# Determining the relative permeability ($\mu$) of the material used in the ferritic wall of HBT-EP

Chris Stoafer
Dept. of Applied Physics and Applied Mathematics
Columbia University

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## Overview

A material was chosen for the ferritic wall on HBT-EP to provide a large effect with a limited thickness of the material. The relative permeability, $\mu$, of the material chosen, HiperCo 50, which is a mix of $\sim$ 49% cobalt, $\sim$ 49% iron, $\sim$ 2% vanadium, with other trace element including nickel. According to manufacturer specifications, the material should have a value $\mu = 8$ with an applied field of $3.5kG$, the nominal toroidal field in HBT-EP.

This document outlines the measurement and analysis performed to test the $\mu$ value of the material.

## Experimental Setup

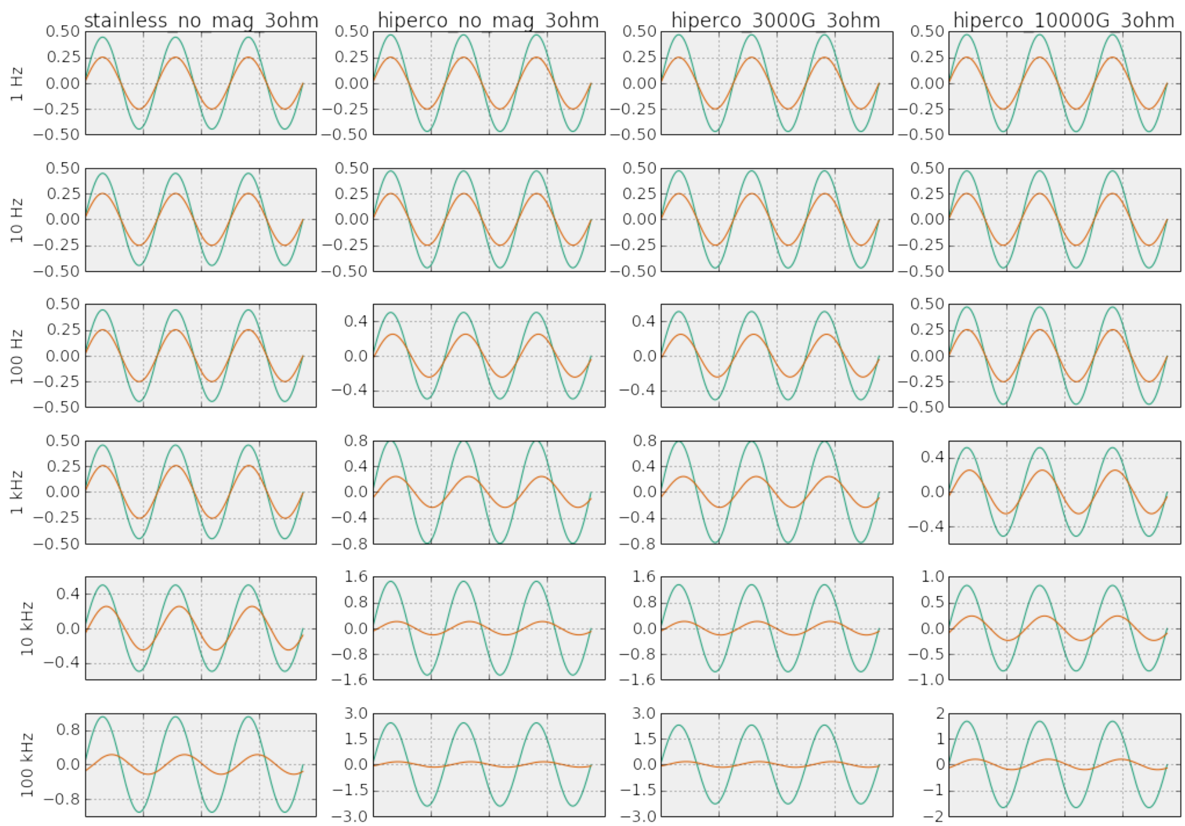
A sample of the HiperCo 50 material was machined to make a rectangular slab 10 cm long, 1.05cm wide, and 0.55 cm thick. With the available materials, the piece had to be put together using 2 pieces 5 cm long and held together with epoxy. A thin wire was then wrapped around the sample for a total of 171 turns over 9.5 cm. A similar sample of stainless steel (316 steel) was also made for comparison. The calculated inductance of a solenoid (with long solenoid approximation) and a relative permeability of 1 was determined to be 0.0223 mH. This will be used to determine $\mu$ of the material.

