How to account for climate impacts in macroeconomic growth models?

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1. Integrated assessment models

The production system in integrated assessment models (IAMs) is often described by an aggregate production function of constant-elasticity of substition type (Bauer et al., 2017)

$$Y(\tau, T) \equiv D^A(T)A(\tau)\left[\xi_K(\theta_K D^K(T)K)^{\rho_Y} + \xi_L(D^L(T)\theta_L L)^{\rho_Y} + \xi_E\theta_{\mathcal{E}}\mathcal{E})^{\rho_Y}\right]^{\frac{1}{\rho_Y}}$$

where τ and T are the slow timescale of the IAM (e.g, decadal) and the global mean temperature, respectively. Further, ξ_K, ξ_L, and ξ_E denote shares of the production factors capital K, labor L, and *energy*, respectively, and θ_K, θ_L, and θ_E denote capital, labor and energy efficiencies. The intertemporal elasticity of substitution is denoted by ρ_Y. Damages to total factor productivity A, capital stock$K$, and labor productivity θ_L are denoted by D^A, D^K, and D^L, respectively. They are only implicitly time dependent via the global mean temperature T.

- TODO: *So far, the capital stock appears to be the only impact channel for which it make sense to derive impact channel specific losses, because capital shock can trigger investment decisions.*

2. Empirical analysis

In this section, we discuss how the damage terms D^K, D^A, and as well as losses to the GDP growth rate can be derived by regressing economic data with hazard indicators derived from the ISMIP impact model simulations. Since, we aim to apply these damages to different IAM realizations, i.e., different scenarios for capital accumulations, etc., it is key to consider relative damages only.

2.1. GDP growth rate losses

Following Bakkensen and Barrage (2018) and Hsiang and Jina (2014), we use panel regressions of the form

$$\begin{aligned} g_{j,t} \equiv \ln(\mathrm{GDP}_{j,t}) - \ln(\mathrm{GDP}_{j,t-1}) &= \gamma_j + \delta_t + \theta_j t + \sum_{l=0}^{L} \beta_l P_{j,t-l} + \epsilon_{j,t} \\ &= g^0_{j,t} + \delta_{j,t} \end{aligned}$$

to capture the effects of climate extremes on county-level growth domestic product $\mathrm{GDP}_{j,t}$, where index j labels the countries, and all time-dependent observables are assumed to vary on the fast (discrete) timescale t of the bio-physical impacts (e.g, annual or monthly). In the above equation, the timeseries of GDP are de-trended log-linearly by accounting for country and year fixed-effects denoted by γ_j, and δ_t, respectively, as well as for country-specific time trends $\theta_j t$. The persistent effects of climate extremes on GDP are measured by the coefficients β_l, where the index $l \in [0, \ldots, L]$ numbers the time-lags up to the maximum lag-time

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L. Further, the $P_{j,t-l}$'s denote the population affected, and standard errors are denoted by $\epsilon_{j,t}$. In the second line of Eq. (1), we have rewritten the GDP growth rate $g_{r,t}$ at time t as the sum of the growth rate $g^0_{j,t} \equiv \gamma_j + \delta_t + \theta_j t + \epsilon_{j,t}$ of a reference system that is not perturbed in the time span $t-L, \ldots, t$, and the cumulative growth rate reduction

$$\delta_{j,t}\left(\{P^i_{j,t-m}\}^L_{m=0}\right) \equiv g_{j,t}\left(\{P^i_{j,t-m}\}^L_{l=0}\right) - g^0_{j,t} = \sum_{l=0}^{L} \beta_l P_{j,t-l} \quad \forall j,t,$$

resulting from a series of hazards $\{P^i_{j,t-m}\}^L_{m=0}$ striking at timepoints $t-L,\ldots,t$.

