x^*)^5$

$\hat{e}^* = c_0(1+\epsilon_0) + c_1(1+\epsilon_1)\, x(1+\epsilon_x) \ldots$

$(x^*)^5$

$(x^*)^2 = x^2(1+\epsilon_x)^2$

$= x^2(1+2\epsilon_x)$

approx

$(x^*)^4 = ((x^*)^2)^2$

$\approx (x^2(1+2\epsilon_x))^2$

$\approx x^4(1+4\epsilon_x)$

$(x^*)^5 = (x^*)^4 x^*$

$x^4(1+4\epsilon_x)\, x(1+\epsilon_x)$

$x^5(1+5\epsilon_x)$

$(x^*)^n \approx x^n(1+n\epsilon_x)$

Proof by Strong induction