The two leads of the windings of the HiperCo 50 sample were connected to a circuit for the measurement. A function generator was used to drive sine waves at various frequencies which was hooked up across the sample with a 2.7$\Omega$ resistor in series. The output voltage of the generator was measured with a scope in addition to the voltage across the resistor, so that time traces of both of these measurements were obtained.

Three measurements (1, 3, and 7) were taken at frequencies in every decade from $10^2$ Hz to $10^5$ Hz (i.e. at 100 Hz, 300 Hz, 700Hz, 1000 Hz ... ). This was done with both the HiperCo 50 and stainless steel samples. These measurements were made on the HiperCo 50 sample with various external magnetic field induced by a large electromagnet. This allows us to test the saturation field and mu at various applied magnetic fields.

# Results

The measurement of the voltage sine wave across both the 2.7 $\Omega$ resistor and the sample solenoid inductor (called the measurement across the circuit), as well as a measurement just across the resistor (called the measurement across the resistor) were taken. A sine wave was fit to each measurement signal using four parameters: amplitude, frequency, phase, and offset. For the analysis, only the amplitude and phase are required. The sine signals of both the circuit (blue) and the resistor (orange) are shown as a function of phase. We see that as the frequency is increased, the amplitude of the resistor measurement decreases relative to the circuit measurement and is also delayed in phase.




# Analysis

We determine the inductance of the sample by using the amplitude of the voltage across the resistor and the voltage across the circuit, as well as the phase difference. The following shows how we determine the inductance from this information. $V_C$ is the voltage measurement across the circuit (both the resistor and inductor) and $V_R$ is the measurement across the resistor. $R_{meas}$ is the resistor we measure across (the 2.7 $\Omega$ resistor), $R_{ind}$ is the resistance of the inductor, which has both a DC and AC component. The DC component is the resistance of the wire used to wind the sample. The AC component of the resistance is due to eddy currents that are generated in the sample by the changing magnetic field from the oscillating current in the solenoid. The eddy currents both reduce the inductance and dissipate energy through heat because of the resistivity of the metal. We can include both of these effects by using both the amplitude and phase information of the measurements. $\omega$ is $2\pi$ times the frequency, and $I$ is the current in the circuit.

$$V_C = (i\omega L + R_{meas} + R_{ind})I$$

$$V_R = R_{meas} I$$

By using $V_R$ to determine $I$ and plug into the equation for $V_C$:

$$\frac{V_C}{V_R} = \frac{1}{R_{meas}}(i\omega L + R_{meas} + R_{ind})$$

We can use the ratio of the amplitudes $A = |V_C/V_R|$ and phase difference $\Delta_\phi$ to determine the inductance.

$$A = \left|\frac{V_C}{V_R}\right| = \sqrt{\left(\frac{V_C}{V_R}\right)\left(\frac{V_C}{V_R}\right)^*} = \frac{1}{R_{meas}}\sqrt{\omega^2 L^2 + (R_{meas} + R_{ind})^2}$$

$$\Delta_\phi = \tan^{-1}\left(\frac{\omega L}{R_{ind} + R_{meas}}\right)$$

Solving for $\omega L$:

$$\omega L = R_{meas}\frac{A\ \tan\Delta_\phi}{\sqrt{1 + \tan^2\Delta_\phi}}$$

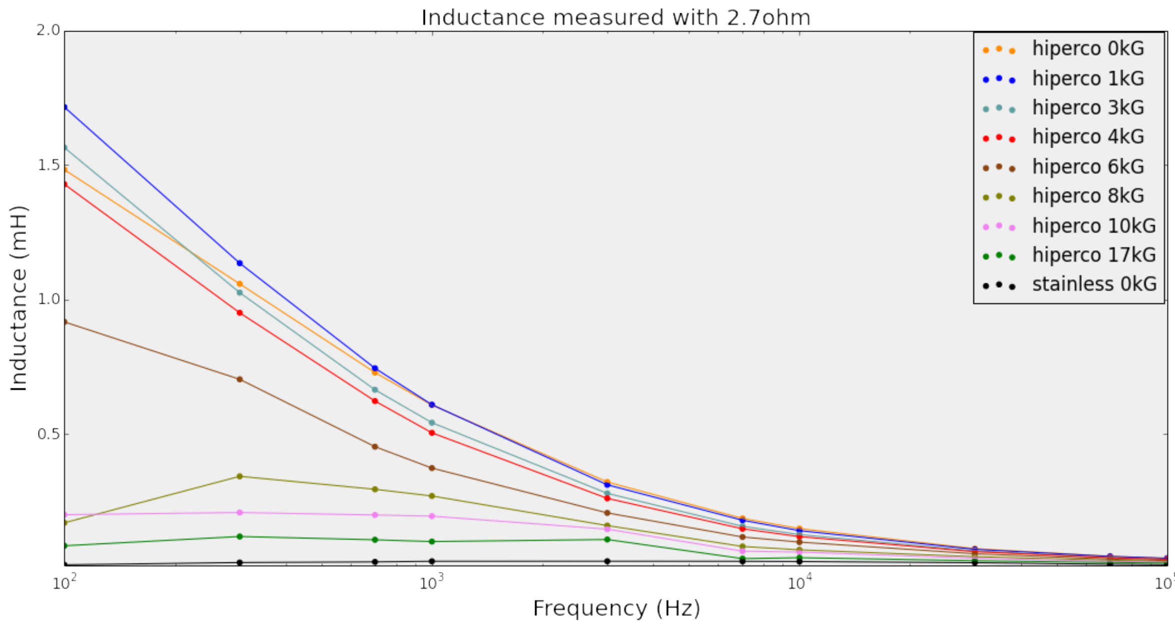
Solving for $L$:

$$L = \frac{R_{meas}}{2\pi f}\frac{A\ \tan\Delta_\phi}{\sqrt{1 + \tan^2\Delta_\phi}}$$

Solving for $R_{ind}$:

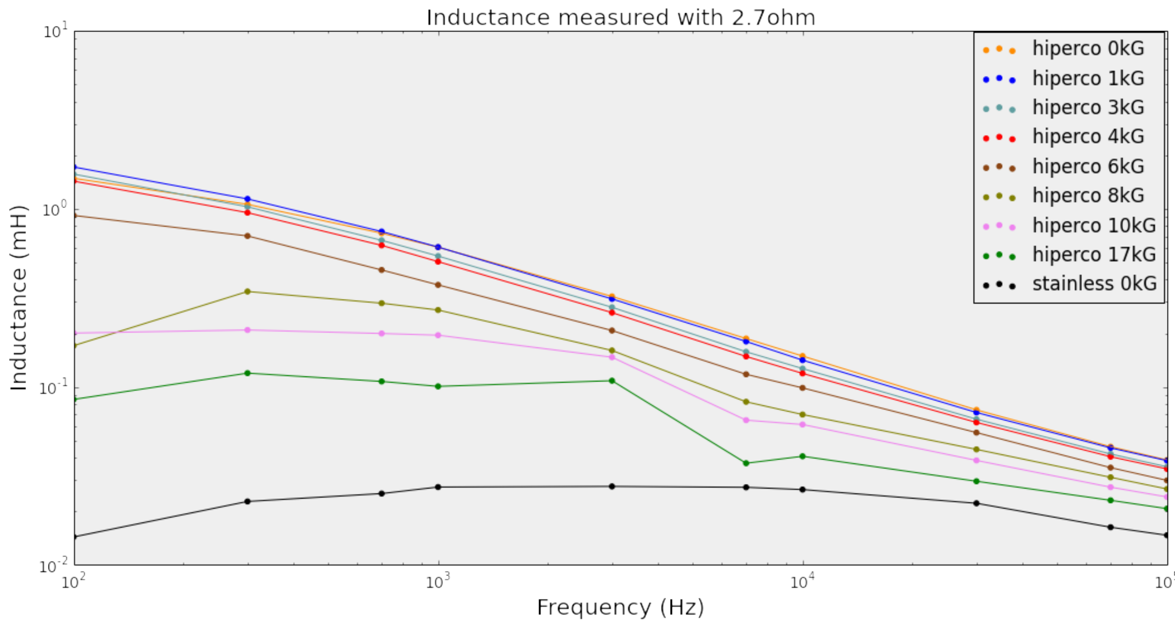
$$R_{ind} = R_{meas}\left(\frac{A}{\sqrt{1 + \tan^2\Delta_\phi}} - 1\right)$$