- TODO: *People affected for each category may have to be rescaled by their variance or category dependent vulnerability factors may have to be introduced.*
- TODO: *The damage notation may be misleading, since $\delta_{j,t}$ are only country dependent due to the impact realisation. The loss coefficients β_l are the same for all countries or groups of countries. However, we may apply the regressions to sub-groups of countries, e.g., developing countries or OECD countries as done in (Berlemann and Wenzel, 2016, 2018).*

From Eqs. (1), it follows that the logarithmic GDP path of the perturbed system may be written as

$$\begin{aligned} \ln(\mathrm{GDP}_{j,t}) &= \ln(\mathrm{GDP}_{j,t-1}) + g^0_{j,t} + \sum_{i=0}^{L} \beta_i P_{j,t-i} \\ &= \ln(\mathrm{GDP}_{j,0}) + \sum_{t'=0}^{t-1}\Big[g^0_{t-t'} + \underbrace{\sum_{i=0}^{L}\beta_i P_{j,t-t'-i}}_{=\delta_{j,t-t'}}\Big] = \sum_{\tilde{t}=1}^{t}\left[g^0_{j,\tilde{t}} + \delta_{j,\tilde{t}}\right] \\ \Leftrightarrow \qquad \langle g\rangle_t &\equiv \ln(\mathrm{GDP}_{j,t}) - \ln(\mathrm{GDP}_{j,0}) = \sum_{\tilde{t}=1}^{t}\left[g^0_{j,\tilde{t}} + \delta_{j,\tilde{t}}\right], \end{aligned} \tag{2a}$$

where, in Eq. (2a), we have introduced the temporal averaged GDP growth rate for the time period under consideration.

2.2. Losses in total factor productivity

Following Bakkensen and Barrage (2018), we use panel regressions of the form

$$\begin{aligned} \ln(A_{j,t}) &= \gamma^A_j + \delta^A_t + \theta^A_j t + \sum_{l=0}^{L^A} \beta^A_l P_{j,t-l} + \epsilon^A_{j,t} \\ &= \ln(A^0_{j,t}) + \ln(\delta^A_{j,t}) = \ln(A^0_{j,t}\delta^A_{j,t}) \end{aligned} \tag{3a}$$

to estimate losses in in total factor productivity $A_{j,t}$ (or any other production factor on which extreme events are expected to have persistent impacts). Analogous to Eq. (1), the timeseries of $A_{j,t}$ are de-trended log-linearly by accounting for country and year fixed-effects denoted by γ^A_j, and δ^A_t, respectively, as well as for country-specific time trends $\theta^A_j t$. The persistent effects of climate extremes on TFP are measured by the coefficients β^A_l, where the index $l \in [0,\ldots,L]$ numbers the time-lags up to the maximum lag-time L^A. Standard errors are denoted by $\epsilon^A_{j,t}$. In Eq. (3a), we have rewritten $\ln(A_{j,t})$ as the sum of the logarithmic TFP, $\ln(A^0_{j,t}) \equiv \gamma^A_j + \delta^A_t + \theta^A_j t + \epsilon^A_{j,t}$, of a reference system that is not perturbed during the time span $t-L,\ldots,t$, and the cumulative reduction in logarithmic TFP

$$\ln\left(\delta^A_{j,t}\left(\{P^i_{j,t-m}\}^{L^A}_{m=0}\right)\right) \equiv \ln\left(A_{j,t}\left(\{P^i_{j,t-m}\}^{L^A}_{l=0}\right)\right) - \ln(A^0_{j,t}) = \sum_{l=0}^{L^A} \beta^A_l P_{j,t-l} \quad \forall j,t,$$

resulting from a series of hazards $\{P^i_{j,t-m}\}^L_{m=0}$ striking at timepoints $t-L,\dots,t$.

From Eq. (3a), we see that $A_{j,t}$ may be rewritten as

$$A_{j,t} = A^0_{j,t}\delta^A_{j,t} = A^0_{j,t}\exp\left(\sum_{i=0}^{L}\beta^A_i P_{j,t-i}\right) \quad \forall j,t.$$

2.3. Losses to the capital stock

Losses to the capital stock are assumed to be instantaneous; their persistencies are endogenously calculated by the macroeconomic growth model. They are implemented as relative shocks to the capital stock K,

$$\delta^K_{j.t} \equiv 1 - \frac{\Delta K_{j.t}}{K_{j,t}},$$

where $\Delta K_{j,t} \leq 0$, denotes the absolute shock $K_{j,t}$ at time t.

3. From time to temperature dependent impacts

The ISIMIP2b modeling round includes 2 RCPs (RCP2.6,RCP6.0), 4 GCMs, 1 SSPs (SSP2), and a various number $\mathbb{N}_c$ of global impact models for each event category $c \in \mathbb{C}$. We expect that impacts scale with global mean temperature (determined by the RCP-GCM) combination and with socioeconomic development (determined by the SSP). However, there is a good chance that the temperature scaling is universal, and the dependence of relative impacts on the RCP and also the SSP may be negligible. Thus for each RCP-SSP combination, we obtain $4 \times c$ impact timeseries for each sector c. We first group countries according to the world regions of r the IAM. We then check for a possible scaling of the regional damages with global (annual?) mean temperature or with (regional/national annual?) mean temperature by, separately for each region, regressing the country level damages with mean temperature

$$\delta_{j,t} = \gamma^T_j + \beta^T_0 T_{r,t} + \beta^T_1 T^2_{r,t},$$

where γ^T_j capture country level fixed damage levels, and β^T_0 and β^T_1, denote the coefficients for the linear and quadratic temperature dependence, respectively. For simplicity, we have dropped the superscipts denoting the type of damages (e.g., GDP growth reduction, TFP reductions, and losses to the capital stock).

4. Social Cost of Carbon

- Modularization of economic system, impacts and energy system

An alternative approach to integrate climate impacts into IAMs is to first derive and analytical formula for the social cost of carbon SCC and soft couple it with the IAM, and the climate module.

4.1. Semi-analytic expression for SCC

Let us assume that the each regional economy r growth exponentially with rate $g_{r,t} + \delta_{r,t}$, where $g_{r,t}$ denotes the growth rate of the unperturbed growth path (without climate shocks) and $\delta_{r,t}$ describes climate induced deviations from this path. Regional output $Y_{r,t}$ then obeys the following linear difference equation

$$Y_t \equiv \left[1 + g_{r,t} + \delta_{r,t}\right] Y_{t-1}. \tag{4}$$

The latter may be simplified by a coordinate transformation to a frame growing with the rate $g_{n,t}$ of the unperturbed system

$$\tilde{Y}_t \equiv \prod_{t'=0}^{t} \left[1 + g_{t'}\right]^{-1} Y_{t'}$$

Applying this coordinate transformation to Eq. (4) and dropping the regional index n for simplicity, yields

$$\begin{aligned}
\prod_{t'=0}^{t}(1+g_{t'})\tilde{Y}_t &= \prod_{t'=0}^{t-1}(1+g_{t'})\tilde{Y}_{t-1}(1+g_{t'}+\delta_{t'}) \\
&= \prod_{t'=0}^{t}(1+g_{t'})\tilde{Y}_{t-1}\left(1+\frac{\delta_{t'}}{1+g_{t'}}\right) \\
\Rightarrow \tilde{Y}_t &= \prod_{t'=0}^{t}\left(1+\frac{\delta_{t'}}{1+g_{t'}}\right)Y_0, \qquad (5)
\end{aligned}$$

where we have employed the relation $\tilde{Y}_0 = Y_0$ in the last line. Transforming Eq. (5) back to the original coordinate system then yields

$$Y_t = \underbrace{\prod_{t'=0}^{t}\left(1+\frac{\delta_{t'}}{1+g_{t'}}\right)}_{\equiv D_t} Y_t^0$$

where

$$D_t \equiv \prod_{t'=0}^{t}\left(1+\frac{\delta_{t'}}{1+g_{t'}}\right)$$

and

$$Y_t^0 \equiv \prod_{t'=0}^{t}(1+g_{t'})Y_0$$

denote cumulative climate damages at time t and the unperturbed growth path, respectively.