By using the sine fit parameters of amplitude and phase for each signal, both $A$ and $\Delta_\phi$ were used for find the inductance at each frequency and applied magnetic field. The results are shown below:

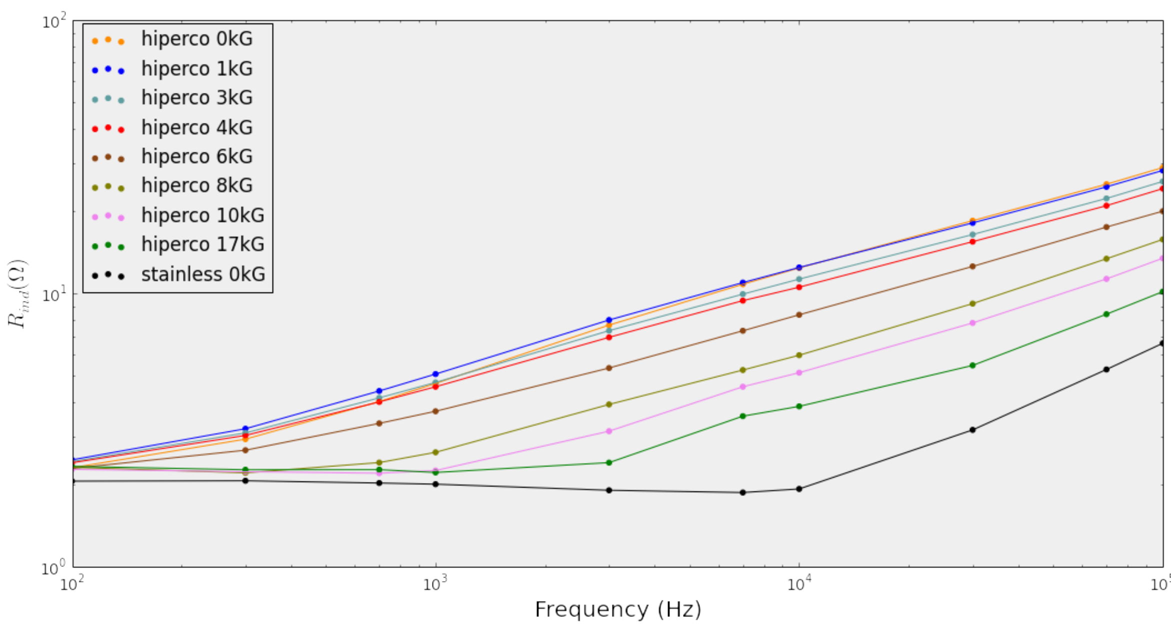



The calculated inductance of a solenoid with these dimensions and a $\mu$ of one is 0.0223 mH, which can be used to compare these inductances and determine a $\mu$ at different applied magnetic fields. These results show a much higher $\mu$ than expected from the manufacturer specifications for $3-4$ kG, showing a result above $\mu \sim 65$ where the expected $\mu$ is 8. The mu for the stainless steel sample is close to 1, as expected.