4.1.1. Regional Langrangian for SCC

- Climate damages fully internal for regional planners
- TODO: *Is this a standard approach to derive the SCC?-¿ Ask Gunnar*
- Investments

$$I_{r,t} \equiv K_{r,t+1} - (1-\delta_K)K_{r,t}$$

- Regional utility

$$u_{r,t}(c_{r,t}) \equiv \sum_r \omega_r \frac{c_{r,t}^{1-\eta}-1}{1-\eta},$$

  where η and ω_r denote the intertemporal elasticity of substitution and the regional Negishi weights, respectively.

$$\begin{aligned}
\mathcal{L}_r(\mathcal{T};E_r,c_r) \equiv \sum_{t'=0}^{T}\sum_{r'} \Big[&\omega_{r'}N_{r',t'}u(c_{r',t'})(1+\rho)^{-t'} \\
&+\lambda_{r',t'}\left[Y_{r',t'}(E_{r',t'})D_{r'}(T_{r',t'}) - c_{r',t'}N_{r',t'} - \left(K_{r',t'+1}-(1-\delta_K)K_{r',t'}\right) - p_{r,t}E_{r,t}\right]\Big],
\end{aligned}$$

where we have introduced the regional tax on carbon emissions $p_{r,t}$.

- Note that damages $D_{r,t} = D_{r,t}(T_{r,t})$ depend only implicitly via the regional temperature on time

The first order conditions then read

- Shadow price of consumption

$$\frac{\partial L_r}{\partial c_{r,t}} = N_{r,t}\omega_r(1+\rho)^{-t}c_{r,t}^{-\eta} - \lambda_{r,t}N_{r,t} \quad \Leftrightarrow \quad \lambda_{r,t} = \omega_r(1+\rho)^{-t}c_{r,t}^{-\eta} \quad \forall\, r,t \tag{6}$$

From Eq. (6), we may derive an expression for $\lambda_{r,t+1}$ in terms of $\lambda_{r,t}$,

$$\lambda_{r,t-1} = \omega_r(1-\rho)^{1-t}c_{r,t-1}^{-\eta} = (1+\rho)\left(\frac{c_{r,t-1}}{c_{r,t}}\right)^{-\eta}\lambda_{r,t},$$

which will permit us to simplify the golden rule of capital accumulation.

- Golden rule for capital accumulation

$$\frac{\partial L_r}{\partial K_{r,t}} = \lambda_{r,t}\left[D_{r,t}\frac{\partial Y_{r,t}}{\partial K_{r,t}} + 1 - \delta_K\right] - \lambda_{r,t-1} = 0$$

$$\Leftrightarrow \quad 1 + \underbrace{D_{r,t}\frac{\partial Y_{r,t}}{\partial K_{r,t}} - \delta_K}_{\equiv r_{r,t}} = (1+\rho)\left(\frac{c_{r,t-1}}{c_{r,t}}\right)^{-\eta} \quad \forall\, r,t \tag{7}$$

- Emission tax determines marginal value of emissions for production

$$\frac{\partial L_r}{\partial E_{r,t}} = \lambda_{r,t}\left[D_{r,t}\frac{\partial Y_{r,t}}{\partial E_{r,t}} - p_{r,t}\right] \quad \Leftrightarrow \quad p_{r,t} = D_{r,t}\frac{\partial Y_{r,t}}{\partial E_{r,t}} \quad \forall\, r,t$$

Also, for deriving the social cost of carbon, we may note the following useful relations

$$\omega_r \approx \frac{c_{r,t}^{\eta}}{N_t} \quad \text{with } N_t \equiv \sum_{r'} c_{r',t}^{\eta} \quad \forall\, r,t$$

$$\lambda_{r',t'} = \frac{(1+\rho)^{-t'}}{N_t}\left(\frac{c_{r',t}}{c_{r',t'}}\right)^{\eta} \quad \forall\, r',t',t \tag{8a}$$

$$\overset{t'\equiv t}{\Rightarrow} \quad \lambda_{r,t} = \lambda_t = \frac{(1+\rho)^{-t}}{N_t} \quad \forall t \tag{8b}$$

4.1.2. Global Langrangian for SCC

$$\mathcal{L}(\mathcal{T};E_r,c_r) \equiv \sum_{t'=0}^{\mathcal{T}}\sum_{r'}\Big[\omega_{r'}N_{r',t'}u\big(c_{r',t'}\big)(1+\rho)^{-t'}$$
$$+\lambda_{r',t'}\left[Y_{r',t'}\big(E_{r',t'}\big)D_{r',t'}\big(T_{r',t'}\big) - c_{r',t'}N_{r',t'} - \big(K_{r',t'+1} - \big(1-\delta_K\big)K_{r',t'}\big)\right]\Big]$$

The first order conditions then read

$$\frac{\partial\mathcal{L}}{\partial E_{r,t}} = \lambda_{r,t}\underbrace{\frac{\partial Y_{r,t}}{\partial E_{r,t}}D_{r,t}}_{=p_{r,t}} + \sum_{t'=0}^{\mathcal{T}}\sum_{r'}\lambda_{r',t'}Y_{r',t'}\frac{\partial D_{r',t'}}{\partial E_{r,t}} = 0 \quad \forall\, r,t$$

$$\begin{aligned}
\Rightarrow \quad p_{r,t} &= -\lambda_{r,t}^{-1}\sum_{t'=0}^{\mathcal{T}}\sum_{r'}\lambda_{r',t'}Y_{r',t'}\frac{\partial D_{r',t'}}{\partial E_{r,t}} \\
&= -\lambda_{r,t}^{-1}\sum_{t'=0}^{\mathcal{T}}\sum_{r'}\lambda_{r',t'}Y_{r',t'}\frac{\partial}{\partial E_{r,t}}\left[\prod_{t''=0}^{t'}\left[1+\frac{\delta_{r',t''}\left(\sum_{\tilde{t}=0}^{t''}E_{r',\tilde{t}}\right)}{1+g_{r,t''}}\right]\right] \\
&= -\lambda_{r,t}^{-1}\sum_{t'=t}^{\mathcal{T}}\sum_{r'}\lambda_{r',t'}Y_{r',t'}\sum_{t''=t}^{t'}(1+g_{n,t''})^{-1}\frac{\partial\delta_{r',t''}}{\partial E_{r,t}}\prod_{\tilde{t}=0,\tilde{t}\neq t''}^{t'}\left[1+\frac{\delta_{r',\tilde{t}}}{1+g_{r,\tilde{t}}}\right] \\
&= -\lambda_{r,t}^{-1}\sum_{t'=t}^{\mathcal{T}}\sum_{r'}\lambda_{r',t'}Y_{r',t'}\underbrace{\prod_{\tilde{t}=0}^{t'}\left[1+\frac{\delta_{r',\tilde{t}}}{1+g_{r,\tilde{t}}}\right]}_{=D_{r',t'}}\sum_{t''=t}^{t'}\underbrace{(1+g_{r',t''}+\delta_{r',t''})^{-1}\frac{\partial\delta_{r',t''}}{\partial T_{r',t''}}}_{\equiv-\Theta_{r',t''}}\underbrace{\frac{\partial T_{r',t''}}{\partial E_{r,t}}}_{\equiv\Delta T_{r',t'',r,t}} \\
&= \sum_{t'=t}^{\mathcal{T}}\sum_{r'}\Phi_{r',t',t}Y^0_{r',t'}D_{r',t'}\sum_{t''=t}^{t'}\Theta_{r',t''}\Delta T_{r',t'',r,t},
\end{aligned}$$

where we have introduced the marginal change of the growth rate with temperature

$$\Theta_{r,t} \equiv -(1+g_{r,t}+\delta_{r,t})^{-1}\frac{\partial\delta_{r,t}}{\partial T_{r,t}}$$

as well as the marginal response of temperature in region response to emissions

$$\Delta T_{r',t',r,t} \equiv \begin{cases}\frac{\partial T_{r',t'}}{\partial E_{r,t}} & \text{for} \quad t' \geq t,\\ 0 & \text{else.}\end{cases}$$

$$\begin{aligned}
\Phi_{r',t',t} \equiv \lambda_{r,t}^{-1}\lambda_{r',t'} \overset{(12a),(12b)}{=} (1+\rho)^{t-t'}\left(\frac{c_{r',t}}{c_{r',t'}}\right)^{\eta} &= (1+\rho)^{-(t'-t)}\prod_{\tilde{t}=t+1}^{t'}\left(\frac{c_{r',\tilde{t}-1}}{c_{r',\tilde{t}}}\right)^{\eta} \qquad \text{for } t \leq t' \\
&\overset{(11)}{=} \begin{cases}1 & \text{for } t = t',\\ \prod_{\tilde{t}=t+1}^{t'}(1+r_{r',\tilde{t}})^{-1} & \text{for } t < t'.\end{cases}
\end{aligned}$$

$$\Phi_{r',t',t} \equiv \lambda_{r,t}^{-1}\lambda_{r',t'} = \begin{cases}1 & \text{for } t = t',\\ \prod_{\tilde{t}=t+1}^{t'}(1+r_{r',\tilde{t}})^{-1} & \text{for } t < t'.\end{cases}$$