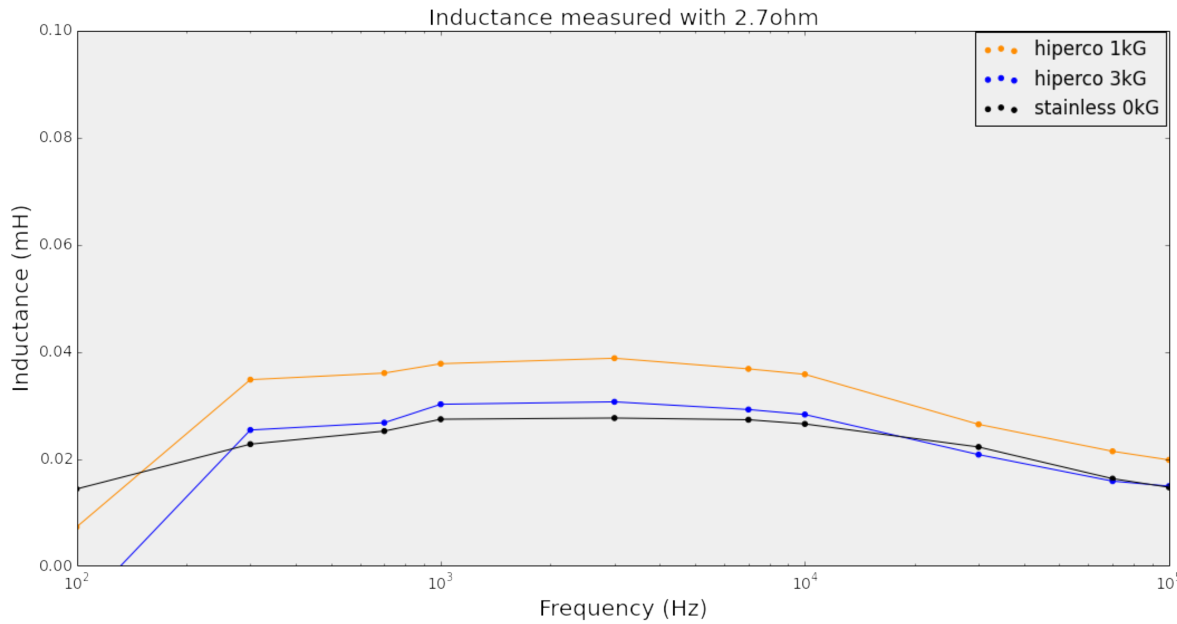
A log plot shows an expected $1/\sqrt{f}$ trend in inductance, especially for the lower applied field cases. This is expected because the skin depth for the eddy currents, which cancel out the magnetic flux changes in the sample. A simplified picture is that only the area outside of the skin depth respond to the changing magnetic field and, thus, affect the inductance. As the skin depth decreases the effective area of the sample decreases. Skin depth decreases as a function of $1/sqrtf$, therefore the inductance decreases as a function of $1/sqrtf$, as seen in the plot below.




We can also look at this effect by plotting $R_{ind}$ versus frequency. We expect $R_{ind}$ be around the DC resistance as measured at lower frequencies (2.4$\Omega$ for HiperCo 50 and 2.1$\Omega$ for stainless). Then the resistance will increase with frequency once the skin depth does not penetrate the entire sample, at which point the resistance will increase at $\sqrt{f}$.




We also performed the test with the sample aligned with the external magnetic field. The maximum field with the magnet bore far apart was 3 kG. We see dramatically different results with this alignment compared with a perpendicular magnetic field alignment, suggesting we don't fully understand how the magnetic material is responding to an external magnetic field.




This implies a smaller $\mu$ than expected at 3 kG and a much smaller saturation field. The field varies less than 10% within the bore of the magnet in positions both parallel with the field and perpendicular to it.

If the analysis and measurements are valid, then some questions arise about how we calculate the $\mu$ of the material. How does the manufacturer test for the specifications that they provide? Are the tests done on thin metal sheets or bulk material? Are these tests valid for the samples that we use?

This is ongoing work and we aim to answer these questions with review and refinement of our understanding of these tests and the material.