- TODO: *SCC appear to decrease with increasing Damages*
- TODO: *Damages are not explicitly time dependent anymore, only indirect dependence via the regional temperature*

4.2. Semi-analytic expression for SCC *with capital losses*

Let us assume that in each region r, capital accumulation can be described by a simple Solow-Swan like equation

$$K_{r,t+1} = (1-\delta_K)\delta^K_{r,t}K_{r,t} + \underbrace{sA^0_{r,t}\delta^K_{r,t}K_{r,t}}_{\equiv Y_{r,t}(K_{r,t})} = \left(1 + \underbrace{sA^0_{r,t} - \delta_K}_{\equiv g^{K,0}_{r,t}} + \tilde{\delta}^K_{r,t}\right)K_{r,t}, \tag{9}$$

$$K_{r,t+1} = (1-\delta_K)\delta^K_{r,t}K_{r,t} + \underbrace{sA^0_{r,t}\delta^K_{r,t}K_{r,t}}_{\equiv Y_{r,t}(K_{r,t})} = \left(1 + \underbrace{sA^0_{r,t} - \delta_K}_{\equiv g^{K,0}_{r,t}} + \tilde{\delta}^K_{r,t}\right) K_{r,t},$$

where we have assumed that the production function $Y_{r,t}(K_{r,t}) \equiv sA^0_{r,t}\delta^K_{r,t}K_{r,t}$ is of $A-K$-type, where are $A^0_{r,t}$ describes TFP. Further, we have introduced rescaled damage terms for damages to the capital stock, $\tilde{\delta}^K_{r,t} \equiv (\delta^K_{r,t} - 1)(1 + g^{K,0}_{r,t})$. This enables us to rewrite the equation for capital accumulation as a linear difference equation with growth rate $g^{K,0}_{r,t} + \tilde{\delta}^K_{r,t} + \tilde{\delta}^A_{r,t}$, where $g^{K,0}_{r,t} \equiv sA^0_{r,t} - \delta_K$ denotes the growth rate of the unperturbed growth path (without climate shocks) and the terms $\tilde{\delta}^K_{r,t}$ describe climate induced deviations from this path.

Analogous to the calculations in in Sec. 4.2, we may solve Eq. (9) which permits to express the perturbed capital stock in terms of the unperturbed one,

$$K_{r,t} = D^{K,A}_{r,t} K^0_{r,t} \quad \forall\, r, t, \tag{10}$$

$$K_{r,t} = \underbrace{\prod_{t'=0}^{t} \left(1 + \tilde{\delta}^K_{r,t}\right)}_{\equiv D^{K,A}_{r,t}} K^0_{r,t},$$

where

$$D^{K,A}_{r,t} \equiv \prod_{t'=0}^{t} \left(1 + \tilde{\delta}^K_{r,t}\right)$$

and

$$K^0_{r,t} \equiv \prod_{t'=0}^{t} \left(1 + g^K_{r,t'}\right) K_{r,0}$$

denote cumulative climate damages to the capital stock at time t and the unperturbed growth of the capital stock, respectively.

4.2.1. Regional Langrangian for SCC *with capital losses*

For the production function, we may assume

$$Y_{r,t} = sA^0_{r,t}(D^K_{r,t}K_{r,0})$$

$$\begin{aligned}\mathcal{L}_r(\mathcal{T}; E_r, c_r) \equiv \sum_{t'=0}^{T} \sum_{r'} \Big[& \omega_{r'} N_{r',t'} u\big(c_{r',t'}\big)(1+\rho)^{-t'} \\ & + \lambda_{r',t'} \left[Y_{t,t}\big(E_{r',t'}\big) - c_{r',t'} N_{r',t'} - \big(D^K_{r',t'+1} K_{r',t'+1} - \big(1-\delta_K\big) D^K_{r',t'} K_{r',t'}\big) - p_{r,t} E_{r,t} \right] \Big],\end{aligned}$$

where we have introduced the regional tax on carbon emissions $p_{r,t}$.

- Note that damages $D^K_{r,t} = D^K_{r,t}(T_{r,t})$ depend only implicitly on time via the regional temperature $T_{r,t}$

The first order conditions then read

- The Shadow price of consumption $\frac{\partial L_r}{\partial c_{r,t}}$ remains unchanged with respect to Eq. (6).

- Golden rule for capital accumulation read

$$
\begin{aligned}
\frac{\partial L_r}{\partial K_{r,t}} = \lambda_{r,t}\left[D^K_{r,t}\frac{\partial(Y_{r,t})}{\partial K_{r,t}} + (1-\delta_K)D^K_{r,t}\right] - \lambda_{r,t-1}D^K_{r,t-1} = 0 \\
\Leftrightarrow \quad \frac{D^K_{r,t}}{D^K_{r,t-1}}\left[1 + \frac{\partial Y_{r,t}}{\partial K_{r,t}} - \delta_K\right] = (1+\rho)\left(\frac{c_{r,t-1}}{c_{r,t}}\right)^{-\eta} \quad \forall\, r,t
\end{aligned}
\tag{11}
$$

TODO: *we may simplify by assuming $D^K_{r,t-1} \approx D^K_{r,t}$*

- Emission tax determines marginal value of emissions for production

$$
\frac{\partial L_r}{\partial E_{r,t}} = \lambda_{r,t}\left[\frac{\partial Y_{r,t}}{\partial E_{r,t}} - p_{r,t}\right] \quad \Leftrightarrow \quad p_{r,t} = \frac{\partial Y_{r,t}}{\partial E_{r,t}} \quad \forall\, r,t
$$

TODO: *Here, it appears strange to assume that production depends on emission but not the capital stock if production is simply $Y = sAK$*

Also, for deriving the social cost of carbon, we may note the following useful relations

$$
\omega_r \approx \frac{c^\eta_{r,t}}{N_t} \quad \text{with } N_t \equiv \sum_{r'} c^\eta_{r',t} \quad \forall\, r,t
$$

$$
\lambda_{r',t'} = \frac{(1+\rho)^{-t'}}{N_t}\left(\frac{c_{r',t}}{c_{r',t'}}\right)^\eta \quad \forall\, r',t',t \tag{12a}
$$

$$
\overset{t'=t}{\Rightarrow} \quad \lambda_{r,t} = \lambda_t = \frac{(1+\rho)^{-t}}{N_t} \quad \forall t \tag{12b}
$$

4.2.2. Global Langrangian for SCC *with losses to the capital stock*

$$
\begin{aligned}
\mathcal{L}(\mathcal{T}; E_r, c_r) \equiv \sum_{t'=0}^{\mathcal{T}}\sum_{r'}\Big[\omega_{r'}N_{r',t'}u\big(c_{r',t'}\big)(1+\rho)^{-t'} \\
+\lambda_{r',t'}\left[D^K_{r',t'}Y_{r',t'}\left(E_{r',t'}\right) - c_{r',t'}N_{r',t'} - \big(D^K_{r',t'+1}\big(T_{r',t'+1}\big)K_{r',t'+1} - \big(1-\delta_K\big)D^K_{r',t'}\left(T_{r',t'}\right)K_{r',t'}\big)\right]\Big]
\end{aligned}
$$

The first order conditions with respect to emissions then reads

$$\frac{\partial \mathcal{L}}{\partial E_{r,t}} = \lambda_{r,t} \underbrace{\frac{\partial Y_{r,t}}{\partial E_{r,t}}}_{=p_{r,t}} + \sum_{t'=0}^{\mathcal{T}} \sum_{r'} \lambda_{r',t'} \left[Y_{r',t'} \frac{\partial D^K_{r',t'}}{\partial E_{r,t}} - \frac{\partial D^K_{r',t'+1}}{\partial E_{r,t}} K_{r',t'+1} + (1-\delta_K) \frac{\partial D^K_{r',t'}}{\partial E_{r,t}} K_{r',t'} \right] = 0 \quad \forall r,t$$

$$\begin{aligned}
\Rightarrow \quad p_{r,t} &= -\lambda_{r,t}^{-1} \sum_{t'=0}^{\mathcal{T}} \sum_{r'} \lambda_{r',t'} \left[(Y_{r',t'} + (1-\delta_K) K_{r',t'}) \frac{\partial D^K_{r',t'}}{\partial E_{r,t}} - K_{r',t'+1} \frac{\partial D^K_{r',t'+1}}{\partial E_{r,t}} \right] \\
&= -\lambda_{r,t}^{-1} \sum_{r'} \left[\sum_{t'=0}^{\mathcal{T}} \lambda_{r',t'} \left[Y_{r',t'} + (1-\delta_K) K_{r',t'} \right] \frac{\partial D^K_{r',t'}}{\partial E_{r,t}} - \sum_{t'=0}^{\mathcal{T}} \lambda_{r',t'} K_{r',t'+1} \frac{\partial D^K_{r',t+1}}{\partial E_{r,t}} \right] \\
&= \lambda_{r,t}^{-1} \sum_{r'} \left[\sum_{t'=t}^{\mathcal{T}} \lambda_{r',t'} \left[Y_{r',t'} + (1-\delta_K) K_{r',t'} \right] D^K_{r',t'} \sum_{\tilde{t}=t}^{t'} \Theta^K_{r',\tilde{t}} \Delta T_{r',\tilde{t},r,t} \right. \\
&\qquad \left. - \sum_{t'=t+1}^{\mathcal{T}} \lambda_{r',t'} K_{r',t'+1} D^K_{r',t'} \sum_{\tilde{t}=t+1}^{t'} \Theta^K_{r',\tilde{t}} \Delta T_{r',\tilde{t},r,t} \right] \\
&= \lambda_{r,t}^{-1} \sum_{r'} \left[\sum_{t'=t}^{\mathcal{T}} \lambda_{r',t'} \left[Y^0_{r',t'} + (1-\delta_K) K^0_{r',t'} \right] D^K_{r',t'} \sum_{\tilde{t}=t}^{t'} \Theta^K_{r',\tilde{t}} \Delta T_{r',\tilde{t},r,t} \right. \\
&\qquad \left. - \sum_{t'=t}^{\mathcal{T}-1} \lambda_{r',t'+1} K^0_{r',t'} D^K_{r',t'+1} \sum_{\tilde{t}=t+1}^{t'-1} \Theta^K_{r',\tilde{t}} \Delta T_{r',\tilde{t},r,t} \right]
\end{aligned}$$

$$\begin{aligned}
p_{r,t} = \sum_{r'} & \left[\sum_{t'=t}^{\mathcal{T}} \Phi^K_{r',t',t} \left[Y^0_{r',t'} + (1-\delta_K) K^0_{r',t'} \right] D^K_{r',t'} \sum_{\tilde{t}=t}^{t'} \Theta^K_{r',\tilde{t}} \Delta T_{r',\tilde{t},r,t} \right. \\
& \left. - \sum_{t'=t}^{\mathcal{T}-1} \Phi^K_{r',t'+1,t} K^0_{r',t'} D^K_{r',t'+1} \sum_{\tilde{t}=t+1}^{t'-1} \Theta^K_{r',\tilde{t}} \Delta T_{r',\tilde{t},r,t} \right]
\end{aligned}$$

where we have introduced the marginal change of the capital growth rate with temperature

$$\Theta^K_{r,t} \equiv -(1 + g^{K,0}_{r,t} + \delta^K_{r,t})^{-1} \frac{\partial \delta^K_{r,t}}{\partial T_{r,t}}.$$

- TODO: *Damages are not explicitly time dependent anymore, only indirect dependence via the regional temperature*
- TODO: *May be simplified by assuming that damages D^K, marginal changes Θ^K, and ΔT vary slow compared to the timescale t, i.e., we may write $D^K_{r,t} \approx D^K_{r,t+1}$ etc. This should actually be a good approximation because all three terms are not directly time dependent and vary only on the slow timescale of the regional temperature.*

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